Computer algebra independent integration tests

Summer 2022 edition

6-Hyperbolic-functions/6.5-Hyperbolic-secant/180-6.5.7-d-hyper-^m-a+b-c-sech-^n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [220]. This is test number [180].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (220)	0.00 (0)
Mathematica	100.00 (220)	0.00 (0)
Fricas	98.64 (217)	1.36 (3)
Maple	81.82 (180)	18.18 (40)
Maxima	66.82 (147)	33.18 (73)
Giac	60.91 (134)	39.09 (86)
Mupad	55.00 (121)	45.00 (99)
Sympy	4.55 (10)	95.45 (210)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

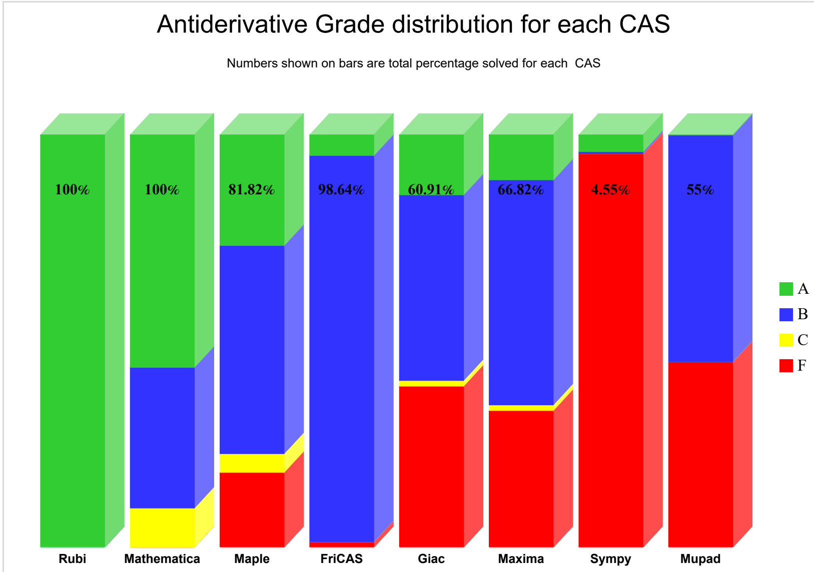
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

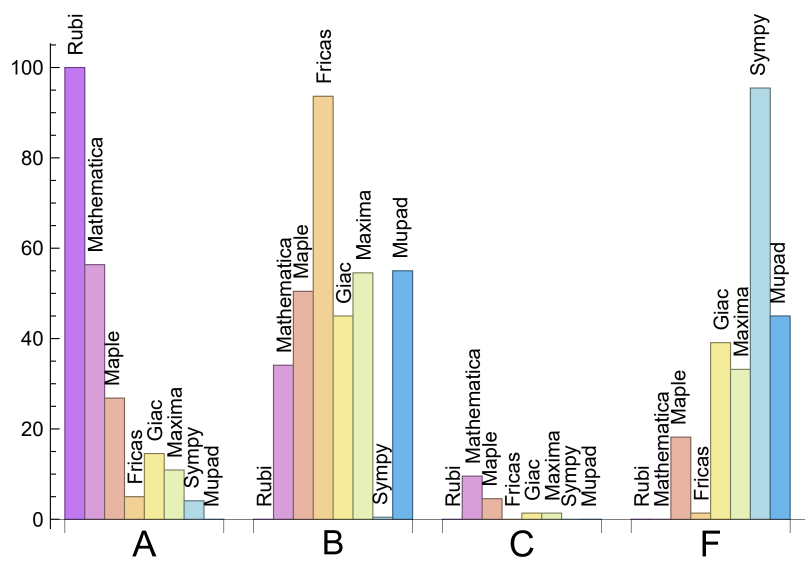
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	56.36	34.09	9.55	0.00
Maple	26.82	50.45	4.55	18.18
Giac	14.55	45.00	1.36	39.09
Maxima	10.91	54.55	1.36	33.18
Fricas	5.00	93.64	0.00	1.36
Sympy	4.09	0.45	0.00	95.45
Mupad	N/A	55.00	0.00	45.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	40	100.00 %	0.00 %	0.00 %
Fricas	3	100.00 %	0.00 %	0.00 %
Giac	86	0.00 %	0.00 %	100.00 %
Maxima	73	98.63 %	0.00 %	1.37 %
Sympy	210	81.90 %	16.19 %	1.90 %
Mupad	99	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

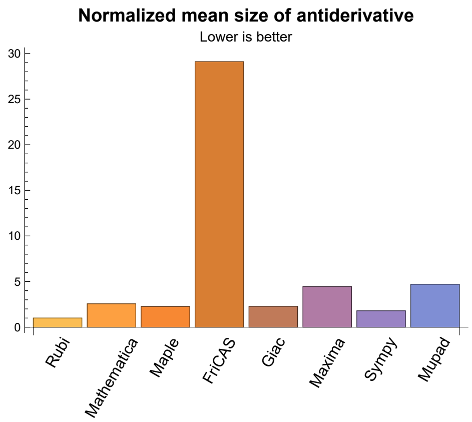
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

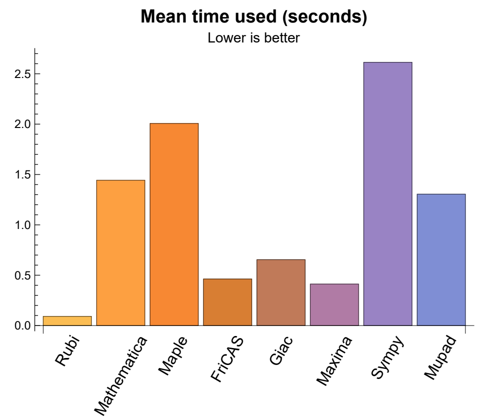
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.09	84.68	1.00	76.00	1.00
Mathematica	1.44	263.59	2.56	130.50	1.78
Maple	2.01	193.69	2.27	173.50	2.37
Maxima	0.41	450.10	4.45	244.00	3.00
Fricas	0.46	3116.78	29.11	1666.00	21.88
Sympy	2.61	86.20	1.80	72.50	1.54
Giac	0.65	191.01	2.29	157.00	2.14
Mupad	1.30	347.68	4.70	238.00	3.79

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
30
25
20
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {25, 26, 27, 33, 34, 35, 36, 40, 41, 42, 43, 44, 46, 47, 48, 66, 68, 69, 71, 91, 147, 151, 153, 155, 157, 158, 160, 162, 164, 166, 168, 169}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

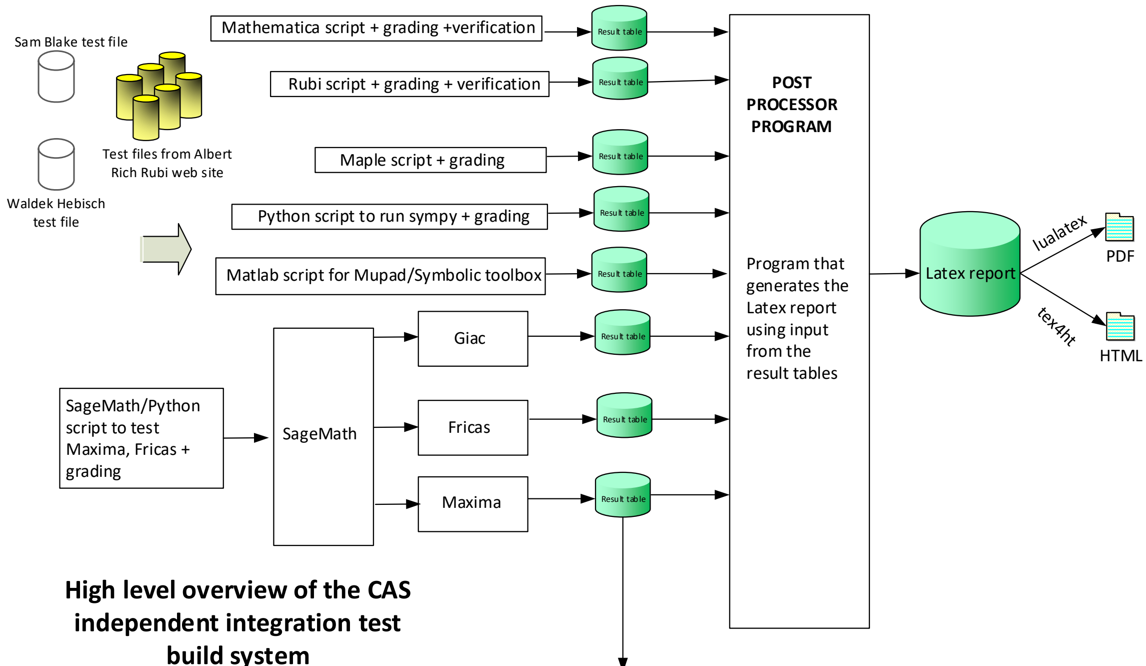
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 15, 18, 20, 21, 23, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 73, 74, 75, 76, 77, 78, 83, 84, 86, 88, 89, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 111, 113, 115, 117, 119, 121, 123, 125, 127, 129, 131, 133, 135, 138, 140, 142, 144, 146, 148, 150, 152, 154, 156, 159, 161, 163, 165, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 182, 183, 184, 185, 186, 187, 188, 190, 192, 193, 194, 195, 197, 201, 202, 203, 204, 208, 209, 210, 211, 212, 214, 216, 217, 219, 220 }

B grade: { 5, 7, 13, 14, 16, 17, 19, 22, 24, 25, 27, 30, 32, 33, 35, 38, 40, 41, 43, 46, 48, 70, 72, 79, 80, 81, 82, 85, 87, 90, 91, 92, 96, 112, 114, 116, 118, 120, 122, 124, 126, 128, 130, 132, 134, 136, 137, 139, 141, 143, 145, 147, 149, 151, 153, 155, 157, 158, 160, 162, 164, 166, 180, 181, 191, 196, 198, 199, 200, 205, 206, 207, 213, 215, 218 }

C grade: { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 66, 68, 69, 71, 108, 110, 168, 169, 189 }

F grade: { }

2.1.3 Maple

A grade: { 1, 3, 4, 5, 6, 8, 9, 11, 12, 13, 16, 17, 20, 21, 28, 29, 31, 36, 37, 39, 44, 47, 49, 50, 51, 52, 56, 57, 59, 65, 67, 92, 103, 105, 106, 107, 109, 110, 111, 113, 115, 116, 117, 119, 121, 123, 125, 127, 129, 131, 133, 142, 152, 163, 180, 189, 198, 207, 216 }

B grade: { 2, 7, 10, 14, 15, 18, 19, 22, 23, 24, 25, 26, 27, 30, 32, 33, 34, 35, 38, 40, 41, 42, 43, 45, 46, 48, 54, 62, 64, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 108, 112, 114, 118, 120, 122, 124, 126, 128, 130, 132, 134, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

C grade: { 53, 55, 58, 60, 61, 63, 66, 68, 69, 71 }

F grade: { 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220 }

2.1.4 Maxima

A grade: { 3, 4, 6, 12, 20, 49, 51, 52, 57, 59, 60, 65, 103, 104, 105, 106, 115, 118, 127, 138, 140, 170, 171, 172 }

B grade: { 1, 2, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 27, 30, 32, 33, 35, 38, 40, 41, 43, 46, 48, 50, 53, 54, 55, 56, 58, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 75, 78, 80, 82, 84, 87, 89, 91, 93, 96, 98, 100, 102, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169 }

C grade: { 173, 174, 175 }

F grade: { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 74, 76, 77, 79, 81, 83, 85, 86, 88, 90, 92, 94, 95, 97, 99, 101, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

2.1.5 FriCAS

A grade: { 1, 3, 4, 49, 50, 51, 57, 59, 65, 171, 172 }

B grade: { 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

C grade: { }

F grade: { 173, 174, 175 }

2.1.6 Sympy

A grade: { 51, 103, 105, 113, 115, 125, 127, 207, 216 }

B grade: { 142 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 106, 107, 108, 109, 110, 111, 112, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220 }

2.1.7 Giac

A grade: { 4, 6, 8, 12, 16, 17, 20, 30, 32, 33, 35, 52, 56, 57, 58, 75, 78, 80, 82, 87, 89, 106, 107, 108, 118, 139, 141, 143, 145, 151, 153, 157 }

B grade: { 1, 2, 3, 5, 7, 9, 10, 11, 13, 14, 15, 18, 19, 21, 22, 23, 24, 25, 27, 38, 40, 41, 43, 46, 48, 49, 50, 51, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 84, 91, 93, 96, 98, 100, 102, 103, 104, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 147, 149, 155, 158, 160, 162, 164, 166, 168, 169, 170, 171, 172, 191, 192 }

C grade: { 173, 174, 175 }

F grade: { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 74, 76, 77, 79, 81, 83, 85, 86, 88, 90, 92, 94, 95, 97, 99, 101, 138, 140, 142, 144, 146, 148, 150, 152, 154, 156, 159, 161, 163, 165, 167, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 152, 163, 180, 189, 198, 207, 216 }

C grade: { }

F grade: { 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **N.S.** in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	54	119	129	114	0	130	73
N.S.	1	1.00	0.77	1.70	1.84	1.63	0.00	1.86	1.04
time (sec)	N/A	0.059	0.239	1.598	0.276	0.441	0.000	0.403	1.609

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	53	111	111	85	0	85	44
N.S.	1	1.00	1.20	2.52	2.52	1.93	0.00	1.93	1.00
time (sec)	N/A	0.035	0.044	1.379	0.287	0.445	0.000	0.404	0.177

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	57	45	62	67	0	92	55
N.S.	1	1.00	1.33	1.05	1.44	1.56	0.00	2.14	1.28
time (sec)	N/A	0.036	0.142	1.004	0.262	0.430	0.000	0.399	1.428

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	26	36	38	0	45	26
N.S.	1	1.00	1.46	1.08	1.50	1.58	0.00	1.88	1.08
time (sec)	N/A	0.019	0.019	0.656	0.272	0.357	0.000	0.411	0.083

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	67	36	80	180	0	72	79
N.S.	1	1.00	2.48	1.33	2.96	6.67	0.00	2.67	2.93
time (sec)	N/A	0.029	0.040	1.105	0.278	0.362	0.000	0.404	1.412

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	37	48	39	91	0	34	34
N.S.	1	1.00	1.37	1.78	1.44	3.37	0.00	1.26	1.26
time (sec)	N/A	0.028	0.057	1.977	0.276	0.590	0.000	0.399	1.376

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	131	151	198	924	0	142	160
N.S.	1	1.00	2.43	2.80	3.67	17.11	0.00	2.63	2.96
time (sec)	N/A	0.049	0.039	1.991	0.276	0.565	0.000	0.412	0.166

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	84	75	187	246	0	80	172
N.S.	1	1.00	1.87	1.67	4.16	5.47	0.00	1.78	3.82
time (sec)	N/A	0.036	0.045	1.789	0.274	0.518	0.000	0.407	1.401

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	153	192	211	342	0	231	269
N.S.	1	1.00	1.34	1.68	1.85	3.00	0.00	2.03	2.36
time (sec)	N/A	0.099	1.135	1.811	0.278	0.535	0.000	0.439	0.270

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	83	174	266	212	0	140	201
N.S.	1	1.00	1.15	2.42	3.69	2.94	0.00	1.94	2.79
time (sec)	N/A	0.063	0.362	1.728	0.298	0.406	0.000	0.423	1.496

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	126	109	160	252	0	144	236
N.S.	1	1.00	1.73	1.49	2.19	3.45	0.00	1.97	3.23
time (sec)	N/A	0.076	0.653	1.664	0.297	0.418	0.000	0.424	0.159

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	59	43	65	133	0	75	45
N.S.	1	1.00	1.31	0.96	1.44	2.96	0.00	1.67	1.00
time (sec)	N/A	0.033	0.126	0.570	0.292	0.397	0.000	0.413	1.466

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	108	72	197	1148	0	139	232
N.S.	1	1.00	2.08	1.38	3.79	22.08	0.00	2.67	4.46
time (sec)	N/A	0.055	0.390	1.183	0.298	0.376	0.000	0.399	1.503

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	109	129	140	284	0	111	215
N.S.	1	1.00	2.18	2.58	2.80	5.68	0.00	2.22	4.30
time (sec)	N/A	0.044	1.210	2.105	0.303	0.369	0.000	0.429	1.487

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	144	318	354	2930	0	228	316
N.S.	1	1.00	1.38	3.06	3.40	28.17	0.00	2.19	3.04
time (sec)	N/A	0.096	1.107	2.398	0.272	0.427	0.000	0.415	1.655

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	151	129	285	408	0	115	115
N.S.	1	1.00	2.01	1.72	3.80	5.44	0.00	1.53	1.53
time (sec)	N/A	0.059	0.906	2.233	0.335	0.362	0.000	0.415	0.213

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	651	294	422	727	0	339	686
N.S.	1	1.00	3.58	1.62	2.32	3.99	0.00	1.86	3.77
time (sec)	N/A	0.162	1.728	2.161	0.286	0.418	0.000	0.453	0.367

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	119	285	489	403	0	193	348
N.S.	1	1.00	1.20	2.88	4.94	4.07	0.00	1.95	3.52
time (sec)	N/A	0.079	0.828	2.066	0.305	0.420	0.000	0.433	0.330

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	143	480	233	443	595	0	278	592
N.S.	1	1.28	4.29	2.08	3.96	5.31	0.00	2.48	5.29
time (sec)	N/A	0.137	1.276	2.001	0.275	0.374	0.000	0.440	0.230

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	93	58	94	276	0	101	288
N.S.	1	1.00	1.45	0.91	1.47	4.31	0.00	1.58	4.50
time (sec)	N/A	0.040	0.177	0.714	0.268	0.352	0.000	0.415	1.497

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	134	118	358	3443	0	228	434
N.S.	1	1.00	1.61	1.42	4.31	41.48	0.00	2.75	5.23
time (sec)	N/A	0.073	0.873	1.287	0.300	0.393	0.000	0.429	1.532

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	380	258	358	622	0	249	644
N.S.	1	1.00	5.43	3.69	5.11	8.89	0.00	3.56	9.20
time (sec)	N/A	0.053	1.905	2.274	0.287	0.351	0.000	0.436	1.479

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	224	550	556	6717	0	341	536
N.S.	1	1.00	1.56	3.82	3.86	46.65	0.00	2.37	3.72
time (sec)	N/A	0.126	2.607	2.582	0.287	0.416	0.000	0.454	1.644

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	213	316	664	955	0	355	745
N.S.	1	1.00	2.05	3.04	6.38	9.18	0.00	3.41	7.16
time (sec)	N/A	0.073	1.690	2.399	0.291	0.347	0.000	0.421	1.602

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	294	357	526	1681	0	220	328
N.S.	1	1.00	2.51	3.05	4.50	14.37	0.00	1.88	2.80
time (sec)	N/A	0.135	1.665	2.204	0.518	0.375	0.000	1.400	2.426

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	372	170	0	1246	0	0	473
N.S.	1	1.00	5.24	2.39	0.00	17.55	0.00	0.00	6.66
time (sec)	N/A	0.070	1.492	2.058	0.000	0.380	0.000	0.000	1.841

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	236	230	352	805	0	132	276
N.S.	1	1.00	3.15	3.07	4.69	10.73	0.00	1.76	3.68
time (sec)	N/A	0.077	0.656	1.904	0.505	0.437	0.000	0.739	2.029

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	328	45	0	595	0	0	42
N.S.	1	1.00	6.98	0.96	0.00	12.66	0.00	0.00	0.89
time (sec)	N/A	0.034	0.689	0.316	0.000	0.362	0.000	0.000	0.145

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	232	65	0	533	0	0	616
N.S.	1	1.00	4.22	1.18	0.00	9.69	0.00	0.00	11.20
time (sec)	N/A	0.060	0.643	1.678	0.000	0.415	0.000	0.000	2.181

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	179	145	100	588	0	75	847
N.S.	1	1.00	3.38	2.74	1.89	11.09	0.00	1.42	15.98
time (sec)	N/A	0.053	0.530	1.747	0.494	0.379	0.000	0.610	2.467

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	338	111	0	1881	0	0	1586
N.S.	1	1.00	3.89	1.28	0.00	21.62	0.00	0.00	18.23
time (sec)	N/A	0.083	1.334	2.167	0.000	0.411	0.000	0.000	3.070

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	216	215	195	1753	0	123	248
N.S.	1	1.00	2.88	2.87	2.60	23.37	0.00	1.64	3.31
time (sec)	N/A	0.077	1.396	1.860	0.506	0.366	0.000	0.571	2.170

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	1080	459	1299	5169	0	323	-1
N.S.	1	1.00	5.57	2.37	6.70	26.64	0.00	1.66	-0.01
time (sec)	N/A	0.199	11.808	2.596	0.551	0.443	0.000	1.273	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	861	264	0	3804	0	0	-1
N.S.	1	1.00	7.55	2.32	0.00	33.37	0.00	0.00	-0.01
time (sec)	N/A	0.100	3.357	2.250	0.000	0.394	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	791	324	696	2925	0	234	-1
N.S.	1	1.00	6.04	2.47	5.31	22.33	0.00	1.79	-0.01
time (sec)	N/A	0.134	9.539	2.246	0.531	0.384	0.000	1.060	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	479	70	0	1780	0	0	71
N.S.	1	1.00	5.70	0.83	0.00	21.19	0.00	0.00	0.85
time (sec)	N/A	0.045	1.873	0.943	0.000	0.400	0.000	0.000	1.645

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	377	167	0	2376	0	0	-1
N.S.	1	1.00	3.81	1.69	0.00	24.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.791	2.325	0.000	0.400	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	220	238	262	2407	0	239	-1
N.S.	1	1.00	2.39	2.59	2.85	26.16	0.00	2.60	-0.01
time (sec)	N/A	0.060	1.757	2.293	0.517	0.403	0.000	0.617	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	462	214	0	6878	0	0	-1
N.S.	1	1.00	3.14	1.46	0.00	46.79	0.00	0.00	-0.01
time (sec)	N/A	0.147	1.697	2.742	0.000	0.534	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	295	324	430	6143	0	253	-1
N.S.	1	1.00	2.40	2.63	3.50	49.94	0.00	2.06	-0.01
time (sec)	N/A	0.143	3.895	2.618	0.573	0.429	0.000	0.630	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	3080	538	2468	12353	0	518	-1
N.S.	1	1.00	12.73	2.22	10.20	51.05	0.00	2.14	-0.00
time (sec)	N/A	0.276	20.184	3.270	0.624	0.580	0.000	1.753	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	1217	348	0	8667	0	0	-1
N.S.	1	1.00	7.90	2.26	0.00	56.28	0.00	0.00	-0.01
time (sec)	N/A	0.155	7.057	2.337	0.000	0.446	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	2544	421	1373	9730	0	370	-1
N.S.	1	1.00	13.60	2.25	7.34	52.03	0.00	1.98	-0.01
time (sec)	N/A	0.208	15.669	2.352	0.579	0.487	0.000	1.649	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	453	84	0	4829	0	0	103
N.S.	1	1.00	3.91	0.72	0.00	41.63	0.00	0.00	0.89
time (sec)	N/A	0.057	6.163	0.950	0.000	0.485	0.000	0.000	1.607

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	440	291	0	8742	0	0	-1
N.S.	1	1.00	2.86	1.89	0.00	56.77	0.00	0.00	-0.01
time (sec)	N/A	0.155	1.654	2.498	0.000	0.497	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	741	299	533	7275	0	347	-1
N.S.	1	1.00	5.88	2.37	4.23	57.74	0.00	2.75	-0.01
time (sec)	N/A	0.073	5.350	2.420	0.558	0.438	0.000	0.864	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	524	350	0	20341	0	0	-1
N.S.	1	1.00	2.46	1.64	0.00	95.50	0.00	0.00	-0.00
time (sec)	N/A	0.250	2.786	2.850	0.000	0.577	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	985	412	782	15161	0	406	-1
N.S.	1	1.00	5.97	2.50	4.74	91.88	0.00	2.46	-0.01
time (sec)	N/A	0.191	4.096	2.611	0.661	0.472	0.000	0.874	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	45	46	97	61	0	116	50
N.S.	1	1.00	0.74	0.75	1.59	1.00	0.00	1.90	0.82
time (sec)	N/A	0.032	0.076	2.678	0.266	0.357	0.000	0.393	0.129

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	50	32	85	41	0	72	34
N.S.	1	1.00	1.67	1.07	2.83	1.37	0.00	2.40	1.13
time (sec)	N/A	0.038	0.015	2.516	0.269	0.350	0.000	0.401	0.085

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	33	37	38	28	60	66	23
N.S.	1	1.00	1.06	1.19	1.23	0.90	1.94	2.13	0.74
time (sec)	N/A	0.021	0.023	1.768	0.299	0.354	8.076	0.405	0.077

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	24	28	93	0	36	62
N.S.	1	1.00	1.46	1.00	1.17	3.88	0.00	1.50	2.58
time (sec)	N/A	0.022	0.016	1.505	0.518	0.367	0.000	0.397	1.436

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	48	105	81	321	0	84	124
N.S.	1	1.00	1.20	2.62	2.02	8.02	0.00	2.10	3.10
time (sec)	N/A	0.019	0.019	1.171	0.482	0.350	0.000	0.395	0.159

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	43	39	62	112	158	0	61	61
N.S.	1	1.43	1.30	2.07	3.73	5.27	0.00	2.03	2.03
time (sec)	N/A	0.028	0.013	1.168	0.277	0.330	0.000	0.395	1.382

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	60	173	184	1112	0	156	283
N.S.	1	1.00	0.86	2.47	2.63	15.89	0.00	2.23	4.04
time (sec)	N/A	0.036	0.077	1.359	0.503	0.378	0.000	0.403	1.383

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	65	71	86	300	343	0	85	292
N.S.	1	1.30	1.42	1.72	6.00	6.86	0.00	1.70	5.84
time (sec)	N/A	0.032	0.013	1.314	0.277	0.333	0.000	0.398	1.435

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	58	57	105	78	0	151	66
N.S.	1	1.00	0.71	0.70	1.28	0.95	0.00	1.84	0.80
time (sec)	N/A	0.066	0.092	2.148	0.286	0.357	0.000	0.398	1.371

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	72	133	105	414	0	94	114
N.S.	1	1.00	1.47	2.71	2.14	8.45	0.00	1.92	2.33
time (sec)	N/A	0.047	0.023	1.981	0.482	0.390	0.000	0.405	0.166

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	52	68	63	80	0	128	65
N.S.	1	1.00	1.11	1.45	1.34	1.70	0.00	2.72	1.38
time (sec)	N/A	0.057	0.108	1.963	0.282	0.388	0.000	0.397	0.164

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	80	144	101	653	0	112	172
N.S.	1	1.00	1.43	2.57	1.80	11.66	0.00	2.00	3.07
time (sec)	N/A	0.050	0.035	1.964	0.505	0.432	0.000	0.391	1.439

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	71	218	201	1372	0	170	303
N.S.	1	1.00	0.79	2.42	2.23	15.24	0.00	1.89	3.37
time (sec)	N/A	0.060	0.099	1.619	0.507	0.359	0.000	0.399	1.518

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	93	157	324	404	0	156	452
N.S.	1	1.00	1.75	2.96	6.11	7.62	0.00	2.94	8.53
time (sec)	N/A	0.048	0.028	1.591	0.294	0.330	0.000	0.403	1.465

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	104	358	348	2946	0	293	569
N.S.	1	1.00	0.81	2.80	2.72	23.02	0.00	2.29	4.45
time (sec)	N/A	0.100	0.171	1.753	0.500	0.386	0.000	0.408	1.570

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	144	198	671	677	0	197	692
N.S.	1	1.00	1.80	2.48	8.39	8.46	0.00	2.46	8.65
time (sec)	N/A	0.056	0.022	1.763	0.280	0.376	0.000	0.407	1.416

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	70	147	130	153	0	177	117
N.S.	1	1.00	0.83	1.75	1.55	1.82	0.00	2.11	1.39
time (sec)	N/A	0.081	0.295	2.141	0.278	0.390	0.000	0.430	1.529

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F(-2)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	483	215	179	1409	0	163	218
N.S.	1	1.00	5.96	2.65	2.21	17.40	0.00	2.01	2.69
time (sec)	N/A	0.064	6.666	1.909	0.522	0.419	0.000	0.425	0.223

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	113	160	270	0	152	221
N.S.	1	1.00	0.89	1.57	2.22	3.75	0.00	2.11	3.07
time (sec)	N/A	0.075	0.358	2.144	0.290	0.365	0.000	0.415	0.167

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	575	257	221	1992	0	199	344
N.S.	1	1.00	6.18	2.76	2.38	21.42	0.00	2.14	3.70
time (sec)	N/A	0.073	7.459	2.302	0.527	0.394	0.000	0.427	0.191

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	1430	403	365	3465	0	310	535
N.S.	1	1.00	9.73	2.74	2.48	23.57	0.00	2.11	3.64
time (sec)	N/A	0.103	9.047	1.819	0.502	0.383	0.000	0.418	1.472

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	319	303	695	816	0	302	978
N.S.	1	1.00	4.31	4.09	9.39	11.03	0.00	4.08	13.22
time (sec)	N/A	0.057	0.994	1.872	0.282	0.339	0.000	0.432	1.495

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	1618	611	556	6114	0	485	931
N.S.	1	1.00	8.26	3.12	2.84	31.19	0.00	2.47	4.75
time (sec)	N/A	0.155	10.223	1.822	0.538	0.405	0.000	0.420	1.599

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	348	361	1245	1190	0	360	1333
N.S.	1	1.00	3.22	3.34	11.53	11.02	0.00	3.33	12.34
time (sec)	N/A	0.066	1.219	1.757	0.292	0.413	0.000	0.425	1.473

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	95	347	526	1713	0	208	260
N.S.	1	1.00	0.81	2.97	4.50	14.64	0.00	1.78	2.22
time (sec)	N/A	0.149	0.365	3.347	0.539	0.410	0.000	1.409	1.998

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	79	209	0	1616	0	0	332
N.S.	1	1.00	1.04	2.75	0.00	21.26	0.00	0.00	4.37
time (sec)	N/A	0.066	0.197	3.032	0.000	0.390	0.000	0.000	2.213

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	229	352	829	0	125	206
N.S.	1	1.00	0.89	3.05	4.69	11.05	0.00	1.67	2.75
time (sec)	N/A	0.081	0.159	3.519	0.489	0.407	0.000	0.879	1.968

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	125	0	718	0	0	277
N.S.	1	1.00	1.00	2.40	0.00	13.81	0.00	0.00	5.33
time (sec)	N/A	0.043	0.082	2.710	0.000	0.385	0.000	0.000	1.739

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	80	0	487	0	0	108
N.S.	1	1.00	1.00	2.22	0.00	13.53	0.00	0.00	3.00
time (sec)	N/A	0.027	0.045	1.783	0.000	0.361	0.000	0.000	1.896

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	102	66	411	0	47	125
N.S.	1	1.00	1.00	2.83	1.83	11.42	0.00	1.31	3.47
time (sec)	N/A	0.041	0.043	1.721	0.509	0.362	0.000	0.531	0.570

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	194	104	0	526	0	0	307
N.S.	1	1.00	3.53	1.89	0.00	9.56	0.00	0.00	5.58
time (sec)	N/A	0.050	0.482	1.708	0.000	0.394	0.000	0.000	1.815

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	182	138	91	645	0	72	166
N.S.	1	1.00	3.50	2.65	1.75	12.40	0.00	1.38	3.19
time (sec)	N/A	0.049	0.480	1.538	0.501	0.384	0.000	0.644	0.475

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	213	159	0	1518	0	0	946
N.S.	1	1.00	2.48	1.85	0.00	17.65	0.00	0.00	11.00
time (sec)	N/A	0.072	1.270	1.947	0.000	0.401	0.000	0.000	3.500

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	214	183	160	1905	0	118	334
N.S.	1	1.00	2.78	2.38	2.08	24.74	0.00	1.53	4.34
time (sec)	N/A	0.062	1.497	1.835	0.510	0.387	0.000	0.605	1.990

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	113	321	0	5842	0	0	-1
N.S.	1	1.00	0.90	2.57	0.00	46.74	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.637	3.106	0.000	0.450	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	103	341	696	3739	0	323	-1
N.S.	1	1.00	0.72	2.37	4.83	25.97	0.00	2.24	-0.01
time (sec)	N/A	0.162	0.933	3.119	0.520	0.451	0.000	1.064	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	234	237	0	3154	0	0	-1
N.S.	1	1.00	2.34	2.37	0.00	31.54	0.00	0.00	-0.01
time (sec)	N/A	0.101	1.252	2.853	0.000	0.426	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	124	201	0	1856	0	0	-1
N.S.	1	1.00	1.51	2.45	0.00	22.63	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.207	2.150	0.000	0.395	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	187	206	150	1489	0	130	-1
N.S.	1	1.00	2.53	2.78	2.03	20.12	0.00	1.76	-0.01
time (sec)	N/A	0.053	0.747	1.610	0.556	0.370	0.000	0.697	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	108	185	0	1570	0	0	-1
N.S.	1	1.00	1.48	2.53	0.00	21.51	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.157	1.710	0.000	0.408	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	88	222	165	1569	0	139	-1
N.S.	1	1.00	1.06	2.67	1.99	18.90	0.00	1.67	-0.01
time (sec)	N/A	0.063	0.132	1.925	0.557	0.400	0.000	0.602	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	282	216	0	2069	0	0	-1
N.S.	1	1.00	2.79	2.14	0.00	20.49	0.00	0.00	-0.01
time (sec)	N/A	0.082	1.742	1.858	0.000	0.429	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	229	250	244	2958	0	225	-1
N.S.	1	1.00	2.27	2.48	2.42	29.29	0.00	2.23	-0.01
time (sec)	N/A	0.104	2.519	1.750	0.581	0.435	0.000	0.602	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	489	271	0	6499	0	0	-1
N.S.	1	1.00	3.20	1.77	0.00	42.48	0.00	0.00	-0.01
time (sec)	N/A	0.160	3.421	1.934	0.000	0.474	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	156	439	1373	11740	0	395	-1
N.S.	1	1.00	0.76	2.15	6.73	57.55	0.00	1.94	-0.00
time (sec)	N/A	0.279	2.798	3.564	0.580	0.511	0.000	1.782	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	292	335	0	9856	0	0	-1
N.S.	1	1.00	1.90	2.18	0.00	64.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	2.455	3.299	0.000	0.502	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	214	308	0	6806	0	0	-1
N.S.	1	1.00	1.51	2.17	0.00	47.93	0.00	0.00	-0.01
time (sec)	N/A	0.101	1.986	2.397	0.000	0.488	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	258	260	353	5109	0	282	-1
N.S.	1	1.00	2.39	2.41	3.27	47.31	0.00	2.61	-0.01
time (sec)	N/A	0.062	1.521	2.129	0.529	0.396	0.000	0.814	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	159	294	0	6037	0	0	-1
N.S.	1	1.00	1.29	2.39	0.00	49.08	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.541	2.082	0.000	0.473	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	250	310	369	5447	0	274	-1
N.S.	1	1.00	2.00	2.48	2.95	43.58	0.00	2.19	-0.01
time (sec)	N/A	0.077	2.263	1.792	0.581	0.435	0.000	1.031	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	125	240	0	5006	0	0	-1
N.S.	1	1.00	1.18	2.26	0.00	47.23	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.214	1.970	0.000	0.399	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	125	328	395	5887	0	302	-1
N.S.	1	1.00	0.87	2.28	2.74	40.88	0.00	2.10	-0.01
time (sec)	N/A	0.111	0.681	1.965	0.606	0.499	0.000	0.995	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	247	314	0	7993	0	0	-1
N.S.	1	1.00	1.61	2.05	0.00	52.24	0.00	0.00	-0.01
time (sec)	N/A	0.148	3.149	1.999	0.000	0.498	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	57	102	92	327	0	108	433
N.S.	1	1.00	1.19	2.12	1.92	6.81	0.00	2.25	9.02
time (sec)	N/A	0.043	0.025	1.917	0.289	0.382	0.000	0.418	1.545

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	45	57	78	1072	80	119	173
N.S.	1	1.00	0.92	1.16	1.59	21.88	1.63	2.43	3.53
time (sec)	N/A	0.040	0.020	1.068	0.568	0.370	0.352	0.429	0.125

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	41	66	42	155	0	72	163
N.S.	1	1.00	1.28	2.06	1.31	4.84	0.00	2.25	5.09
time (sec)	N/A	0.041	0.014	1.602	0.290	0.404	0.000	0.404	1.503

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	27	359	42	83	72
N.S.	1	1.00	1.00	0.93	0.93	12.38	1.45	2.86	2.48
time (sec)	N/A	0.020	0.019	0.634	0.281	0.381	0.153	0.398	1.470

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	24	23	36	0	23	23
N.S.	1	1.00	1.00	1.60	1.53	2.40	0.00	1.53	1.53
time (sec)	N/A	0.010	0.004	1.412	0.268	0.361	0.000	0.404	1.395

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	44	24	65	69	0	55	167
N.S.	1	1.00	1.57	0.86	2.32	2.46	0.00	1.96	5.96
time (sec)	N/A	0.039	0.031	1.465	0.493	0.385	0.000	0.395	0.172

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	41	43	47	39	0	30	25
N.S.	1	1.00	2.28	2.39	2.61	2.17	0.00	1.67	1.39
time (sec)	N/A	0.042	0.026	1.757	0.285	0.367	0.000	0.411	0.111

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	52	37	108	378	0	84	76
N.S.	1	1.00	1.68	1.19	3.48	12.19	0.00	2.71	2.45
time (sec)	N/A	0.042	0.124	2.366	0.273	0.373	0.000	0.423	1.414

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	49	64	170	140	0	70	161
N.S.	1	1.00	1.44	1.88	5.00	4.12	0.00	2.06	4.74
time (sec)	N/A	0.046	0.019	2.116	0.271	0.381	0.000	0.427	1.492

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	62	67	251	1099	0	120	179
N.S.	1	1.00	1.22	1.31	4.92	21.55	0.00	2.35	3.51
time (sec)	N/A	0.058	0.183	1.768	0.258	0.399	0.000	0.450	0.110

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	395	272	649	721	0	278	1022
N.S.	1	1.00	5.13	3.53	8.43	9.36	0.00	3.61	13.27
time (sec)	N/A	0.073	0.788	2.441	0.280	0.397	0.000	0.446	0.183

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	107	94	333	2591	129	244	349
N.S.	1	1.00	1.39	1.22	4.32	33.65	1.68	3.17	4.53
time (sec)	N/A	0.067	0.202	1.416	0.468	0.376	0.865	0.451	1.522

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	281	190	325	435	0	196	513
N.S.	1	1.00	4.76	3.22	5.51	7.37	0.00	3.32	8.69
time (sec)	N/A	0.070	0.590	2.230	0.277	0.350	0.000	0.426	0.142

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	81	42	55	1180	63	162	182
N.S.	1	1.00	1.69	0.88	1.15	24.58	1.31	3.38	3.79
time (sec)	N/A	0.035	0.094	0.668	0.268	0.365	0.416	0.413	1.689

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	106	67	120	176	0	79	163
N.S.	1	1.00	2.65	1.68	3.00	4.40	0.00	1.98	4.08
time (sec)	N/A	0.023	0.262	1.528	0.260	0.382	0.000	0.402	1.458

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	84	50	161	665	0	150	308
N.S.	1	1.00	1.58	0.94	3.04	12.55	0.00	2.83	5.81
time (sec)	N/A	0.057	0.172	1.585	0.471	0.411	0.000	0.417	0.296

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	82	77	71	106	0	68	60
N.S.	1	1.00	2.28	2.14	1.97	2.94	0.00	1.89	1.67
time (sec)	N/A	0.062	0.521	2.379	0.281	0.464	0.000	0.410	1.422

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	82	64	206	637	0	148	240
N.S.	1	1.00	1.49	1.16	3.75	11.58	0.00	2.69	4.36
time (sec)	N/A	0.065	0.142	1.707	0.484	0.432	0.000	0.452	0.220

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	160	94	268	201	0	100	183
N.S.	1	1.00	3.48	2.04	5.83	4.37	0.00	2.17	3.98
time (sec)	N/A	0.064	0.560	2.160	0.272	0.391	0.000	0.452	1.424

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	77	84	282	1252	0	150	207
N.S.	1	1.00	1.48	1.62	5.42	24.08	0.00	2.88	3.98
time (sec)	N/A	0.067	0.163	1.776	0.267	0.696	0.000	0.458	1.466

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	256	164	613	425	0	170	511
N.S.	1	1.00	4.00	2.56	9.58	6.64	0.00	2.66	7.98
time (sec)	N/A	0.067	0.752	2.566	0.284	0.695	0.000	0.449	1.439

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	107	132	696	2548	0	219	377
N.S.	1	1.00	1.24	1.53	8.09	29.63	0.00	2.55	4.38
time (sec)	N/A	0.095	0.359	1.813	0.276	0.477	0.000	0.492	1.530

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	301	469	1453	1323	0	475	1834
N.S.	1	1.00	2.74	4.26	13.21	12.03	0.00	4.32	16.67
time (sec)	N/A	0.088	3.840	2.724	0.276	0.382	0.000	0.453	1.623

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	128	131	652	4658	178	387	573
N.S.	1	1.00	1.24	1.27	6.33	45.22	1.73	3.76	5.56
time (sec)	N/A	0.069	0.506	1.723	0.478	0.427	1.722	0.477	1.638

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	479	353	788	881	0	359	1133
N.S.	1	1.00	5.21	3.84	8.57	9.58	0.00	3.90	12.32
time (sec)	N/A	0.083	1.208	2.395	0.284	0.377	0.000	0.432	0.213

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	100	59	85	2519	87	271	347
N.S.	1	1.00	1.41	0.83	1.20	35.48	1.23	3.82	4.89
time (sec)	N/A	0.042	0.204	0.953	0.258	0.374	0.868	0.433	0.215

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	268	164	332	470	0	182	502
N.S.	1	1.00	3.67	2.25	4.55	6.44	0.00	2.49	6.88
time (sec)	N/A	0.036	0.624	1.697	0.277	0.408	0.000	0.407	1.442

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	114	86	300	2376	0	283	360
N.S.	1	1.00	1.36	1.02	3.57	28.29	0.00	3.37	4.29
time (sec)	N/A	0.080	0.443	1.833	0.494	0.382	0.000	0.425	1.638

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	126	192	172	359	0	135	234
N.S.	1	1.00	2.07	3.15	2.82	5.89	0.00	2.21	3.84
time (sec)	N/A	0.072	1.213	2.670	0.270	0.488	0.000	0.433	0.142

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	110	110	314	1701	0	247	324
N.S.	1	1.00	1.36	1.36	3.88	21.00	0.00	3.05	4.00
time (sec)	N/A	0.087	0.855	2.143	0.489	0.441	0.000	0.461	1.723

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	343	178	366	354	0	158	260
N.S.	1	1.00	5.72	2.97	6.10	5.90	0.00	2.63	4.33
time (sec)	N/A	0.073	1.146	2.651	0.277	0.359	0.000	0.462	1.479

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	101	119	422	1830	0	227	384
N.S.	1	1.00	1.25	1.47	5.21	22.59	0.00	2.80	4.74
time (sec)	N/A	0.087	0.613	1.979	0.493	0.474	0.000	0.515	1.636

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	303	207	826	521	0	213	547
N.S.	1	1.00	4.39	3.00	11.97	7.55	0.00	3.09	7.93
time (sec)	N/A	0.076	0.717	2.826	0.277	0.375	0.000	0.482	1.603

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	98	149	727	2632	0	242	411
N.S.	1	1.00	1.27	1.94	9.44	34.18	0.00	3.14	5.34
time (sec)	N/A	0.087	0.529	2.127	0.274	0.478	0.000	0.550	1.614

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	455	310	703	941	0	334	1083
N.S.	1	1.00	4.10	2.79	6.33	8.48	0.00	3.01	9.76
time (sec)	N/A	0.053	1.107	1.859	0.291	0.386	0.000	0.418	0.196

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	724	507	1277	1652	0	537	1952
N.S.	1	1.00	4.44	3.11	7.83	10.13	0.00	3.29	11.98
time (sec)	N/A	0.073	6.381	1.958	0.287	0.400	0.000	0.437	0.317

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	98	183	131	736	0	0	421
N.S.	1	1.00	1.40	2.61	1.87	10.51	0.00	0.00	6.01
time (sec)	N/A	0.083	0.216	1.907	0.478	0.437	0.000	0.000	1.812

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	196	184	637	683	0	94	183
N.S.	1	1.00	3.32	3.12	10.80	11.58	0.00	1.59	3.10
time (sec)	N/A	0.121	0.792	2.342	0.566	0.394	0.000	1.064	1.873

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	132	77	112	0	0	238
N.S.	1	1.00	0.91	2.93	1.71	2.49	0.00	0.00	5.29
time (sec)	N/A	0.061	0.083	2.153	0.473	0.435	0.000	0.000	1.650

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	174	144	291	419	0	66	105
N.S.	1	1.00	3.78	3.13	6.33	9.11	0.00	1.43	2.28
time (sec)	N/A	0.094	0.244	2.447	0.499	0.439	0.000	1.159	0.426

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	26	36	51	76	114	0	51
N.S.	1	1.00	1.13	1.57	2.22	3.30	4.96	0.00	2.22
time (sec)	N/A	0.024	0.133	1.054	0.269	0.383	2.616	0.000	0.345

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	172	142	83	436	0	64	470
N.S.	1	1.00	3.74	3.09	1.80	9.48	0.00	1.39	10.22
time (sec)	N/A	0.035	0.169	2.268	0.491	0.411	0.000	0.465	2.161

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	42	125	100	115	0	0	228
N.S.	1	1.00	0.91	2.72	2.17	2.50	0.00	0.00	4.96
time (sec)	N/A	0.061	0.082	2.199	0.277	0.416	0.000	0.000	1.817

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	193	183	429	749	0	92	977
N.S.	1	1.00	3.11	2.95	6.92	12.08	0.00	1.48	15.76
time (sec)	N/A	0.125	0.804	2.562	0.495	0.419	0.000	1.032	3.389

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	100	171	187	862	0	0	523
N.S.	1	1.00	1.37	2.34	2.56	11.81	0.00	0.00	7.16
time (sec)	N/A	0.086	0.171	3.033	0.282	0.478	0.000	0.000	2.069

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	380	252	1435	2705	0	164	779
N.S.	1	1.00	4.37	2.90	16.49	31.09	0.00	1.89	8.95
time (sec)	N/A	0.197	2.222	3.098	0.590	0.447	0.000	1.604	4.146

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	109	209	154	853	0	0	-1
N.S.	1	1.00	1.43	2.75	2.03	11.22	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.314	2.360	0.478	0.462	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	228	259	1053	1479	0	173	-1
N.S.	1	1.00	2.51	2.85	11.57	16.25	0.00	1.90	-0.01
time (sec)	N/A	0.138	1.462	2.785	0.657	0.465	0.000	1.811	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	81	178	108	485	0	0	-1
N.S.	1	1.00	1.59	3.49	2.12	9.51	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.522	2.319	0.287	0.394	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	326	236	597	1846	0	137	-1
N.S.	1	1.00	3.84	2.78	7.02	21.72	0.00	1.61	-0.01
time (sec)	N/A	0.120	3.066	2.300	0.536	0.554	0.000	1.213	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	79	62	106	476	0	0	53
N.S.	1	1.00	1.61	1.27	2.16	9.71	0.00	0.00	1.08
time (sec)	N/A	0.043	0.413	0.952	0.272	0.436	0.000	0.000	1.730

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	221	254	187	1690	0	163	-1
N.S.	1	1.00	2.38	2.73	2.01	18.17	0.00	1.75	-0.01
time (sec)	N/A	0.068	1.406	2.425	0.496	0.504	0.000	0.506	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	115	206	209	1031	0	0	-1
N.S.	1	1.00	1.39	2.48	2.52	12.42	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.207	2.681	0.284	0.475	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	268	288	1070	3624	0	273	-1
N.S.	1	1.00	2.21	2.38	8.84	29.95	0.00	2.26	-0.01
time (sec)	N/A	0.202	1.907	2.707	0.550	0.454	0.000	1.194	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	130	260	384	3624	0	0	-1
N.S.	1	1.00	1.18	2.36	3.49	32.95	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.877	3.270	0.302	0.588	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	350	365	2961	9849	0	293	-1
N.S.	1	1.00	2.17	2.27	18.39	61.17	0.00	1.82	-0.01
time (sec)	N/A	0.288	3.367	3.152	0.770	0.486	0.000	1.879	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	515	389	3239	5463	0	354	-1
N.S.	1	1.00	3.48	2.63	21.89	36.91	0.00	2.39	-0.01
time (sec)	N/A	0.221	3.824	3.076	1.131	0.512	0.000	3.259	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	136	226	206	1741	0	0	-1
N.S.	1	1.00	1.77	2.94	2.68	22.61	0.00	0.00	-0.01
time (sec)	N/A	0.084	1.412	2.520	0.278	0.423	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	1317	343	2201	6464	0	281	-1
N.S.	1	1.00	9.47	2.47	15.83	46.50	0.00	2.02	-0.01
time (sec)	N/A	0.195	9.751	2.707	0.822	0.458	0.000	2.301	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	131	236	209	1753	0	0	-1
N.S.	1	1.00	1.62	2.91	2.58	21.64	0.00	0.00	-0.01
time (sec)	N/A	0.085	1.008	2.444	0.292	0.408	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	1317	335	1255	7158	0	296	-1
N.S.	1	1.00	9.47	2.41	9.03	51.50	0.00	2.13	-0.01
time (sec)	N/A	0.175	9.549	2.609	0.622	0.455	0.000	1.845	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	129	84	193	1666	0	0	94
N.S.	1	1.00	1.77	1.15	2.64	22.82	0.00	0.00	1.29
time (sec)	N/A	0.058	1.329	1.118	0.279	0.403	0.000	0.000	1.604

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	301	355	402	6538	0	327	-1
N.S.	1	1.00	2.06	2.43	2.75	44.78	0.00	2.24	-0.01
time (sec)	N/A	0.141	4.077	2.525	0.526	0.476	0.000	0.668	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	155	292	419	4132	0	0	-1
N.S.	1	1.00	1.19	2.25	3.22	31.78	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.685	2.884	0.301	0.631	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	1769	394	1971	11606	0	389	-1
N.S.	1	1.00	9.72	2.16	10.83	63.77	0.00	2.14	-0.01
time (sec)	N/A	0.281	6.075	3.105	0.684	0.518	0.000	1.758	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	172	354	692	10255	0	0	-1
N.S.	1	1.00	1.13	2.33	4.55	67.47	0.00	0.00	-0.01
time (sec)	N/A	0.170	1.237	3.374	0.314	0.916	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	3334	468	4920	24263	0	469	-1
N.S.	1	1.00	14.37	2.02	21.21	104.58	0.00	2.02	-0.00
time (sec)	N/A	0.355	7.163	3.474	1.114	0.638	0.000	2.506	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	1405	545	718	17283	0	594	-1
N.S.	1	1.00	6.79	2.63	3.47	83.49	0.00	2.87	-0.00
time (sec)	N/A	0.243	6.594	2.604	0.556	0.580	0.000	0.534	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	120	33	183	0	72	-1
N.S.	1	1.00	0.86	4.14	1.14	6.31	0.00	2.48	-0.03
time (sec)	N/A	0.021	0.018	1.493	0.485	0.385	0.000	0.391	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	79	13	18	0	26	-1
N.S.	1	1.00	1.00	5.64	0.93	1.29	0.00	1.86	-0.07
time (sec)	N/A	0.015	0.005	1.500	0.479	0.364	0.000	0.389	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	79	22	18	0	31	-1
N.S.	1	1.00	1.00	5.64	1.57	1.29	0.00	2.21	-0.07
time (sec)	N/A	0.017	0.008	1.115	0.496	0.409	0.000	0.391	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	27	123	33	0	0	83	-1
N.S.	1	1.00	0.79	3.62	0.97	0.00	0.00	2.44	-0.03
time (sec)	N/A	0.019	0.010	1.437	0.488	0.000	0.000	0.393	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	81	13	0	0	31	-1
N.S.	1	1.00	1.00	5.06	0.81	0.00	0.00	1.94	-0.06
time (sec)	N/A	0.014	0.005	1.443	0.479	0.000	0.000	0.393	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	81	22	0	0	37	-1
N.S.	1	1.00	1.00	5.06	1.38	0.00	0.00	2.31	-0.06
time (sec)	N/A	0.015	0.008	1.346	0.500	0.000	0.000	0.391	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	114	0	0	4594	0	0	-1
N.S.	1	1.00	1.37	0.00	0.00	55.35	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.486	1.937	0.000	0.647	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	192	0	0	8852	0	0	-1
N.S.	1	1.00	1.54	0.00	0.00	70.82	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.307	1.658	0.000	0.682	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	90	0	0	2394	0	0	-1
N.S.	1	1.00	1.53	0.00	0.00	40.58	0.00	0.00	-0.02
time (sec)	N/A	0.071	0.248	1.208	0.000	0.464	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	150	0	0	4316	0	0	-1
N.S.	1	1.00	1.72	0.00	0.00	49.61	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.195	0.983	0.000	0.504	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	90	43	0	1608	0	0	32
N.S.	1	1.00	2.25	1.08	0.00	40.20	0.00	0.00	0.80
time (sec)	N/A	0.043	0.182	0.296	0.000	0.489	0.000	0.000	1.710

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	134	0	0	2949	0	0	-1
N.S.	1	1.00	2.27	0.00	0.00	49.98	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.495	1.239	0.000	0.435	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	111	0	0	3597	0	0	-1
N.S.	1	1.00	1.98	0.00	0.00	64.23	0.00	0.00	-0.02
time (sec)	N/A	0.073	0.075	1.528	0.000	0.464	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	75	0	0	1303	0	0	-1
N.S.	1	1.00	1.56	0.00	0.00	27.15	0.00	0.00	-0.02
time (sec)	N/A	0.120	0.364	1.553	0.000	0.502	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	156	0	0	5247	0	0	-1
N.S.	1	1.00	1.88	0.00	0.00	63.22	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.346	1.725	0.000	0.523	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	149	0	0	2341	0	0	-1
N.S.	1	1.00	1.77	0.00	0.00	27.87	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.401	1.704	0.000	0.491	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	191	0	0	12548	0	0	-1
N.S.	1	1.00	1.53	0.00	0.00	100.38	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.615	1.691	0.000	0.686	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	129	0	0	4226	0	0	-1
N.S.	1	1.00	1.70	0.00	0.00	55.61	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.586	1.110	0.000	0.629	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	197	0	0	8582	0	0	-1
N.S.	1	1.00	1.58	0.00	0.00	68.66	0.00	0.00	-0.01
time (sec)	N/A	0.236	0.557	0.915	0.000	0.689	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	65	58	0	2312	0	0	45
N.S.	1	1.00	1.14	1.02	0.00	40.56	0.00	0.00	0.79
time (sec)	N/A	0.057	0.076	0.215	0.000	0.575	0.000	0.000	3.359

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	152	0	0	4140	0	0	-1
N.S.	1	1.00	1.73	0.00	0.00	47.05	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.179	1.216	0.000	0.520	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	159	0	0	4123	0	134	-1
N.S.	1	1.00	2.27	0.00	0.00	58.90	0.00	1.91	-0.01
time (sec)	N/A	0.089	0.333	1.556	0.000	0.605	0.000	0.598	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	144	0	0	3349	0	222	-1
N.S.	1	1.00	1.78	0.00	0.00	41.35	0.00	2.74	-0.01
time (sec)	N/A	0.151	0.223	1.497	0.000	0.490	0.000	0.558	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	280	0	0	12452	0	0	-1
N.S.	1	1.00	1.65	0.00	0.00	73.25	0.00	0.00	-0.01
time (sec)	N/A	0.135	8.223	2.826	0.000	0.895	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	109	0	0	2678	0	0	-1
N.S.	1	1.00	1.65	0.00	0.00	40.58	0.00	0.00	-0.02
time (sec)	N/A	0.093	0.335	1.747	0.000	0.487	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	169	0	0	4569	0	0	-1
N.S.	1	1.00	1.88	0.00	0.00	50.77	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.360	1.706	0.000	0.581	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	105	0	0	1650	0	0	-1
N.S.	1	1.00	2.50	0.00	0.00	39.29	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.136	1.441	0.000	0.466	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	107	0	0	2856	0	0	-1
N.S.	1	1.00	1.78	0.00	0.00	47.60	0.00	0.00	-0.02
time (sec)	N/A	0.130	0.122	1.429	0.000	0.489	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	70	30	0	1430	0	0	19
N.S.	1	1.00	2.80	1.20	0.00	57.20	0.00	0.00	0.76
time (sec)	N/A	0.037	0.050	0.684	0.000	0.435	0.000	0.000	1.667

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	62	0	0	1059	0	0	-1
N.S.	1	1.00	2.14	0.00	0.00	36.52	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.029	1.690	0.000	0.399	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	124	0	0	3663	0	0	-1
N.S.	1	1.00	2.21	0.00	0.00	65.41	0.00	0.00	-0.02
time (sec)	N/A	0.073	0.165	1.683	0.000	0.476	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	94	0	0	1365	0	0	-1
N.S.	1	1.00	1.77	0.00	0.00	25.75	0.00	0.00	-0.02
time (sec)	N/A	0.129	0.078	1.802	0.000	0.452	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	159	0	0	6475	0	0	-1
N.S.	1	1.00	1.77	0.00	0.00	71.94	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.514	1.827	0.000	0.619	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	129	0	0	3360	0	0	-1
N.S.	1	1.00	1.90	0.00	0.00	49.41	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.320	1.677	0.000	0.540	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	169	0	0	5170	0	0	-1
N.S.	1	1.00	1.97	0.00	0.00	60.12	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.256	1.555	0.000	0.602	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	103	0	0	2194	0	0	-1
N.S.	1	1.00	2.10	0.00	0.00	44.78	0.00	0.00	-0.02
time (sec)	N/A	0.077	0.147	1.300	0.000	0.445	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	128	0	0	1733	0	0	-1
N.S.	1	1.00	2.51	0.00	0.00	33.98	0.00	0.00	-0.02
time (sec)	N/A	0.140	0.505	1.304	0.000	0.426	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	98	46	0	2034	44	0	35
N.S.	1	1.00	2.28	1.07	0.00	47.30	1.02	0.00	0.81
time (sec)	N/A	0.052	0.235	0.507	0.000	0.465	3.371	0.000	1.741

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	107	0	0	2095	0	0	-1
N.S.	1	1.00	1.88	0.00	0.00	36.75	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.558	1.181	0.000	0.507	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	155	0	0	6939	0	0	-1
N.S.	1	1.00	1.96	0.00	0.00	87.84	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.422	1.540	0.000	0.572	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	120	0	0	3941	0	0	-1
N.S.	1	1.00	1.36	0.00	0.00	44.78	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.299	1.624	0.000	0.517	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	178	0	0	11939	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	101.18	0.00	0.00	-0.01
time (sec)	N/A	0.229	0.486	1.573	0.000	0.868	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	126	0	0	5184	0	0	-1
N.S.	1	1.00	1.66	0.00	0.00	68.21	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.371	1.534	0.000	0.627	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	290	0	0	3559	0	0	-1
N.S.	1	1.00	3.22	0.00	0.00	39.54	0.00	0.00	-0.01
time (sec)	N/A	0.172	1.377	1.552	0.000	0.634	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	124	0	0	4644	0	0	-1
N.S.	1	1.00	1.82	0.00	0.00	68.29	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.620	1.329	0.000	0.597	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	290	0	0	4989	0	0	-1
N.S.	1	1.00	3.30	0.00	0.00	56.69	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.767	1.303	0.000	0.618	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	112	67	0	3994	65	0	50
N.S.	1	1.00	1.81	1.08	0.00	64.42	1.05	0.00	0.81
time (sec)	N/A	0.060	0.466	0.523	0.000	0.606	7.702	0.000	3.069

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	130	0	0	6299	0	0	-1
N.S.	1	1.00	1.37	0.00	0.00	66.31	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.301	1.161	0.000	0.583	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	242	0	0	18563	0	0	-1
N.S.	1	1.00	2.22	0.00	0.00	170.30	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.730	1.643	0.000	0.954	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	155	0	0	11205	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	84.25	0.00	0.00	-0.01
time (sec)	N/A	0.247	0.540	1.682	0.000	0.986	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	330	0	0	18565	0	0	-1
N.S.	1	1.00	1.80	0.00	0.00	101.45	0.00	0.00	-0.01
time (sec)	N/A	0.138	4.949	2.991	0.000	1.658	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [9] had the largest ratio of [23]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.00	21	0.238
2	A	3	2	1.00	21	0.095
3	A	4	4	1.00	21	0.190
4	A	3	2	1.00	19	0.105
5	A	3	3	1.00	19	0.158
6	A	3	2	1.00	21	0.095
7	A	4	4	1.00	21	0.190
8	A	3	2	1.00	21	0.095
9	A	6	5	1.00	23	0.217
10	A	3	2	1.00	23	0.087
11	A	5	5	1.00	23	0.217
12	A	3	2	1.00	21	0.095
13	A	4	3	1.00	21	0.143
14	A	3	2	1.00	23	0.087
15	A	5	5	1.00	23	0.217
16	A	3	2	1.00	23	0.087
17	A	6	5	1.00	23	0.217
18	A	3	2	1.00	23	0.087
19	A	6	5	1.28	23	0.217
20	A	3	2	1.00	21	0.095
21	A	4	3	1.00	21	0.143
22	A	3	2	1.00	23	0.087
23	A	5	4	1.00	23	0.174
24	A	3	2	1.00	23	0.087
25	A	6	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	4	4	1.00	23	0.174
27	A	5	5	1.00	23	0.217
28	A	3	3	1.00	21	0.143
29	A	4	4	1.00	21	0.190
30	A	3	3	1.00	23	0.130
31	A	5	5	1.00	23	0.217
32	A	4	4	1.00	23	0.174
33	A	7	6	1.00	23	0.261
34	A	5	4	1.00	23	0.174
35	A	6	6	1.00	23	0.261
36	A	4	4	1.00	21	0.190
37	A	5	5	1.00	21	0.238
38	A	4	4	1.00	23	0.174
39	A	6	6	1.00	23	0.261
40	A	5	4	1.00	23	0.174
41	A	8	6	1.00	23	0.261
42	A	6	5	1.00	23	0.217
43	A	7	6	1.00	23	0.261
44	A	5	4	1.00	21	0.190
45	A	6	6	1.00	21	0.286
46	A	5	4	1.00	23	0.174
47	A	7	7	1.00	23	0.304
48	A	6	5	1.00	23	0.217
49	A	3	3	1.00	21	0.143
50	A	3	2	1.00	21	0.095
51	A	2	2	1.00	21	0.095
52	A	2	2	1.00	19	0.105
53	A	2	2	1.00	19	0.105
54	A	3	3	1.43	21	0.143
55	A	3	3	1.00	21	0.143
56	A	3	2	1.30	21	0.095
57	A	4	4	1.00	23	0.174
58	A	4	3	1.00	23	0.130
59	A	5	4	1.00	23	0.174
60	A	5	4	1.00	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	4	1.00	21	0.190
62	A	3	2	1.00	23	0.087
63	A	5	5	1.00	23	0.217
64	A	3	2	1.00	23	0.087
65	A	6	5	1.00	23	0.217
66	A	5	4	1.00	23	0.174
67	A	5	4	1.00	23	0.174
68	A	6	5	1.00	21	0.238
69	A	5	5	1.00	21	0.238
70	A	3	2	1.00	23	0.087
71	A	6	6	1.00	23	0.261
72	A	3	2	1.00	23	0.087
73	A	6	6	1.00	23	0.261
74	A	4	3	1.00	23	0.130
75	A	5	5	1.00	23	0.217
76	A	3	3	1.00	21	0.143
77	A	2	2	1.00	21	0.095
78	A	2	2	1.00	23	0.087
79	A	4	4	1.00	23	0.174
80	A	3	3	1.00	23	0.130
81	A	5	5	1.00	23	0.217
82	A	4	3	1.00	23	0.130
83	A	5	4	1.00	23	0.174
84	A	6	6	1.00	23	0.261
85	A	5	4	1.00	21	0.190
86	A	3	3	1.00	21	0.143
87	A	3	3	1.00	23	0.130
88	A	3	3	1.00	23	0.130
89	A	3	3	1.00	23	0.130
90	A	5	5	1.00	23	0.217
91	A	5	4	1.00	23	0.174
92	A	6	6	1.00	23	0.261
93	A	7	6	1.00	23	0.261
94	A	6	5	1.00	21	0.238
95	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	3	1.00	23	0.130
97	A	4	4	1.00	23	0.174
98	A	4	4	1.00	23	0.174
99	A	4	3	1.00	23	0.130
100	A	4	4	1.00	23	0.174
101	A	6	6	1.00	23	0.261
102	A	4	3	1.00	21	0.143
103	A	4	3	1.00	21	0.143
104	A	4	3	1.00	21	0.143
105	A	3	2	1.00	19	0.105
106	A	3	2	1.00	12	0.167
107	A	4	3	1.00	19	0.158
108	A	4	3	1.00	21	0.143
109	A	4	3	1.00	21	0.143
110	A	4	3	1.00	21	0.143
111	A	4	3	1.00	21	0.143
112	A	4	3	1.00	23	0.130
113	A	4	3	1.00	23	0.130
114	A	4	3	1.00	23	0.130
115	A	4	3	1.00	21	0.143
116	A	4	3	1.00	14	0.214
117	A	4	3	1.00	21	0.143
118	A	4	3	1.00	23	0.130
119	A	4	3	1.00	23	0.130
120	A	4	3	1.00	23	0.130
121	A	4	3	1.00	23	0.130
122	A	4	3	1.00	23	0.130
123	A	5	4	1.00	23	0.174
124	A	4	3	1.00	23	0.130
125	A	5	4	1.00	23	0.174
126	A	4	3	1.00	23	0.130
127	A	4	3	1.00	21	0.143
128	A	4	3	1.00	14	0.214
129	A	4	3	1.00	21	0.143
130	A	4	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	3	1.00	23	0.130
132	A	4	3	1.00	23	0.130
133	A	4	3	1.00	23	0.130
134	A	4	3	1.00	23	0.130
135	A	4	3	1.00	23	0.130
136	A	4	3	1.00	14	0.214
137	A	4	3	1.00	14	0.214
138	A	4	3	1.00	23	0.130
139	A	6	6	1.00	23	0.261
140	A	4	3	1.00	23	0.130
141	A	5	5	1.00	23	0.217
142	A	2	2	1.00	21	0.095
143	A	3	3	1.00	14	0.214
144	A	4	3	1.00	21	0.143
145	A	6	6	1.00	23	0.261
146	A	4	3	1.00	23	0.130
147	A	7	7	1.00	23	0.304
148	A	4	3	1.00	23	0.130
149	A	6	6	1.00	23	0.261
150	A	4	3	1.00	23	0.130
151	A	6	6	1.00	23	0.261
152	A	4	3	1.00	21	0.143
153	A	5	5	1.00	14	0.357
154	A	4	3	1.00	21	0.143
155	A	7	7	1.00	23	0.304
156	A	4	3	1.00	23	0.130
157	A	8	7	1.00	23	0.304
158	A	7	7	1.00	23	0.304
159	A	4	3	1.00	23	0.130
160	A	7	7	1.00	23	0.304
161	A	4	3	1.00	23	0.130
162	A	7	7	1.00	23	0.304
163	A	4	3	1.00	21	0.143
164	A	6	6	1.00	14	0.429
165	A	4	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	8	8	1.00	23	0.348
167	A	4	3	1.00	23	0.130
168	A	9	8	1.00	23	0.348
169	A	7	6	1.00	14	0.429
170	A	4	4	1.00	12	0.333
171	A	3	3	1.00	12	0.250
172	A	3	3	1.00	12	0.250
173	A	4	4	1.00	10	0.400
174	A	3	3	1.00	10	0.300
175	A	3	3	1.00	10	0.300
176	A	7	6	1.00	17	0.353
177	A	9	9	1.00	17	0.529
178	A	6	6	1.00	17	0.353
179	A	8	8	1.00	17	0.471
180	A	5	5	1.00	15	0.333
181	A	6	6	1.00	12	0.500
182	A	7	5	1.00	15	0.333
183	A	6	6	1.00	17	0.353
184	A	8	6	1.00	17	0.353
185	A	7	7	1.00	17	0.412
186	A	9	7	1.00	17	0.412
187	A	7	6	1.00	17	0.353
188	A	9	9	1.00	17	0.529
189	A	6	5	1.00	15	0.333
190	A	7	7	1.00	12	0.583
191	A	8	6	1.00	15	0.400
192	A	8	8	1.00	17	0.471
193	A	8	8	1.00	16	0.500
194	A	6	5	1.00	17	0.294
195	A	8	8	1.00	17	0.471
196	A	5	5	1.00	17	0.294
197	A	7	7	1.00	17	0.412
198	A	4	4	1.00	15	0.267
199	A	3	3	1.00	12	0.250
200	A	7	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	6	6	1.00	17	0.353
202	A	8	6	1.00	17	0.353
203	A	6	5	1.00	17	0.294
204	A	8	8	1.00	17	0.471
205	A	5	5	1.00	17	0.294
206	A	5	5	1.00	17	0.294
207	A	5	5	1.00	15	0.333
208	A	4	4	1.00	12	0.333
209	A	8	6	1.00	15	0.400
210	A	7	7	1.00	17	0.412
211	A	9	9	1.00	17	0.529
212	A	6	5	1.00	17	0.294
213	A	7	7	1.00	17	0.412
214	A	6	6	1.00	17	0.353
215	A	7	7	1.00	17	0.412
216	A	6	5	1.00	15	0.333
217	A	6	6	1.00	12	0.500
218	A	9	7	1.00	15	0.467
219	A	8	8	1.00	17	0.471
220	A	7	6	1.00	16	0.375

Chapter 3

Listing of integrals

Local contents

3.1 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)\sinh^4(c+dx)\,dx$

Optimal. Leaf size=70

$$\frac{3}{8}(a-4b)x-\frac{(5a-4b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a\cosh^3(c+dx)\sinh(c+dx)}{4d}+\frac{b\tanh(c+dx)}{d}$$

```
[Out] 3/8*(a-4*b)*x-1/8*(5*a-4*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a*cosh(d*x+c)^3*s
inh(d*x+c)/d+b*tanh(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4217, 466, 1171, 396, 212}

$$-\frac{(5a-4b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3}{8}x(a-4b)+\frac{a\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^4,x]
```

```
[Out] (3*(a - 4*b)*x)/8 - ((5*a - 4*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (a*Co
sh[c + d*x]^3*Sinh[c + d*x])/(4*d) + (b*Tanh[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int (a + b\text{sech}^2(c+dx))\sinh^4(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+b-bx^2)}{(1-x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{-a-4ax^2+4bx^4}{(1-x^2)^2}\,dx, x, \tanh(\right.}{4d}$$

$$= -\frac{(5a-4b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a\cosh^3(c+dx)\sinh(c+}{4d}$$

$$= -\frac{(5a-4b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a\cosh^3(c+dx)\sinh(c+}{4d}$$

$$= \frac{3}{8}(a-4b)x - \frac{(5a-4b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a\cosh^3(c+}{}$$

Mathematica [A]
time = 0.24, size = 54, normalized size = 0.77

$$\frac{12(a-4b)(c+dx) - 8(a-b)\sinh(2(c+dx)) + a\sinh(4(c+dx)) + 32b\tanh(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^4,x]
```

```
[Out] (12*(a - 4*b)*(c + d*x) - 8*(a - b)*Sinh[2*(c + d*x)] + a*Sinh[4*(c + d*x)]
 + 32*b*Tanh[c + d*x])/(32*d)
```

Maple [A]
time = 1.60, size = 119, normalized size = 1.70

method	result	size
risch	$\frac{3ax}{8}-\frac{3bx}{2}+\frac{a\,\mathrm{e}^{4dx+4c}}{64d}-\frac{\mathrm{e}^{2dx+2c}a}{8d}+\frac{\mathrm{e}^{2dx+2c}b}{8d}+\frac{\mathrm{e}^{-2dx-2c}a}{8d}-\frac{\mathrm{e}^{-2dx-2c}b}{8d}-\frac{a\,\mathrm{e}^{-4dx-4c}}{64d}-\frac{2b}{d(1+\mathrm{e}^{2dx+2c})}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a*x-3/2*b*x+1/64*a/d*exp(4*d*x+4*c)-1/8/d*exp(2*d*x+2*c)*a+1/8/d*exp(2*
d*x+2*c)*b+1/8/d*exp(-2*d*x-2*c)*a-1/8/d*exp(-2*d*x-2*c)*b-1/64*a/d*exp(-4*
d*x-4*c)-2*b/d/(1+exp(2*d*x+2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(64) = 128.
time = 0.28, size = 129, normalized size = 1.84

$$\frac{1}{64}\,a\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{8}\,b\left(\frac{12\,(dx+c)}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}-\frac{17\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/8*b*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17
*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c))))
```

Fricas [A]
time = 0.44, size = 114, normalized size = 1.63

$$\frac{a\sinh\left(dx+c\right)^{5}+\left(10\,a\cosh\left(dx+c\right)^{2}-7\,a+8\,b\right)\sinh\left(dx+c\right)^{3}+8\left(3\left(a-4\,b\right)dx-8\,b\right)\cosh\left(dx+c\right)+\left(5\,a\cosh\left(dx+c\right)^{4}-3\left(7\,a-8\,b\right)\cosh\left(dx+c\right)^{2}-8\,a+72\,b\right)\sinh\left(dx+c\right)}{64\,d\cosh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/64*(a*sinh(d*x + c)^5 + (10*a*cosh(d*x + c)^2 - 7*a + 8*b)*sinh(d*x + c)^
3 + 8*(3*(a - 4*b)*d*x - 8*b)*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 - 3*(7*a
 - 8*b)*cosh(d*x + c)^2 - 8*a + 72*b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^{2}\left(c+dx\right)\right)\sinh^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(64) = 128$.
time = 0.40, size = 130, normalized size = 1.86

$$\frac{24\,(dx+c)(a-4\,b)+ae^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}+8\,be^{(2\,dx+2\,c)}-\left(18\,ae^{(4\,dx+4\,c)}-72\,be^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}+8\,be^{(2\,dx+2\,c)}+a\right)e^{(-4\,dx-4\,c)}-\frac{128\,b}{e^{(2\,dx+2\,c)}+1}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/64*(24*(d*x + c)*(a - 4*b) + a*e^(4*d*x + 4*c) - 8*a*e^(2*d*x + 2*c) + 8*
b*e^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) - 72*b*e^(4*d*x + 4*c) - 8*a*e^(2
*d*x + 2*c) + 8*b*e^(2*d*x + 2*c) + a)*e^(-4*d*x - 4*c) - 128*b/(e^(2*d*x +
 2*c) + 1))/d
```

Mupad [B]
time = 1.61, size = 73, normalized size = 1.04

$$\frac{3\,a\,x}{8}-\frac{3\,b\,x}{2}-\frac{a\,\mathrm{sinh}\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a\,\mathrm{sinh}\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{b\,\mathrm{sinh}\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{b\,\mathrm{sinh}\left(c+d\,x\right)}{d\,\mathrm{cosh}\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (3*a*x)/8 - (3*b*x)/2 - (a*sinh(2*c + 2*d*x))/(4*d) + (a*sinh(4*c + 4*d*x))
/(32*d) + (b*sinh(2*c + 2*d*x))/(4*d) + (b*sinh(c + d*x))/(d*cosh(c + d*x))
```

3.2 $\int \left(a+b\text{sech}^2(c+dx)\right)\sinh^3(c+dx)\,dx$

Optimal. Leaf size=44

$$-\frac{(a-b)\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}+\frac{b\text{sech}(c+dx)}{d}$$

```
[Out] -(a-b)*cosh(d*x+c)/d+1/3*a*cosh(d*x+c)^3/d+b*sech(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4218, 459}

$$-\frac{(a-b)\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}+\frac{b\text{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^3,x]
```

```
[Out] -(((a - b)*Cosh[c + d*x])/d) + (a*Cosh[c + d*x]^3)/(3*d) + (b*Sech[c + d*x]
)/d
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\text{sech}^2(c+dx)\right)\sinh^3(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)}{x^2}\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(a\left(1-\frac{b}{a}\right)+\frac{b}{x^2}-ax^2\right)\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{(a-b)\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}+\frac{b\text{sech}(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 1.20

$$-\frac{3a\cosh(c+dx)}{4d}+\frac{b\cosh(c+dx)}{d}+\frac{a\cosh(3(c+dx))}{12d}+\frac{b\operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^3,x]
```

```
[Out] (-3*a*Cosh[c + d*x])/(4*d) + (b*Cosh[c + d*x])/d + (a*Cosh[3*(c + d*x)])/(1
2*d) + (b*Sech[c + d*x])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(42) = 84$.
time = 1.38, size = 111, normalized size = 2.52

method	result	size
risch	$\frac{a\,e^{3dx+3c}}{24d}-\frac{3a\,e^{dx+c}}{8d}+\frac{b\,e^{dx+c}}{2d}-\frac{3\,e^{-dx-c}a}{8d}+\frac{e^{-dx-c}b}{2d}+\frac{a\,e^{-3dx-3c}}{24d}+\frac{2b\,e^{dx+c}}{d(1+e^{2dx+2c})}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*a/d*exp(3*d*x+3*c)-3/8*a/d*exp(d*x+c)+1/2*b/d*exp(d*x+c)-3/8/d*exp(-d*
x-c)*a+1/2/d*exp(-d*x-c)*b+1/24*a/d*exp(-3*d*x-3*c)+2/d*b*exp(d*x+c)/(1+exp
(2*d*x+2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(42) = 84$.
time = 0.29, size = 111, normalized size = 2.52

$$\frac{1}{24}a\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{1}{2}b\left(\frac{e^{(-dx-c)}}{d}+\frac{5\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-dx-c)}+e^{(-3\,dx-3\,c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*a*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x
- 3*c)/d) + 1/2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x -
c) + e^(-3*d*x - 3*c))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(42) = 84$.
time = 0.45, size = 85, normalized size = 1.93

$$\frac{a\cosh(dx+c)^4+a\sinh(dx+c)^4-4\,(2\,a-3\,b)\cosh(dx+c)^2+2\,(3\,a\cosh(dx+c)^2-4\,a+6\,b)\sinh(dx+c)^2-9\,a+36\,b}{24\,d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 4*(2*a - 3*b)*cosh(d*x + c)^2
 + 2*(3*a*cosh(d*x + c)^2 - 4*a + 6*b)*sinh(d*x + c)^2 - 9*a + 36*b)/(d*cos
h(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^{2}\left(c + dx\right)\right)\sinh^{3}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(42) = 84.
time = 0.40, size = 85, normalized size = 1.93

$$\frac{a\left(e^{(dx+c)} + e^{(-dx-c)}\right)^{3} - 12\,a\left(e^{(dx+c)} + e^{(-dx-c)}\right) + 12\,b\left(e^{(dx+c)} + e^{(-dx-c)}\right) + \frac{48\,b}{e^{(dx+c)} + e^{(-dx-c)}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/24*(a*(e^(d*x + c) + e^(-d*x - c))^3 - 12*a*(e^(d*x + c) + e^(-d*x - c))
+ 12*b*(e^(d*x + c) + e^(-d*x - c)) + 48*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

Mupad [**B**]
time = 0.18, size = 44, normalized size = 1.00

$$\frac{a\cosh(c + d\,x)^{3}}{3\,d} - \frac{\cosh(c + d\,x)\ (a - b)}{d} + \frac{b}{d\cosh\left(c + d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (a*cosh(c + d*x)^3)/(3*d) - (cosh(c + d*x)*(a - b))/d + b/(d*cosh(c + d*x))
```

3.3 $\int (a + b\,\mathbf{sech}^2(c + dx)) \sinh^2(c + dx)\, dx$

Optimal. Leaf size=43

$$-\frac{1}{2}(a-2b)x + \frac{a\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{b\tanh(c+dx)}{d}$$

```
[Out] -1/2*(a-2*b)*x+1/2*a*cosh(d*x+c)*sinh(d*x+c)/d-b*tanh(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4217, 466, 396, 212}

$$-\frac{1}{2}x(a-2b) + \frac{a\sinh(c+dx)\cosh(c+dx)}{2d} - \frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^2,x]
```

```
[Out] -1/2*((a - 2*b)*x) + (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (b*Tanh[c + d*
x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int (a+b\text{sech}^2(c+dx))\sinh^2(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+b-bx^2)}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{a\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{a-2bx^2}{1-x^2}\,dx, x, \tanh(c+d$$
$$= \frac{a\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{b\tanh(c+dx)}{d} - \frac{(a-2b)\text{Subst}($$
$$= -\frac{1}{2}(a-2b)x + \frac{a\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{b\tanh(c+dx)}{d}$$

Mathematica [A]
time = 0.14, size = 57, normalized size = 1.33

$$\frac{a(-c-dx)}{2d} + \frac{b\tanh^{-1}(\tanh(c+dx))}{d} + \frac{a\sinh(2(c+dx))}{4d} - \frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x]^2,x]
```

```
[Out] (a*(-c - d*x))/(2*d) + (b*ArcTanh[Tanh[c + d*x]])/d + (a*Sinh[2*(c + d*x)])
/(4*d) - (b*Tanh[c + d*x])/d
```

Maple [A]
time = 1.00, size = 45, normalized size = 1.05

method	result	size
derivativedivides	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b(dx+c-\tanh(dx+c))}{d}$	45
default	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b(dx+c-\tanh(dx+c))}{d}$	45
risch	$-\frac{ax}{2}+bx+\frac{e^{2dx+2c}a}{8d}-\frac{e^{-2dx-2c}a}{8d}+\frac{2b}{d(1+e^{2dx+2c})}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b*(d*x+c-tanh(d*x+c)))
```

Maxima [**A**]
time = 0.26, size = 62, normalized size = 1.44

$$-\frac{1}{8}a\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + b\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + b*(x + c/d - 2/(d*(
e^(-2*d*x - 2*c) + 1)))
```

Fricas [**A**]
time = 0.43, size = 67, normalized size = 1.56

$$\frac{a\sinh(dx+c)^3 - 4((a-2b)dx - 2b)\cosh(dx+c) + (3a\cosh(dx+c)^2 + a - 8b)\sinh(dx+c)}{8d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(a*sinh(d*x + c)^3 - 4*((a - 2*b)*d*x - 2*b)*cosh(d*x + c) + (3*a*cosh(
d*x + c)^2 + a - 8*b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{sech}^2(c+dx))\sinh^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(39) = 78$.
time = 0.40, size = 92, normalized size = 2.14

$$-\frac{4(dx+c)(a-2b) - ae^{(2dx+2c)} - \frac{ae^{(4dx+4c)} - 2be^{(4dx+4c)} + 14be^{(2dx+2c)} - a}{e^{(4dx+4c)} + e^{(2dx+2c)}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*(d*x + c)*(a - 2*b) - a*e^(2*d*x + 2*c) - (a*e^(4*d*x + 4*c) - 2*b*
e^(4*d*x + 4*c) + 14*b*e^(2*d*x + 2*c) - a)/(e^(4*d*x + 4*c) + e^(2*d*x + 2
*c)))/d
```

Mupad [B]
time = 1.43, size = 55, normalized size = 1.28

$$\frac{a\cosh(c+d\,x)\,\sinh(c+d\,x)}{2\,d}-\frac{\frac{a\,d\,x}{2}-b\,d\,x}{d}-\frac{b\sinh(c+d\,x)}{d\cosh\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (a*cosh(c + d*x)*sinh(c + d*x))/(2*d) - ((a*d*x)/2 - b*d*x)/d - (b*sinh(c +
 d*x))/(d*cosh(c + d*x))
```

3.4 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)\sinh(c+dx)\,dx$

Optimal. Leaf size=24

$$\frac{a\cosh(c+dx)}{d}-\frac{b\text{sech}(c+dx)}{d}$$

```
[Out] a*cosh(d*x+c)/d-b*sech(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4218, 14}

$$\frac{a\cosh(c+dx)}{d}-\frac{b\text{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x],x]
```

```
[Out] (a*Cosh[c + d*x])/d - (b*Sech[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\text{sech}^2(c+dx)\right)\sinh(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2}\,dx,x,\cosh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(a+\frac{b}{x^2}\right)\,dx,x,\cosh(c+dx)\right)}{d}\\
&= \frac{a\cosh(c+dx)}{d}-\frac{b\text{sech}(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.46

$$\frac{a\cosh(c)\cosh(dx)}{d} - \frac{b\text{sech}(c+dx)}{d} + \frac{a\sinh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Sinh[c + d*x],x]
```

```
[Out] (a*Cosh[c]*Cosh[d*x])/d - (b*Sech[c + d*x])/d + (a*Sinh[c]*Sinh[d*x])/d
```

Maple [A]
time = 0.66, size = 26, normalized size = 1.08

method	result	size
derivativedivides	$-\frac{b\,\text{sech}(dx+c)-\frac{a}{\text{sech}(dx+c)}}{d}$	26
default	$-\frac{b\,\text{sech}(dx+c)-\frac{a}{\text{sech}(dx+c)}}{d}$	26
risch	$\frac{a\,e^{dx+c}}{2d}+\frac{e^{-dx-c}a}{2d}-\frac{2b\,e^{dx+c}}{d(1+e^{2dx+2c})}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*sinh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(b*sech(d*x+c)-a/sech(d*x+c))
```

Maxima [A]
time = 0.27, size = 36, normalized size = 1.50

$$\frac{a\cosh\left(dx+c\right)}{d} - \frac{2\,b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*cosh(d*x + c)/d - 2*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

Fricas [A]
time = 0.36, size = 38, normalized size = 1.58

$$\frac{a\cosh\left(dx+c\right)^{2}+a\sinh\left(dx+c\right)^{2}+a-2\,b}{2\,d\cosh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a - 2*b)/(d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2\left(c + dx\right)\right)\sinh\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*sinh(d*x+c),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sinh(c + d*x), x)
```

Giac [A]
time = 0.41, size = 45, normalized size = 1.88

$$\frac{a\left(e^{(dx+c)} + e^{(-dx-c)}\right) - \frac{4\,b}{e^{(dx+c)}+e^{(-dx-c)}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*(e^(d*x + c) + e^(-d*x - c)) - 4*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

Mupad [B]
time = 0.08, size = 26, normalized size = 1.08

$$-\frac{b - a\cosh(c + d\,x)^2}{d\cosh\left(c + d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] -(b - a*cosh(c + d*x)^2)/(d*cosh(c + d*x))
```

3.5 $\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=27

$$-\frac{(a+b)\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\operatorname{sech}(c+dx)}{d}$$

```
[Out] -(a+b)*arctanh(cosh(d*x+c))/d+b*sech(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4218, 464, 212}

$$\frac{b\operatorname{sech}(c+dx)}{d}-\frac{(a+b)\tanh^{-1}(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -(((a + b)*ArcTanh[Cosh[c + d*x]])/d) + (b*Sech[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{b\operatorname{sech}(c+dx)}{d} - \frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{(a+b)\tanh^{-1}(\cosh(c+dx))}{d} + \frac{b\operatorname{sech}(c+dx)}{d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 67 vs. $2(27) = 54$.
time = 0.04, size = 67, normalized size = 2.48

$$-\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{b\operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a*Log[Cosh[c/2 + (d*x)/2]])/d) + (a*Log[Sinh[c/2 + (d*x)/2]])/d + (b*Log
[Tanh[(c + d*x)/2]])/d + (b*Sech[c + d*x])/d
```

Maple [**A**]
time = 1.10, size = 36, normalized size = 1.33

method	result	size
derivativedivides	$\frac{-2a\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\left(\frac{1}{\cosh(dx+c)}-2\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	36
default	$\frac{-2a\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\left(\frac{1}{\cosh(dx+c)}-2\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	36
risch	$\frac{2b\,\mathrm{e}^{dx+c}}{d(1+\mathrm{e}^{2dx+2c})}-\frac{a\ln(\mathrm{e}^{dx+c}+1)}{d}-\frac{\ln(\mathrm{e}^{dx+c}+1)b}{d}+\frac{a\ln(\mathrm{e}^{dx+c}-1)}{d}+\frac{\ln(\mathrm{e}^{dx+c}-1)b}{d}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a*arctanh(exp(d*x+c))+b*(1/cosh(d*x+c)-2*arctanh(exp(d*x+c))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(27) = 54$.
time = 0.28, size = 80, normalized size = 2.96

$$-b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} - \frac{2\,e^{(-dx-c)}}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right) + \frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(
e^(-2*d*x - 2*c) + 1))) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(27) = 54$.
time = 0.36, size = 180, normalized size = 6.67

$$\frac{2\,b\cosh\left(dx+c\right)-\left(\left(a+b\right)\cosh\left(dx+c\right)^{2}+2\,\left(a+b\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(a+b\right)\sinh\left(dx+c\right)^{2}+a+b\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)+\left(\left(a+b\right)\cosh\left(dx+c\right)^{2}+2\,\left(a+b\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(a+b\right)\sinh\left(dx+c\right)^{2}+a+b\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)+2\,b\sinh\left(dx+c\right)}{d\cosh\left(dx+c\right)^{2}+2\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+d\sinh\left(dx+c\right)^{2}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] (2*b*cosh(d*x + c) - ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sin
h(d*x + c) + (a + b)*sinh(d*x + c)^2 + a + b)*log(cosh(d*x + c) + sinh(d*x
+ c) + 1) + ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c
) + (a + b)*sinh(d*x + c)^2 + a + b)*log(cosh(d*x + c) + sinh(d*x + c) - 1)
 + 2*b*sinh(d*x + c))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c)
+ d*sinh(d*x + c)^2 + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^{2}\left(c + dx\right)\right)\operatorname{csch}\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*csch(c + d*x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(27) = 54$.
time = 0.40, size = 72, normalized size = 2.67

$$-\frac{(a+b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-(a+b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\,b}{e^{(dx+c)}+e^{(-dx-c)}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*((a + b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (a + b)*log(e^(d*x + c)
 + e^(-d*x - c) - 2) - 4*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

Mupad [B]
time = 1.41, size = 79, normalized size = 2.93

$$\frac{b}{d\cosh\left(c+d\,x\right)}-\frac{2\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\left(a\,\sqrt{-d^2}+b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^2+2\,a\,b+b^2}}\right)\,\sqrt{a^2+2\,a\,b+b^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)/sinh(c + d*x),x)
```

```
[Out] b/(d*cosh(c + d*x)) - (2*atan((exp(d*x)*exp(c)*(a*(-d^2)^(1/2) + b*(-d^2)^(
1/2)))/(d*(2*a*b + a^2 + b^2)^(1/2)))*(2*a*b + a^2 + b^2)^(1/2))/(-d^2)^(1/
2)
```

3.6 $\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=27

$$-\frac{(a+b)\coth(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}$$

```
[Out] -(a+b)*coth(d*x+c)/d-b*tanh(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4217, 14}

$$-\frac{(a+b)\coth(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -(((a + b)*Coth[c + d*x])/d) - (b*Tanh[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a+b-bx^2}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(-b+\frac{a+b}{x^2}\right)\,dx,x,\tanh(c+dx)\right)}{d}\\
&= -\frac{(a+b)\coth(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 37, normalized size = 1.37

$$-\frac{a\coth(c+dx)}{d}-\frac{b\coth(c+dx)}{d}-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
[Out] -((a*Coth[c + d*x])/d) - (b*Coth[c + d*x])/d - (b*Tanh[c + d*x])/d
```

Maple [A]
time = 1.98, size = 48, normalized size = 1.78

method	result	size
risch	$-\frac{2\left(a\,e^{2dx+2c}+a+2b\right)}{d\left(1+e^{2dx+2c}\right)\left(e^{2dx+2c}-1\right)}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] -2*(a*exp(2*d*x+2*c)+a+2*b)/d/(1+exp(2*d*x+2*c))/(exp(2*d*x+2*c)-1)
```

Maxima [A]
time = 0.28, size = 39, normalized size = 1.44

$$\frac{2\,a}{d(e^{(-2\,dx-2\,c)}-1)}+\frac{4\,b}{d(e^{(-4\,dx-4\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
[Out] 2*a/(d*(e^(-2*d*x - 2*c) - 1)) + 4*b/(d*(e^(-4*d*x - 4*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(27) = 54.
time = 0.59, size = 91, normalized size = 3.37

$$-\frac{4\left((a+b)\cosh\left(dx+c\right)-b\sinh\left(dx+c\right)\right)}{d\cosh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+d\sinh\left(dx+c\right)^{3}-d\cosh\left(dx+c\right)+\left(3\,d\cosh\left(dx+c\right)^{2}+d\right)\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
[Out] -4*((a + b)*cosh(d*x + c) - b*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(
d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(
d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2\left(c + dx\right)\right)\operatorname{csch}^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*csch(c + d*x)**2, x)
```

Giac [A]
time = 0.40, size = 34, normalized size = 1.26

$$-\frac{2\left(ae^{(2\,dx+2\,c)} + a + 2\,b\right)}{d(e^{(4\,dx+4\,c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2*(a*e^(2*d*x + 2*c) + a + 2*b)/(d*(e^(4*d*x + 4*c) - 1))
```

Mupad [B]
time = 1.38, size = 34, normalized size = 1.26

$$-\frac{2\left(a + 2\,b + a\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)/sinh(c + d*x)^2,x)
```

```
[Out] -(2*(a + 2*b + a*exp(2*c + 2*d*x)))/(d*(exp(4*c + 4*d*x) - 1))
```

3.7 $\int \mathrm{csch}^3(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=54

$$\frac{(a+3b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{(a+b)\coth(c+dx)\mathrm{csch}(c+dx)}{2d}-\frac{b\mathrm{sech}(c+dx)}{d}$$

```
[Out] 1/2*(a+3*b)*arctanh(cosh(d*x+c))/d-1/2*(a+b)*coth(d*x+c)*csch(d*x+c)/d-b*se
ch(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4218, 467, 464, 212}

$$\frac{(a+3b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{(a+b)\coth(c+dx)\mathrm{csch}(c+dx)}{2d}-\frac{b\mathrm{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 3*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - ((a + b)*Coth[c + d*x]*Csch[c +
d*x])/(2*d) - (b*Sech[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
```

```
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx=\frac{\operatorname{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)^2}\,dx,x,\cosh(c+dx)\right)}{d}$$
$$=-\frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{\operatorname{Subst}\left(\int \frac{-2b-(a+b)x^2}{x^2(1-x^2)}\,dx,x\right.}{2d}$$
$$=-\frac{(a+b)\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{b\operatorname{sech}(c+dx)}{d}+\frac{(a+3b)}{}$$
$$=\frac{(a+3b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{(a+b)\coth(c+dx)\operatorname{csch}(c+}{2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 131 vs. 2(54) = 108.
time = 0.04, size = 131, normalized size = 2.43

$$-\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{3b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/8*(a*Csch[(c + d*x)/2]^2)/d - (b*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Tan
h[(c + d*x)/2]])/(2*d) - (3*b*Log[Tanh[(c + d*x)/2]])/(2*d) - (a*Sech[(c +
d*x)/2]^2)/(8*d) - (b*Sech[(c + d*x)/2]^2)/(8*d) - (b*Sech[c + d*x])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(50) = 100.
time = 1.99, size = 151, normalized size = 2.80

method	result

risch	$-\frac{e^{dx+c}\left(a\,e^{4dx+4c}+3b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+3b\right)}{d\left(e^{2dx+2c}-1\right)^{2}\left(1+e^{2dx+2c}\right)}-\frac{a\ln\left(e^{dx+c}-1\right)}{2d}-\frac{3\ln\left(e^{dx+c}-1\right)b}{2d}+\frac{a\ln\left(e^{dx+c}+1\right)}{2d}+\frac{3\ln(e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -exp(d*x+c)*(a*exp(4*d*x+4*c)+3*b*exp(4*d*x+4*c)+2*a*exp(2*d*x+2*c)-2*b*exp
(2*d*x+2*c)+a+3*b)/d/(exp(2*d*x+2*c)-1)^2/(1+exp(2*d*x+2*c))-1/2*a/d*ln(exp
(d*x+c)-1)-3/2/d*ln(exp(d*x+c)-1)*b+1/2*a/d*ln(exp(d*x+c)+1)+3/2/d*ln(exp(d
*x+c)+1)*b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(50) = 100$.
time = 0.28, size = 198, normalized size = 3.67

$$\frac{1}{2}\,b\left(\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3\,e^{(-dx-c)}-2\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}\right)}{d(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}-e^{(-6\,dx-6\,c)}-1)}\right)+\frac{1}{2}\,a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x
 - c) - 2*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(e^(-2*d*x - 2*c) + e^(
-4*d*x - 4*c) - e^(-6*d*x - 6*c) - 1))) + 1/2*a*(log(e^(-d*x - c) + 1)/d -
log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(50) = 100$.
time = 0.57, size = 924, normalized size = 17.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a + 3*b)*cosh(d*x + c)^5 + 10*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c
)^4 + 2*(a + 3*b)*sinh(d*x + c)^5 + 4*(a - b)*cosh(d*x + c)^3 + 4*(5*(a + 3
*b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^3 + 4*(5*(a + 3*b)*cosh(d*x + c)
^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a + 3*b)*cosh(d*x + c) -
 ((a + 3*b)*cosh(d*x + c)^6 + 6*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (
a + 3*b)*sinh(d*x + c)^6 - (a + 3*b)*cosh(d*x + c)^4 + (15*(a + 3*b)*cosh(d
*x + c)^2 - a - 3*b)*sinh(d*x + c)^4 + 4*(5*(a + 3*b)*cosh(d*x + c)^3 - (a
+ 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a + 3*b)*cosh(d*x + c)^2 + (15*(a
+ 3*b)*cosh(d*x + c)^4 - 6*(a + 3*b)*cosh(d*x + c)^2 - a - 3*b)*sinh(d*x +
```

```
c)^2 + 2*(3*(a + 3*b)*cosh(d*x + c)^5 - 2*(a + 3*b)*cosh(d*x + c)^3 - (a +
3*b)*cosh(d*x + c))*sinh(d*x + c) + a + 3*b)*log(cosh(d*x + c) + sinh(d*x +
 c) + 1) + ((a + 3*b)*cosh(d*x + c)^6 + 6*(a + 3*b)*cosh(d*x + c)*sinh(d*x
+ c)^5 + (a + 3*b)*sinh(d*x + c)^6 - (a + 3*b)*cosh(d*x + c)^4 + (15*(a + 3
*b)*cosh(d*x + c)^2 - a - 3*b)*sinh(d*x + c)^4 + 4*(5*(a + 3*b)*cosh(d*x +
c)^3 - (a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a + 3*b)*cosh(d*x + c)^2
 + (15*(a + 3*b)*cosh(d*x + c)^4 - 6*(a + 3*b)*cosh(d*x + c)^2 - a - 3*b)*s
inh(d*x + c)^2 + 2*(3*(a + 3*b)*cosh(d*x + c)^5 - 2*(a + 3*b)*cosh(d*x + c)
^3 - (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + a + 3*b)*log(cosh(d*x + c) +
sinh(d*x + c) - 1) + 2*(5*(a + 3*b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x +
c)^2 + a + 3*b)*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(
d*x + c)^5 + d*sinh(d*x + c)^6 - d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2
- d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x +
 c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 - d
)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 - d*cosh(d
*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2\left(c + dx\right)\right)\operatorname{csch}^3\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
[Out] Integral((a + b*sech(c + d*x)**2)*csch(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(50) = 100.
time = 0.41, size = 142, normalized size = 2.63

$$\frac{(a+3b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-(a+3b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+3b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-8b\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-4e^{(dx+c)}-4e^{(-dx-c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
[Out] 1/4*((a + 3*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (a + 3*b)*log(e^(d*x +
 c) + e^(-d*x - c) - 2) - 4*(a*(e^(d*x + c) + e^(-d*x - c))^2 + 3*b*(e^(d*x
 + c) + e^(-d*x - c))^2 - 8*b)/((e^(d*x + c) + e^(-d*x - c))^3 - 4*e^(d*x +
 c) - 4*e^(-d*x - c)))/d
```

Mupad [B]
time = 0.17, size = 160, normalized size = 2.96

$$\frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(a\sqrt{-d^2}+3b\sqrt{-d^2}\right)}{d\sqrt{a^2+6ab+9b^2}}\right)\sqrt{a^2+6ab+9b^2}}{\sqrt{-d^2}}-\frac{e^{c+dx}\,(a+b)}{d\,\left(e^{2c+2dx}-1\right)}-\frac{2\,e^{c+dx}\,(a+b)}{d\,\left(e^{4c+4dx}-2\,e^{2c+2dx}+1\right)}-\frac{2\,b\,e^{c+dx}}{d\,\left(e^{2c+2dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a*(-d^2)^(1/2) + 3*b*(-d^2)^(1/2)))/(d*(6*a*b + a^2
 + 9*b^2)^(1/2)))*(6*a*b + a^2 + 9*b^2)^(1/2))/(-d^2)^(1/2) - (exp(c + d*x)
*(a + b))/(d*(exp(2*c + 2*d*x) - 1)) - (2*exp(c + d*x)*(a + b))/(d*(exp(4*c
 + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (2*b*exp(c + d*x))/(d*(exp(2*c + 2*d
*x) + 1))
```

3.8 $\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=45

$$\frac{(a+2b)\coth(c+dx)}{d}-\frac{(a+b)\coth^3(c+dx)}{3d}+\frac{b\tanh(c+dx)}{d}$$

```
[Out] (a+2*b)*coth(d*x+c)/d-1/3*(a+b)*coth(d*x+c)^3/d+b*tanh(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {4217, 459}

$$-\frac{(a+b)\coth^3(c+dx)}{3d}+\frac{(a+2b)\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b)*Coth[c + d*x])/d - ((a + b)*Coth[c + d*x]^3)/(3*d) + (b*Tanh[c +
 d*x])/d
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx&=\frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+b-bx^2)}{x^4}\,dx,x,\tanh(c+dx)\right)}{d}\\&=\frac{\operatorname{Subst}\left(\int \left(b+\frac{a+b}{x^4}+\frac{-a-2b}{x^2}\right)\,dx,x,\tanh(c+dx)\right)}{d}\\&=\frac{(a+2b)\coth(c+dx)}{d}-\frac{(a+b)\coth^3(c+dx)}{3d}+\frac{b\tanh(c+dx}{d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 84, normalized size = 1.87

$$\frac{2a\coth(c+dx)}{3d}+\frac{5b\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}-\frac{b\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (2*a*Coth[c + d*x])/(3*d) + (5*b*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Cs
ch[c + d*x]^2)/(3*d) - (b*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) + (b*Tanh[c
+ d*x])/d
```

Maple [A]
time = 1.79, size = 75, normalized size = 1.67

method	result	size
risch	$-\frac{4\left(3a\,\mathrm{e}^{4dx+4c}+2a\,\mathrm{e}^{2dx+2c}+8b\,\mathrm{e}^{2dx+2c}-a-4b\right)}{3d\left(\mathrm{e}^{2dx+2c}-1\right)^{3}\left(1+\mathrm{e}^{2dx+2c}\right)}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(3*a*exp(4*d*x+4*c)+2*a*exp(2*d*x+2*c)+8*b*exp(2*d*x+2*c)-a-4*b)/d/(ex
p(2*d*x+2*c)-1)^3/(1+exp(2*d*x+2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(43) = 86$.
time = 0.27, size = 187, normalized size = 4.16

$$\frac{4}{3}\,a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)+\frac{16}{3}\,b\left(\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}-1)}-\frac{1}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) - 1))) + 16/3*b*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*
e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^
(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(43) = 86$.
time = 0.52, size = 246, normalized size = 5.47

$$-\frac{8\left((a-2\,b)\cosh\left(dx+c\right)^{2}+4\,(a+b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(a-2\,b)\sinh\left(dx+c\right)^{2}+a+4\,b\right)}{3\left(d\cosh\left(dx+c\right)^{6}+6\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+d\sinh\left(dx+c\right)^{6}-2\,d\cosh\left(dx+c\right)^{4}+\left(15\,d\cosh\left(dx+c\right)^{2}-2\,d\right)\sinh\left(dx+c\right)^{4}+4\left(5\,d\cosh\left(dx+c\right)^{3}-2\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}-d\cosh\left(dx+c\right)^{2}+\left(15\,d\cosh\left(dx+c\right)^{4}-12\,d\cosh\left(dx+c\right)^{2}-d\right)\sinh\left(dx+c\right)^{2}+2\left(3\,d\cosh\left(dx+c\right)^{5}-4\,d\cosh\left(dx+c\right)^{3}+d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")

[Out] -8/3*((a - 2*b)*cosh(d*x + c)^2 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a - 2*b)*sinh(d*x + c)^2 + a + 4*b)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 2*d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 2*d*cosh(d*x + c))*sinh(d*x + c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + 2*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{sech}^{2}\left(c + dx\right)\right) \operatorname{csch}^{4}\left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4*(a+b*sech(d*x+c)**2),x)

[Out] Integral((a + b*sech(c + d*x)**2)*csch(c + d*x)**4, x)

Giac [A]
time = 0.41, size = 80, normalized size = 1.78

$$-\frac{2\left(\frac{3\,b}{e^{(2\,dx+2\,c)}+1} - \frac{3\,be^{(4\,dx+4\,c)} - 6\,ae^{(2\,dx+2\,c)} - 12\,be^{(2\,dx+2\,c)} + 2\,a + 5\,b}{\left(e^{(2\,dx+2\,c)}-1\right)^{3}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")

[Out] -2/3*(3*b/(e^(2*d*x + 2*c) + 1) - (3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 12*b*e^(2*d*x + 2*c) + 2*a + 5*b)/(e^(2*d*x + 2*c) - 1)^3)/d

Mupad [B]
time = 1.40, size = 172, normalized size = 3.82

$$\frac{\frac{2\,b}{3\,d} + \frac{2\,b\,e^{4\,c+4\,d\,x}}{3\,d} - \frac{4\,e^{2\,c+2\,d\,x}\,(2\,a+3\,b)}{3\,d}}{3\,e^{2\,c+2\,d\,x} - 3\,e^{4\,c+4\,d\,x} + e^{6\,c+6\,d\,x} - 1} - \frac{\frac{2\,(2\,a+3\,b)}{3\,d} - \frac{2\,b\,e^{2\,c+2\,d\,x}}{3\,d}}{e^{4\,c+4\,d\,x} - 2\,e^{2\,c+2\,d\,x} + 1} + \frac{2\,b}{3\,d\,\left(e^{2\,c+2\,d\,x} - 1\right)} - \frac{2\,b}{d\,\left(e^{2\,c+2\,d\,x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d*x)^2)/sinh(c + d*x)^4,x)

[Out] ((2*b)/(3*d) + (2*b*exp(4*c + 4*d*x))/(3*d) - (4*exp(2*c + 2*d*x)*(2*a + 3*b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((2*(2*a + 3*b))/(3*d) - (2*b*exp(2*c + 2*d*x))/(3*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) + (2*b)/(3*d*(exp(2*c + 2*d*x) - 1)) - (2*b)/(d*(exp(2*c + 2*d*x) + 1))

3.9 $\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2 \sinh^4(c+dx)\,dx$

Optimal. Leaf size=114

$$\frac{1}{8}\left(3a^2-24ab+8b^2\right)x-\frac{a(a-8b)\cosh(c+dx)\sinh(c+dx)}{8d}-\frac{\left(a^2-8ab+4b^2\right)\tanh(c+dx)}{4d}+\frac{a^2\sinh^4(c+d}{4}$$

```
[Out] 1/8*(3*a^2-24*a*b+8*b^2)*x-1/8*a*(a-8*b)*cosh(d*x+c)*sinh(d*x+c)/d-1/4*(a^2
-8*a*b+4*b^2)*tanh(d*x+c)/d+1/4*a^2*sinh(d*x+c)^4*tanh(d*x+c)/d-1/3*b^2*tan
h(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4217, 474, 466, 1167, 212}

$$-\frac{\left(a^2-8ab+4b^2\right)\tanh(c+dx)}{4d}+\frac{1}{8}x\left(3a^2-24ab+8b^2\right)+\frac{a^2\sinh^4(c+dx)\tanh(c+dx)}{4d}-\frac{a(a-8b)\sinh(c+dx)\cosh(c+dx)}{8d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^4,x]
```

```
[Out] ((3*a^2 - 24*a*b + 8*b^2)*x)/8 - (a*(a - 8*b)*Cosh[c + d*x]*Sinh[c + d*x])/
(8*d) - ((a^2 - 8*a*b + 4*b^2)*Tanh[c + d*x])/(4*d) + (a^2*Sinh[c + d*x]^4*
Tanh[c + d*x])/(4*d) - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
```

```
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2 \sinh^4(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b-bx^2\right)^2}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{a^2\sinh^4(c+dx)\tanh(c+dx)}{4d} - \frac{\text{Subst}\left(\int \frac{x^4\left(5a^2-4(a+b)^2+4b^2x^2\right)}{\left(1-x^2\right)^2}\right.}{4d}$$

$$= -\frac{a(a-8b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a^2\sinh^4(c+dx)\tanh(}{4d}$$

$$= -\frac{a(a-8b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{a^2\sinh^4(c+dx)\tanh(}{4d}$$

$$= -\frac{a(a-8b)\cosh(c+dx)\sinh(c+dx)}{8d} - \frac{\left(a^2-8ab+4b^2\right)\tanh}{4d}$$

$$= \frac{1}{8}\left(3a^2-24ab+8b^2\right)x - \frac{a(a-8b)\cosh(c+dx)\sinh(c+dx)}{8d}$$

Mathematica [A]
time = 1.14, size = 153, normalized size = 1.34

$$\frac{\left(b+a\cosh^2(c+dx)\right)^2\operatorname{sech}^3(c+dx)\left(32b^2\operatorname{sech}(c)\sinh(dx)+64(3a-2b)b\cosh^2(c+dx)\operatorname{sech}(c)\sinh(dx)+3\cosh^3(c+dx)\left(4\left(3a^2-24ab+8b^2\right)dx-8a(a-2b)\sinh(2(c+dx))+a^2\sinh(4(c+dx))\right)+32b^2\cosh(c+dx)\tanh(c)\right)}{24d(a+2b+a\cosh(2(c+dx)))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^4,x]

[Out] ((b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(32*b^2*Sech[c]*Sinh[d*x] + 64*(3*a - 2*b)*b*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + 3*Cosh[c + d*x]^3*(4*(3*a^2 - 24*a*b + 8*b^2)*d*x - 8*a*(a - 2*b)*Sinh[2*(c + d*x)] + a^2*Sinh[4*(c + d*x)]) + 32*b^2*Cosh[c + d*x]*Tanh[c]))/(24*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)

Maple [A]
time = 1.81, size = 192, normalized size = 1.68

method	result
risch	$\frac{3a^2x}{8} - 3abx + b^2x + \frac{a^2 e^{4dx+4c}}{64d} + \frac{a\, e^{2dx+2c} b}{4d} - \frac{a^2 e^{2dx+2c}}{8d} - \frac{a\, e^{-2dx-2c} b}{4d} + \frac{a^2 e^{-2dx-2c}}{8d} - \frac{a^2 e^{-4dx-4c}}{64d} - \frac{4b(3a\, e^{4d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x,method=_RETURNVERBOSE)

[Out] 3/8*a^2*x-3*a*b*x+b^2*x+1/64*a^2/d*exp(4*d*x+4*c)+1/4*a/d*exp(2*d*x+2*c)*b-1/8*a^2/d*exp(2*d*x+2*c)-1/4*a/d*exp(-2*d*x-2*c)*b+1/8*a^2/d*exp(-2*d*x-2*c)-1/64*a^2/d*exp(-4*d*x-4*c)-4/3*b*(3*a*exp(4*d*x+4*c)-3*b*exp(4*d*x+4*c)+6*a*exp(2*d*x+2*c)-3*b*exp(2*d*x+2*c)+3*a-2*b)/d/(1+exp(2*d*x+2*c))^3

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(104) = 208$.
time = 0.28, size = 211, normalized size = 1.85

$$\frac{1}{64}a^2\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) + \frac{1}{3}b^2\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2\right)}{d\left(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1\right)}\right) - \frac{1}{4}ab\left(\frac{12(dx+c)}{d} + \frac{e^{(-2dx-2c)}}{d} - \frac{17e^{(-2dx-2c)} + 1}{d\left(e^{(-2dx-2c)} + e^{(-4dx-4c)}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x, algorithm="maxima")

[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) + 1/3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) - 1/4*a*b*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c))))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(104) = 208$.
time = 0.54, size = 342, normalized size = 3.00

$$\frac{3a^2\sinh(dx+c)^7 + 3\left(21a^2\cosh(dx+c)^2 - 5a^2 + 16ab\right)\sinh(dx+c)^5 + 8\left(3\left(3a^2 - 24ab + 8b^2\right)dx - 48ab + 32b^2\right)\cosh(dx+c)^3 + 24\left(3\left(3a^2 - 24ab + 8b^2\right)dx - 48ab + 32b^2\right)\cosh(dx+c)\sinh(dx+c)^2 + \left(105a^2\cosh(dx+c)^4 - 30\left(5a^2 - 16ab\right)\cosh(dx+c)^2 - 63a^2 + 528ab - 256b^2\right)\sinh(dx+c)^3 + 24\left(3\left(3a^2 - 24ab + 8b^2\right)dx - 48ab + 32b^2\right)\cosh(dx+c) + 3\left(7a^2\cosh(dx+c)^6 - 5\left(5a^2 - 16ab\right)\cosh(dx+c)^4 - \left(63a^2 - 528ab + 256b^2\right)\cosh(dx+c)^2 - 15a^2 + 160ab\right)\sinh(dx+c)}{192\left(d\cosh(dx+c)^3 + 3d\cosh(dx+c)\sinh(dx+c)^2 + 3d\cosh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x, algorithm="fricas")

```
[Out] 1/192*(3*a^2*sinh(d*x + c)^7 + 3*(21*a^2*cosh(d*x + c)^2 - 5*a^2 + 16*a*b)*
sinh(d*x + c)^5 + 8*(3*(3*a^2 - 24*a*b + 8*b^2)*d*x - 48*a*b + 32*b^2)*cosh
(d*x + c)^3 + 24*(3*(3*a^2 - 24*a*b + 8*b^2)*d*x - 48*a*b + 32*b^2)*cosh(d*
x + c)*sinh(d*x + c)^2 + (105*a^2*cosh(d*x + c)^4 - 30*(5*a^2 - 16*a*b)*cos
h(d*x + c)^2 - 63*a^2 + 528*a*b - 256*b^2)*sinh(d*x + c)^3 + 24*(3*(3*a^2 -
 24*a*b + 8*b^2)*d*x - 48*a*b + 32*b^2)*cosh(d*x + c) + 3*(7*a^2*cosh(d*x +
 c)^6 - 5*(5*a^2 - 16*a*b)*cosh(d*x + c)^4 - (63*a^2 - 528*a*b + 256*b^2)*c
osh(d*x + c)^2 - 15*a^2 + 160*a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*
cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(104) = 208$.
time = 0.44, size = 231, normalized size = 2.03

$$\frac{3\,a^2e^{(4\,dx+4\,c)} - 24\,a^2e^{(2\,dx+2\,c)} + 48\,abe^{(2\,dx+2\,c)} + 24\,(3\,a^2 - 24\,ab + 8\,b^2)(dx + c) - 3\,\left(18\,a^2e^{(4\,dx+4\,c)} - 144\,abe^{(4\,dx+4\,c)} + 48\,b^2e^{(4\,dx+4\,c)} - 8\,a^2e^{(2\,dx+2\,c)} + 16\,abe^{(2\,dx+2\,c)} + a^2\right)e^{(-4\,dx-4\,c)} - \frac{256\,(3\,abe^{(4\,dx+4\,c)} - 3\,b^2e^{(4\,dx+4\,c)} + 6\,abe^{(2\,dx+2\,c)} - 3\,b^2e^{(2\,dx+2\,c)} + 3\,ab - 2\,b^2)}{\left(e^{(2\,dx+2\,c)} + 1\right)^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*e^(4*d*x + 4*c) - 24*a^2*e^(2*d*x + 2*c) + 48*a*b*e^(2*d*x + 2
*c) + 24*(3*a^2 - 24*a*b + 8*b^2)*(d*x + c) - 3*(18*a^2*e^(4*d*x + 4*c) - 1
44*a*b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) + 1
6*a*b*e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c) - 256*(3*a*b*e^(4*d*x + 4*c)
- 3*b^2*e^(4*d*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) - 3*b^2*e^(2*d*x + 2*c) + 3
*a*b - 2*b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B]
time = 0.27, size = 269, normalized size = 2.36

$$x\,\left(\frac{3\,a^2}{8} - 3\,a\,b + b^2\right) - \frac{\frac{4\,\left(a\,b-b^2\right)}{3\,d} + \frac{4\,\mathrm{e}^{4\,c+4\,d\,x}\,\left(a\,b-b^2\right)}{3\,d} + \frac{8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x} + 3\,\mathrm{e}^{4\,c+4\,d\,x} + \mathrm{e}^{6\,c+6\,d\,x} + 1} - \frac{\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(a\,b-b^2\right)}{3\,d} + \frac{4\,a\,b}{3\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x} + \mathrm{e}^{4\,c+4\,d\,x} + 1} - \frac{4\,\left(a\,b - b^2\right)}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x} + 1\right)} + \frac{\mathrm{e}^{2\,c+2\,d\,x}\,\left(2\,a\,b - a^2\right)}{8\,d} - \frac{a^2\,\mathrm{e}^{-4\,c-4\,d\,x}}{64\,d} + \frac{a^2\,\mathrm{e}^{4\,c+4\,d\,x}}{64\,d} + \frac{a\,\mathrm{e}^{-2\,c-2\,d\,x}\,\left(a - 2\,b\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] x*((3*a^2)/8 - 3*a*b + b^2) - ((4*(a*b - b^2))/(3*d) + (4*exp(4*c + 4*d*x)*
(a*b - b^2))/(3*d) + (8*a*b*exp(2*c + 2*d*x))/(3*d))/(3*exp(2*c + 2*d*x) +
```

```
3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((4*exp(2*c + 2*d*x)*(a*b - b^
2))/(3*d) + (4*a*b)/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (4
*(a*b - b^2))/(3*d*(exp(2*c + 2*d*x) + 1)) + (exp(2*c + 2*d*x)*(2*a*b - a^2
))/(8*d) - (a^2*exp(- 4*c - 4*d*x))/(64*d) + (a^2*exp(4*c + 4*d*x))/(64*d)
+ (a*exp(- 2*c - 2*d*x)*(a - 2*b))/(8*d)
```

3.10 $\int (a+b\,\mathrm{sech}^2(c+dx))^2 \sinh^3(c+dx)\,dx$

Optimal. Leaf size=72

$$-\frac{a(a-2b)\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}+\frac{(2a-b)b\,\mathrm{sech}(c+dx)}{d}+\frac{b^2\mathrm{sech}^3(c+dx)}{3d}$$

```
[Out] -a*(a-2*b)*cosh(d*x+c)/d+1/3*a^2*cosh(d*x+c)^3/d+(2*a-b)*b*sech(d*x+c)/d+1/
3*b^2*sech(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4218, 459}

$$\frac{a^2\cosh^3(c+dx)}{3d}-\frac{a(a-2b)\cosh(c+dx)}{d}+\frac{b(2a-b)\mathrm{sech}(c+dx)}{d}+\frac{b^2\mathrm{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^3,x]
```

```
[Out] -((a*(a - 2*b)*Cosh[c + d*x])/d) + (a^2*Cosh[c + d*x]^3)/(3*d) + ((2*a - b)
*b*Sech[c + d*x])/d + (b^2*Sech[c + d*x]^3)/(3*d)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \sinh^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^2}{x^4}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(a(a-2b)+\frac{b^2}{x^4}+\frac{(2a-b)b}{x^2}-a^2x^2\right)\,dx, x, \cosh(c+dx$$

$$= -\frac{a(a-2b)\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}+\frac{(2a-b)b\text{sech}(c}{d}$$

Mathematica [A]
time = 0.36, size = 83, normalized size = 1.15

$$\frac{\left(-26a^2+168ab-16b^2-3(11a^2-64ab+16b^2)\cosh(2(c+dx))-6a(a-4b)\cosh(4(c+dx))+a^2\cosh(6(c+dx))\right)\text{sech}^3(c+dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^3,x]
```

```
[Out] ((-26*a^2 + 168*a*b - 16*b^2 - 3*(11*a^2 - 64*a*b + 16*b^2)*Cosh[2*(c + d*x
)] - 6*a*(a - 4*b)*Cosh[4*(c + d*x)] + a^2*Cosh[6*(c + d*x)])*Sech[c + d*x]
^3)/(96*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(68) = 136.
time = 1.73, size = 174, normalized size = 2.42

method	result
risch	$\frac{a^2 e^{3dx+3c}}{24d}-\frac{3a^2 e^{dx+c}}{8d}+\frac{ab\, e^{dx+c}}{d}-\frac{3a^2 e^{-dx-c}}{8d}+\frac{a\, e^{-dx-c} b}{d}+\frac{a^2 e^{-3dx-3c}}{24d}+\frac{2\, e^{dx+c} b\left(6a\, e^{4dx+4c}-3b\, e^{4dx+4c}+12a\, e^{2dx+}\right.}{3d\left(1+e^{2dx+2c}\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*a^2/d*exp(3*d*x+3*c)-3/8*a^2/d*exp(d*x+c)+a*b/d*exp(d*x+c)-3/8*a^2/d*e
xp(-d*x-c)+a/d*exp(-d*x-c)*b+1/24*a^2/d*exp(-3*d*x-3*c)+2/3*exp(d*x+c)*b*(6
*a*exp(4*d*x+4*c)-3*b*exp(4*d*x+4*c)+12*a*exp(2*d*x+2*c)-2*b*exp(2*d*x+2*c)
+6*a-3*b)/d/(1+exp(2*d*x+2*c))^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(68) = 136.
time = 0.30, size = 266, normalized size = 3.69

$$\frac{1}{24}a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+ab\left(\frac{e^{(-dx-c)}}{d}+\frac{5\,e^{(-2\,dx-2\,c)}+1}{d\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}\right)-\frac{2}{3}b^2\left(\frac{3\,e^{(-dx-c)}}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}+\frac{2\,e^{(-3\,dx-3\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}+\frac{3\,e^{(-5\,dx-5\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) + a*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x -
c) + e^(-3*d*x - 3*c)))) - 2/3*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c) +
 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*(3*e^(
-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5*d*x -
 5*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(68) = 136$.
time = 0.41, size = 212, normalized size = 2.94

$$\frac{a^2 \cosh(dx+c)^6 + a^2 \sinh(dx+c)^6 - 6\,(a^2 - 4\,ab)\cosh(dx+c)^4 + 3\left(5\,a^2\cosh(dx+c)^2 - 2\,a^2 + 8\,ab\right)\sinh(dx+c)^4 - 3\,(11\,a^2 - 64\,ab + 16\,b^2)\cosh(dx+c)^2 + 3\left(5\,a^2\cosh(dx+c)^4 - 12\,(a^2 - 4\,ab)\cosh(dx+c)^2 - 11\,a^2 + 64\,ab - 16\,b^2\right)\sinh(dx+c)^2 - 26\,a^2 + 168\,ab - 16\,b^2}{24\left(d\cosh(dx+c)^3 + 3\,d\cosh(dx+c)\sinh(dx+c)^2 + 3\,d\cosh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(a^2*cosh(d*x + c)^6 + a^2*sinh(d*x + c)^6 - 6*(a^2 - 4*a*b)*cosh(d*x
+ c)^4 + 3*(5*a^2*cosh(d*x + c)^2 - 2*a^2 + 8*a*b)*sinh(d*x + c)^4 - 3*(11*
a^2 - 64*a*b + 16*b^2)*cosh(d*x + c)^2 + 3*(5*a^2*cosh(d*x + c)^4 - 12*(a^2
 - 4*a*b)*cosh(d*x + c)^2 - 11*a^2 + 64*a*b - 16*b^2)*sinh(d*x + c)^2 - 26*
a^2 + 168*a*b - 16*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c
)^2 + 3*d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c+dx)\right)^2 \sinh^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sinh(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(68) = 136$.
time = 0.42, size = 140, normalized size = 1.94

$$\frac{a^2\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3 - 12\,a^2\left(e^{(dx+c)} + e^{(-dx-c)}\right) + 24\,ab\left(e^{(dx+c)} + e^{(-dx-c)}\right) + \frac{16\left(6\,ab\left(e^{(dx+c)} + e^{(-dx-c)}\right)^2 - 3\,b^2\left(e^{(dx+c)} + e^{(-dx-c)}\right)^2 + 4\,b^2\right)}{\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12*a^2*(e^(d*x + c) + e^(-d*x -
c)) + 24*a*b*(e^(d*x + c) + e^(-d*x - c)) + 16*(6*a*b*(e^(d*x + c) + e^(-d*
x - c))^2 - 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 4*b^2)/(e^(d*x + c) + e^
(-d*x - c))^3)/d
```

Mupad [B]
time = 1.50, size = 201, normalized size = 2.79

$$\frac{e^{c+dx}(8ab-3a^2)}{8d}+\frac{e^{-c-dx}(8ab-3a^2)}{8d}+\frac{a^2e^{-3c-3dx}}{24d}+\frac{a^2e^{3c+3dx}}{24d}-\frac{8b^2e^{c+dx}}{3d(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1)}+\frac{2e^{c+dx}(2ab-b^2)}{d(e^{2c+2dx}+1)}+\frac{8b^2e^{c+dx}}{3d(2e^{2c+2dx}+e^{4c+4dx}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (exp(c + d*x)*(8*a*b - 3*a^2))/(8*d) + (exp(- c - d*x)*(8*a*b - 3*a^2))/(8*
d) + (a^2*exp(- 3*c - 3*d*x))/(24*d) + (a^2*exp(3*c + 3*d*x))/(24*d) - (8*b
^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c +
6*d*x) + 1)) + (2*exp(c + d*x)*(2*a*b - b^2))/(d*(exp(2*c + 2*d*x) + 1)) +
(8*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.11 $\int \left(a+b\,\text{sech}^2(c+dx)\right)^2 \sinh^2(c+dx)\,dx$

Optimal. Leaf size=73

$$-\frac{1}{2}a(a-4b)x+\frac{a(a-4b)\tanh(c+dx)}{2d}+\frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] -1/2*a*(a-4*b)*x+1/2*a*(a-4*b)*tanh(d*x+c)/d+1/2*a^2*sinh(d*x+c)^2*tanh(d*x
+c)/d+1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4217, 474, 470, 327, 212}

$$\frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d}+\frac{a(a-4b)\tanh(c+dx)}{2d}-\frac{1}{2}ax(a-4b)+\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^2,x]
```

```
[Out] -1/2*(a*(a - 4*b)*x) + (a*(a - 4*b)*Tanh[c + d*x])/(2*d) + (a^2*Sinh[c + d*
x]^2*Tanh[c + d*x])/(2*d) + (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2\sinh^2(c+dx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2\left(a+b-bx^2\right)^2}{\left(1-x^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d}-\frac{\operatorname{Subst}\left(\int \frac{x^2\left(3a^2-2(a+b)^2+2b^2x^2\right)}{1-x^2}\,dx\right.}{2d}\\ &= \frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d}+\frac{b^2\tanh^3(c+dx)}{3d}-\frac{(a(a-4b))}{}\\ &= \frac{a(a-4b)\tanh(c+dx)}{2d}+\frac{a^2\sinh^2(c+dx)\tanh(c+dx)}{2d}+\frac{b^2\tan}{}\\ &= -\frac{1}{2}a(a-4b)x+\frac{a(a-4b)\tanh(c+dx)}{2d}+\frac{a^2\sinh^2(c+dx)\tanh}{2d}\end{aligned}$$

Mathematica [A]
time = 0.65, size = 126, normalized size = 1.73

$$\frac{\left(b+a\cosh^2(c+dx)\right)^2\operatorname{sech}^3(c+dx)\left(-4b^2\operatorname{sech}(c)\sinh(dx)-4(6a-b)b\cosh^2(c+dx)\operatorname{sech}(c)\sinh(dx)+3a\cosh^3(c+dx)(-2(a-4b)dx+a\sinh(2(c+dx)))-4b^2\cosh(c+dx)\tanh(c)\right)}{3d(a+2b+a\cosh(2(c+dx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x]^2,x]
```

```
[Out] ((b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(-4*b^2*Sech[c]*Sinh[d*x] - 4*(6
*a - b)*b*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + 3*a*Cosh[c + d*x]^3*(-2*(a -
4*b)*d*x + a*Sinh[2*(c + d*x)]) - 4*b^2*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2
*b + a*Cosh[2*(c + d*x)])^2)
```

Maple [A]
time = 1.66, size = 109, normalized size = 1.49

method	result	size
risch	$-\frac{a^{2} x}{2}+2 a b x+\frac{a^{2} {\mathrm e}^{2 d x +2 c}}{8 d}-\frac{a^{2} {\mathrm e}^{-2 d x -2 c}}{8 d}+\frac{2 b \left(6 a \,{\mathrm e}^{4 d x +4 c}-3 b \,{\mathrm e}^{4 d x +4 c}+12 a \,{\mathrm e}^{2 d x +2 c}+6 a -b \right)}{3 d \left(1+{\mathrm e}^{2 d x +2 c}\right)^{3}}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*x+2*a*b*x+1/8*a^2/d*exp(2*d*x+2*c)-1/8*a^2/d*exp(-2*d*x-2*c)+2/3*b
*(6*a*exp(4*d*x+4*c)-3*b*exp(4*d*x+4*c)+12*a*exp(2*d*x+2*c)+6*a-b)/d/(1+exp
(2*d*x+2*c))^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(65) = 130.
time = 0.30, size = 160, normalized size = 2.19

$$-\frac{1}{8} a^{2} \left(4 x - \frac{e^{\left(2 d x + 2 c\right)}}{d} + \frac{e^{\left(-2 d x - 2 c\right)}}{d}\right) + 2 a b \left(x + \frac{c}{d} - \frac{2}{d \left(e^{\left(-2 d x - 2 c\right)} + 1\right)}\right) + \frac{2}{3} b^{2} \left(\frac{3 e^{\left(-4 d x - 4 c\right)}}{d \left(3 e^{\left(-2 d x - 2 c\right)} + 3 e^{\left(-4 d x - 4 c\right)} + e^{\left(-6 d x - 6 c\right)} + 1\right)} + \frac{1}{d \left(3 e^{\left(-2 d x - 2 c\right)} + 3 e^{\left(-4 d x - 4 c\right)} + e^{\left(-6 d x - 6 c\right)} + 1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 2*a*b*(x + c/d -
2/(d*(e^(-2*d*x - 2*c) + 1))) + 2/3*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x
 - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x -
 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(65) = 130.
time = 0.42, size = 252, normalized size = 3.45

$$\frac{3 a^{2} \sinh \left(d x + c\right)^{5} - 4 \left(3 \left(a^{2} - 4 a b\right) d x - 12 a b + 2 b^{2}\right) \cosh \left(d x + c\right)^{3} - 12 \left(3 \left(a^{2} - 4 a b\right) d x - 12 a b + 2 b^{2}\right) \cosh \left(d x + c\right) \sinh \left(d x + c\right)^{2} + \left(30 a^{2} \cosh \left(d x + c\right)^{2} + 9 a^{2} - 48 a b + 8 b^{2}\right) \sinh \left(d x + c\right)^{3} - 12 \left(3 \left(a^{2} - 4 a b\right) d x - 12 a b + 2 b^{2}\right) \cosh \left(d x + c\right) + 3 \left(5 a^{2} \cosh \left(d x + c\right)^{4} + \left(9 a^{2} - 48 a b + 8 b^{2}\right) \cosh \left(d x + c\right)^{2} + 2 a^{2} - 16 a b - 8 b^{2}\right) \sinh \left(d x + c\right)}{24 \left(d \cosh \left(d x + c\right)^{3} + 3 d \cosh \left(d x + c\right) \sinh \left(d x + c\right)^{2} + 3 d \cosh \left(d x + c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*a^2*sinh(d*x + c)^5 - 4*(3*(a^2 - 4*a*b)*d*x - 12*a*b + 2*b^2)*cosh
(d*x + c)^3 - 12*(3*(a^2 - 4*a*b)*d*x - 12*a*b + 2*b^2)*cosh(d*x + c)*sinh(
d*x + c)^2 + (30*a^2*cosh(d*x + c)^2 + 9*a^2 - 48*a*b + 8*b^2)*sinh(d*x + c
)^3 - 12*(3*(a^2 - 4*a*b)*d*x - 12*a*b + 2*b^2)*cosh(d*x + c) + 3*(5*a^2*co
sh(d*x + c)^4 + (9*a^2 - 48*a*b + 8*b^2)*cosh(d*x + c)^2 + 2*a^2 - 16*a*b -
 8*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)
^2 + 3*d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^2\sinh^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sinh(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(65) = 130.
time = 0.42, size = 144, normalized size = 1.97

$$\frac{3\,a^2e^{(2\,dx+2\,c)}-12\,(a^2-4\,ab)(dx+c)+3\left(2\,a^2e^{(2\,dx+2\,c)}-8\,abe^{(2\,dx+2\,c)}-a^2\right)e^{(-2\,dx-2\,c)}+\frac{16\left(6\,abe^{(4\,dx+4\,c)}-3\,b^2e^{(4\,dx+4\,c)}+12\,abe^{(2\,dx+2\,c)}+6\,ab-b^2\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*e^(2*d*x + 2*c) - 12*(a^2 - 4*a*b)*(d*x + c) + 3*(2*a^2*e^(2*d*
x + 2*c) - 8*a*b*e^(2*d*x + 2*c) - a^2)*e^(-2*d*x - 2*c) + 16*(6*a*b*e^(4*d
*x + 4*c) - 3*b^2*e^(4*d*x + 4*c) + 12*a*b*e^(2*d*x + 2*c) + 6*a*b - b^2)/(
e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B]
time = 0.16, size = 236, normalized size = 3.23

$$\frac{\frac{2\,(b^2+2\,a\,b)}{3\,d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(2\,a\,b-b^2)}{3\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}+x\left(2\,a\,b-\frac{a^2}{2}\right)+\frac{\frac{2\,(2\,a\,b-b^2)}{3\,d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(b^2+2\,a\,b)}{3\,d}+\frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\,(2\,a\,b-b^2)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}+\frac{2\,(2\,a\,b-b^2)}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{a^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d}+\frac{a^2\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] ((2*(2*a*b + b^2))/(3*d) + (2*exp(2*c + 2*d*x)*(2*a*b - b^2))/(3*d))/(2*exp
(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + x*(2*a*b - a^2/2) + ((2*(2*a*b - b^
2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d) + (2*exp(4*c + 4*d*x)*
(2*a*b - b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c +
6*d*x) + 1) + (2*(2*a*b - b^2))/(3*d*(exp(2*c + 2*d*x) + 1)) - (a^2*exp(- 2
*c - 2*d*x))/(8*d) + (a^2*exp(2*c + 2*d*x))/(8*d)
```

3.12 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^2 \sinh(c+dx)\,dx$

Optimal. Leaf size=45

$$\frac{a^2\cosh(c+dx)}{d}-\frac{2ab\,\mathrm{sech}(c+dx)}{d}-\frac{b^2\mathrm{sech}^3(c+dx)}{3d}$$

```
[Out] a^2*cosh(d*x+c)/d-2*a*b*sech(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4218, 276}

$$\frac{a^2\cosh(c+dx)}{d}-\frac{2ab\,\mathrm{sech}(c+dx)}{d}-\frac{b^2\mathrm{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x],x]
```

```
[Out] (a^2*Cosh[c + d*x])/d - (2*a*b*Sech[c + d*x])/d - (b^2*Sech[c + d*x]^3)/(3*
d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^2 \sinh(c+dx)\,dx &= \frac{\mathrm{Subst}\left(\int \frac{(b+ax^2)^2}{x^4}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= \frac{\mathrm{Subst}\left(\int \left(a^2+\frac{b^2}{x^4}+\frac{2ab}{x^2}\right)\,dx, x, \cosh(c+dx)\right)}{d}\\
&= \frac{a^2\cosh(c+dx)}{d}-\frac{2ab\,\mathrm{sech}(c+dx)}{d}-\frac{b^2\mathrm{sech}^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 59, normalized size = 1.31

$$\frac{(9a^2 - 24ab - 8b^2 + 12a(a-2b)\cosh(2(c+dx)) + 3a^2\cosh(4(c+dx)))\operatorname{sech}^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Sinh[c + d*x],x]
```

```
[Out] ((9*a^2 - 24*a*b - 8*b^2 + 12*a*(a - 2*b)*Cosh[2*(c + d*x)] + 3*a^2*Cosh[4*
(c + d*x)])*Sech[c + d*x]^3)/(24*d)
```

Maple [A]
time = 0.57, size = 43, normalized size = 0.96

method	result	size
derivativedivides	$-\frac{\frac{b^2\operatorname{sech}(dx+c)^3}{3}+2ab\operatorname{sech}(dx+c)-\frac{a^2}{\operatorname{sech}(dx+c)}}{d}$	43
default	$-\frac{\frac{b^2\operatorname{sech}(dx+c)^3}{3}+2ab\operatorname{sech}(dx+c)-\frac{a^2}{\operatorname{sech}(dx+c)}}{d}$	43
risch	$\frac{a^2 e^{dx+c}}{2d}+\frac{a^2 e^{-dx-c}}{2d}-\frac{4 e^{dx+c} b(3a e^{4dx+4c}+6a e^{2dx+2c}+2b e^{2dx+2c}+3a)}{3d(1+e^{2dx+2c})^3}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(1/3*b^2*sech(d*x+c)^3+2*a*b*sech(d*x+c)-a^2/sech(d*x+c))
```

Maxima [A]
time = 0.29, size = 65, normalized size = 1.44

$$\frac{a^2\cosh(dx+c)}{d} - \frac{4ab}{d(e^{(dx+c)} + e^{(-dx-c)})} - \frac{8b^2}{3d(e^{(dx+c)} + e^{(-dx-c)})^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a^2*cosh(d*x + c)/d - 4*a*b/(d*(e^(d*x + c) + e^(-d*x - c))) - 8/3*b^2/(d*(
e^(d*x + c) + e^(-d*x - c))^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(43) = 86.
time = 0.40, size = 133, normalized size = 2.96

$$\frac{3a^2\cosh(dx+c)^4 + 3a^2\sinh(dx+c)^4 + 12(a^2-2ab)\cosh(dx+c)^2 + 6(3a^2\cosh(dx+c)^2 + 2a^2 - 4ab)\sinh(dx+c)^2 + 9a^2 - 24ab - 8b^2}{6(d\cosh(dx+c)^3 + 3d\cosh(dx+c)\sinh(dx+c)^2 + 3d\cosh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*cosh(d*x + c)^4 + 3*a^2*sinh(d*x + c)^4 + 12*(a^2 - 2*a*b)*cosh(
d*x + c)^2 + 6*(3*a^2*cosh(d*x + c)^2 + 2*a^2 - 4*a*b)*sinh(d*x + c)^2 + 9*
a^2 - 24*a*b - 8*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^
2 + 3*d*cosh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2\left(c + dx\right)\right)^2 \sinh\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*sinh(d*x+c),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sinh(c + d*x), x)
```

Giac [**A**]
time = 0.41, size = 75, normalized size = 1.67

$$\frac{3\,a^2\left(e^{(dx+c)} + e^{(-dx-c)}\right) - \frac{8\left(3\,ab\left(e^{(dx+c)} + e^{(-dx-c)}\right)^2 + 2\,b^2\right)}{\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/6*(3*a^2*(e^(d*x + c) + e^(-d*x - c)) - 8*(3*a*b*(e^(d*x + c) + e^(-d*x -
 c))^2 + 2*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d
```

Mupad [**B**]
time = 1.47, size = 45, normalized size = 1.00

$$\frac{a^2\cosh(c + d\,x)}{d} - \frac{\frac{b^2}{3} + 2\,a\,b\cosh(c + d\,x)^2}{d\cosh\left(c + d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (a^2*cosh(c + d*x))/d - (b^2/3 + 2*a*b*cosh(c + d*x)^2)/(d*cosh(c + d*x)^3)
```

3.13 $\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=52

$$-\frac{(a+b)^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

```
[Out] -(a+b)^2*arctanh(cosh(d*x+c))/d+b*(2*a+b)*sech(d*x+c)/d+1/3*b^2*sech(d*x+c)
^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4218, 472, 213}

$$\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}-\frac{(a+b)^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -(((a + b)^2*ArcTanh[Cosh[c + d*x]])/d) + (b*(2*a + b)*Sech[c + d*x])/d + (
b^2*Sech[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = -\frac{\operatorname{Subst}\left(\int \frac{(b+ax^2)^2}{x^4(1-x^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(\frac{b^2}{x^4}+\frac{b(2a+b)}{x^2}-\frac{(a+b)^2}{-1+x^2}\right)dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{b(2a+b)\operatorname{sech}(c+dx)}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}+\frac{(a+b)^2\operatorname{Subst}\left(\int \frac{}{-1}\right.}{}$$
$$= -\frac{(a+b)^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}+\frac{b^2\mathrm{s}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(52) = 104.
time = 0.39, size = 108, normalized size = 2.08

$$-\frac{4\left(b+a\cosh^2(c+dx)\right)^2\left(-b^2-3b(2a+b)\cosh^2(c+dx)+3(a+b)^2\cosh^3(c+dx)\left(\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\operatorname{sech}^3(c+dx)}{3d(a+2b+a\cosh(2(c+dx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (-4*(b + a*Cosh[c + d*x]^2)^2*(-b^2 - 3*b*(2*a + b)*Cosh[c + d*x]^2 + 3*(a
+ b)^2*Cosh[c + d*x]^3*(Log[Cosh[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]]))*S
ech[c + d*x]^3)/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

Maple [A]
time = 1.18, size = 72, normalized size = 1.38

method	result
derivativedivides	$\frac{-2a^2\operatorname{arctanh}\left(e^{dx+c}\right)+2ab\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b^2\left(\frac{1}{3\cosh(dx+c)^3}+\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)}{d}$
default	$\frac{-2a^2\operatorname{arctanh}\left(e^{dx+c}\right)+2ab\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b^2\left(\frac{1}{3\cosh(dx+c)^3}+\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)}{d}$
risch	$\frac{2b\,e^{dx+c}\left(6a\,e^{4dx+4c}+3b\,e^{4dx+4c}+12a\,e^{2dx+2c}+10b\,e^{2dx+2c}+6a+3b\right)}{3d\left(1+e^{2dx+2c}\right)^3}-\frac{a^2\ln\left(e^{dx+c}+1\right)}{d}-\frac{2ab\ln\left(e^{dx+c}+1\right)}{d}-\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*(1/cosh(d*x+c)-2*arctanh(exp(d*x+c)))
+b^2*(1/3/cosh(d*x+c)^3+1/cosh(d*x+c)-2*arctanh(exp(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(50) = 100$.
time = 0.30, size = 197, normalized size = 3.79

$$-\frac{1}{3}\,b^2\left(\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\,\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,\left(3\,e^{(-dx-c)}+10\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}\right)}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)-2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right)+\frac{a^2\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d - 2*(3*e^(-
d*x - c) + 10*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(3*e^(-2*d*x - 2*c)
 + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) - 2*a*b*(log(e^(-d*x - c) +
 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(e^(-2*d*x - 2*c) + 1))
) + a^2*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(50) = 100$.
time = 0.38, size = 1148, normalized size = 22.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*(2*a*b + b^2)*cosh(d*x + c)^5 + 30*(2*a*b + b^2)*cosh(d*x + c)*sinh(
d*x + c)^4 + 6*(2*a*b + b^2)*sinh(d*x + c)^5 + 4*(6*a*b + 5*b^2)*cosh(d*x +
 c)^3 + 4*(15*(2*a*b + b^2)*cosh(d*x + c)^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^
3 + 12*(5*(2*a*b + b^2)*cosh(d*x + c)^3 + (6*a*b + 5*b^2)*cosh(d*x + c))*si
nh(d*x + c)^2 + 6*(2*a*b + b^2)*cosh(d*x + c) - 3*((a^2 + 2*a*b + b^2)*cosh
(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 +
2*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(5
*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 +
 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 +
 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2
 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 6*((a^2 + 2*a*b + b^2
)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b +
b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) +
 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b +
 b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*
a*b + b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(
a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b +
```

```
 b^2 + 6*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x
 + c) + sinh(d*x + c) - 1) + 6*(5*(2*a*b + b^2)*cosh(d*x + c)^4 + 2*(6*a*b
+ 5*b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^6 +
 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 + 3*d*cosh(d*x + c)^
4 + 3*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 +
3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x
+ c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 +
2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^2 \mathrm{csch}\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(50) = 100.
time = 0.40, size = 139, normalized size = 2.67

$$-\frac{3\left(a^2 + 2ab + b^2\right)\log\left(e^{(dx+c)} + e^{(-dx-c)} + 2\right) - 3\left(a^2 + 2ab + b^2\right)\log\left(e^{(dx+c)} + e^{(-dx-c)} - 2\right) - \frac{4\left(6ab\left(e^{(dx+c)} + e^{(-dx-c)}\right)^2 + 3b^2\left(e^{(dx+c)} + e^{(-dx-c)}\right)^2 + 4b^2\right)}{\left(e^{(dx+c)} + e^{(-dx-c)}\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*(a^2 + 2*a*b + b^2)*log(e^(d*x + c) + e^(-d*x - c) + 2) - 3*(a^2 +
2*a*b + b^2)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 4*(6*a*b*(e^(d*x + c) +
e^(-d*x - c))^2 + 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 4*b^2)/(e^(d*x + c
) + e^(-d*x - c))^3)/d
```

Mupad [B]
time = 1.50, size = 232, normalized size = 4.46

$$\frac{2\,e^{c+d\,x}\,\left(b^2+2\,a\,b\right)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{8\,b^2\,e^{c+d\,x}}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}-\frac{2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^c\,\left(a^2\,\sqrt{-d^2}+b^2\,\sqrt{-d^2}+2\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4}}\right)\,\sqrt{a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4}}{\sqrt{-d^2}}+\frac{8\,b^2\,e^{c+d\,x}}{3\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2/sinh(c + d*x),x)
```

```
[Out] (2*exp(c + d*x)*(2*a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1)) - (8*b^2*exp(c +
d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)
) - (2*atan((exp(d*x)*exp(c)*(a^2*(-d^2)^(1/2) + b^2*(-d^2)^(1/2) + 2*a*b*(
-d^2)^(1/2)))/(d*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^(1/2)))*(4*a*b
^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^(1/2))/(-d^2)^(1/2) + (8*b^2*exp(c +
d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.14 $\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx$

Optimal. Leaf size=50

$$-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{2b(a+b)\tanh(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] -(a+b)^2*coth(d*x+c)/d-2*b*(a+b)*tanh(d*x+c)/d+1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$,
Rules used = {4217, 276}

$$-\frac{2b(a+b)\tanh(c+dx)}{d}-\frac{(a+b)^2\coth(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -(((a + b)^2*Coth[c + d*x])/d) - (2*b*(a + b)*Tanh[c + d*x])/d + (b^2*Tanh[
c + d*x]^3)/(3*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-bx^2)^2}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(-2b(a+b)+\frac{(a+b)^2}{x^2}+b^2x^2\right)dx,x,\tanh(c+dx)\right)}{d}\\ &= -\frac{(a+b)^2\coth(c+dx)}{d}-\frac{2b(a+b)\tanh(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(50) = 100.
time = 1.21, size = 109, normalized size = 2.18

$$-\frac{4\left(b+a\cosh^2(c+dx)\right)^2\operatorname{sech}^3(c+dx)\left(b^2\operatorname{sech}(c)\sinh(dx)+\cosh^2(c+dx)\left(-3(a+b)^2\coth(c+dx)\operatorname{csch}(c)+b(6a+5b)\operatorname{sech}(c)\right)\sinh(dx)+b^2\cosh(c+dx)\tanh(c)\right)}{3d(a+2b+a\cosh(2(c+dx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (-4*(b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(b^2*Sech[c]*Sinh[d*x] + Cosh
[c + d*x]^2*(-3*(a + b)^2*Coth[c + d*x]*Csch[c] + b*(6*a + 5*b)*Sech[c])*Si
nh[d*x] + b^2*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^
2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(48) = 96.
time = 2.10, size = 129, normalized size = 2.58

method	result	size
risch	$-\frac{2\left(3a^2 e^{6dx+6c}+9a^2 e^{4dx+4c}+12ab\,e^{4dx+4c}+9a^2 e^{2dx+2c}+24ab\,e^{2dx+2c}+16b^2 e^{2dx+2c}+3a^2+12ab+8b^2\right)}{3d\left(1+e^{2dx+2c}\right)^3\left(e^{2dx+2c}-1\right)}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(3*a^2*exp(6*d*x+6*c)+9*a^2*exp(4*d*x+4*c)+12*a*b*exp(4*d*x+4*c)+9*a^2
*exp(2*d*x+2*c)+24*a*b*exp(2*d*x+2*c)+16*b^2*exp(2*d*x+2*c)+3*a^2+12*a*b+8*
b^2)/d/(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(48) = 96.
time = 0.30, size = 140, normalized size = 2.80

$$-\frac{16}{3}b^2\left(\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}-2e^{(-6dx-6c)}-e^{(-8dx-8c)}+1)}+\frac{1}{d(2e^{(-2dx-2c)}-2e^{(-6dx-6c)}-e^{(-8dx-8c)}+1)}\right)+\frac{2a^2}{d(e^{(-2dx-2c)}-1)}+\frac{8ab}{d(e^{(-4dx-4c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -16/3*b^2*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) -
 e^(-8*d*x - 8*c) + 1)) + 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) - e
^(-8*d*x - 8*c) + 1))) + 2*a^2/(d*(e^(-2*d*x - 2*c) - 1)) + 8*a*b/(d*(e^(-4
*d*x - 4*c) - 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(48) = 96.
time = 0.37, size = 284, normalized size = 5.68

$$-\frac{4\left((3a^2+6ab+4b^2)\cosh(dx+c)^3+3(3a^2+6ab+4b^2)\cosh(dx+c)\sinh(dx+c)^2-2(3ab+2b^2)\sinh(dx+c)^3+(9a^2+18ab+8b^2)\cosh(dx+c)-2\left(3(3ab+2b^2)\cosh(dx+c)^2+3ab+4b^2\right)\sinh(dx+c)\right)}{3\left(d\cosh(dx+c)^5+5d\cosh(dx+c)\sinh(dx+c)^4+d\sinh(dx+c)^5+d\cosh(dx+c)^3+\left(10d\cosh(dx+c)^2+3d\right)\sinh(dx+c)^3+\left(10d\cosh(dx+c)^3+3d\cosh(dx+c)\right)\sinh(dx+c)^2-2d\cosh(dx+c)+\left(5d\cosh(dx+c)^4+9d\cosh(dx+c)^2+2d\right)\sinh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -4/3*((3*a^2 + 6*a*b + 4*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 6*a*b + 4*b^2)*c
osh(d*x + c)*sinh(d*x + c)^2 - 2*(3*a*b + 2*b^2)*sinh(d*x + c)^3 + (9*a^2 +
 18*a*b + 8*b^2)*cosh(d*x + c) - 2*(3*(3*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a
*b + 4*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x
+ c)^4 + d*sinh(d*x + c)^5 + d*cosh(d*x + c)^3 + (10*d*cosh(d*x + c)^2 + 3*
d)*sinh(d*x + c)^3 + (10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x +
c)^2 - 2*d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + 2*d
)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2\left(c + dx\right)\right)^2 \operatorname{csch}^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(48) = 96.
time = 0.43, size = 111, normalized size = 2.22

$$-\frac{2\left(\frac{3\left(a^2+2ab+b^2\right)}{e^{(2dx+2c)}-1} - \frac{6abe^{(4dx+4c)}+3b^2e^{(4dx+4c)}+12abe^{(2dx+2c)}+12b^2e^{(2dx+2c)}+6ab+5b^2}{\left(e^{(2dx+2c)}+1\right)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*(a^2 + 2*a*b + b^2)/(e^(2*d*x + 2*c) - 1) - (6*a*b*e^(4*d*x + 4*c)
+ 3*b^2*e^(4*d*x + 4*c) + 12*a*b*e^(2*d*x + 2*c) + 12*b^2*e^(2*d*x + 2*c) +
 6*a*b + 5*b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B]
time = 1.49, size = 215, normalized size = 4.30

$$\frac{\frac{2\,\left(3\,b^2+2\,a\,b\right)}{3\,d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(b^2+2\,a\,b\right)}{3\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}+\frac{\frac{2\,\left(b^2+2\,a\,b\right)}{3\,d}+\frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\,\left(b^2+2\,a\,b\right)}{3\,d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(3\,b^2+2\,a\,b\right)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}-\frac{2\,\left(a^2+2\,a\,b+b^2\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}+\frac{2\,\left(b^2+2\,a\,b\right)}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2/sinh(c + d*x)^2,x)
```

```
[Out] ((2*(2*a*b + 3*b^2))/(3*d) + (2*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d))/(2*e
xp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((2*(2*a*b + b^2))/(3*d) + (2*exp
(4*c + 4*d*x)*(2*a*b + b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b + 3*b^2))/(
3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2
*(2*a*b + a^2 + b^2))/(d*(exp(2*c + 2*d*x) - 1)) + (2*(2*a*b + b^2))/(3*d*(
exp(2*c + 2*d*x) + 1))
```

3.15 $\int \mathrm{csch}^3(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=104

$$\frac{(a+b)(a+5b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{(3a^2+6ab+5b^2)\coth(c+dx)\mathrm{csch}(c+dx)}{6d}-\frac{b(6a+5b)\mathrm{sech}(c+dx)}{3d}$$

```
[Out] 1/2*(a+b)*(a+5*b)*arctanh(cosh(d*x+c))/d-1/6*(3*a^2+6*a*b+5*b^2)*coth(d*x+c
)*csch(d*x+c)/d-1/3*b*(6*a+5*b)*sech(d*x+c)/d+1/3*b^2*csch(d*x+c)^2*sech(d*
x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4218, 473, 467, 464, 212}

$$-\frac{(3a^2+6ab+5b^2)\coth(c+dx)\mathrm{csch}(c+dx)}{6d}-\frac{b(6a+5b)\mathrm{sech}(c+dx)}{3d}+\frac{(a+b)(a+5b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b^2\mathrm{csch}^2(c+dx)\mathrm{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*(a + 5*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - ((3*a^2 + 6*a*b + 5*b^2)
*Coth[c + d*x]*Csch[c + d*x])/(6*d) - (b*(6*a + 5*b)*Sech[c + d*x])/(3*d) +
 (b^2*Csch[c + d*x]^2*Sech[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
```

```
a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^4\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{b^2\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{3d} + \frac{\operatorname{Subst}\left(\int \frac{b(6a+5b)+3a^2x^2}{x^2\left(1-x^2\right)^2}\,dx, x, c\right.}{3d} \\
&= -\frac{\left(3a^2+6ab+5b^2\right)\coth(c+dx)\operatorname{csch}(c+dx)}{6d} + \frac{b^2\operatorname{csch}^2(c+d}{} \\
&= -\frac{\left(3a^2+6ab+5b^2\right)\coth(c+dx)\operatorname{csch}(c+dx)}{6d} - \frac{b(6a+5b)\operatorname{sec}}{3d} \\
&= \frac{(a+b)(a+5b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{\left(3a^2+6ab+5b^2\right)\operatorname{cot}}{6}
\end{aligned}$$

Mathematica [A]
time = 1.11, size = 144, normalized size = 1.38

$$-\frac{\left(b+a\cosh^2(c+dx)\right)^2\left(8b^2+48b(a+b)\cosh^2(c+dx)+3(a+b)\cosh^3(c+dx)\left((a+b)\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)-4(a+5b)\left(\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)+(a+b)\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)\right)\right)\operatorname{sech}^3(c+dx)}{6d(a+2b+a\cosh(2(c+dx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/6*((b + a*Cosh[c + d*x]^2)^2*(8*b^2 + 48*b*(a + b)*Cosh[c + d*x]^2 + 3*(
a + b)*Cosh[c + d*x]^3*((a + b)*Csch[(c + d*x)/2]^2 - 4*(a + 5*b)*(Log[Cosh
[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]]) + (a + b)*Sech[(c + d*x)/2]^2))*Se
ch[c + d*x]^3)/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(96) = 192$.
time = 2.40, size = 318, normalized size = 3.06

method	result
risch	$-\frac{e^{dx+c}\left(3a^2e^{8dx+8c}+18ab\,e^{8dx+8c}+15b^2e^{8dx+8c}+12a^2e^{6dx+6c}+24ab\,e^{6dx+6c}+20b^2e^{6dx+6c}+18a^2e^{4dx+4c}+12ab\,e^{4dx+4c}-22b^2e^{4dx}\right.}{3d\left(1+e^{2dx+2c}\right)^3\left(e^{2dx+2c}-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*exp(d*x+c)*(3*a^2*exp(8*d*x+8*c)+18*a*b*exp(8*d*x+8*c)+15*b^2*exp(8*d*
x+8*c)+12*a^2*exp(6*d*x+6*c)+24*a*b*exp(6*d*x+6*c)+20*b^2*exp(6*d*x+6*c)+18
*a^2*exp(4*d*x+4*c)+12*a*b*exp(4*d*x+4*c)-22*b^2*exp(4*d*x+4*c)+12*a^2*exp(
2*d*x+2*c)+24*a*b*exp(2*d*x+2*c)+20*b^2*exp(2*d*x+2*c)+3*a^2+18*a*b+15*b^2)
/d/(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^2-1/2*a^2/d*ln(exp(d*x+c)-1)-3*a
*b/d*ln(exp(d*x+c)-1)-5/2/d*ln(exp(d*x+c)-1)*b^2+1/2*a^2/d*ln(exp(d*x+c)+1)
+3*a*b/d*ln(exp(d*x+c)+1)+5/2/d*ln(exp(d*x+c)+1)*b^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(96) = 192$.
time = 0.27, size = 354, normalized size = 3.40

$$\frac{1}{6}b^2\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(15\,e^{(-dx-c)}+20\,e^{(-3\,dx-3\,c)}-22\,e^{(-5\,dx-5\,c)}+20\,e^{(-7\,dx-7\,c)}+15\,e^{(-9\,dx-9\,c)}\right)}{d(e^{(-2\,dx-2\,c)}-2\,e^{(-4\,dx-4\,c)}-2\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}\right)+ab\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3\,e^{(-dx-c)}-2\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}\right)}{d(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}-e^{(-6\,dx-6\,c)}-1)}\right)+\frac{1}{2}a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d - 2*(15*e^
(-d*x - c) + 20*e^(-3*d*x - 3*c) - 22*e^(-5*d*x - 5*c) + 20*e^(-7*d*x - 7*c
) + 15*e^(-9*d*x - 9*c))/(d*(e^(-2*d*x - 2*c) - 2*e^(-4*d*x - 4*c) - 2*e^(-
6*d*x - 6*c) + e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a*b*(3*log(e^
(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 2*e^(-3
*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) -
 e^(-6*d*x - 6*c) - 1))) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x -
 c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e
^(-4*d*x - 4*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2930 vs. $2(96) = 192$.

time = 0.43, size = 2930, normalized size = 28.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^9 + 54*(a^2 + 6*a*b + 5*b^2)*co
sh(d*x + c)*sinh(d*x + c)^8 + 6*(a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^9 + 8*(
3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^7 + 8*(27*(a^2 + 6*a*b + 5*b^2)*cosh(d
*x + c)^2 + 3*a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^7 + 56*(9*(a^2 + 6*a*b + 5
*b^2)*cosh(d*x + c)^3 + (3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c
)^6 + 4*(9*a^2 + 6*a*b - 11*b^2)*cosh(d*x + c)^5 + 4*(189*(a^2 + 6*a*b + 5*
b^2)*cosh(d*x + c)^4 + 42*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + 9*a^2 +
 6*a*b - 11*b^2)*sinh(d*x + c)^5 + 4*(189*(a^2 + 6*a*b + 5*b^2)*cosh(d*x +
c)^5 + 70*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + 5*(9*a^2 + 6*a*b - 11*b
^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 8*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c
)^3 + 8*(63*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 35*(3*a^2 + 6*a*b + 5*b
^2)*cosh(d*x + c)^4 + 5*(9*a^2 + 6*a*b - 11*b^2)*cosh(d*x + c)^2 + 3*a^2 +
6*a*b + 5*b^2)*sinh(d*x + c)^3 + 8*(27*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^
7 + 21*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 + 5*(9*a^2 + 6*a*b - 11*b^2)
*cosh(d*x + c)^3 + 3*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^2
 + 6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c) - 3*((a^2 + 6*a*b + 5*b^2)*cosh(d*
x + c)^10 + 10*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^2 +
 6*a*b + 5*b^2)*sinh(d*x + c)^10 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^8 +
(45*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + a^2 + 6*a*b + 5*b^2)*sinh(d*x +
 c)^8 + 8*(15*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + (a^2 + 6*a*b + 5*b^2)
*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 +
 2*(105*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 + 14*(a^2 + 6*a*b + 5*b^2)*co
sh(d*x + c)^2 - a^2 - 6*a*b - 5*b^2)*sinh(d*x + c)^6 + 4*(63*(a^2 + 6*a*b +
 5*b^2)*cosh(d*x + c)^5 + 14*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 - 3*(a^2
 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + 6*a*b + 5*b^2)*
cosh(d*x + c)^4 + 2*(105*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 35*(a^2 +
6*a*b + 5*b^2)*cosh(d*x + c)^4 - 15*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 -
 a^2 - 6*a*b - 5*b^2)*sinh(d*x + c)^4 + 8*(15*(a^2 + 6*a*b + 5*b^2)*cosh(d*
x + c)^7 + 7*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 - 5*(a^2 + 6*a*b + 5*b^2
)*cosh(d*x + c)^3 - (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + (45*(a^2 + 6*a*b + 5*b^2)*cosh(d*x
+ c)^8 + 28*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 - 30*(a^2 + 6*a*b + 5*b^2
)*cosh(d*x + c)^4 - 12*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 + a^2 + 6*a*b
+ 5*b^2)*sinh(d*x + c)^2 + a^2 + 6*a*b + 5*b^2 + 2*(5*(a^2 + 6*a*b + 5*b^2)
*cosh(d*x + c)^9 + 4*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^7 - 6*(a^2 + 6*a*b
 + 5*b^2)*cosh(d*x + c)^5 - 4*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + (a^2
+ 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
 + c) + 1) + 3*((a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^10 + 10*(a^2 + 6*a*b +
```

```
5*b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^
10 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^8 + (45*(a^2 + 6*a*b + 5*b^2)*cosh
(d*x + c)^2 + a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^8 + 8*(15*(a^2 + 6*a*b + 5
*b^2)*cosh(d*x + c)^3 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^
7 - 2*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 2*(105*(a^2 + 6*a*b + 5*b^2)*
cosh(d*x + c)^4 + 14*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 - a^2 - 6*a*b -
5*b^2)*sinh(d*x + c)^6 + 4*(63*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 + 14*(
a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 - 3*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c
))*sinh(d*x + c)^5 - 2*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 + 2*(105*(a^2
+ 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 35*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4
 - 15*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 - a^2 - 6*a*b - 5*b^2)*sinh(d*x
 + c)^4 + 8*(15*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^7 + 7*(a^2 + 6*a*b + 5*
b^2)*cosh(d*x + c)^5 - 5*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 - (a^2 + 6*a
*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x
 + c)^2 + (45*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^8 + 28*(a^2 + 6*a*b + 5*b
^2)*cosh(d*x + c)^6 - 30*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 - 12*(a^2 +
6*a*b + 5*b^2)*cosh(d*x + c)^2 + a^2 + 6*a*b + 5*b^2)*sinh(d*x + c)^2 + a^2
 + 6*a*b + 5*b^2 + 2*(5*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^9 + 4*(a^2 + 6*
a*b + 5*b^2)*cosh(d*x + c)^7 - 6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^5 - 4*
(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^3 + (a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)
)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(27*(a^2 + 6*a*
b + 5*b^2)*cosh(d*x + c)^8 + 28*(3*a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^6 + 1
0*(9*a^2 + 6*a*b - 11*b^2)*cosh(d*x + c)^4 + 12*(3*a^2 + 6*a*b + 5*b^2)*cos
h(d*x + c)^2 + 3*a^2 + 18*a*b + 15*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^10
+ 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + d*cosh(d*x + c)
^8 + (45*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 +
 d*cosh(d*x + c))*sinh(d*x + c)^7 - 2*d*cosh(d*x + c)^6 + 2*(105*d*cosh(d*x
 + c)^4 + 14*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x + c)
^5 + 14*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^{2}\left(c + dx\right)\right)^{2} \operatorname{csch}^{3}\left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(96) = 192.
time = 0.41, size = 228, normalized size = 2.19

$$\frac{3\left(a^{2}+6\,ab+5\,b^{2}\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-3\left(a^{2}+6\,ab+5\,b^{2}\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{12\left(a^{2}\left(e^{(dx+c)}+e^{(-dx-c)}\right)+2\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)+b^{2}\left(e^{(dx+c)}+e^{(-dx-c)}\right)\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{2}-4}-\frac{16\left(3\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{2}+3\,b^{2}\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{2}+2\,b^{2}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{3}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*(a^2 + 6*a*b + 5*b^2)*log(e^(d*x + c) + e^(-d*x - c) + 2) - 3*(a^2
+ 6*a*b + 5*b^2)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 12*(a^2*(e^(d*x + c)
 + e^(-d*x - c)) + 2*a*b*(e^(d*x + c) + e^(-d*x - c)) + b^2*(e^(d*x + c) +
e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4) - 16*(3*a*b*(e^(d*x + c
) + e^(-d*x - c))^2 + 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 2*b^2)/(e^(d*x
 + c) + e^(-d*x - c))^3)/d
```

Mupad [B]
time = 1.65, size = 316, normalized size = 3.04

$$\frac{\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a^2\,\sqrt{-d^2}+5\,b^2\,\sqrt{-d^2}+6\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^4+12\,a^3\,b+46\,a^2\,b^2+60\,a\,b^3+25\,b^4}}\right)\,\sqrt{a^4+12\,a^3\,b+46\,a^2\,b^2+60\,a\,b^3+25\,b^4}}{\sqrt{-d^2}}-\frac{e^{c+d\,x}\,\left(a^2+2\,a\,b+b^2\right)}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}+\frac{8\,b^2\,e^{c+d\,x}}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}-\frac{2\,e^{c+d\,x}\,\left(a^2+2\,a\,b+b^2\right)}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{4\,e^{c+d\,x}\,\left(b^2+a\,b\right)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{8\,b^2\,e^{c+d\,x}}{3\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a^2*(-d^2)^(1/2) + 5*b^2*(-d^2)^(1/2) + 6*a*b*(-d^2
)^(1/2)))/(d*(60*a*b^3 + 12*a^3*b + a^4 + 25*b^4 + 46*a^2*b^2)^(1/2)))*(60*
a*b^3 + 12*a^3*b + a^4 + 25*b^4 + 46*a^2*b^2)^(1/2))/(-d^2)^(1/2) - (exp(c
+ d*x)*(2*a*b + a^2 + b^2))/(d*(exp(2*c + 2*d*x) - 1)) + (8*b^2*exp(c + d*x
))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) -
 (2*exp(c + d*x)*(2*a*b + a^2 + b^2))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*
d*x) + 1)) - (4*exp(c + d*x)*(a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1)) - (8*b
^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.16 $\int \mathbf{csch}^4(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=75

$$\frac{(a+b)(a+3b)\coth(c+dx)}{d}-\frac{(a+b)^2\coth^3(c+dx)}{3d}+\frac{b(2a+3b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] (a+b)*(a+3*b)*coth(d*x+c)/d-1/3*(a+b)^2*coth(d*x+c)^3/d+b*(2*a+3*b)*tanh(d*
x+c)/d-1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4217, 459}

$$\frac{b(2a+3b)\tanh(c+dx)}{d}-\frac{(a+b)^2\coth^3(c+dx)}{3d}+\frac{(a+b)(a+3b)\coth(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*(a + 3*b)*Coth[c + d*x])/d - ((a + b)^2*Coth[c + d*x]^3)/(3*d) + (
b*(2*a + 3*b)*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+b-bx^2)^2}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(b(2a+3b)+\frac{(a+b)^2}{x^4}+\frac{(-a-3b)(a+b)}{x^2}-b^2x^2\right)dx, x, \tan\right.}{d}$$
$$= \frac{(a+b)(a+3b)\coth(c+dx)}{d} - \frac{(a+b)^2\coth^3(c+dx)}{3d} + \frac{b(2a}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 151 vs. $2(75) = 150$.
time = 0.91, size = 151, normalized size = 2.01

$$-\frac{\operatorname{csch}(2c)\operatorname{csch}^3(2(c+dx))\left(8a(a+2b)\sinh(2c)-6(a+2b)^2\sinh(2dx)-3a^2\sinh(2(c+dx))-6ab\sinh(2(c+dx))+a^2\sinh(6(c+dx))+2ab\sinh(6(c+dx))+3a^2\sinh(4c+2dx)+a^2\sinh(4c+6dx)+8ab\sinh(4c+6dx)+8b^2\sinh(4c+6dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/6*(Csch[2*c]*Csch[2*(c + d*x)]^3*(8*a*(a + 2*b)*Sinh[2*c] - 6*(a + 2*b)^
2*Sinh[2*d*x] - 3*a^2*Sinh[2*(c + d*x)] - 6*a*b*Sinh[2*(c + d*x)] + a^2*Sin
h[6*(c + d*x)] + 2*a*b*Sinh[6*(c + d*x)] + 3*a^2*Sinh[4*c + 2*d*x] + a^2*Si
nh[4*c + 6*d*x] + 8*a*b*Sinh[4*c + 6*d*x] + 8*b^2*Sinh[4*c + 6*d*x]))/d
```

Maple [A]
time = 2.23, size = 129, normalized size = 1.72

method	result	size
risch	$-\frac{4\left(3a^2e^{8dx+8c}+8a^2e^{6dx+6c}+16ab\,e^{6dx+6c}+6a^2e^{4dx+4c}+24ab\,e^{4dx+4c}+24b^2e^{4dx+4c}-a^2-8ab-8b^2\right)}{3d\left(1+e^{2dx+2c}\right)^3\left(e^{2dx+2c}-1\right)^3}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(3*a^2*exp(8*d*x+8*c)+8*a^2*exp(6*d*x+6*c)+16*a*b*exp(6*d*x+6*c)+6*a^2
*exp(4*d*x+4*c)+24*a*b*exp(4*d*x+4*c)+24*b^2*exp(4*d*x+4*c)-a^2-8*a*b-8*b^2
)/d/(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(71) = 142$.
time = 0.33, size = 285, normalized size = 3.80

$$\frac{4}{3}a^2\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}-\frac{1}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+\frac{32}{3}ab\left(\frac{2e^{(-2dx-2c)}}{d\left(2e^{(-2dx-2c)}-2e^{(-6dx-6c)}+e^{(-8dx-8c)}-1\right)}-\frac{1}{d\left(2e^{(-2dx-2c)}-2e^{(-6dx-6c)}+e^{(-8dx-8c)}-1\right)}\right)+\frac{32}{3}b^2\left(\frac{3e^{(-4dx-4c)}}{d\left(3e^{(-4dx-4c)}-3e^{(-8dx-8c)}+e^{(-12dx-12c)}-1\right)}-\frac{1}{d\left(3e^{(-4dx-4c)}-3e^{(-8dx-8c)}+e^{(-12dx-12c)}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1))) + 32/3*a*b*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c)
- 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) -
2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1))) + 32/3*b^2*(3*e^(-4*d*x - 4*c)
/(d*(3*e^(-4*d*x - 4*c) - 3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)) - 1
/(d*(3*e^(-4*d*x - 4*c) - 3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(71) = 142$.
time = 0.36, size = 408, normalized size = 5.44

$$-\frac{8\left(\left(a^{2}-4\,ab-4\,b^{2}\right)\cosh\left(dx+c\right)^{4}+8\left(a^{2}+2\,ab+2\,b^{2}\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+\left(a^{2}-4\,ab-4\,b^{2}\right)\sinh\left(dx+c\right)^{4}+4\left(a^{2}+2\,ab\right)\cosh\left(dx+c\right)^{2}+2\left(3\left(a^{2}-4\,ab-4\,b^{2}\right)\cosh\left(dx+c\right)^{2}+2\,a^{2}+4\,ab\right)\sinh\left(dx+c\right)^{2}+3\,a^{2}+12\,ab+12\,b^{2}+8\left(\left(a^{2}+2\,ab+2\,b^{2}\right)\cosh\left(dx+c\right)^{3}+\left(a^{2}+2\,ab\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)}{3\left(d\cosh\left(dx+c\right)^{8}+56\,d\cosh\left(dx+c\right)^{3}\sinh\left(dx+c\right)^{5}+28\,d\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)^{6}+8\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{7}+d\sinh\left(dx+c\right)^{8}-4\,d\cosh\left(dx+c\right)^{4}+2\left(35\,d\cosh\left(dx+c\right)^{4}-2\,d\right)\sinh\left(dx+c\right)^{4}+8\left(7\,d\cosh\left(dx+c\right)^{5}-d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+4\left(7\,d\cosh\left(dx+c\right)^{6}-6\,d\cosh\left(dx+c\right)^{2}\right)\sinh\left(dx+c\right)^{2}+8\left(d\cosh\left(dx+c\right)^{7}-d\cosh\left(dx+c\right)^{3}\right)\sinh\left(dx+c\right)+3\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -8/3*((a^2 - 4*a*b - 4*b^2)*cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + 2*b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 - 4*a*b - 4*b^2)*sinh(d*x + c)^4 + 4*(a^2 +
 2*a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 - 4*a*b - 4*b^2)*cosh(d*x + c)^2 + 2*a^
2 + 4*a*b)*sinh(d*x + c)^2 + 3*a^2 + 12*a*b + 12*b^2 + 8*((a^2 + 2*a*b + 2*
b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(
d*x + c)^8 + 56*d*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^2*si
nh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d
*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 2*d)*sinh(d*x + c)^4 + 8*(7*d*
cosh(d*x + c)^5 - d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*d*cosh(d*x + c)^6
 - 6*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - d*cosh(d*x
 + c)^3)*sinh(d*x + c) + 3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^{2}\left(c+dx\right)\right)^{2}\operatorname{csch}^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*csch(c + d*x)**4, x)
```

Giac [A]
time = 0.41, size = 115, normalized size = 1.53

$$-\frac{4\left(3\,a^{2}e^{\left(8\,dx+8\,c\right)}+8\,a^{2}e^{\left(6\,dx+6\,c\right)}+16\,abe^{\left(6\,dx+6\,c\right)}+6\,a^{2}e^{\left(4\,dx+4\,c\right)}+24\,abe^{\left(4\,dx+4\,c\right)}+24\,b^{2}e^{\left(4\,dx+4\,c\right)}-a^{2}-8\,ab-8\,b^{2}\right)}{3\,d\left(e^{\left(4\,dx+4\,c\right)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")

[Out] -4/3*(3*a^2*e^(8*d*x + 8*c) + 8*a^2*e^(6*d*x + 6*c) + 16*a*b*e^(6*d*x + 6*c) + 6*a^2*e^(4*d*x + 4*c) + 24*a*b*e^(4*d*x + 4*c) + 24*b^2*e^(4*d*x + 4*c) - a^2 - 8*a*b - 8*b^2)/(d*(e^(4*d*x + 4*c) - 1)^3)

Mupad [B]
time = 0.21, size = 115, normalized size = 1.53

$$-\frac{4\left(6a^2 e^{4c+4dx}-a^2-8b^2-8ab+8a^2 e^{6c+6dx}+3a^2 e^{8c+8dx}+24b^2 e^{4c+4dx}+24ab e^{4c+4dx}+16ab e^{6c+6dx}\right)}{3d\left(e^{4c+4dx}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d*x)^2)^2/sinh(c + d*x)^4,x)

[Out] -(4*(6*a^2*exp(4*c + 4*d*x) - a^2 - 8*b^2 - 8*a*b + 8*a^2*exp(6*c + 6*d*x) + 3*a^2*exp(8*c + 8*d*x) + 24*b^2*exp(4*c + 4*d*x) + 24*a*b*exp(4*c + 4*d*x) + 16*a*b*exp(6*c + 6*d*x)))/(3*d*(exp(4*c + 4*d*x) - 1)^3)

3.17 $\int \left(a+b\,\mathbf{sech}^2(c+dx)\right)^3 \sinh^4(c+dx)\,dx$

Optimal. Leaf size=182

$$\frac{3}{8}a\left(a^2-12ab+8b^2\right)x-\frac{3a(a^2-12ab+8b^2)\tanh(c+dx)}{8d}+\frac{b(6a^2-23ab-8b^2)\tanh^3(c+dx)}{8d}-\frac{3(5a-16b)}{}$$

```
[Out] 3/8*a*(a^2-12*a*b+8*b^2)*x-3/8*a*(a^2-12*a*b+8*b^2)*tanh(d*x+c)/d+1/8*b*(6*
a^2-23*a*b-8*b^2)*tanh(d*x+c)^3/d-3/40*(5*a-16*b)*b^2*tanh(d*x+c)^5/d-3/8*(
a-2*b)*sinh(d*x+c)^2*tanh(d*x+c)*(a+b-b*tanh(d*x+c)^2)^2/d+1/4*cosh(d*x+c)*
sinh(d*x+c)^3*(a+b-b*tanh(d*x+c)^2)^3/d
```

Rubi [A]
time = 0.16, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4217, 478, 591, 584, 212}

$$\frac{b(6a^2-23ab-8b^2)\tanh^3(c+dx)}{8d}-\frac{3a(a^2-12ab+8b^2)\tanh(c+dx)}{8d}+\frac{3}{8}ax\left(a^2-12ab+8b^2\right)-\frac{3b^2(5a-16b)\tanh^5(c+dx)}{40d}-\frac{3(a-2b)\sinh^2(c+dx)\tanh(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^2}{8d}+\frac{\sinh^3(c+dx)\cosh(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^4,x]
```

```
[Out] (3*a*(a^2 - 12*a*b + 8*b^2)*x)/8 - (3*a*(a^2 - 12*a*b + 8*b^2)*Tanh[c + d*x
])/(8*d) + (b*(6*a^2 - 23*a*b - 8*b^2)*Tanh[c + d*x]^3)/(8*d) - (3*(5*a - 1
6*b)*b^2*Tanh[c + d*x]^5)/(40*d) - (3*(a - 2*b)*Sinh[c + d*x]^2*Tanh[c + d*
x]*(a + b - b*Tanh[c + d*x]^2)^2)/(8*d) + (Cosh[c + d*x]*Sinh[c + d*x]^3*(a
 + b - b*Tanh[c + d*x]^2)^3)/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3\sinh^4(c+dx)\,dx=\frac{\text{Subst}\left(\int\frac{x^4\left(a+b-bx^2\right)^3}{\left(1-x^2\right)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{\cosh(c+dx)\sinh^3(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^3}{4d}-\frac{\text{Subst}}{}$$

$$=-\frac{3(a-2b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b-b\tanh^2(c+dx\right.}{8d}$$

$$=-\frac{3(a-2b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b-b\tanh^2(c+dx\right.}{8d}$$

$$=-\frac{3a(a^2-12ab+8b^2)\tanh(c+dx)}{8d}+\frac{b(6a^2-23ab-8b^2)\tan}{8d}$$

$$=\frac{3}{8}a\left(a^2-12ab+8b^2\right)x-\frac{3a(a^2-12ab+8b^2)\tanh(c+dx)}{8d}+$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 651 vs. 2(182) = 364.

time = 1.73, size = 651, normalized size = 3.58

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^4,x]
```

```
[Out] ((b + a*Cosh[c + d*x]^2)^3*Sech[c]*Sech[c + d*x]^5*(1200*a*(a^2 - 12*a*b +
8*b^2)*d*x*Cosh[d*x] + 1200*a*(a^2 - 12*a*b + 8*b^2)*d*x*Cosh[2*c + d*x] +
600*a^3*d*x*Cosh[2*c + 3*d*x] - 7200*a^2*b*d*x*Cosh[2*c + 3*d*x] + 4800*a*b
^2*d*x*Cosh[2*c + 3*d*x] + 600*a^3*d*x*Cosh[4*c + 3*d*x] - 7200*a^2*b*d*x*C
osh[4*c + 3*d*x] + 4800*a*b^2*d*x*Cosh[4*c + 3*d*x] + 120*a^3*d*x*Cosh[4*c
+ 5*d*x] - 1440*a^2*b*d*x*Cosh[4*c + 5*d*x] + 960*a*b^2*d*x*Cosh[4*c + 5*d*
x] + 120*a^3*d*x*Cosh[6*c + 5*d*x] - 1440*a^2*b*d*x*Cosh[6*c + 5*d*x] + 960
*a*b^2*d*x*Cosh[6*c + 5*d*x] - 180*a^3*Sinh[d*x] + 12120*a^2*b*Sinh[d*x] -
14080*a*b^2*Sinh[d*x] + 1280*b^3*Sinh[d*x] - 180*a^3*Sinh[2*c + d*x] - 7080
*a^2*b*Sinh[2*c + d*x] + 11520*a*b^2*Sinh[2*c + d*x] - 310*a^3*Sinh[2*c + 3
*d*x] + 8760*a^2*b*Sinh[2*c + 3*d*x] - 8960*a*b^2*Sinh[2*c + 3*d*x] - 310*a
^3*Sinh[4*c + 3*d*x] - 840*a^2*b*Sinh[4*c + 3*d*x] + 3840*a*b^2*Sinh[4*c +
3*d*x] - 640*b^3*Sinh[4*c + 3*d*x] - 150*a^3*Sinh[4*c + 5*d*x] + 2520*a^2*b
*Sinh[4*c + 5*d*x] - 2560*a*b^2*Sinh[4*c + 5*d*x] + 128*b^3*Sinh[4*c + 5*d*
x] - 150*a^3*Sinh[6*c + 5*d*x] + 600*a^2*b*Sinh[6*c + 5*d*x] - 15*a^3*Sinh[
6*c + 7*d*x] + 120*a^2*b*Sinh[6*c + 7*d*x] - 15*a^3*Sinh[8*c + 7*d*x] + 120
*a^2*b*Sinh[8*c + 7*d*x] + 5*a^3*Sinh[8*c + 9*d*x] + 5*a^3*Sinh[10*c + 9*d*
x]))/(1280*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [A]
time = 2.16, size = 294, normalized size = 1.62

method	result
risch	$\frac{3a^3x}{8} - \frac{9a^2bx}{2} + 3a\,b^2x + \frac{a^3 e^{4dx+4c}}{64d} - \frac{a^3 e^{2dx+2c}}{8d} + \frac{3a^2 e^{2dx+2c} b}{8d} + \frac{a^3 e^{-2dx-2c}}{8d} - \frac{3a^2 e^{-2dx-2c} b}{8d} - \frac{a^3 e^{-4dx-4c}}{64d} - 2b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a^3*x-9/2*a^2*b*x+3*a*b^2*x+1/64*a^3/d*exp(4*d*x+4*c)-1/8*a^3/d*exp(2*d
*x+2*c)+3/8*a^2/d*exp(2*d*x+2*c)*b+1/8*a^3/d*exp(-2*d*x-2*c)-3/8*a^2/d*exp(
-2*d*x-2*c)*b-1/64*a^3/d*exp(-4*d*x-4*c)-2/5*b*(15*a^2*exp(8*d*x+8*c)-30*a*
b*exp(8*d*x+8*c)+5*b^2*exp(8*d*x+8*c)+60*a^2*exp(6*d*x+6*c)-90*a*b*exp(6*d*
x+6*c)+90*a^2*exp(4*d*x+4*c)-110*a*b*exp(4*d*x+4*c)+10*b^2*exp(4*d*x+4*c)+6
0*a^2*exp(2*d*x+2*c)-70*a*b*exp(2*d*x+2*c)+15*a^2-20*a*b+b^2)/d/(1+exp(2*d*
x+2*c))^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(176) = 352.

time = 0.29, size = 422, normalized size = 2.32

$$\frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+ab^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)-\frac{3}{8}a^2b\left(\frac{12(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2dx-2c)}+1}{d(e^{(-2dx-2c)}+e^{(-4dx-4c)})}\right)+\frac{2}{5}b^3\left(\frac{10e^{(-4dx-4c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{5e^{(-8dx-8c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{1}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + a*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3
*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*
d*x - 6*c) + 1))) - 3/8*a^2*b*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^
(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)))) + 2/5*b^3*(1
0*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*
d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-8*d*x -
8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5
*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
- 10*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(176) = 352$.
time = 0.42, size = 727, normalized size = 3.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/320*(5*a^3*sinh(d*x + c)^9 + 15*(12*a^3*cosh(d*x + c)^2 - a^3 + 8*a^2*b)*
sinh(d*x + c)^7 - 8*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3 - 12*a^2*b + 8
*a*b^2)*d*x)*cosh(d*x + c)^5 - 40*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3
- 12*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 + (630*a^3*cosh(d*
x + c)^4 - 150*a^3 + 1560*a^2*b - 1280*a*b^2 + 64*b^3 - 315*(a^3 - 8*a^2*b)
*cosh(d*x + c)^2)*sinh(d*x + c)^5 - 40*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*
(a^3 - 12*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 5*(84*a^3*cosh(d*x + c)^6
 - 105*(a^3 - 8*a^2*b)*cosh(d*x + c)^4 - 62*a^3 + 792*a^2*b - 512*a*b^2 - 6
4*b^3 - 4*(75*a^3 - 780*a^2*b + 640*a*b^2 - 32*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^3 - 40*(2*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3 - 12*a^2*b + 8*a
*b^2)*d*x)*cosh(d*x + c)^3 + 3*(120*a^2*b - 160*a*b^2 + 8*b^3 - 15*(a^3 - 1
2*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 - 80*(120*a^2*b - 16
0*a*b^2 + 8*b^3 - 15*(a^3 - 12*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c) + 5*(9*a
^3*cosh(d*x + c)^8 - 21*(a^3 - 8*a^2*b)*cosh(d*x + c)^6 - 2*(75*a^3 - 780*a
^2*b + 640*a*b^2 - 32*b^3)*cosh(d*x + c)^4 - 36*a^3 + 504*a^2*b - 256*a*b^2
 + 128*b^3 - 6*(31*a^3 - 396*a^2*b + 256*a*b^2 + 32*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*
```

```
cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)
^2 + 10*d*cosh(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c)**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.45, size = 339, normalized size = 1.86

$$\frac{5\,a^{3}e^{\left(4\,dx+4\,c\right)}-40\,a^{3}e^{\left(2\,dx+2\,c\right)}+120\,a^{2}be^{\left(2\,dx+2\,c\right)}+120\,\left(a^{3}-12\,a^{2}b+8\,ab^{2}\right)\left(dx+c\right)-5\,\left(18\,a^{3}e^{\left(4\,dx+4\,c\right)}-216\,a^{2}be^{\left(4\,dx+4\,c\right)}+144\,ab^{2}e^{\left(4\,dx+4\,c\right)}-8\,a^{3}e^{\left(2\,dx+2\,c\right)}+24\,a^{2}be^{\left(2\,dx+2\,c\right)}+a^{3}\right)e^{\left(-4\,dx-4\,c\right)}-\frac{128\,\left(15\,a^{2}be^{\left(8\,dx+8\,c\right)}-30\,ab^{2}e^{\left(8\,dx+8\,c\right)}+5\,b^{3}e^{\left(8\,dx+8\,c\right)}+60\,a^{2}be^{\left(6\,dx+6\,c\right)}-90\,ab^{2}e^{\left(6\,dx+6\,c\right)}+90\,a^{2}be^{\left(4\,dx+4\,c\right)}-110\,ab^{2}e^{\left(4\,dx+4\,c\right)}+10\,b^{3}e^{\left(4\,dx+4\,c\right)}+60\,a^{2}be^{\left(2\,dx+2\,c\right)}-70\,ab^{2}e^{\left(2\,dx+2\,c\right)}+15\,a^{2}b-20\,ab^{2}+b^{3}\right)}{{\left(e^{\left(2\,dx+2\,c\right)}+1\right)}^{5}}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/320*(5*a^3*e^(4*d*x + 4*c) - 40*a^3*e^(2*d*x + 2*c) + 120*a^2*b*e^(2*d*x
 + 2*c) + 120*(a^3 - 12*a^2*b + 8*a*b^2)*(d*x + c) - 5*(18*a^3*e^(4*d*x + 4*
c) - 216*a^2*b*e^(4*d*x + 4*c) + 144*a*b^2*e^(4*d*x + 4*c) - 8*a^3*e^(2*d*x
 + 2*c) + 24*a^2*b*e^(2*d*x + 2*c) + a^3)*e^(-4*d*x - 4*c) - 128*(15*a^2*b*
e^(8*d*x + 8*c) - 30*a*b^2*e^(8*d*x + 8*c) + 5*b^3*e^(8*d*x + 8*c) + 60*a^2
*b*e^(6*d*x + 6*c) - 90*a*b^2*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) -
110*a*b^2*e^(4*d*x + 4*c) + 10*b^3*e^(4*d*x + 4*c) + 60*a^2*b*e^(2*d*x + 2*
c) - 70*a*b^2*e^(2*d*x + 2*c) + 15*a^2*b - 20*a*b^2 + b^3)/(e^(2*d*x + 2*c)
 + 1)^5)/d
```

Mupad [B]
time = 0.37, size = 686, normalized size = 3.77

$$\frac{\frac{2\,\left(-3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}-\frac{6\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b-2\,a\,b^2+b^3\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(-3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}-\frac{2\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(3\,a^2\,b-6\,a\,b^2+b^3\right)}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{\frac{2\,\left(3\,a^2\,b-6\,a\,b^2+b^3\right)}{5\,d}-\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(-3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}+\frac{12\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2\,b-2\,a\,b^2+b^3\right)}{5\,d}-\frac{8\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(-3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(3\,a^2\,b-6\,a\,b^2+b^3\right)}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}+\frac{\frac{2\,\left(-3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}-\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b-6\,a\,b^2+b^3\right)}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{\frac{2\,\left(3\,a^2\,b-2\,a\,b^2+b^3\right)}{5\,d}-\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(-3\,a^2\,b+3\,a\,b^2+b^3\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2\,b-6\,a\,b^2+b^3\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}+\frac{3\,a\,x\,\left(a^2-12\,a\,b+8\,b^2\right)}{8}-\frac{a^3\,{\mathrm{e}}^{-4\,c-4\,d\,x}}{64\,d}+\frac{a^3\,{\mathrm{e}}^{4\,c+4\,d\,x}}{64\,d}-\frac{2\,\left(3\,a^2\,b-6\,a\,b^2+b^3\right)}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}+\frac{a^2\,{\mathrm{e}}^{-2\,c-2\,d\,x}\,\left(a-3\,b\right)}{8\,d}-\frac{a^2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a-3\,b\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] ((2*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) - (6*exp(2*c + 2*d*x)*(3*a^2*b - 2*a*b
^2 + b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) - (
2*exp(6*c + 6*d*x)*(3*a^2*b - 6*a*b^2 + b^3))/(5*d))/(4*exp(2*c + 2*d*x) +
6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(3*a^
2*b - 6*a*b^2 + b^3))/(5*d) - (8*exp(2*c + 2*d*x)*(3*a*b^2 - 3*a^2*b + b^3)
)/(5*d) + (12*exp(4*c + 4*d*x)*(3*a^2*b - 2*a*b^2 + b^3))/(5*d) - (8*exp(6*
c + 6*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) + (2*exp(8*c + 8*d*x)*(3*a^2*b
```

```
- 6*a*b^2 + b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp
(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) + ((2*(3*a*b^2
 - 3*a^2*b + b^3))/(5*d) - (2*exp(2*c + 2*d*x)*(3*a^2*b - 6*a*b^2 + b^3))/(
5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(3*a^2*b - 2*a*b^2
+ b^3))/(5*d) - (4*exp(2*c + 2*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) + (2*e
xp(4*c + 4*d*x)*(3*a^2*b - 6*a*b^2 + b^3))/(5*d))/(3*exp(2*c + 2*d*x) + 3*e
xp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + (3*a*x*(a^2 - 12*a*b + 8*b^2))/8
- (a^3*exp(- 4*c - 4*d*x))/(64*d) + (a^3*exp(4*c + 4*d*x))/(64*d) - (2*(3*a
^2*b - 6*a*b^2 + b^3))/(5*d*(exp(2*c + 2*d*x) + 1)) + (a^2*exp(- 2*c - 2*d*
x)*(a - 3*b))/(8*d) - (a^2*exp(2*c + 2*d*x)*(a - 3*b))/(8*d)
```

3.18 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^3 \sinh^3(c+dx)\,dx$

Optimal. Leaf size=99

$$-\frac{a^2(a-3b)\cosh(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d}+\frac{3a(a-b)b\,\mathrm{sech}(c+dx)}{d}+\frac{(3a-b)b^2\mathrm{sech}^3(c+dx)}{3d}+\frac{b^3\mathrm{sech}^5(c+$$

```
[Out] -a^2*(a-3*b)*cosh(d*x+c)/d+1/3*a^3*cosh(d*x+c)^3/d+3*a*(a-b)*b*sech(d*x+c)/
d+1/3*(3*a-b)*b^2*sech(d*x+c)^3/d+1/5*b^3*sech(d*x+c)^5/d
```

Rubi [A]
time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4218, 459}

$$\frac{a^3\cosh^3(c+dx)}{3d}-\frac{a^2(a-3b)\cosh(c+dx)}{d}+\frac{b^2(3a-b)\mathrm{sech}^3(c+dx)}{3d}+\frac{3ab(a-b)\mathrm{sech}(c+dx)}{d}+\frac{b^3\mathrm{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^3,x]
```

```
[Out] -((a^2*(a - 3*b)*Cosh[c + d*x])/d) + (a^3*Cosh[c + d*x]^3)/(3*d) + (3*a*(a
- b)*b*Sech[c + d*x])/d + ((3*a - b)*b^2*Sech[c + d*x]^3)/(3*d) + (b^3*Sech
[c + d*x]^5)/(5*d)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int (a+b\text{sech}^2(c+dx))^3 \sinh^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^3}{x^6}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(a^2(a-3b)+\frac{b^3}{x^6}+\frac{(3a-b)b^2}{x^4}+\frac{3a(a-b)b}{x^2}-a^3x^2\right)dx, x$$
$$= -\frac{a^2(a-3b)\cosh(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d}+\frac{3a(a-b)b\text{sech}}{d}$$

Mathematica [A]
time = 0.83, size = 119, normalized size = 1.20

$$\frac{4(b+a\cosh^2(c+dx))^3\left(6b^3+10(3a-b)b^2\cosh^2(c+dx)+90a(a-b)b\cosh^4(c+dx)+5a^2\cosh^6(c+dx)(-5a+18b+a\cosh(2(c+dx)))\right)\text{sech}^5(c+dx)}{15d(a+2b+a\cosh(2(c+dx)))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^3,x]
```

```
[Out] (4*(b + a*Cosh[c + d*x]^2)^3*(6*b^3 + 10*(3*a - b)*b^2*Cosh[c + d*x]^2 + 90
*a*(a - b)*b*Cosh[c + d*x]^4 + 5*a^2*Cosh[c + d*x]^6*(-5*a + 18*b + a*Cosh[
2*(c + d*x)]))*Sech[c + d*x]^5)/(15*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(93) = 186$.
time = 2.07, size = 285, normalized size = 2.88

method	result
risch	$\frac{e^{3dx+3c}a^3}{24d}-\frac{3\,e^{dx+c}a^3}{8d}+\frac{3\,e^{dx+c}a^2b}{2d}-\frac{3\,e^{-dx-c}a^3}{8d}+\frac{3\,e^{-dx-c}a^2b}{2d}+\frac{e^{-3dx-3c}a^3}{24d}+\frac{2\,e^{dx+c}b(45a^2e^{8dx+8c}-45ab\,e^{8dx+8c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*exp(3*d*x+3*c)*a^3-3/8/d*exp(d*x+c)*a^3+3/2/d*exp(d*x+c)*a^2*b-3/8/d
*exp(-d*x-c)*a^3+3/2/d*exp(-d*x-c)*a^2*b+1/24/d*exp(-3*d*x-3*c)*a^3+2/15*ex
p(d*x+c)*b*(45*a^2*exp(8*d*x+8*c)-45*a*b*exp(8*d*x+8*c)+180*a^2*exp(6*d*x+6
*c)-120*a*b*exp(6*d*x+6*c)-20*b^2*exp(6*d*x+6*c)+270*a^2*exp(4*d*x+4*c)-150
*a*b*exp(4*d*x+4*c)+8*b^2*exp(4*d*x+4*c)+180*a^2*exp(2*d*x+2*c)-120*a*b*exp
(2*d*x+2*c)-20*b^2*exp(2*d*x+2*c)+45*a^2-45*a*b)/d/(1+exp(2*d*x+2*c))^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(93) = 186$.
time = 0.31, size = 489, normalized size = 4.94

$$\frac{1}{24}a^3\left(\frac{e^{(3dx+3c)}}{d}-\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}+\frac{e^{(-3dx-3c)}}{d}\right)+\frac{3}{2}a^2b\left(\frac{e^{(-dx-c)}}{d}+\frac{5e^{(-2dx-2c)}+1}{d(e^{(-dx-c)}+e^{(-3dx-3c)})}\right)-2ab^2\left(\frac{3e^{(-dx-c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{2e^{(-3dx-3c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{3e^{(-5dx-5c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)-\frac{8}{15}b^3\left(\frac{5e^{(-3dx-3c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}-\frac{2e^{(-5dx-5c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{5e^{(-7dx-7c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*a^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) + 3/2*a^2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-
d*x - c) + e^(-3*d*x - 3*c)))) - 2*a*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x -
2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*
(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5
*d*x - 5*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)
+ 1))) - 8/15*b^3*(5*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1
)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-7*d
*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c
) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(93) = 186$.
time = 0.42, size = 403, normalized size = 4.07

$$\frac{5\,a^3\cosh\left(dx+c\right)^8+5\,a^3\sinh\left(dx+c\right)^8-20\,\left(a^3-9\,a^2b\right)\cosh\left(dx+c\right)^6+20\,\left(7\,a^3\cosh\left(dx+c\right)^2-a^3+9\,a^2b\right)\sinh\left(dx+c\right)^6-20\,\left(11\,a^3-90\,a^2b+36\,ab^2\right)\cosh\left(dx+c\right)^4+10\,\left(35\,a^3\cosh\left(dx+c\right)^4-22\,a^3+180\,a^2b-72\,ab^2-30\,\left(a^3-9\,a^2b\right)\cosh\left(dx+c\right)^2\right)\sinh\left(dx+c\right)^4-425\,a^3+3960\,a^2b-1200\,ab^2+64\,b^3-20\,\left(31\,a^3-279\,a^2b+96\,ab^2+16\,b^3\right)\cosh\left(dx+c\right)^2+20\,\left(7\,a^3\cosh\left(dx+c\right)^6-15\,\left(a^3-9\,a^2b\right)\cosh\left(dx+c\right)^4-31\,a^3+279\,a^2b-96\,ab^2-16\,b^3-6\,\left(11\,a^3-90\,a^2b+36\,ab^2\right)\cosh\left(dx+c\right)^2\right)\sinh\left(dx+c\right)^2}{120\,\left(d\cosh\left(dx+c\right)^5+5\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+5\,d\cosh\left(dx+c\right)^3+5\,\left(2\,d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2+10\,d\cosh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(5*a^3*cosh(d*x + c)^8 + 5*a^3*sinh(d*x + c)^8 - 20*(a^3 - 9*a^2*b)*c
osh(d*x + c)^6 + 20*(7*a^3*cosh(d*x + c)^2 - a^3 + 9*a^2*b)*sinh(d*x + c)^6
 - 20*(11*a^3 - 90*a^2*b + 36*a*b^2)*cosh(d*x + c)^4 + 10*(35*a^3*cosh(d*x
+ c)^4 - 22*a^3 + 180*a^2*b - 72*a*b^2 - 30*(a^3 - 9*a^2*b)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 - 425*a^3 + 3960*a^2*b - 1200*a*b^2 + 64*b^3 - 20*(31*a^3
 - 279*a^2*b + 96*a*b^2 + 16*b^3)*cosh(d*x + c)^2 + 20*(7*a^3*cosh(d*x + c)
^6 - 15*(a^3 - 9*a^2*b)*cosh(d*x + c)^4 - 31*a^3 + 279*a^2*b - 96*a*b^2 - 1
6*b^3 - 6*(11*a^3 - 90*a^2*b + 36*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2)/
(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^
3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh
(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(93) = 186.
time = 0.43, size = 193, normalized size = 1.95

$$\frac{5\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-60\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)+180\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+\frac{16\left(45\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4-45\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+60\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-20\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+48\,b^3\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/120*(5*a^3*(e^(d*x + c) + e^(-d*x - c))^3 - 60*a^3*(e^(d*x + c) + e^(-d*x
 - c)) + 180*a^2*b*(e^(d*x + c) + e^(-d*x - c)) + 16*(45*a^2*b*(e^(d*x + c)
 + e^(-d*x - c))^4 - 45*a*b^2*(e^(d*x + c) + e^(-d*x - c))^4 + 60*a*b^2*(e^
(d*x + c) + e^(-d*x - c))^2 - 20*b^3*(e^(d*x + c) + e^(-d*x - c))^2 + 48*b^
3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

Mupad [**B**]
time = 0.33, size = 348, normalized size = 3.52

$$\frac{a^3\,e^{-3c-3dx}}{24\,d}+\frac{a^3\,e^{3c+3dx}}{24\,d}-\frac{3\,a^2\,e^{-c-dx}\,(a-4\,b)}{8\,d}+\frac{8\,e^{c+dx}\,(3\,a\,b^2-b^3)}{3\,d\,(2\,e^{2c+2dx}+e^{4c+4dx}+1)}-\frac{64\,b^3\,e^{c+dx}}{5\,d\,(4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1)}-\frac{8\,e^{c+dx}\,(15\,a\,b^2-17\,b^3)}{15\,d\,(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1)}+\frac{32\,b^3\,e^{c+dx}}{5\,d\,(5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1)}-\frac{6\,e^{c+dx}\,(a\,b^2-a^2\,b)}{d\,(e^{2c+2dx}+1)}-\frac{3\,a^2\,e^{c+dx}\,(a-4\,b)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (a^3*exp(- 3*c - 3*d*x))/(24*d) + (a^3*exp(3*c + 3*d*x))/(24*d) - (3*a^2*ex
p(- c - d*x)*(a - 4*b))/(8*d) + (8*exp(c + d*x)*(3*a*b^2 - b^3))/(3*d*(2*ex
p(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (64*b^3*exp(c + d*x))/(5*d*(4*exp
(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x)
+ 1)) - (8*exp(c + d*x)*(15*a*b^2 - 17*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*
exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (32*b^3*exp(c + d*x))/(5*d*(5*e
xp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8
*d*x) + exp(10*c + 10*d*x) + 1)) - (6*exp(c + d*x)*(a*b^2 - a^2*b))/(d*(exp
(2*c + 2*d*x) + 1)) - (3*a^2*exp(c + d*x)*(a - 4*b))/(8*d)
```

3.19 $\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh^2(c+dx)\,dx$

Optimal. Leaf size=112

$$-\frac{1}{2}a^2(a-6b)x+\frac{a^3}{4d(1-\tanh(c+dx))}-\frac{3a^2b\tanh(c+dx)}{d}+\frac{b^2(3a+b)\tanh^3(c+dx)}{3d}-\frac{b^3\tanh^5(c+dx)}{5d}-\frac{a^3}{4d(}$$

```
[Out] -1/2*a^2*(a-6*b)*x+1/4*a^3/d/(1-tanh(d*x+c))-3*a^2*b*tanh(d*x+c)/d+1/3*b^2*
(3*a+b)*tanh(d*x+c)^3/d-1/5*b^3*tanh(d*x+c)^5/d-1/4*a^3/d/(1+tanh(d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4217, 478, 542, 396, 212}

$$-\frac{b(81a^2-28ab-4b^2)\tanh(c+dx)}{30d}-\frac{1}{2}a^2x(a-6b)-\frac{7b\tanh(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^2}{10d}-\frac{b(33a-2b)\tanh(c+dx)\left(a-b\tanh^2(c+dx)+b\right)}{30d}+\frac{\sinh(c+dx)\cosh(c+dx)\left(a-b\tanh^2(c+dx)+b\right)^3}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^2,x]
```

```
[Out] -1/2*(a^2*(a - 6*b)*x) - (b*(81*a^2 - 28*a*b - 4*b^2)*Tanh[c + d*x])/(30*d)
 - ((33*a - 2*b)*b*Tanh[c + d*x]*(a + b - b*Tanh[c + d*x]^2))/(30*d) - (7*b
*Tanh[c + d*x]*(a + b - b*Tanh[c + d*x]^2)^2)/(10*d) + (Cosh[c + d*x]*Sinh[
c + d*x]*(a + b - b*Tanh[c + d*x]^2)^3)/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
```

```
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh^2(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b-bx^2\right)^3}{\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^3}{2d} - \frac{\text{Subst}}{}$$

$$= -\frac{7b\tanh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^2}{10d} + \frac{\cosh(c+dx)\,\text{si}}{}$$

$$= -\frac{(33a-2b)b\tanh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)}{30d} - \frac{7b\tan}{}$$

$$= -\frac{b(81a^2-28ab-4b^2)\tanh(c+dx)}{30d} - \frac{(33a-2b)b\tanh(c+d}{}$$

$$= -\frac{1}{2}a^2(a-6b)x - \frac{b(81a^2-28ab-4b^2)\tanh(c+dx)}{30d} - \frac{(33a-}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 480 vs. 2(112) = 224.
time = 1.28, size = 480, normalized size = 4.29

$$\frac{\text{sech}(c)\text{sech}^5(c+dx)\left(-600a^2(a-6b)dx\cosh(dx)-600a^2(a-6b)dx\cosh(2c+dx)-300a^3dx\cosh(2c+3dx)+1800a^2bdx\cosh(2c+3dx)-300a^3dx\cosh(4c+3dx)+1800a^2bdx\cosh(4c+3dx)-60a^3dx\cosh(4c+5dx)+360a^2bdx\cosh(4c+5dx)-60a^3dx\cosh(6c+5dx)+360a^2bdx\cosh(6c+5dx)+75a^3\sinh(dx)-4320a^2b\sinh(dx)+960ab^2\sinh(dx)-160b^3\sinh(dx)+75a^3\sinh(2c+dx)+2880a^2b\sinh(2c+dx)-1440ab^2\sinh(2c+dx)-480b^3\sinh(2c+dx)+135a^3\sinh(2c+3dx)-2880a^2b\sinh(2c+3dx)+480ab^2\sinh(2c+3dx)+160b^3\sinh(2c+3dx)+135a^3\sinh(4c+3dx)+720a^2b\sinh(4c+3dx)-720ab^2\sinh(4c+3dx)+75a^3\sinh(4c+5dx)-720a^2b\sinh(4c+5dx)+240ab^2\sinh(4c+5dx)+32b^3\sinh(4c+5dx)+75a^3\sinh(6c+5dx)+15a^3\sinh(6c+7dx)+15a^3\sinh(8c+7dx)\right)}{3840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x]^2,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^5*(-600*a^2*(a - 6*b)*d*x*Cosh[d*x] - 600*a^2*(a - 6
*b)*d*x*Cosh[2*c + d*x] - 300*a^3*d*x*Cosh[2*c + 3*d*x] + 1800*a^2*b*d*x*Co
sh[2*c + 3*d*x] - 300*a^3*d*x*Cosh[4*c + 3*d*x] + 1800*a^2*b*d*x*Cosh[4*c +
 3*d*x] - 60*a^3*d*x*Cosh[4*c + 5*d*x] + 360*a^2*b*d*x*Cosh[4*c + 5*d*x] -
60*a^3*d*x*Cosh[6*c + 5*d*x] + 360*a^2*b*d*x*Cosh[6*c + 5*d*x] + 75*a^3*Sin
h[d*x] - 4320*a^2*b*Sinh[d*x] + 960*a*b^2*Sinh[d*x] - 160*b^3*Sinh[d*x] + 7
5*a^3*Sinh[2*c + d*x] + 2880*a^2*b*Sinh[2*c + d*x] - 1440*a*b^2*Sinh[2*c +
d*x] - 480*b^3*Sinh[2*c + d*x] + 135*a^3*Sinh[2*c + 3*d*x] - 2880*a^2*b*Sin
h[2*c + 3*d*x] + 480*a*b^2*Sinh[2*c + 3*d*x] + 160*b^3*Sinh[2*c + 3*d*x] +
135*a^3*Sinh[4*c + 3*d*x] + 720*a^2*b*Sinh[4*c + 3*d*x] - 720*a*b^2*Sinh[4*
c + 3*d*x] + 75*a^3*Sinh[4*c + 5*d*x] - 720*a^2*b*Sinh[4*c + 5*d*x] + 240*a
*b^2*Sinh[4*c + 5*d*x] + 32*b^3*Sinh[4*c + 5*d*x] + 75*a^3*Sinh[6*c + 5*d*x
] + 15*a^3*Sinh[6*c + 7*d*x] + 15*a^3*Sinh[8*c + 7*d*x]))/(3840*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(102) = 204$.
time = 2.00, size = 233, normalized size = 2.08

method	result
risch	$-\frac{a^{3} x}{2}+3 a^{2} b x+\frac{a^{3} {\mathrm e}^{2 d x +2 c}}{8 d}-\frac{a^{3} {\mathrm e}^{-2 d x -2 c}}{8 d}+\frac{2 b \left(45 a^{2} {\mathrm e}^{8 d x +8 c}-45 a b \,{\mathrm e}^{8 d x +8 c}+180 a^{2} {\mathrm e}^{6 d x +6 c}-90 a b \,{\mathrm e}^{6 d x +6 c}-30 b^{2} {\mathrm e}^{6 d x +6 c}+27}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^3*x+3*a^2*b*x+1/8*a^3/d*exp(2*d*x+2*c)-1/8*a^3/d*exp(-2*d*x-2*c)+2/1
5*b*(45*a^2*exp(8*d*x+8*c)-45*a*b*exp(8*d*x+8*c)+180*a^2*exp(6*d*x+6*c)-90*
a*b*exp(6*d*x+6*c)-30*b^2*exp(6*d*x+6*c)+270*a^2*exp(4*d*x+4*c)-60*a*b*exp(
4*d*x+4*c)+10*b^2*exp(4*d*x+4*c)+180*a^2*exp(2*d*x+2*c)-30*a*b*exp(2*d*x+2*
c)-10*b^2*exp(2*d*x+2*c)+45*a^2-15*a*b-2*b^2)/d/(1+exp(2*d*x+2*c))^5
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(100) = 200$.
time = 0.28, size = 443, normalized size = 3.96

$$-\frac{1}{8}a^{3}\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+3a^{2}b\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)+\frac{4}{15}b^{3}\left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}-\frac{5e^{(-4dx-4c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{15e^{(-6dx-6c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{1}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)+2ab^{2}\left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 3*a^2*b*(x + c/d
- 2/(d*(e^(-2*d*x - 2*c) + 1))) + 4/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*
```

```
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
- 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x -
 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))
 + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5
*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a*b^2*(3*e^(-4*d*x - 4*c)
/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(
d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 595 vs. $2(100) = 200$.
time = 0.37, size = 595, normalized size = 5.31

$$\frac{15a^3\sinh(dx+c)^7 + 4(90a^2b - 30ab^2 - 4b^3 - 15(a^3 - 6a^2b)dx)\cosh(dx+c)^5 + 20(90a^2b - 30ab^2 - 4b^3 - 15(a^3 - 6a^2b)dx)\cosh(dx+c)\sinh(dx+c)^4 + (315a^3\cosh(dx+c)^2 + 75a^3 - 360a^2b + 120ab^2 + 16b^3)\sinh(dx+c)^5 + 20(90a^2b - 30ab^2 - 4b^3 - 15(a^3 - 6a^2b)dx)\cosh(dx+c)^3 + 5(105a^3\cosh(dx+c)^4 + 27a^3 - 216a^2b - 24ab^2 + 16b^3 + 2(75a^3 - 360a^2b + 120ab^2 + 16b^3)\cosh(dx+c)^2)\sinh(dx+c)^3 + 20(2(90a^2b - 30ab^2 - 4b^3 - 15(a^3 - 6a^2b)dx)\cosh(dx+c)^3 + 3(90a^2b - 30ab^2 - 4b^3 - 15(a^3 - 6a^2b)dx)\cosh(dx+c))\sinh(dx+c)^2 + 40(90a^2b - 30ab^2 - 4b^3 - 15(a^3 - 6a^2b)dx)\cosh(dx+c) + 5(21a^3\cosh(dx+c)^6 + (75a^3 - 360a^2b + 120ab^2 + 16b^3)\cosh(dx+c)^4 + 15a^3 - 144a^2b - 48ab^2 - 64b^3 + 3(27a^3 - 216a^2b - 24ab^2 + 16b^3)\cosh(dx+c)^2)\sinh(dx+c)}{120(d\cosh(dx+c)^5 + 5d\cosh(dx+c)\sinh(dx+c)^4 + 5d\cosh(dx+c)^3 + 5(2d\cosh(dx+c)^3 + 3d\cosh(dx+c))\sinh(dx+c)^2 + 10d\cosh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/120*(15*a^3*sinh(d*x + c)^7 + 4*(90*a^2*b - 30*a*b^2 - 4*b^3 - 15*(a^3 -
6*a^2*b)*d*x)*cosh(d*x + c)^5 + 20*(90*a^2*b - 30*a*b^2 - 4*b^3 - 15*(a^3 -
 6*a^2*b)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 + (315*a^3*cosh(d*x + c)^2 + 7
5*a^3 - 360*a^2*b + 120*a*b^2 + 16*b^3)*sinh(d*x + c)^5 + 20*(90*a^2*b - 30
*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(d*x + c)^3 + 5*(105*a^3*cosh(
d*x + c)^4 + 27*a^3 - 216*a^2*b - 24*a*b^2 + 16*b^3 + 2*(75*a^3 - 360*a^2*b
 + 120*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 20*(2*(90*a^2*b -
 30*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(d*x + c)^3 + 3*(90*a^2*b -
 30*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2
+ 40*(90*a^2*b - 30*a*b^2 - 4*b^3 - 15*(a^3 - 6*a^2*b)*d*x)*cosh(d*x + c) +
 5*(21*a^3*cosh(d*x + c)^6 + (75*a^3 - 360*a^2*b + 120*a*b^2 + 16*b^3)*cosh
(d*x + c)^4 + 15*a^3 - 144*a^2*b - 48*a*b^2 - 64*b^3 + 3*(27*a^3 - 216*a^2*
b - 24*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 +
 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x
+ c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(100) = 200.
time = 0.44, size = 278, normalized size = 2.48

$$\frac{15\,a^3e^{(2\,dx+2\,c)} - 60\,(a^3 - 6\,a^2b)(dx+c) + 15\,\left(2\,a^3e^{(2\,dx+2\,c)} - 12\,a^2be^{(2\,dx+2\,c)} - a^3\right)e^{(-2\,dx-2\,c)} + \frac{16\,\left(45\,a^2be^{(8\,dx+8\,c)} - 45\,ab^2e^{(8\,dx+8\,c)} + 180\,a^2be^{(6\,dx+6\,c)} - 90\,ab^2e^{(6\,dx+6\,c)} - 30\,b^3e^{(6\,dx+6\,c)} + 270\,a^2be^{(4\,dx+4\,c)} - 60\,ab^2e^{(4\,dx+4\,c)} + 10\,b^3e^{(4\,dx+4\,c)} + 180\,a^2be^{(2\,dx+2\,c)} - 30\,ab^2e^{(2\,dx+2\,c)} - 10\,b^3e^{(2\,dx+2\,c)} + 45\,a^2b - 15\,ab^2 - 2\,b^3\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/120*(15*a^3*e^(2*d*x + 2*c) - 60*(a^3 - 6*a^2*b)*(d*x + c) + 15*(2*a^3*e^
(2*d*x + 2*c) - 12*a^2*b*e^(2*d*x + 2*c) - a^3)*e^(-2*d*x - 2*c) + 16*(45*a
^2*b*e^(8*d*x + 8*c) - 45*a*b^2*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6*c)
 - 90*a*b^2*e^(6*d*x + 6*c) - 30*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x +
 4*c) - 60*a*b^2*e^(4*d*x + 4*c) + 10*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*
d*x + 2*c) - 30*a*b^2*e^(2*d*x + 2*c) - 10*b^3*e^(2*d*x + 2*c) + 45*a^2*b -
 15*a*b^2 - 2*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 0.23, size = 592, normalized size = 5.29

$$\frac{\frac{2\,\left(9\,a^2\,b+3\,a\,b^2+4\,b^3\right)}{15\,d}-\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a\,b^2-a^2\,b\right)}{5\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b-b^3\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}+\frac{\frac{2\,\left(3\,a^2\,b-b^3\right)}{5\,d}-\frac{6\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a\,b^2-a^2\,b\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(9\,a^2\,b+3\,a\,b^2+4\,b^3\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2\,b-b^3\right)}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}+\frac{\frac{2\,\left(3\,a^2\,b-b^3\right)}{5\,d}-\frac{6\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a\,b^2-a^2\,b\right)}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}+\frac{\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(9\,a^2\,b+3\,a\,b^2+4\,b^3\right)}{5\,d}-\frac{6\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(a\,b^2-a^2\,b\right)}{5\,d}-\frac{6\,\left(a\,b^2-a^2\,b\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b-b^3\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(3\,a^2\,b-b^3\right)}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}-\frac{6\,\left(a\,b^2-a^2\,b\right)}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{a^2\,x\,\left(a-6\,b\right)}{2}-\frac{a^3\,{\mathrm{e}}^{-2\,c-2\,d\,x}}{8\,d}+\frac{a^3\,{\mathrm{e}}^{2\,c+2\,d\,x}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] ((2*(3*a*b^2 + 9*a^2*b + 4*b^3))/(15*d) - (6*exp(4*c + 4*d*x)*(a*b^2 - a^2*
b))/(5*d) + (4*exp(2*c + 2*d*x)*(3*a^2*b - b^3))/(5*d))/(3*exp(2*c + 2*d*x)
 + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((2*(3*a^2*b - b^3))/(5*d)
- (6*exp(6*c + 6*d*x)*(a*b^2 - a^2*b))/(5*d) + (2*exp(2*c + 2*d*x)*(3*a*b^2
 + 9*a^2*b + 4*b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(3*a^2*b - b^3))/(5*d))/(4
*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d
*x) + 1) + ((2*(3*a^2*b - b^3))/(5*d) - (6*exp(2*c + 2*d*x)*(a*b^2 - a^2*b)
)/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((4*exp(4*c + 4*d*x)
*(3*a*b^2 + 9*a^2*b + 4*b^3))/(5*d) - (6*exp(8*c + 8*d*x)*(a*b^2 - a^2*b))/
(5*d) - (6*(a*b^2 - a^2*b))/(5*d) + (8*exp(2*c + 2*d*x)*(3*a^2*b - b^3))/(5
*d) + (8*exp(6*c + 6*d*x)*(3*a^2*b - b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*
exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10
*d*x) + 1) - (6*(a*b^2 - a^2*b))/(5*d*(exp(2*c + 2*d*x) + 1)) - (a^2*x*(a -
 6*b))/2 - (a^3*exp(- 2*c - 2*d*x))/(8*d) + (a^3*exp(2*c + 2*d*x))/(8*d)
```

3.20 $\int \left(a+b\text{sech}^2(c+dx)\right)^3 \sinh(c+dx)\,dx$

Optimal. Leaf size=64

$$\frac{a^3\cosh(c+dx)}{d}-\frac{3a^2b\text{sech}(c+dx)}{d}-\frac{ab^2\text{sech}^3(c+dx)}{d}-\frac{b^3\text{sech}^5(c+dx)}{5d}$$

```
[Out] a^3*cosh(d*x+c)/d-3*a^2*b*sech(d*x+c)/d-a*b^2*sech(d*x+c)^3/d-1/5*b^3*sech(
d*x+c)^5/d
```

Rubi [A]

time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4218, 276}

$$\frac{a^3\cosh(c+dx)}{d}-\frac{3a^2b\text{sech}(c+dx)}{d}-\frac{ab^2\text{sech}^3(c+dx)}{d}-\frac{b^3\text{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x],x]
```

```
[Out] (a^3*Cosh[c + d*x])/d - (3*a^2*b*Sech[c + d*x])/d - (a*b^2*Sech[c + d*x]^3)
/d - (b^3*Sech[c + d*x]^5)/(5*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\text{sech}^2(c+dx)\right)^3\sinh(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{(b+ax^2)^3}{x^6}\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a^3+\frac{b^3}{x^6}+\frac{3ab^2}{x^4}+\frac{3a^2b}{x^2}\right)dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{a^3\cosh(c+dx)}{d}-\frac{3a^2b\text{sech}(c+dx)}{d}-\frac{ab^2\text{sech}^3(c+dx)}{d}-\frac{b^3\text{se}}{}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 93, normalized size = 1.45

$$\frac{8\left(b+a\cosh^2(c+dx)\right)^3\left(-b^3-5ab^2\cosh^2(c+dx)-15a^2b\cosh^4(c+dx)+5a^3\cosh^6(c+dx)\right)\operatorname{sech}^5(c+dx)}{5d(a+2b+a\cosh(2(c+dx)))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Sinh[c + d*x],x]
```

```
[Out] (8*(b + a*Cosh[c + d*x]^2)^3*(-b^3 - 5*a*b^2*Cosh[c + d*x]^2 - 15*a^2*b*Cos
h[c + d*x]^4 + 5*a^3*Cosh[c + d*x]^6)*Sech[c + d*x]^5)/(5*d*(a + 2*b + a*Co
sh[2*(c + d*x)])^3)
```

Maple [A]
time = 0.71, size = 58, normalized size = 0.91

method	result
derivativedivides	$-\frac{\frac{b^3\operatorname{sech}(dx+c)^5}{5}+a\,b^2\operatorname{sech}(dx+c)^3+3a^2b\operatorname{sech}(dx+c)-\frac{a^3}{\operatorname{sech}(dx+c)}}{d}$
default	$-\frac{\frac{b^3\operatorname{sech}(dx+c)^5}{5}+a\,b^2\operatorname{sech}(dx+c)^3+3a^2b\operatorname{sech}(dx+c)-\frac{a^3}{\operatorname{sech}(dx+c)}}{d}$
risch	$\frac{\mathrm{e}^{dx+c}a^3}{2d}+\frac{\mathrm{e}^{-dx-c}a^3}{2d}-\frac{2\,\mathrm{e}^{dx+c}b\left(15a^2\mathrm{e}^{8dx+8c}+60a^2\mathrm{e}^{6dx+6c}+20ab\,\mathrm{e}^{6dx+6c}+90a^2\mathrm{e}^{4dx+4c}+40ab\,\mathrm{e}^{4dx+4c}+16b^2\mathrm{e}^{4dx+4}\right.}{5d\left(1+\mathrm{e}^{2dx+2c}\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(1/5*b^3*sech(d*x+c)^5+a*b^2*sech(d*x+c)^3+3*a^2*b*sech(d*x+c)-a^3/sec
h(d*x+c))
```

Maxima [A]
time = 0.27, size = 94, normalized size = 1.47

$$\frac{a^3\cosh(dx+c)}{d}-\frac{6\,a^2b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)}-\frac{8\,ab^2}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}-\frac{32\,b^3}{5\,d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a^3*cosh(d*x + c)/d - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*x - c))) - 8*a*b^2/(d
*(e^(d*x + c) + e^(-d*x - c))^3) - 32/5*b^3/(d*(e^(d*x + c) + e^(-d*x - c))
^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(62) = 124.
time = 0.35, size = 276, normalized size = 4.31

$$\frac{5\,a^3\cosh(dx+c)^6+5\,a^3\sinh(dx+c)^6+30\,(a^3-2\,a^2b)\cosh(dx+c)^4+15\left(5\,a^3\cosh(dx+c)^2+2\,a^3-4\,a^2b\right)\sinh(dx+c)^4+50\,a^3-180\,a^2b-80\,ab^2-32\,b^3+5\,(15\,a^3-48\,a^2b-16\,ab^2)\cosh(dx+c)^2+5\left(15\,a^3\cosh(dx+c)^4+15\,a^3-48\,a^2b-16\,ab^2+36\,(a^3-2\,a^2b)\cosh(dx+c)^2\right)\sinh(dx+c)^2}{10\left(d\cosh(dx+c)^5+5\,d\cosh(dx+c)\sinh(dx+c)^4+5\,d\cosh(dx+c)^3+5\left(2\,d\cosh(dx+c)^3+3\,d\cosh(dx+c)\right)\sinh(dx+c)^2+10\,d\cosh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x, algorithm="fricas")

[Out] 1/10*(5*a^3*cosh(d*x + c)^6 + 5*a^3*sinh(d*x + c)^6 + 30*(a^3 - 2*a^2*b)*co
sh(d*x + c)^4 + 15*(5*a^3*cosh(d*x + c)^2 + 2*a^3 - 4*a^2*b)*sinh(d*x + c)^
4 + 50*a^3 - 180*a^2*b - 80*a*b^2 - 32*b^3 + 5*(15*a^3 - 48*a^2*b - 16*a*b^
2)*cosh(d*x + c)^2 + 5*(15*a^3*cosh(d*x + c)^4 + 15*a^3 - 48*a^2*b - 16*a*b
^2 + 36*(a^3 - 2*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^
5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)^3\sinh\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)**2)**3*sinh(d*x+c),x)

[Out] Integral((a + b*sech(c + d*x)**2)**3*sinh(c + d*x), x)

Giac [A]
time = 0.41, size = 101, normalized size = 1.58

$$\frac{5\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)-\frac{4\left(15\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+20\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+16\,b^3\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3*sinh(d*x+c),x, algorithm="giac")

[Out] 1/10*(5*a^3*(e^(d*x + c) + e^(-d*x - c)) - 4*(15*a^2*b*(e^(d*x + c) + e^(-d
*x - c))^4 + 20*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 16*b^3)/(e^(d*x + c)
 + e^(-d*x - c))^5)/d

Mupad [B]
time = 1.50, size = 288, normalized size = 4.50

$$\frac{a^3\,\mathrm{e}^{c+d\,x}}{2\,d}+\frac{a^3\,\mathrm{e}^{-c-d\,x}}{2\,d}+\frac{64\,b^3\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}+\frac{8\,\mathrm{e}^{c+d\,x}\,\left(5\,a\,b^2-4\,b^3\right)}{5\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}-\frac{32\,b^3\,\mathrm{e}^{c+d\,x}}{5\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1\right)}-\frac{6\,a^2\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{8\,a\,b^2\,\mathrm{e}^{c+d\,x}}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)

[Out] (a^3*exp(c + d*x))/(2*d) + (a^3*exp(- c - d*x))/(2*d) + (64*b^3*exp(c + d*x
))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp

```
(8*c + 8*d*x) + 1)) + (8*exp(c + d*x)*(5*a*b^2 - 4*b^3))/(5*d*(3*exp(2*c +
2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (32*b^3*exp(c + d*x)
)/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*
exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (6*a^2*b*exp(c + d*x))/(d*(ex
p(2*c + 2*d*x) + 1)) - (8*a*b^2*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + exp(
4*c + 4*d*x) + 1))
```

3.21 $\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=83

$$-\frac{(a+b)^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+b)\operatorname{sech}^3(c+dx)}{3d}+\frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

```
[Out] -(a+b)^3*arctanh(cosh(d*x+c))/d+b*(3*a^2+3*a*b+b^2)*sech(d*x+c)/d+1/3*b^2*(
3*a+b)*sech(d*x+c)^3/d+1/5*b^3*sech(d*x+c)^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4218, 472, 213}

$$\frac{b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+b)\operatorname{sech}^3(c+dx)}{3d}-\frac{(a+b)^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -(((a + b)^3*ArcTanh[Cosh[c + d*x]])/d) + (b*(3*a^2 + 3*a*b + b^2)*Sech[c +
 d*x])/d + (b^2*(3*a + b)*Sech[c + d*x]^3)/(3*d) + (b^3*Sech[c + d*x]^5)/(5
*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(b+ax^2\right)^3}{x^6(1-x^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{Subst}\left(\int \left(\frac{b^3}{x^6}+\frac{b^2(3a+b)}{x^4}+\frac{b(3a^2+3ab+b^2)}{x^2}-\frac{(a+b)^3}{-1+x^2}\right)\,dx, x, \cosh(c\right.}{d}$$

$$= \frac{b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)}{d}+\frac{b^2(3a+b)\operatorname{sech}^3(c+dx)}{3d}+\frac{b^3\mathrm{se}}{}$$

$$= -\frac{(a+b)^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(3a^2+3ab+b^2)\operatorname{sech}(c+dx}{d}$$

Mathematica [**A**]
time = 0.87, size = 134, normalized size = 1.61

$$-\frac{8\left(b+a\cosh^2(c+dx)\right)^3\left(-3b^3-5b^2(3a+b)\cosh^2(c+dx)-15b(3a^2+3ab+b^2)\cosh^4(c+dx)+15(a+b)^3\cosh^5(c+dx)\left(\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\operatorname{sech}^5(c+dx)}{15d(a+2b+a\cosh(2(c+dx)))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-8*(b + a*Cosh[c + d*x]^2)^3*(-3*b^3 - 5*b^2*(3*a + b)*Cosh[c + d*x]^2 - 1
5*b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x]^4 + 15*(a + b)^3*Cosh[c + d*x]^5*(L
og[Cosh[(c + d*x)/2]] - Log[Sinh[(c + d*x)/2]]))*Sech[c + d*x]^5)/(15*d*(a
+ 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [**A**]
time = 1.29, size = 118, normalized size = 1.42

method	result
derivativedivides	$\frac{-2a^3\operatorname{arctanh}(\mathrm{e}^{dx+c})+3a^2b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}(\mathrm{e}^{dx+c})\right)+3a\,b^2\left(\frac{1}{3\cosh(dx+c)^3}+\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}(\mathrm{e}^{dx+c})\right)+b^3\left(\right.}{d}$
default	$\frac{-2a^3\operatorname{arctanh}(\mathrm{e}^{dx+c})+3a^2b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}(\mathrm{e}^{dx+c})\right)+3a\,b^2\left(\frac{1}{3\cosh(dx+c)^3}+\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}(\mathrm{e}^{dx+c})\right)+b^3\left(\right.}{d}$
risch	$\frac{2b\,\mathrm{e}^{dx+c}\left(45a^2\mathrm{e}^{8dx+8c}+45ab\,\mathrm{e}^{8dx+8c}+15b^2\mathrm{e}^{8dx+8c}+180a^2\mathrm{e}^{6dx+6c}+240ab\,\mathrm{e}^{6dx+6c}+80b^2\mathrm{e}^{6dx+6c}+270a^2\mathrm{e}^{4dx+4c}+390ab\right.}{15d\left(1+\mathrm{e}^{2dx+2c}\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*(1/cosh(d*x+c)-2*arctanh(exp(d*x+c)
))+3*a*b^2*(1/3/cosh(d*x+c)^3+1/cosh(d*x+c)-2*arctanh(exp(d*x+c)))+b^3*(1/5
/cosh(d*x+c)^5+1/3/cosh(d*x+c)^3+1/cosh(d*x+c)-2*arctanh(exp(d*x+c))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(79) = 158$.
time = 0.30, size = 358, normalized size = 4.31

$$-\frac{1}{15}\,b^3\left(\frac{15\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\,\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(15\,e^{(-dx-c)}+80\,e^{(-3\,dx-3\,c)}+178\,e^{(-5\,dx-5\,c)}+80\,e^{(-7\,dx-7\,c)}+15\,e^{(-9\,dx-9\,c)}\right)}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}\right)-ab^2\left(\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\,\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(3\,e^{(-dx-c)}+10\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}\right)}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)-3\,a^2b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right)+\frac{a^3\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/15*b^3*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d - 2*(15*
e^(-d*x - c) + 80*e^(-3*d*x - 3*c) + 178*e^(-5*d*x - 5*c) + 80*e^(-7*d*x -
7*c) + 15*e^(-9*d*x - 9*c))/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) - a*b^
2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d - 2*(3*e^(-d*x - c
) + 10*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(3*e^(-2*d*x - 2*c) + 3*e^
(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) - 3*a^2*b*(log(e^(-d*x - c) + 1)/d
 - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(e^(-2*d*x - 2*c) + 1))) + a
^3*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3443 vs. $2(79) = 158$.
time = 0.39, size = 3443, normalized size = 41.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 270*(3*a^2*b + 3*a*b^2
 + b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + 30*(3*a^2*b + 3*a*b^2 + b^3)*sinh(d
*x + c)^9 + 40*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^7 + 40*(9*a^2*b +
 12*a*b^2 + 4*b^3 + 27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^7 + 280*(9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (9*a^2*b + 12*a
*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(135*a^2*b + 195*a*b^2 + 8
9*b^3)*cosh(d*x + c)^5 + 4*(945*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 +
 135*a^2*b + 195*a*b^2 + 89*b^3 + 210*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^5 + 20*(189*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^
5 + 70*(9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + (135*a^2*b + 195*a*b^
2 + 89*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 40*(9*a^2*b + 12*a*b^2 + 4*b^3
)*cosh(d*x + c)^3 + 40*(63*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(
9*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 9*a^2*b + 12*a*b^2 + 4*b^3 +
(135*a^2*b + 195*a*b^2 + 89*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 40*(27*
(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 21*(9*a^2*b + 12*a*b^2 + 4*b^3)
*cosh(d*x + c)^5 + (135*a^2*b + 195*a*b^2 + 89*b^3)*cosh(d*x + c)^3 + 3*(9*
a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 30*(3*a^2*b + 3*
```

a*b^2 + b^3)*cosh(d*x + c) - 15*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^10 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 40*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 10*(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 10*(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 40*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2 + 5*(9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 10*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 15*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^10 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 40*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 10*(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 10*(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 40*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^

```
3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2 + 5*(9*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(d*x + c)^6 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*
a^2*b + 3*a*b^2 + b^3 + 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^2 + 10*((a^3 + 3*a^2*b + 3*a*b^2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^3\operatorname{csch}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(79) = 158.
time = 0.43, size = 228, normalized size = 2.75

$$-\frac{15\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-15\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(45a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+45ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+15b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+60ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+20b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+48b^3\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/30*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(e^(d*x + c) + e^(-d*x - c) +
2) - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(e^(d*x + c) + e^(-d*x - c) - 2)
 - 4*(45*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 + 45*a*b^2*(e^(d*x + c) + e^(
-d*x - c))^4 + 15*b^3*(e^(d*x + c) + e^(-d*x - c))^4 + 60*a*b^2*(e^(d*x + c
) + e^(-d*x - c))^2 + 20*b^3*(e^(d*x + c) + e^(-d*x - c))^2 + 48*b^3)/(e^(d
*x + c) + e^(-d*x - c))^5)/d
```

Mupad [B]
time = 1.53, size = 434, normalized size = 5.23

$$\frac{2\,e^{c+d\,x}\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^c\,\left(a^3\,\sqrt{-d^2}+b^3\,\sqrt{-d^2}+3\,a\,b^2\,\sqrt{-d^2}+3\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6+6\,a^5\,b+15\,a^4\,b^2+20\,a^3\,b^3+15\,a^2\,b^4+6\,a\,b^5+b^6}}\right)\,\sqrt{a^6+6\,a^5\,b+15\,a^4\,b^2+20\,a^3\,b^3+15\,a^2\,b^4+6\,a\,b^5+b^6}}{\sqrt{-d^2}}-\frac{64\,b^3\,e^{c+d\,x}}{5\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}-\frac{8\,e^{c+d\,x}\,\left(15\,a\,b^2-7\,b^3\right)}{15\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}+\frac{32\,b^3\,e^{c+d\,x}}{5\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}+\frac{8\,e^{c+d\,x}\,\left(b^3+3\,a\,b^2\right)}{3\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/sinh(c + d*x),x)
```

```
[Out] (2*exp(c + d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (2*
atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2) + b^3*(-d^2)^(1/2) + 3*a*b^2*(-d^2)
^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/(d*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*
b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2)))*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*
a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2))/(-d^2)^(1/2) - (64*b^3*exp(c + d*
```

```
x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + ex
p(8*c + 8*d*x) + 1)) - (8*exp(c + d*x)*(15*a*b^2 - 7*b^3))/(15*d*(3*exp(2*c
 + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (32*b^3*exp(c + d
*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) +
 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (8*exp(c + d*x)*(3*a*b^2 +
 b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.22 $\int \mathrm{csch}^2(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=70

$$-\frac{(a+b)^3\coth(c+dx)}{d}-\frac{3b(a+b)^2\tanh(c+dx)}{d}+\frac{b^2(a+b)\tanh^3(c+dx)}{d}-\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[Out] -(a+b)^3*coth(d*x+c)/d-3*b*(a+b)^2*tanh(d*x+c)/d+b^2*(a+b)*tanh(d*x+c)^3/d-1/5*b^3*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4217, 276}

$$\frac{b^2(a+b)\tanh^3(c+dx)}{d}-\frac{3b(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)^3\coth(c+dx)}{d}-\frac{b^3\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -(((a + b)^3*Coth[c + d*x])/d) - (3*b*(a + b)^2*Tanh[c + d*x])/d + (b^2*(a + b)*Tanh[c + d*x]^3)/d - (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \mathrm{csch}^2(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx &= \frac{\mathrm{Subst}\left(\int \frac{\left(a+b-bx^2\right)^3}{x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\mathrm{Subst}\left(\int \left(-3b(a+b)^2+\frac{(a+b)^3}{x^2}+3b^2(a+b)x^2-b^3x^4\right)\,dx,x,\right.}{d}\\ &= -\frac{(a+b)^3\coth(c+dx)}{d}-\frac{3b(a+b)^2\tanh(c+dx)}{d}+\frac{b^2(a+b)}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 380 vs. 2(70) = 140.
time = 1.90, size = 380, normalized size = 5.43

$$-\frac{\coth(c+dx)\operatorname{csch}(c)\operatorname{sech}(c)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\left(10a(5a^2+12ab+8b^2)\sinh(2c)-10(5a^3+18a^2b+20ab^2+8b^3)\sinh(2dx)-25a^3\sinh(2(c+dx))+50ab^2\sinh(2(c+dx))+30b^3\sinh(2(c+dx))-20a^3\sinh(4(c+dx))+40ab^2\sinh(4(c+dx))+24b^3\sinh(4(c+dx))-5a^3\sinh(6(c+dx))+10ab^2\sinh(6(c+dx))+6b^3\sinh(6(c+dx))-25a^3\sinh(2(c+2dx))-120a^2b\sinh(2(c+2dx))-160ab^2\sinh(2(c+2dx))-64b^3\sinh(2(c+2dx))+25a^3\sinh(4c+2dx)+30a^2b\sinh(4c+2dx)+5a^3\sinh(6c+4dx)-5a^3\sinh(4c+6dx)-30a^2b\sinh(4c+6dx)-40ab^2\sinh(4c+6dx)-16b^3\sinh(4c+6dx)\right)}{40d(a+2b+a\cosh(2(c+dx)))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/40*(Coth[c + d*x]*Csch[c]*Sech[c]*(a + b*Sech[c + d*x]^2)^3*(10*a*(5*a^2
 + 12*a*b + 8*b^2)*Sinh[2*c] - 10*(5*a^3 + 18*a^2*b + 20*a*b^2 + 8*b^3)*Sin
h[2*d*x] - 25*a^3*Sinh[2*(c + d*x)] + 50*a*b^2*Sinh[2*(c + d*x)] + 30*b^3*S
inh[2*(c + d*x)] - 20*a^3*Sinh[4*(c + d*x)] + 40*a*b^2*Sinh[4*(c + d*x)] +
24*b^3*Sinh[4*(c + d*x)] - 5*a^3*Sinh[6*(c + d*x)] + 10*a*b^2*Sinh[6*(c + d
*x)] + 6*b^3*Sinh[6*(c + d*x)] - 25*a^3*Sinh[2*(c + 2*d*x)] - 120*a^2*b*Sin
h[2*(c + 2*d*x)] - 160*a*b^2*Sinh[2*(c + 2*d*x)] - 64*b^3*Sinh[2*(c + 2*d*x
)] + 25*a^3*Sinh[4*c + 2*d*x] + 30*a^2*b*Sinh[4*c + 2*d*x] + 5*a^3*Sinh[6*c
 + 4*d*x] - 5*a^3*Sinh[4*c + 6*d*x] - 30*a^2*b*Sinh[4*c + 6*d*x] - 40*a*b^2
*Sinh[4*c + 6*d*x] - 16*b^3*Sinh[4*c + 6*d*x]))/(d*(a + 2*b + a*Cosh[2*(c +
 d*x)])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(68) = 136.
time = 2.27, size = 258, normalized size = 3.69

method	result
risch	$-\frac{2\left(5a^3e^{10dx+10c}+25a^3e^{8dx+8c}+30a^2b\,e^{8dx+8c}+50a^3e^{6dx+6c}+120a^2b\,e^{6dx+6c}+80a\,b^2e^{6dx+6c}+50a^3e^{4dx+4c}+180a^2b\,e^{4dx+4c}+200a\right.}{5d\left(1+e^{2dx+2c}\right)^5\left(e^{2dx}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(5*a^3*exp(10*d*x+10*c)+25*a^3*exp(8*d*x+8*c)+30*a^2*b*exp(8*d*x+8*c)+
50*a^3*exp(6*d*x+6*c)+120*a^2*b*exp(6*d*x+6*c)+80*a*b^2*exp(6*d*x+6*c)+50*a
^3*exp(4*d*x+4*c)+180*a^2*b*exp(4*d*x+4*c)+200*a*b^2*exp(4*d*x+4*c)+80*b^3*
exp(4*d*x+4*c)+25*a^3*exp(2*d*x+2*c)+120*a^2*b*exp(2*d*x+2*c)+160*a*b^2*exp
(2*d*x+2*c)+64*b^3*exp(2*d*x+2*c)+5*a^3+30*a^2*b+40*a*b^2+16*b^3)/d/(1+exp(
2*d*x+2*c))^5/(exp(2*d*x+2*c)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(68) = 136.
time = 0.29, size = 358, normalized size = 5.11

$$-\frac{32}{5}b^3\left(\frac{4e^{(-2dx-2c)}}{d(4e^{(-2dx-2c)}+5e^{(-4dx-4c)}-5e^{(-8dx-8c)}-4e^{(-10dx-10c)}-e^{(-12dx-12c)}+1)}+\frac{5e^{(-4dx-4c)}}{d(4e^{(-2dx-2c)}+5e^{(-4dx-4c)}-5e^{(-8dx-8c)}-4e^{(-10dx-10c)}-e^{(-12dx-12c)}+1)}+\frac{1}{d(4e^{(-2dx-2c)}+5e^{(-4dx-4c)}-5e^{(-8dx-8c)}-4e^{(-10dx-10c)}-e^{(-12dx-12c)}+1)}\right)-16ab^2\left(\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}-2e^{(-6dx-6c)}-e^{(-8dx-8c)}+1)}+\frac{1}{d(2e^{(-2dx-2c)}-2e^{(-6dx-6c)}-e^{(-8dx-8c)}+1)}\right)+\frac{2a^3}{d(e^{(-2dx-2c)}-1)}+\frac{12a^2b}{d(e^{(-4dx-4c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -32/5*b^3*(4*e^(-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) -
 5*e^(-8*d*x - 8*c) - 4*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) + 1)) + 5*e
^(-4*d*x - 4*c)/(d*(4*e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) - 5*e^(-8*d*x -
 8*c) - 4*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) + 1)) + 1/(d*(4*e^(-2*d*x
 - 2*c) + 5*e^(-4*d*x - 4*c) - 5*e^(-8*d*x - 8*c) - 4*e^(-10*d*x - 10*c) -
e^(-12*d*x - 12*c) + 1))) - 16*a*b^2*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x -
2*c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1)) + 1/(d*(2*e^(-2*d*x - 2*
c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1))) + 2*a^3/(d*(e^(-2*d*x - 2
*c) - 1)) + 12*a^2*b/(d*(e^(-4*d*x - 4*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(68) = 136$.
time = 0.35, size = 622, normalized size = 8.89

$$-\frac{4\left(\left(5a^{3}+15a^{2}b+20ab^{2}+8b^{3}\right)\cosh\left(dx+c\right)^{5}+5\left(5a^{3}+15a^{2}b+20ab^{2}+8b^{3}\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}-\left(15a^{2}b+20ab^{2}+8b^{3}\right)\sinh\left(dx+c\right)^{5}+\left(25a^{3}+75a^{2}b+80ab^{2}+32b^{3}\right)\cosh\left(dx+c\right)^{3}-\left(45a^{2}b+80ab^{2}+32b^{3}+10\left(15a^{2}b+20ab^{2}+8b^{3}\right)\cosh\left(dx+c\right)^{2}\right)\sinh\left(dx+c\right)^{3}+\left(10\left(5a^{3}+15a^{2}b+20ab^{2}+8b^{3}\right)\cosh\left(dx+c\right)^{3}+3\left(25a^{3}+75a^{2}b+80ab^{2}+32b^{3}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+10\left(5a^{3}+15a^{2}b+14ab^{2}+4b^{3}\right)\cosh\left(dx+c\right)-\left(5\left(15a^{2}b+20ab^{2}+8b^{3}\right)\cosh\left(dx+c\right)^{4}+30a^{2}b+60ab^{2}+40b^{3}+3\left(45a^{2}b+80ab^{2}+32b^{3}\right)\cosh\left(dx+c\right)^{2}\right)\sinh\left(dx+c\right)\right)}{5\left(d\cosh\left(dx+c\right)^{7}+7d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{6}+d\sinh\left(dx+c\right)^{7}+3d\cosh\left(dx+c\right)^{5}+\left(21d\cosh\left(dx+c\right)^{2}+5d\right)\sinh\left(dx+c\right)^{5}+5\left(7d\cosh\left(dx+c\right)^{3}+3d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{4}+d\cosh\left(dx+c\right)^{3}+\left(35d\cosh\left(dx+c\right)^{4}+50d\cosh\left(dx+c\right)^{2}+9d\right)\sinh\left(dx+c\right)^{3}+3\left(7d\cosh\left(dx+c\right)^{5}+10d\cosh\left(dx+c\right)^{3}+d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}-5d\cosh\left(dx+c\right)+\left(7d\cosh\left(dx+c\right)^{6}+25d\cosh\left(dx+c\right)^{4}+27d\cosh\left(dx+c\right)^{2}+5d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/5*((5*a^3 + 15*a^2*b + 20*a*b^2 + 8*b^3)*cosh(d*x + c)^5 + 5*(5*a^3 + 15
*a^2*b + 20*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^2*b + 20*a
*b^2 + 8*b^3)*sinh(d*x + c)^5 + (25*a^3 + 75*a^2*b + 80*a*b^2 + 32*b^3)*cos
h(d*x + c)^3 - (45*a^2*b + 80*a*b^2 + 32*b^3 + 10*(15*a^2*b + 20*a*b^2 + 8*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (10*(5*a^3 + 15*a^2*b + 20*a*b^2 +
8*b^3)*cosh(d*x + c)^3 + 3*(25*a^3 + 75*a^2*b + 80*a*b^2 + 32*b^3)*cosh(d*x
 + c))*sinh(d*x + c)^2 + 10*(5*a^3 + 15*a^2*b + 14*a*b^2 + 4*b^3)*cosh(d*x
+ c) - (5*(15*a^2*b + 20*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 30*a^2*b + 60*a*b
^2 + 40*b^3 + 3*(45*a^2*b + 80*a*b^2 + 32*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c
)^7 + 3*d*cosh(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 +
5*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x +
c)^3 + (35*d*cosh(d*x + c)^4 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3
+ 3*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
 + c)^2 - 5*d*cosh(d*x + c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 +
 27*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^{2}\left(c+dx\right)\right)^{3}\operatorname{csch}^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(68) = 136.
time = 0.44, size = 249, normalized size = 3.56

$$-\frac{2\left(\frac{5\left(a^3+3a^2b+3ab^2+b^3\right)}{e^{(2dx+2c)}-1}-\frac{15a^2be^{(8dx+8c)}+15ab^2e^{(8dx+8c)}+5b^3e^{(8dx+8c)}+60a^2be^{(6dx+6c)}+90ab^2e^{(6dx+6c)}+30b^3e^{(6dx+6c)}+90a^2be^{(4dx+4c)}+160ab^2e^{(4dx+4c)}+80b^3e^{(4dx+4c)}+60a^2be^{(2dx+2c)}+110ab^2e^{(2dx+2c)}+50b^3e^{(2dx+2c)}+15a^2b+25ab^2+11b^3}{\left(e^{(2dx+2c)}+1\right)^5}\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/5*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(e^(2*d*x + 2*c) - 1) - (15*a^2*b*e
^(8*d*x + 8*c) + 15*a*b^2*e^(8*d*x + 8*c) + 5*b^3*e^(8*d*x + 8*c) + 60*a^2*
b*e^(6*d*x + 6*c) + 90*a*b^2*e^(6*d*x + 6*c) + 30*b^3*e^(6*d*x + 6*c) + 90*
a^2*b*e^(4*d*x + 4*c) + 160*a*b^2*e^(4*d*x + 4*c) + 80*b^3*e^(4*d*x + 4*c)
+ 60*a^2*b*e^(2*d*x + 2*c) + 110*a*b^2*e^(2*d*x + 2*c) + 50*b^3*e^(2*d*x +
2*c) + 15*a^2*b + 25*a*b^2 + 11*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 1.48, size = 644, normalized size = 9.20

$$\frac{\frac{2\left(3a^2b+6ab^2+2b^3\right)}{5d}+\frac{2e^{2c+2dx}\left(3a^2b+3ab^2+b^3\right)}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1}+\frac{\frac{2\left(3a^2b+6ab^2+2b^3\right)}{5d}+\frac{2e^{6c+6dx}\left(3a^2b+3ab^2+b^3\right)}{5d}+\frac{6e^{2c+2dx}\left(3a^2b+7ab^2+5b^3\right)}{5d}+\frac{6e^{4c+4dx}\left(3a^2b+6ab^2+2b^3\right)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}+\frac{\frac{2\left(3a^2b+3ab^2+b^3\right)}{5d}+\frac{2e^{8c+8dx}\left(3a^2b+3ab^2+b^3\right)}{5d}+\frac{8e^{2c+2dx}\left(3a^2b+6ab^2+2b^3\right)}{5d}+\frac{12e^{4c+4dx}\left(3a^2b+7ab^2+5b^3\right)}{5d}+\frac{8e^{6c+6dx}\left(3a^2b+6ab^2+2b^3\right)}{5d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}+\frac{\frac{2\left(3a^2b+7ab^2+5b^3\right)}{5d}+\frac{2e^{4c+4dx}\left(3a^2b+3ab^2+b^3\right)}{5d}+\frac{4e^{2c+2dx}\left(3a^2b+6ab^2+2b^3\right)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}-\frac{2\left(a^3+3a^2b+3ab^2+b^3\right)}{d\left(e^{2c+2dx}-1\right)}+\frac{2\left(3a^2b+3ab^2+b^3\right)}{5d\left(e^{2c+2dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/sinh(c + d*x)^2,x)
```

```
[Out] ((2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (2*exp(2*c + 2*d*x)*(3*a*b^2 + 3*a
^2*b + b^3))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((2*(6*a*
b^2 + 3*a^2*b + 2*b^3))/(5*d) + (2*exp(6*c + 6*d*x)*(3*a*b^2 + 3*a^2*b + b^
3))/(5*d) + (6*exp(2*c + 2*d*x)*(7*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (6*exp
(4*c + 4*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d))/(4*exp(2*c + 2*d*x) + 6*e
xp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + ((2*(3*a*b^2
 + 3*a^2*b + b^3))/(5*d) + (2*exp(8*c + 8*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(
5*d) + (8*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (12*exp(4*c
 + 4*d*x)*(7*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (8*exp(6*c + 6*d*x)*(6*a*b^2
 + 3*a^2*b + 2*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*
exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) + ((2*(7*a*
b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(4*c + 4*d*x)*(3*a*b^2 + 3*a^2*b + b^
3))/(5*d) + (4*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d))/(3*exp(
2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*(3*a*b^2 + 3
*a^2*b + a^3 + b^3))/(d*(exp(2*c + 2*d*x) - 1)) + (2*(3*a*b^2 + 3*a^2*b + b
^3))/(5*d*(exp(2*c + 2*d*x) + 1))
```

3.23 $\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=144

$$\frac{(a+b)^2(a+7b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{(a+b)^2(a+7b)\operatorname{sech}(c+dx)}{2d}-\frac{b(6a^2+15ab+7b^2)\operatorname{sech}^3(c+dx)}{6d}-$$

```
[Out] 1/2*(a+b)^2*(a+7*b)*arctanh(cosh(d*x+c))/d-1/2*(a+b)^2*(a+7*b)*sech(d*x+c)/
d-1/6*b*(6*a^2+15*a*b+7*b^2)*sech(d*x+c)^3/d-1/10*b^2*(5*a+7*b)*sech(d*x+c)
^5/d-1/2*(a+b)*(b+a*cosh(d*x+c)^2)^2*csch(d*x+c)^2*sech(d*x+c)^5/d
```

Rubi [A]

time = 0.13, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4218, 479, 584, 213}

$$-\frac{b(6a^2+15ab+7b^2)\operatorname{sech}^3(c+dx)}{6d}-\frac{b^2(5a+7b)\operatorname{sech}^5(c+dx)}{10d}-\frac{(a+b)^2(a+7b)\operatorname{sech}(c+dx)}{2d}+\frac{(a+b)^2(a+7b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{(a+b)\operatorname{csch}^2(c+dx)\operatorname{sech}^5(c+dx)\left(a\cosh^2(c+dx)+b\right)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^2*(a + 7*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - ((a + b)^2*(a + 7*b)*S
ech[c + d*x])/(2*d) - (b*(6*a^2 + 15*a*b + 7*b^2)*Sech[c + d*x]^3)/(6*d) -
(b^2*(5*a + 7*b)*Sech[c + d*x]^5)/(10*d) - ((a + b)*(b + a*Cosh[c + d*x]^2)
^2*Csch[c + d*x]^2*Sech[c + d*x]^5)/(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 584

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
```

```
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{(b+ax^2)^3}{x^6(1-x^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\left(b+a\cosh^2(c+dx)\right)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^5(c+dx)}{2d} + \cdots$$

$$= -\frac{(a+b)\left(b+a\cosh^2(c+dx)\right)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^5(c+dx)}{2d} + \cdots$$

$$= -\frac{(a+b)^2(a+7b)\operatorname{sech}(c+dx)}{2d} - \frac{b(6a^2+15ab+7b^2)\operatorname{sech}^3(c+\cdots}{6d}$$

$$= \frac{(a+b)^2(a+7b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{(a+b)^2(a+7b)\operatorname{sech}(\cdots}{2d}$$

Mathematica [A]
time = 2.61, size = 224, normalized size = 1.56

$$-\frac{\left(b+a\cosh^2(c+dx)\right)^3\operatorname{sech}^6(c+dx)\left(\left(150a^3+270a^2b-30ab^2-206b^3+10(9a^3+45a^2b+75ab^2+35b^3)\cosh(4(c+dx))+15(a+b)^2(a+7b)\cosh(6(c+dx))\right)\coth(c+dx)\operatorname{csch}(c+dx)-480(a+b)^2(a+7b)\cosh^6(c+dx)\left(\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)+\frac{3}{4}(75a^3+195a^2b+165ab^2+77b^3)\operatorname{csch}^3(c+dx)\sinh(4(c+dx))\right)}{120d(a+2b+a\cosh(2(c+dx)))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/120*((b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^6*((150*a^3 + 270*a^2*b - 3
0*a*b^2 - 206*b^3 + 10*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*Cosh[4*(c + d
*x)] + 15*(a + b)^2*(a + 7*b)*Cosh[6*(c + d*x)])*Coth[c + d*x]*Csch[c + d*x
] - 480*(a + b)^2*(a + 7*b)*Cosh[c + d*x]^6*(Log[Cosh[(c + d*x)/2]] - Log[S
inh[(c + d*x)/2]]) + (3*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*Csch[c +
d*x]^3*Sinh[4*(c + d*x)])/4))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(134) = 268.
time = 2.58, size = 550, normalized size = 3.82

method	result
risch	$-\frac{e^{dx+c}(225a\,b^2+135a^2b\,e^{12dx+12c}+225a\,b^2e^{12dx+12c}+450a^2b\,e^{10dx+10c}+750a\,b^2e^{10dx+10c}+585a^2b\,e^{4dx+4c}+450a^2b\,e^{2dx+2c}+13}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*exp(d*x+c)*(225*a*b^2+135*a^2*b*exp(12*d*x+12*c)+225*a*b^2*exp(12*d*x
+12*c)+450*a^2*b*exp(10*d*x+10*c)+750*a*b^2*exp(10*d*x+10*c)+585*a^2*b*exp(
4*d*x+4*c)+450*a^2*b*exp(2*d*x+2*c)+135*a^2*b+15*a^3+105*b^3+495*a*b^2*exp(
8*d*x+8*c)-60*a*b^2*exp(6*d*x+6*c)+495*a*b^2*exp(4*d*x+4*c)+585*a^2*b*exp(8
*d*x+8*c)+750*a*b^2*exp(2*d*x+2*c)+540*a^2*b*exp(6*d*x+6*c)+90*a^3*exp(2*d*
x+2*c)+15*a^3*exp(12*d*x+12*c)+105*b^3*exp(12*d*x+12*c)+90*a^3*exp(10*d*x+1
0*c)+350*b^3*exp(2*d*x+2*c)+231*b^3*exp(8*d*x+8*c)+225*a^3*exp(4*d*x+4*c)+2
31*b^3*exp(4*d*x+4*c)+350*b^3*exp(10*d*x+10*c)+225*a^3*exp(8*d*x+8*c)+300*a
^3*exp(6*d*x+6*c)-412*b^3*exp(6*d*x+6*c))/d/(1+exp(2*d*x+2*c))^5/(exp(2*d*x
+2*c)-1)^2-1/2*a^3/d*ln(exp(d*x+c)-1)-9/2*a^2*b/d*ln(exp(d*x+c)-1)-15/2/d*l
n(exp(d*x+c)-1)*a*b^2-7/2/d*ln(exp(d*x+c)-1)*b^3+1/2*a^3/d*ln(exp(d*x+c)+1)
+9/2*a^2*b/d*ln(exp(d*x+c)+1)+15/2/d*ln(exp(d*x+c)+1)*a*b^2+7/2/d*ln(exp(d*
x+c)+1)*b^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(134) = 268$.
time = 0.29, size = 556, normalized size = 3.86

$$\frac{1}{30}b^3\left(\frac{105\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{105\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(105e^{(-dx-c)}+350e^{(-3dx-3c)}+231e^{(-5dx-5c)}-412e^{(-7dx-7c)}+231e^{(-9dx-9c)}+350e^{(-11dx-11c)}+105e^{(-13dx-13c)}\right)}{d(3e^{(-2dx-2c)}+e^{(-4dx-4c)}-5e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}+3e^{(-12dx-12c)}+e^{(-14dx-14c)}+1)}\right)+\frac{1}{2}ab^2\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\left(15e^{(-dx-c)}+20e^{(-3dx-3c)}-22e^{(-5dx-5c)}+20e^{(-7dx-7c)}+15e^{(-9dx-9c)}\right)}{d(e^{(-2dx-2c)}-2e^{(-4dx-4c)}-2e^{(-6dx-6c)}+e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)+\frac{3}{2}a^2b\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3e^{(-dx-c)}-2e^{(-3dx-3c)}+3e^{(-5dx-5c)}\right)}{d(e^{(-2dx-2c)}+e^{(-4dx-4c)}-e^{(-6dx-6c)}-1)}\right)+\frac{1}{2}a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3dx-3c)}\right)}{d(2e^{(-2dx-2c)}-e^{(-4dx-4c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/30*b^3*(105*log(e^(-d*x - c) + 1)/d - 105*log(e^(-d*x - c) - 1)/d - 2*(10
5*e^(-d*x - c) + 350*e^(-3*d*x - 3*c) + 231*e^(-5*d*x - 5*c) - 412*e^(-7*d*
x - 7*c) + 231*e^(-9*d*x - 9*c) + 350*e^(-11*d*x - 11*c) + 105*e^(-13*d*x -
 13*c))/(d*(3*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) - 5*e^(-6*d*x - 6*c) - 5*
e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 3*e^(-12*d*x - 12*c) + e^(-14*d*x -
 14*c) + 1))) + 1/2*a*b^2*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c)
 - 1)/d - 2*(15*e^(-d*x - c) + 20*e^(-3*d*x - 3*c) - 22*e^(-5*d*x - 5*c) +
20*e^(-7*d*x - 7*c) + 15*e^(-9*d*x - 9*c))/(d*(e^(-2*d*x - 2*c) - 2*e^(-4*d
*x - 4*c) - 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)
)) + 3/2*a^2*b*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(
3*e^(-d*x - c) - 2*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/(d*(e^(-2*d*x - 2
*c) + e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c) - 1))) + 1/2*a^3*(log(e^(-d*x - c
) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d
*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6717 vs. $2(134) = 268$.
time = 0.42, size = 6717, normalized size = 46.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/30*(30*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^13 + 390*(a^3 +
9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)*sinh(d*x + c)^12 + 30*(a^3 + 9*a^
2*b + 15*a*b^2 + 7*b^3)*sinh(d*x + c)^13 + 20*(9*a^3 + 45*a^2*b + 75*a*b^2
+ 35*b^3)*cosh(d*x + c)^11 + 20*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3 + 117
*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 220
*(39*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + (9*a^3 + 45*a^2*b
 + 75*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + 6*(75*a^3 + 195*a^2
*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^9 + 2*(10725*(a^3 + 9*a^2*b + 15*a*b
^2 + 7*b^3)*cosh(d*x + c)^4 + 225*a^3 + 585*a^2*b + 495*a*b^2 + 231*b^3 + 5
50*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9
+ 6*(6435*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 550*(9*a^3 +
 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^3 + 9*(75*a^3 + 195*a^2*b + 16
5*a*b^2 + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 + 8*(75*a^3 + 135*a^2*b -
15*a*b^2 - 103*b^3)*cosh(d*x + c)^7 + 8*(6435*(a^3 + 9*a^2*b + 15*a*b^2 + 7
*b^3)*cosh(d*x + c)^6 + 825*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x
 + c)^4 + 75*a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3 + 27*(75*a^3 + 195*a^2*b
+ 165*a*b^2 + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 8*(6435*(a^3 + 9*a
^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^7 + 1155*(9*a^3 + 45*a^2*b + 75*a*b^
2 + 35*b^3)*cosh(d*x + c)^5 + 63*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*
cosh(d*x + c)^3 + 7*(75*a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^6 + 6*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*cosh(d*x +
c)^5 + 6*(6435*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 + 1540*(9
*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 126*(75*a^3 + 195*a^
2*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^4 + 75*a^3 + 195*a^2*b + 165*a*b^2
+ 77*b^3 + 28*(75*a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^5 + 2*(10725*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^9
 + 3300*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^7 + 378*(75*a^
3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^5 + 140*(75*a^3 + 135*a^2
*b - 15*a*b^2 - 103*b^3)*cosh(d*x + c)^3 + 15*(75*a^3 + 195*a^2*b + 165*a*b
^2 + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 20*(9*a^3 + 45*a^2*b + 75*a*b
^2 + 35*b^3)*cosh(d*x + c)^3 + 4*(2145*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*c
osh(d*x + c)^10 + 825*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^
8 + 126*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^6 + 70*(75*
a^3 + 135*a^2*b - 15*a*b^2 - 103*b^3)*cosh(d*x + c)^4 + 45*a^3 + 225*a^2*b
+ 375*a*b^2 + 175*b^3 + 15*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^3 + 4*(585*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh
```

```
(d*x + c)^11 + 275*(9*a^3 + 45*a^2*b + 75*a*b^2 + 35*b^3)*cosh(d*x + c)^9 +
 54*(75*a^3 + 195*a^2*b + 165*a*b^2 + 77*b^3)*cosh(d*x + c)^7 + 42*(75*a^3
+ 135*a^2*b - 15*a*b^2 - 103*b^3)*cosh(d*x + c)^5 + 15*(75*a^3 + 195*a^2*b
+ 165*a*b^2 + 77*b^3)*cosh(d*x + c)^3 + 15*(9*a^3 + 45*a^2*b + 75*a*b^2 + 3
5*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 30*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^
3)*cosh(d*x + c) - 15*((a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^14
+ 14*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)*sinh(d*x + c)^13 + (a
^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*sinh(d*x + c)^14 + 3*(a^3 + 9*a^2*b + 15*a
*b^2 + 7*b^3)*cosh(d*x + c)^12 + (3*a^3 + 27*a^2*b + 45*a*b^2 + 21*b^3 + 91
*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 4*(
91*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + 9*(a^3 + 9*a^2*b +
15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 + (a^3 + 9*a^2*b + 15*a*b
^2 + 7*b^3)*cosh(d*x + c)^10 + (1001*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cos
h(d*x + c)^4 + a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3 + 198*(a^3 + 9*a^2*b + 15*a
*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(1001*(a^3 + 9*a^2*b +
15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 330*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*
cosh(d*x + c)^3 + 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(
d*x + c)^9 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 + (3003*(
a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + 1485*(a^3 + 9*a^2*b + 1
5*a*b^2 + 7*b^3)*cosh(d*x + c)^4 - 5*a^3 - 45*a^2*b - 75*a*b^2 - 35*b^3 + 4
5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(
429*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^7 + 297*(a^3 + 9*a^2*b
 + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^5 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3
)*cosh(d*x + c)^3 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c))*sin
h(d*x + c)^7 - 5*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + (3003
*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^8 + 2772*(a^3 + 9*a^2*b +
 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + 210*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)
*cosh(d*x + c)^4 - 5*a^3 - 45*a^2*b - 75*a*b^2 - 35*b^3 - 140*(a^3 + 9*a^2*
b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 2*(1001*(a^3 + 9*a
^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^9 + 1188*(a^3 + 9*a^2*b + 15*a*b^2 +
 7*b^3)*cosh(d*x + c)^7 + 126*(a^3 + 9*a^2*b + ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^3\operatorname{csch}^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(134) = 268.
time = 0.45, size = 341, normalized size = 2.37

$$\frac{15\left(a^3+9a^2b+15ab^2+7b^3\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-15\left(a^3+9a^2b+15ab^2+7b^3\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{60\left(a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)+3a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+3ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}-\frac{8\left(45a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+90ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+45b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+60ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+40b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+48b^3\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/60*(15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*log(e^(d*x + c) + e^(-d*x - c)
+ 2) - 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*log(e^(d*x + c) + e^(-d*x - c)
 - 2) - 60*(a^3*(e^(d*x + c) + e^(-d*x - c)) + 3*a^2*b*(e^(d*x + c) + e^(-d
*x - c)) + 3*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + b^3*(e^(d*x + c) + e^(-d*
x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4) - 8*(45*a^2*b*(e^(d*x + c) +
e^(-d*x - c))^4 + 90*a*b^2*(e^(d*x + c) + e^(-d*x - c))^4 + 45*b^3*(e^(d*x
+ c) + e^(-d*x - c))^4 + 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 + 40*b^3*(
e^(d*x + c) + e^(-d*x - c))^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

Mupad [B]
time = 1.64, size = 536, normalized size = 3.72

$$\frac{\mathrm{atan}\left(\frac{e^{dx}\,e^{c}\,\left(a^3\,\sqrt{-d^2}+7\,b^3\,\sqrt{-d^2}+15\,a\,b^2\,\sqrt{-d^2}+9\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6+18\,a^5\,b+111\,a^4\,b^2+284\,a^3\,b^3+351\,a^2\,b^4+210\,a\,b^5+49\,b^6}}\right)\,\sqrt{a^6+18\,a^5\,b+111\,a^4\,b^2+284\,a^3\,b^3+351\,a^2\,b^4+210\,a\,b^5+49\,b^6}}{\sqrt{-d^2}}-\frac{2\,e^{c+dx}\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{d\,\left(e^{4c+4dx}-2\,e^{2c+2dx}+1\right)}-\frac{8\,e^{c+dx}\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d\,\left(2\,e^{2c+2dx}+e^{4c+4dx}+1\right)}-\frac{6\,e^{c+dx}\,\left(a^2\,b+2\,a\,b^2+b^3\right)}{d\,\left(e^{2c+2dx}+1\right)}+\frac{64\,b^3\,e^{c+dx}}{5\,d\,\left(4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1\right)}+\frac{8\,e^{c+dx}\,\left(15\,a\,b^2-2\,b^3\right)}{15\,d\,\left(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1\right)}-\frac{32\,b^3\,e^{c+dx}}{5\,d\,\left(5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1\right)}-\frac{e^{c+dx}\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{d\,\left(e^{2c+2dx}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2) + 7*b^3*(-d^2)^(1/2) + 15*a*b^2*(-
d^2)^(1/2) + 9*a^2*b*(-d^2)^(1/2)))/(d*(210*a*b^5 + 18*a^5*b + a^6 + 49*b^6
 + 351*a^2*b^4 + 284*a^3*b^3 + 111*a^4*b^2)^(1/2)))*(210*a*b^5 + 18*a^5*b +
 a^6 + 49*b^6 + 351*a^2*b^4 + 284*a^3*b^3 + 111*a^4*b^2)^(1/2))/(-d^2)^(1/2
) - (2*exp(c + d*x)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(d*(exp(4*c + 4*d*x) -
 2*exp(2*c + 2*d*x) + 1)) - (8*exp(c + d*x)*(3*a*b^2 + 2*b^3))/(3*d*(2*exp(
2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (6*exp(c + d*x)*(2*a*b^2 + a^2*b +
b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c +
2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) +
 (8*exp(c + d*x)*(15*a*b^2 - 2*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c
+ 4*d*x) + exp(6*c + 6*d*x) + 1)) - (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c +
 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) +
exp(10*c + 10*d*x) + 1)) - (exp(c + d*x)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(
d*(exp(2*c + 2*d*x) - 1))
```

3.24 $\int \mathrm{csch}^4(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=104

$$\frac{(a+b)^2(a+4b)\coth(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{3b(a+b)(a+2b)\tanh(c+dx)}{d}-\frac{b^2(3a+4b)\tanh^3(}{3d}$$

```
[Out] (a+b)^2*(a+4*b)*coth(d*x+c)/d-1/3*(a+b)^3*coth(d*x+c)^3/d+3*b*(a+b)*(a+2*b)
*tanh(d*x+c)/d-1/3*b^2*(3*a+4*b)*tanh(d*x+c)^3/d+1/5*b^3*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4217, 459}

$$-\frac{b^2(3a+4b)\tanh^3(c+dx)}{3d}+\frac{3b(a+b)(a+2b)\tanh(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{(a+b)^2(a+4b)\coth(c+dx)}{d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^2*(a + 4*b)*Coth[c + d*x])/d - ((a + b)^3*Coth[c + d*x]^3)/(3*d) +
 (3*b*(a + b)*(a + 2*b)*Tanh[c + d*x])/d - (b^2*(3*a + 4*b)*Tanh[c + d*x]^3
)/(3*d) + (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+b-bx^2)^3}{x^4}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(3b(a+b)(a+2b)+\frac{(a+b)^3}{x^4}-\frac{(a+b)^2(a+4b)}{x^2}-b^2(3a+4b\right.\right.}{d}$$

$$= \frac{(a+b)^2(a+4b)\coth(c+dx)}{d} - \frac{(a+b)^3\coth^3(c+dx)}{3d} + \frac{3b(a\ }{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 213 vs. 2(104) = 208.
time = 1.69, size = 213, normalized size = 2.05

$$-\frac{8\left(b+a\cosh^2(c+dx)\right)^3\operatorname{sech}^5(c+dx)\left(-3b^3\cosh(c+dx)+\cosh^3(c+dx)\left(-b^2(15a+14b)+5(a+b)^3\coth^2(c)\coth^2(c+dx)\right)-3b^3\operatorname{csch}(c)\sinh(dx)+\cosh^4(c+dx)\left(-b(45a^2+120ab+73b^2)+5(a+b)^2(2a+11b)\coth(c)\coth(c+dx)\right)\operatorname{csch}(c)\sinh(dx)-\cosh^2(c+dx)\left(b^2(15a+14b)+5(a+b)^3\coth(c)\coth^3(c+dx)\right)\operatorname{csch}(c)\sinh(dx)\right)\tanh(c)}{15d(a+2b+a\cosh(2(c+dx)))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-8*(b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^5*(-3*b^3*Cosh[c + d*x] + Cosh[
c + d*x]^3*(-(b^2*(15*a + 14*b)) + 5*(a + b)^3*Coth[c]^2*Coth[c + d*x]^2) -
 3*b^3*Csch[c]*Sinh[d*x] + Cosh[c + d*x]^4*(-(b*(45*a^2 + 120*a*b + 73*b^2)
) + 5*(a + b)^2*(2*a + 11*b)*Coth[c]*Coth[c + d*x])*Csch[c]*Sinh[d*x] - Cos
h[c + d*x]^2*(b^2*(15*a + 14*b) + 5*(a + b)^3*Coth[c]*Coth[c + d*x]^3)*Csch
[c]*Sinh[d*x])*Tanh[c])/(15*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(98) = 196.
time = 2.40, size = 316, normalized size = 3.04

method	result
risch	$-\frac{4\left(15a^3 e^{12dx+12c}+70a^3 e^{10dx+10c}+120a^2 b\, e^{10dx+10c}+125a^3 e^{8dx+8c}+420a^2 b\, e^{8dx+8c}+360a\, b^2 e^{8dx+8c}+100a^3 e^{6dx+6c}+480a^2 b\, e^{6dx}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4/15*(15*a^3*exp(12*d*x+12*c)+70*a^3*exp(10*d*x+10*c)+120*a^2*b*exp(10*d*x
+10*c)+125*a^3*exp(8*d*x+8*c)+420*a^2*b*exp(8*d*x+8*c)+360*a*b^2*exp(8*d*x+
8*c)+100*a^3*exp(6*d*x+6*c)+480*a^2*b*exp(6*d*x+6*c)+720*a*b^2*exp(6*d*x+6*
c)+384*b^3*exp(6*d*x+6*c)+25*a^3*exp(4*d*x+4*c)+120*a^2*b*exp(4*d*x+4*c)+24
0*a*b^2*exp(4*d*x+4*c)+128*b^3*exp(4*d*x+4*c)-10*a^3*exp(2*d*x+2*c)-120*a^2
*b*exp(2*d*x+2*c)-240*a*b^2*exp(2*d*x+2*c)-128*b^3*exp(2*d*x+2*c)-5*a^3-60*
a^2*b-120*a*b^2-64*b^3)/d/(1+exp(2*d*x+2*c))^5/(exp(2*d*x+2*c)-1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 664 vs. $2(98) = 196$.
time = 0.29, size = 664, normalized size = 6.38

$$\frac{4}{3}a^3\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}-\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)+\frac{256}{15}b^3\left(\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}-2e^{(-4dx-4c)}-6e^{(-6dx-6c)}+6e^{(-10dx-10c)}+2e^{(-12dx-12c)}-2e^{(-14dx-14c)}-e^{(-16dx-16c)}+1)}-\frac{2e^{(-4dx-4c)}}{d(2e^{(-2dx-2c)}-2e^{(-4dx-4c)}-6e^{(-6dx-6c)}+6e^{(-10dx-10c)}+2e^{(-12dx-12c)}-2e^{(-14dx-14c)}-e^{(-16dx-16c)}+1)}-\frac{6e^{(-6dx-6c)}}{d(2e^{(-2dx-2c)}-2e^{(-4dx-4c)}-6e^{(-6dx-6c)}+6e^{(-10dx-10c)}+2e^{(-12dx-12c)}-2e^{(-14dx-14c)}-e^{(-16dx-16c)}+1)}+\frac{1}{d(2e^{(-2dx-2c)}-2e^{(-4dx-4c)}-6e^{(-6dx-6c)}+6e^{(-10dx-10c)}+2e^{(-12dx-12c)}-2e^{(-14dx-14c)}-e^{(-16dx-16c)}+1)}\right)+16a^2b\left(\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}-2e^{(-6dx-6c)}+e^{(-8dx-8c)}-1)}-\frac{1}{d(2e^{(-2dx-2c)}-2e^{(-6dx-6c)}+e^{(-8dx-8c)}-1)}\right)+32ab^2\left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-4dx-4c)}-3e^{(-8dx-8c)}+e^{(-12dx-12c)}-1)}-\frac{1}{d(3e^{(-4dx-4c)}-3e^{(-8dx-8c)}+e^{(-12dx-12c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1))) + 256/15*b^3*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c
) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x - 10*c) + 2*e^(-
12*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) + 1)) - 2*e^(-4*
d*x - 4*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c)
 + 6*e^(-10*d*x - 10*c) + 2*e^(-12*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(
-16*d*x - 16*c) + 1)) - 6*e^(-6*d*x - 6*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-4
*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x - 10*c) + 2*e^(-12*d*x - 12
*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) + 1)) + 1/(d*(2*e^(-2*d*x -
 2*c) - 2*e^(-4*d*x - 4*c) - 6*e^(-6*d*x - 6*c) + 6*e^(-10*d*x - 10*c) + 2*
e^(-12*d*x - 12*c) - 2*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) + 1))) + 16*
a^2*b*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) + e^(
-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) + e^(-8
*d*x - 8*c) - 1))) + 32*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-4*d*x - 4*c) -
3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)) - 1/(d*(3*e^(-4*d*x - 4*c) -
3*e^(-8*d*x - 8*c) + e^(-12*d*x - 12*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 955 vs. $2(98) = 196$.
time = 0.35, size = 955, normalized size = 9.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -8/15*((5*a^3 - 30*a^2*b - 60*a*b^2 - 32*b^3)*cosh(d*x + c)^6 + 12*(5*a^3 +
 15*a^2*b + 30*a*b^2 + 16*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^3 - 30*
a^2*b - 60*a*b^2 - 32*b^3)*sinh(d*x + c)^6 + 2*(15*a^3 - 60*a*b^2 - 32*b^3)
*cosh(d*x + c)^4 + (30*a^3 - 120*a*b^2 - 64*b^3 + 15*(5*a^3 - 30*a^2*b - 60
*a*b^2 - 32*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(5*a^3 + 15*a^2*b
+ 30*a*b^2 + 16*b^3)*cosh(d*x + c)^3 + 4*(5*a^3 + 15*a^2*b + 15*a*b^2 + 8*b
^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 50*a^3 + 240*a^2*b + 360*a*b^2 + 192*b
^3 + (75*a^3 + 270*a^2*b + 300*a*b^2 + 64*b^3)*cosh(d*x + c)^2 + (15*(5*a^3
 - 30*a^2*b - 60*a*b^2 - 32*b^3)*cosh(d*x + c)^4 + 75*a^3 + 270*a^2*b + 300
```

```
*a*b^2 + 64*b^3 + 12*(15*a^3 - 60*a*b^2 - 32*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 4*(3*(5*a^3 + 15*a^2*b + 30*a*b^2 + 16*b^3)*cosh(d*x + c)^5 + 8*(
5*a^3 + 15*a^2*b + 15*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (25*a^3 + 75*a^2*b +
 30*a*b^2 - 32*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*
d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 2*d*cosh(d*x + c)^8
+ (45*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 +
2*d*cosh(d*x + c))*sinh(d*x + c)^7 - 3*d*cosh(d*x + c)^6 + (210*d*cosh(d*x
+ c)^4 + 56*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^6 + 2*(126*d*cosh(d*x +
c)^5 + 56*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 - 8*d*cosh
(d*x + c)^4 + (210*d*cosh(d*x + c)^6 + 140*d*cosh(d*x + c)^4 - 45*d*cosh(d*
x + c)^2 - 8*d)*sinh(d*x + c)^4 + 4*(30*d*cosh(d*x + c)^7 + 28*d*cosh(d*x +
 c)^5 - 5*d*cosh(d*x + c)^3 - 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 2*d*cosh
(d*x + c)^2 + (45*d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^6 - 45*d*cosh(d*x
+ c)^4 - 48*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c)
^9 + 8*d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 - 8*d*cosh(d*x + c)^3 - 2*d*
cosh(d*x + c))*sinh(d*x + c) + 6*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^3 \mathrm{csch}^4\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*csch(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(98) = 196.
time = 0.42, size = 355, normalized size = 3.41

$$\frac{2\left(\frac{5\left(9a^2be^{(4dx+4c)}+18ab^2e^{(4dx+4c)}+9b^3e^{(4dx+4c)}-6a^3e^{(2dx+2c)}-36a^2be^{(2dx+2c)}-54ab^2e^{(2dx+2c)}-24b^3e^{(2dx+2c)}+2a^3+15a^2b+24ab^2+11b^3\right)}{\left(e^{(2dx+2c)}-1\right)^3}-\frac{45a^2be^{(8dx+8c)}+90ab^2e^{(8dx+8c)}+45b^3e^{(8dx+8c)}+180a^2be^{(6dx+6c)}+450ab^2e^{(6dx+6c)}+240b^3e^{(6dx+6c)}+270a^2be^{(4dx+4c)}+750ab^2e^{(4dx+4c)}+490b^3e^{(4dx+4c)}+180a^2be^{(2dx+2c)}+510ab^2e^{(2dx+2c)}+320b^3e^{(2dx+2c)}+45a^2b+120ab^2+73b^3}{\left(e^{(2dx+2c)}+1\right)^5}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 2/15*(5*(9*a^2*b*e^(4*d*x + 4*c) + 18*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*
x + 4*c) - 6*a^3*e^(2*d*x + 2*c) - 36*a^2*b*e^(2*d*x + 2*c) - 54*a*b^2*e^(2
*d*x + 2*c) - 24*b^3*e^(2*d*x + 2*c) + 2*a^3 + 15*a^2*b + 24*a*b^2 + 11*b^3
)/(e^(2*d*x + 2*c) - 1)^3 - (45*a^2*b*e^(8*d*x + 8*c) + 90*a*b^2*e^(8*d*x +
 8*c) + 45*b^3*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6*c) + 450*a*b^2*e^(6
*d*x + 6*c) + 240*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) + 750*a*b
^2*e^(4*d*x + 4*c) + 490*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) +
510*a*b^2*e^(2*d*x + 2*c) + 320*b^3*e^(2*d*x + 2*c) + 45*a^2*b + 120*a*b^2
+ 73*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 1.60, size = 745, normalized size = 7.16

$$\frac{6\,\left(a^2\,b+2\,a\,b^2+b^3\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{\frac{2\,\left(3\,a^2\,b+9\,a\,b^2+5\,b^3\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a^2\,b+2\,a\,b^2+b^3\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2\,b+9\,a\,b^2+5\,b^3\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(9\,a^2\,b+30\,a\,b^2+25\,b^3\right)}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{\frac{6\,\left(a^2\,b+2\,a\,b^2+b^3\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(a^2\,b+2\,a\,b^2+b^3\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b+9\,a\,b^2+5\,b^3\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(3\,a^2\,b+9\,a\,b^2+5\,b^3\right)}{5\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(9\,a^2\,b+30\,a\,b^2+25\,b^3\right)}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}-\frac{\frac{2\,\left(9\,a^2\,b+30\,a\,b^2+25\,b^3\right)}{15\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a^2\,b+2\,a\,b^2+b^3\right)}{5\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2\,b+9\,a\,b^2+5\,b^3\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}-\frac{4\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{\frac{2\,\left(3\,a^2\,b+9\,a\,b^2+5\,b^3\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2\,b+2\,a\,b^2+b^3\right)}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{6\,\left(a^2\,b+2\,a\,b^2+b^3\right)}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{8\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{3\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/sinh(c + d*x)^4,x)
```

```
[Out] (6*(2*a*b^2 + a^2*b + b^3))/(d*(exp(2*c + 2*d*x) - 1)) - ((2*(9*a*b^2 + 3*a
^2*b + 5*b^3))/(5*d) + (6*exp(6*c + 6*d*x)*(2*a*b^2 + a^2*b + b^3))/(5*d) +
 (6*exp(4*c + 4*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(2*c + 2*d*
x)*(30*a*b^2 + 9*a^2*b + 25*b^3))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c +
4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((6*(2*a*b^2 + a^2*b
+ b^3))/(5*d) + (6*exp(8*c + 8*d*x)*(2*a*b^2 + a^2*b + b^3))/(5*d) + (8*exp
(2*c + 2*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (8*exp(6*c + 6*d*x)*(9*a
*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (4*exp(4*c + 4*d*x)*(30*a*b^2 + 9*a^2*b +
25*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*
d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(30*a*b^2 + 9*a^2
*b + 25*b^3))/(15*d) + (6*exp(4*c + 4*d*x)*(2*a*b^2 + a^2*b + b^3))/(5*d) +
 (4*exp(2*c + 2*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d))/(3*exp(2*c + 2*d*x
) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (4*(3*a*b^2 + 3*a^2*b + a^
3 + b^3))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - ((2*(9*a*b^2 +
3*a^2*b + 5*b^3))/(5*d) + (6*exp(2*c + 2*d*x)*(2*a*b^2 + a^2*b + b^3))/(5*d
))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (6*(2*a*b^2 + a^2*b + b^3)
)/(5*d*(exp(2*c + 2*d*x) + 1)) - (8*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(3*d*(
3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1))
```

3.25 $\int \frac{\sinh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{(3a^2+12ab+8b^2)\,x}{8a^3}-\frac{\sqrt{b}\,(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d}-\frac{(5a+4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d}+\frac{\cosh^3(c$$

[Out] 1/8*(3*a^2+12*a*b+8*b^2)*x/a^3-1/8*(5*a+4*b)*cosh(d*x+c)*sinh(d*x+c)/a^2/d+1/4*cosh(d*x+c)^3*sinh(d*x+c)/a/d-(a+b)^(3/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/a^3/d

Rubi [A]

time = 0.14, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4217, 481, 541, 536, 212, 214}

$$-\frac{\sqrt{b}\,(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d}-\frac{(5a+4b)\sinh(c+dx)\cosh(c+dx)}{8a^2d}+\frac{x(3a^2+12ab+8b^2)}{8a^3}+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((3*a^2 + 12*a*b + 8*b^2)*x)/(8*a^3) - (Sqrt[b]*(a + b)^(3/2)*ArcTanh[(Sqrt
[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a^3*d) - ((5*a + 4*b)*Cosh[c + d*x]*Sinh[
c + d*x])/(8*a^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{\operatorname{Subst}\left(\int \frac{a+b+(4a+3b)x^2}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= -\frac{(5a+4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{(5a+4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{(b(a+b}{}$$

$$= \frac{(3a^2+12ab+8b^2)\,x}{8a^3} - \frac{\sqrt{b}\,(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d} - \frac{(5a+4b)\cosh}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 294 vs. $2(117) = 234$.

time = 1.66, size = 294, normalized size = 2.51

$$-\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^2(c+dx)\left(\sqrt{b}\,(3a^3+34a^2b+64ab^2+32b^3)\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\,\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(\cosh(2c)-\sinh(2c))-\sqrt{b(\cosh(c)-\sinh(c))^4}\left(a^2(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)+\sqrt{b}\,\sqrt{a+b}\,(-2a^2c+12a^2dx+48abdx+32b^2dx-8a(a+b)\sinh(2(c+dx))+a^2\sinh(4(c+dx)))\right)\right)}{64a^3\sqrt{b}\,\sqrt{a+b}\,d\,(a+b\operatorname{sech}^2(c+dx))\,\sqrt{b(\cosh(c)-\sinh(c))^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/64*((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[b]*(3*a^3 + 34
*a^2*b + 64*a*b^2 + 32*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a
+ 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sin
h[c])^4])]*(Cosh[2*c] - Sinh[2*c]) - Sqrt[b*(Cosh[c] - Sinh[c])^4]*(a^2*(3*
a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]] + Sqrt[b]*Sqrt[a + b]
*(-2*a^2*c + 12*a^2*d*x + 48*a*b*d*x + 32*b^2*d*x - 8*a*(a + b)*Sinh[2*(c +
 d*x)] + a^2*Sinh[4*(c + d*x)]))))/(a^3*Sqrt[b]*Sqrt[a + b]*d*(a + b*Sech[c
 + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(103) = 206$.

time = 2.20, size = 357, normalized size = 3.05

method	result
risch	$\frac{3x}{8a}+\frac{3bx}{2a^2}+\frac{x\,b^2}{a^3}+\frac{e^{4dx+4c}}{64ad}-\frac{e^{2dx+2c}}{8ad}-\frac{e^{2dx+2c}b}{8a^2d}+\frac{e^{-2dx-2c}}{8ad}+\frac{e^{-2dx-2c}b}{8a^2d}-\frac{e^{-4dx-4c}}{64ad}+\frac{\sqrt{ab+b^2}}{}$
derivativedivides	$\frac{2b\left(a^2+2ab+b^2\right)\left(\frac{\ln\left(-\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}-\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\,\mathrm{t}\right.}{4\sqrt{b}\,\sqrt{a}}\right.}{a^3}$
default	$\frac{2b\left(a^2+2ab+b^2\right)\left(\frac{\ln\left(-\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}-\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\,\mathrm{t}\right.}{4\sqrt{b}\,\sqrt{a}}\right.}{a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^3*b*(a^2+2*a*b+b^2)*(1/4/b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(
1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))-1/4/b^(1/2)/(a+
b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)
+(a+b)^(1/2)))+1/4/a/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/a/(tanh(1/2*d*x+1/2*c)-1
)^3-1/8*(a+4*b)/a^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/8*(3*a+4*b)/a^2/(tanh(1/2*d
*x+1/2*c)-1)+1/8/a^3*(-3*a^2-12*a*b-8*b^2)*ln(tanh(1/2*d*x+1/2*c)-1)-1/4/a/
```

```
(tanh(1/2*d*x+1/2*c)+1)^4+1/2/a/(tanh(1/2*d*x+1/2*c)+1)^3-1/8*(-a-4*b)/a^2/
(tanh(1/2*d*x+1/2*c)+1)^2-1/8*(3*a+4*b)/a^2/(tanh(1/2*d*x+1/2*c)+1)+1/8*(3*
a^2+12*a*b+8*b^2)/a^3*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(103) = 206$.
time = 0.52, size = 526, normalized size = 4.50

$$\frac{3\,b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,\sqrt{(a+b)b}\,ad}+\frac{3\,(dx+c)}{8\,ad}-\frac{(8\,be^{(-2\,dx-2\,c)}-a)e^{(4\,dx+4\,c)}}{64\,a^2d}-\frac{e^{(2\,dx+2\,c)}}{8\,ad}+\frac{e^{(-2\,dx-2\,c)}}{8\,ad}+\frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{4\,a^2d}-\frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{4\,a^2d}-\frac{(ab+2\,b^2)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}\,a^2d}+\frac{(ab+2\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}\,a^2d}+\frac{(ab+2\,b^2)(dx+c)}{2\,a^3d}+\frac{8\,be^{(-2\,dx-2\,c)}-ae^{(-4\,dx-4\,c)}}{64\,a^2d}+\frac{(a^2b+8\,ab^2+8\,b^3)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,\sqrt{(a+b)b}\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 3/16*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + 3/8*(d*x + c
)/(a*d) - 1/64*(8*b*e^(-2*d*x - 2*c) - a)*e^(4*d*x + 4*c)/(a^2*d) - 1/8*e^(
2*d*x + 2*c)/(a*d) + 1/8*e^(-2*d*x - 2*c)/(a*d) + 1/4*b*log(a*e^(4*d*x + 4*
c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^2*d) - 1/4*b*log(2*(a + 2*b)*e^(-2
*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d) - 1/8*(a*b + 2*b^2)*log((a*e^
(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b +
 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) + 1/8*(a*b + 2*b^2)*log((a*e^(
-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b
+ 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) + 1/2*(a*b + 2*b^2)*(d*x + c)
/(a^3*d) + 1/64*(8*b*e^(-2*d*x - 2*c) - a*e^(-4*d*x - 4*c))/(a^2*d) + 1/16*
(a^2*b + 8*a*b^2 + 8*b^3)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b
)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*
a^3*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(103) = 206$.
time = 0.38, size = 1681, normalized size = 14.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh
(d*x + c)^8 + 8*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b
)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*sinh(d*x + c)
^6 + 8*(7*a^2*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c)^
5 + 2*(35*a^2*cosh(d*x + c)^4 + 4*(3*a^2 + 12*a*b + 8*b^2)*d*x - 60*(a^2 +
a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2
 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c) - 20*(a^2 + a*b)*cosh(d*x + c)^3)*sinh
(d*x + c)^3 + 8*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12
*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c
```

```
)^4 + 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 32*((a + b)*cosh(d*x + c)^4 + 4*(a +
 b)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a + b)*cosh(d*x + c)^2*sinh(d*x + c)
^2 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4)*sqr
t(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3
 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*
x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh
(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x +
c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(
a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh
(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*
b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d
*x + c) + a)) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 + 12*a*b + 8*b^2)*d
*x*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c)^5 + 2*(a^2 + a*b)*cosh(d*x
 + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)^3*sinh
(d*x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*d*cosh(d*x + c)
*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4), 1/64*(a^2*cosh(d*x + c)^8 + 8*a^
2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 8*(3*a^2 + 12*a*b +
 8*b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh
(d*x + c)^2 - 2*a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 - 6
*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cosh(d*x + c)^4 + 4
*(3*a^2 + 12*a*b + 8*b^2)*d*x - 60*(a^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2 + 12*a*b + 8*b^2)*d*x*cosh(d*x +
 c) - 20*(a^2 + a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a^2 + a*b)*cosh(
d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12*(3*a^2 + 12*a*b + 8*b^2)*d*x*cos
h(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c)^4 + 2*a^2 + 2*a*b)*sinh(d*x + c
)^2 - 64*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)^3*sinh(d*x + c)
 + 6*(a + b)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a + b)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a + b)*sinh(d*x + c)^4)*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(
d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)
*sqrt(-a*b - b^2)/(a*b + b^2)) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 +
12*a*b + 8*b^2)*d*x*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c)^5 + 2*(a^
2 + a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cos
h(d*x + c)^3*sinh(d*x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^
3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(103) = 206.
time = 1.40, size = 220, normalized size = 1.88

$$\frac{\frac{8\,(3\,a^2+12\,ab+8\,b^2)(dx+c)}{a^3}+\frac{ae^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}-8\,be^{(2\,dx+2\,c)}}{a^2}-\frac{(18\,a^2e^{(4\,dx+4\,c)}+72\,abe^{(4\,dx+4\,c)}+48\,b^2e^{(4\,dx+4\,c)}-8\,a^2e^{(2\,dx+2\,c)}-8\,abe^{(2\,dx+2\,c)}+a^2)e^{(-4\,dx-4\,c)}}{a^3}-\frac{64\,(a^2b+2\,ab^2+b^3)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^3}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(8*(3*a^2 + 12*a*b + 8*b^2)*(d*x + c)/a^3 + (a*e^(4*d*x + 4*c) - 8*a*e
^(2*d*x + 2*c) - 8*b*e^(2*d*x + 2*c))/a^2 - (18*a^2*e^(4*d*x + 4*c) + 72*a*
b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) - 8*a*b*
e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c)/a^3 - 64*(a^2*b + 2*a*b^2 + b^3)*ar
ctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*
a^3))/d
```

Mupad [B]
time = 2.43, size = 328, normalized size = 2.80

$$\frac{x\,(3\,a^2+12\,a\,b+8\,b^2)}{8\,a^3}-\frac{\mathrm{e}^{-4\,c-4\,d\,x}}{64\,a\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}}{64\,a\,d}+\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,(a+b)}{8\,a^2\,d}-\frac{\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)}{8\,a^2\,d}+\frac{\sqrt{b}\,\ln\left(\frac{4\,b\,(a+b)^3\,(2\,a\,b+a^2+a^2\,\mathrm{e}^{2\,c+2\,d\,x}+8\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}+8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x})}{a^8}-\frac{8\,b^{3/2}\,(a+b)^{7/2}\,(a+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}+4\,b\,\mathrm{e}^{2\,c+2\,d\,x})}{a^8}\right)\,(a+b)^{3/2}}{2\,a^3\,d}-\frac{\sqrt{b}\,\ln\left(\frac{8\,b^{3/2}\,(a+b)^{7/2}\,(a+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}+4\,b\,\mathrm{e}^{2\,c+2\,d\,x})}{a^8}+\frac{4\,b\,(a+b)^3\,(2\,a\,b+a^2+a^2\,\mathrm{e}^{2\,c+2\,d\,x}+8\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}+8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x})}{a^8}\right)\,(a+b)^{3/2}}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (x*(12*a*b + 3*a^2 + 8*b^2))/(8*a^3) - exp(- 4*c - 4*d*x)/(64*a*d) + exp(4*
c + 4*d*x)/(64*a*d) + (exp(- 2*c - 2*d*x)*(a + b))/(8*a^2*d) - (exp(2*c + 2
*d*x)*(a + b))/(8*a^2*d) + (b^(1/2)*log((4*b*(a + b)^3*(2*a*b + a^2 + a^2*e
xp(2*c + 2*d*x) + 8*b^2*exp(2*c + 2*d*x) + 8*a*b*exp(2*c + 2*d*x)))/a^8 - (
8*b^(3/2)*(a + b)^(7/2)*(a + 2*a*exp(2*c + 2*d*x) + 4*b*exp(2*c + 2*d*x)))/
a^8)*(a + b)^(3/2))/(2*a^3*d) - (b^(1/2)*log((8*b^(3/2)*(a + b)^(7/2)*(a +
2*a*exp(2*c + 2*d*x) + 4*b*exp(2*c + 2*d*x)))/a^8 + (4*b*(a + b)^3*(2*a*b +
 a^2 + a^2*exp(2*c + 2*d*x) + 8*b^2*exp(2*c + 2*d*x) + 8*a*b*exp(2*c + 2*d*
x)))/a^8)*(a + b)^(3/2))/(2*a^3*d)
```

3.26 $\int \frac{\sinh^3(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=71

$$\frac{\sqrt{b}\,(a+b)\text{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{a^{5/2}d}-\frac{(a+b)\cosh(c+dx)}{a^2d}+\frac{\cosh^3(c+dx)}{3ad}$$

```
[Out] -(a+b)*cosh(d*x+c)/a^2/d+1/3*cosh(d*x+c)^3/a/d+(a+b)*arctan(cosh(d*x+c)*a^(
1/2)/b^(1/2))*b^(1/2)/a^(5/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4218, 470, 327, 211}

$$\frac{\sqrt{b}\,(a+b)\text{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{a^{5/2}d}-\frac{(a+b)\cosh(c+dx)}{a^2d}+\frac{\cosh^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (Sqrt[b]*(a + b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(a^(5/2)*d) - ((a
 + b)*Cosh[c + d*x])/(a^2*d) + Cosh[c + d*x]^3/(3*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^2(1-x^2)}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= \frac{\cosh^3(c+dx)}{3ad} - \frac{(a+b)\operatorname{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{ad}\\
&= -\frac{(a+b)\cosh(c+dx)}{a^2d} + \frac{\cosh^3(c+dx)}{3ad} + \frac{(b(a+b))\operatorname{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cosh(c
\end{aligned}
$$

$$
= \frac{\sqrt{b}\,(a+b)\tan^{-1}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{a^{5/2}d} - \frac{(a+b)\cosh(c+dx)}{a^2d} + \frac{\cosh^3(c+dx)}{3ad}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.49, size = 372, normalized size = 5.24

$$
\frac{(a+2b+a\cosh(2(c+dx)))\left(3(a^2+8ab+8b^2)\operatorname{ArcTan}\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)+3(a^2+8ab+8b^2)\operatorname{ArcTan}\left(\frac{\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)-3a^2\left(\operatorname{ArcTan}\left(\frac{\sqrt{a}-i\sqrt{a+b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\operatorname{ArcTan}\left(\frac{\sqrt{a}+i\sqrt{a+b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)-6\sqrt{a}\sqrt{b}(3a+4b)\cosh(c+dx)+2a^{3/2}\sqrt{b}\cosh(3(c+dx))\right)}{48a^{5/2}\sqrt{b}\,d\left(b+a\cosh^2(c+dx)\right)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*(3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((Sqrt[a]
- I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c
]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt
[b]] + 3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[
c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*
Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - 3*a^2*(ArcTan[(Sqrt[
a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[(Sqrt[a] + I*Sqrt[a
 + b]*Tanh[(c + d*x)/2])/Sqrt[b]]) - 6*Sqrt[a]*Sqrt[b]*(3*a + 4*b)*Cosh[c +
 d*x] + 2*a^(3/2)*Sqrt[b]*Cosh[3*(c + d*x)]))/(48*a^(5/2)*Sqrt[b]*d*(b + a*
Cosh[c + d*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(61) = 122$.
time = 2.06, size = 170, normalized size = 2.39

method	result
derivativedivides	$\frac{\frac{1}{3a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{2b+a}{2a^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(a+b)b\arctan\left(\frac{2(a+b)\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a-2b}{4\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}-\frac{}{3a(}}{d}$
default	$\frac{\frac{1}{3a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{2b+a}{2a^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(a+b)b\arctan\left(\frac{2(a+b)\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a-2b}{4\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}-\frac{}{3a(}}{d}$
risch	$\frac{e^{3dx+3c}}{24ad}-\frac{3\,e^{dx+c}}{8ad}-\frac{e^{dx+c}b}{2a^{2}d}-\frac{3\,e^{-dx-c}}{8ad}-\frac{e^{-dx-c}b}{2a^{2}d}+\frac{e^{-3dx-3c}}{24ad}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}\,e^{dx+c}}{a}+\right.}{2a^{2}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3/a/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/a/(tanh(1/2*d*x+1/2*c)+1)^2-1/2*(2
*b+a)/a^2/(tanh(1/2*d*x+1/2*c)+1)+(a+b)*b/a^2/(a*b)^(1/2)*arctan(1/4*(2*(a+
b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))-1/3/a/(tanh(1/2*d*x+1/2*c)-1
)^3-1/2/a/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/a^2*(-2*b-a)/(tanh(1/2*d*x+1/2*c)-1
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*(3*a*e^(4*c) + 4*b*e^(4*c))*e^(4*d*x) + 3*(3*a*e^(2*c) + 4*b*e^(2*
c))*e^(2*d*x) - a*e^(6*d*x + 6*c) - a)*e^(-3*d*x - 3*c)/(a^2*d) + 1/8*integ
rate(16*((a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(d*x
))/(a^3*e^(4*d*x + 4*c) + a^3 + 2*(a^3*e^(2*c) + 2*a^2*b*e^(2*c))*e^(2*d*x)
), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(61) = 122$.
time = 0.38, size = 1246, normalized size = 17.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x +
 c)^6 - 3*(3*a + 4*b)*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - 3*a - 4*b)
*sinh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c))*si
nh(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*
(3*a + 4*b)*cosh(d*x + c)^2 - 3*a - 4*b)*sinh(d*x + c)^2 + 12*((a + b)*cosh
(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a + b)*cosh(d*x
+ c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3)*sqrt(-b/a)*log((a*cosh(d*x
+ c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b
)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(
a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x
+ c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x
 + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*
x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2
*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4
*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 6*(a*c
osh(d*x + c)^5 - 2*(3*a + 4*b)*cosh(d*x + c)^3 - (3*a + 4*b)*cosh(d*x + c))
*sinh(d*x + c) + a)/(a^2*d*cosh(d*x + c)^3 + 3*a^2*d*cosh(d*x + c)^2*sinh(d
*x + c) + 3*a^2*d*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*d*sinh(d*x + c)^3), 1
/24*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x + c
)^6 - 3*(3*a + 4*b)*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - 3*a - 4*b)*s
inh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c))*sinh
(d*x + c)^3 - 3*(3*a + 4*b)*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*(3
*a + 4*b)*cosh(d*x + c)^2 - 3*a - 4*b)*sinh(d*x + c)^2 - 24*((a + b)*cosh(d
*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a + b)*cosh(d*x +
c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3)*sqrt(b/a)*arctan(1/2*(a*cosh(
d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4
*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a
)/b) + 24*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2*sinh(d*x + c
) + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3)*sqrt
(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + 6*(a*co
sh(d*x + c)^5 - 2*(3*a + 4*b)*cosh(d*x + c)^3 - (3*a + 4*b)*cosh(d*x + c))*
sinh(d*x + c) + a)/(a^2*d*cosh(d*x + c)^3 + 3*a^2*d*cosh(d*x + c)^2*sinh(d*
x + c) + 3*a^2*d*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*d*sinh(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.84, size = 473, normalized size = 6.66

$$\frac{e^{-3c-3dx}}{24ad}-\frac{\left(2\,\mathrm{atan}\left(\frac{a^6\,e^{dx}\,e^c\left(\frac{4\left(2a^4bd\sqrt{a^2b+2ab^2+b^3}+2a^2b^3d\sqrt{a^2b+2ab^2+b^3}+4a^3b^2d\sqrt{a^2b+2ab^2+b^3}\right)}{a^{11}d^2(a+b)}+\frac{2\left(b^4\sqrt{a^5d^2}+3a^2b^2\sqrt{a^5d^2}+3ab^3\sqrt{a^5d^2}+a^3b\sqrt{a^5d^2}\right)}{a^8d\sqrt{b(a+b)^2}\sqrt{a^5d^2}}\right)\sqrt{a^5d^2}}{4a^2b+8ab^2+4b^3}+\frac{2e^{3c}e^{3dx}\left(b^4\sqrt{a^5d^2}+3a^2b^2\sqrt{a^5d^2}+3ab^3\sqrt{a^5d^2}+a^3b\sqrt{a^5d^2}\right)}{a^2d\sqrt{b(a+b)^2}\left(4a^2b+8ab^2+4b^3\right)}\right)-2\,\mathrm{atan}\left(\frac{e^{dx}\,e^c(a+b)\sqrt{a^5d^2}}{2a^2d\sqrt{b(a+b)^2}}\right)\right)\sqrt{a^2b+2ab^2+b^3}}{2\sqrt{a^5d^2}}+\frac{e^{3c+3dx}}{24ad}-\frac{e^{c+dx}(3a+4b)}{8a^2d}-\frac{e^{-c-dx}(3a+4b)}{8a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] exp(- 3*c - 3*d*x)/(24*a*d) - ((2*atan((a^6*exp(d*x)*exp(c)*((4*(2*a^4*b*d*
(2*a*b^2 + a^2*b + b^3)^(1/2) + 2*a^2*b^3*d*(2*a*b^2 + a^2*b + b^3)^(1/2) +
 4*a^3*b^2*d*(2*a*b^2 + a^2*b + b^3)^(1/2)))/(a^11*d^2*(a + b)) + (2*(b^4*(
a^5*d^2)^(1/2) + 3*a^2*b^2*(a^5*d^2)^(1/2) + 3*a*b^3*(a^5*d^2)^(1/2) + a^3*
b*(a^5*d^2)^(1/2)))/(a^8*d*(b*(a + b)^2)^(1/2)*(a^5*d^2)^(1/2)))*(a^5*d^2)^
(1/2))/(8*a*b^2 + 4*a^2*b + 4*b^3) + (2*exp(3*c)*exp(3*d*x)*(b^4*(a^5*d^2)^
(1/2) + 3*a^2*b^2*(a^5*d^2)^(1/2) + 3*a*b^3*(a^5*d^2)^(1/2) + a^3*b*(a^5*d^
2)^(1/2)))/(a^2*d*(b*(a + b)^2)^(1/2)*(8*a*b^2 + 4*a^2*b + 4*b^3))) - 2*ata
n((exp(d*x)*exp(c)*(a + b)*(a^5*d^2)^(1/2))/(2*a^2*d*(b*(a + b)^2)^(1/2)))
*(2*a*b^2 + a^2*b + b^3)^(1/2))/(2*(a^5*d^2)^(1/2)) + exp(3*c + 3*d*x)/(24*
a*d) - (exp(c + d*x)*(3*a + 4*b))/(8*a^2*d) - (exp(- c - d*x)*(3*a + 4*b))/
(8*a^2*d)
```

3.27 $\int \frac{\sinh^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=75

$$-\frac{(a+2b)x}{2a^2}+\frac{\sqrt{b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2ad}$$

```
[Out] -1/2*(a+2*b)*x/a^2+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d+arctanh(b^(1/2)*tanh(d*x
+c)/(a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4217, 482, 536, 212, 214}

$$\frac{\sqrt{b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d}-\frac{x(a+2b)}{2a^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/2*((a + 2*b)*x)/a^2 + (Sqrt[b]*Sqrt[a + b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x
])/Sqrt[a + b]])/(a^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} - \frac{\operatorname{Subst}\left(\int \frac{a+b+bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{2ad} \\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{(b(a+b))\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{a^2d} - \\
&= -\frac{(a+2b)x}{2a^2} + \frac{\sqrt{b}\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 236 vs. 2(75) = 150.
time = 0.66, size = 236, normalized size = 3.15

$$
\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^2(c+dx)\left(-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}\,d}+\frac{-4(a+2b)x+\frac{(a^2+8ab+8b^2)\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(\cosh(2c)-\sinh(2c))}{\sqrt{a+b}\,d\sqrt{b(\cosh(c)-\sinh(c))^4}}+\frac{2a\cosh(2dx)\sinh(2c)}{d}+\frac{2a\cosh(2c)\sinh(2dx)}{d}}{a^2}\right)}{16\left(a+b\operatorname{sech}^2(c+dx)\right)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(-(ArcTanh[(Sqrt[b]*Tanh[c
 + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)) + (-4*(a + 2*b)*x + ((a^2 +
8*a*b + 8*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d
*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(C
osh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (2*a
*Cosh[2*d*x]*Sinh[2*c])/d + (2*a*Cosh[2*c]*Sinh[2*d*x])/d)/a^2))/(16*(a + b
*Sech[c + d*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(63) = 126$.
time = 1.90, size = 230, normalized size = 3.07

method	result
risch	$-\frac{bx}{a^{2}}-\frac{x}{2a}+\frac{{\mathrm e}^{2dx+2c}}{8ad}-\frac{{\mathrm e}^{-2dx-2c}}{8ad}+\frac{\sqrt{ab+b^{2}}\,\ln\left({\mathrm e}^{2dx+2c}-\frac{-a+2\sqrt{ab+b^{2}}-2b}{a}\right)}{2d\,a^{2}}-\frac{\sqrt{ab+b^{2}}\,\ln\left(\right.}{}$
derivativedivides	$-\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(-2b-a\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^{2}}+\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.}$
default	$-\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(-2b-a\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^{2}}+\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{1}{2a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/a/(tanh(1/2*d*x+1/2*c)+1)+1/2/a^2
*(-2*b-a)*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/a/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/a/(
tanh(1/2*d*x+1/2*c)-1)+1/2*(2*b+a)/a^2*ln(tanh(1/2*d*x+1/2*c)-1)-2*b/a^2*(a
+b)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1
/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*t
anh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(63) = 126$.
time = 0.50, size = 352, normalized size = 4.69

$$-\frac{b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,\sqrt{(a+b)b}\,ad}-\frac{dx+c}{2\,ad}+\frac{e^{(2\,dx+2\,c)}}{8\,ad}-\frac{e^{(-2\,dx-2\,c)}}{8\,ad}-\frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{4\,a^{2}d}+\frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{4\,a^{2}d}+\frac{(ab+2\,b^{2})\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}\,a^{2}d}-\frac{(ab+2\,b^{2})\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) - 1/2*(d*x + c
```

```
)/(a*d) + 1/8*e^(2*d*x + 2*c)/(a*d) - 1/8*e^(-2*d*x - 2*c)/(a*d) - 1/4*b*lo
g(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^2*d) + 1/4*b*log(
2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d) + 1/8*(a*b +
 2*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x +
 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) - 1/8*(a*b +
2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(63) = 126$.
time = 0.44, size = 805, normalized size = 10.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a + 2*b)*d*x*cosh(d*x + c)^2 - a*cosh(d*x + c)^4 - 4*a*cosh(d*x +
 c)*sinh(d*x + c)^3 - a*sinh(d*x + c)^4 + 2*(2*(a + 2*b)*d*x - 3*a*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 - 4*sqrt(a*b + b^2)*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh
(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x
+ c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*
a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*
x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*
x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)
*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a
*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2
*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*(2*(a + 2*b)*d*x*cosh(d*x + c) -
 a*cosh(d*x + c)^3)*sinh(d*x + c) + a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cos
h(d*x + c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2), -1/8*(4*(a + 2*b)*d*x*co
sh(d*x + c)^2 - a*cosh(d*x + c)^4 - 4*a*cosh(d*x + c)*sinh(d*x + c)^3 - a*s
inh(d*x + c)^4 + 2*(2*(a + 2*b)*d*x - 3*a*cosh(d*x + c)^2)*sinh(d*x + c)^2
- 8*sqrt(-a*b - b^2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x +
c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(2*(a +
 2*b)*d*x*cosh(d*x + c) - a*cosh(d*x + c)^3)*sinh(d*x + c) + a)/(a^2*d*cosh
(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c + dx)}{a + b\,\mathrm{sech}^2(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(63) = 126$.
time = 0.74, size = 132, normalized size = 1.76

$$-\frac{\frac{4\,(dx+c)(a+2\,b)}{a^2}-\frac{e^{(2\,dx+2\,c)}}{a}-\frac{\left(2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}-a\right)e^{(-2\,dx-2\,c)}}{a^2}-\frac{8\,(ab+b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*(d*x + c)*(a + 2*b)/a^2 - e^(2*d*x + 2*c)/a - (2*a*e^(2*d*x + 2*c)
+ 4*b*e^(2*d*x + 2*c) - a)*e^(-2*d*x - 2*c)/a^2 - 8*(a*b + b^2)*arctan(1/2*
(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a^2))/d
```

Mupad [B]
time = 2.03, size = 276, normalized size = 3.68

$$\frac{\mathrm{e}^{2\,c+2\,d\,x}}{8\,a\,d}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}}{8\,a\,d}-\frac{x\,(a+2\,b)}{2\,a^2}-\frac{\sqrt{b}\,\ln\left(2\,a\,b+a^2+a^2\,\mathrm{e}^{2\,c+2\,d\,x}+8\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}-2\,a\,\sqrt{b}\,\sqrt{a+b}-8\,b^{3/2}\,\mathrm{e}^{2\,c+2\,d\,x}\,\sqrt{a+b}+8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}-4\,a\,\sqrt{b}\,\mathrm{e}^{2\,c+2\,d\,x}\,\sqrt{a+b}\right)\,\sqrt{a+b}}{2\,a^2\,d}+\frac{\sqrt{b}\,\ln\left(2\,a\,b+a^2+a^2\,\mathrm{e}^{2\,c+2\,d\,x}+8\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}+2\,a\,\sqrt{b}\,\sqrt{a+b}+8\,b^{3/2}\,\mathrm{e}^{2\,c+2\,d\,x}\,\sqrt{a+b}+8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}+4\,a\,\sqrt{b}\,\mathrm{e}^{2\,c+2\,d\,x}\,\sqrt{a+b}\right)\,\sqrt{a+b}}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] exp(2*c + 2*d*x)/(8*a*d) - exp(- 2*c - 2*d*x)/(8*a*d) - (x*(a + 2*b))/(2*a^
2) - (b^(1/2)*log(2*a*b + a^2 + a^2*exp(2*c + 2*d*x) + 8*b^2*exp(2*c + 2*d*
x) - 2*a*b^(1/2)*(a + b)^(1/2) - 8*b^(3/2)*exp(2*c + 2*d*x)*(a + b)^(1/2) +
 8*a*b*exp(2*c + 2*d*x) - 4*a*b^(1/2)*exp(2*c + 2*d*x)*(a + b)^(1/2))*(a +
b)^(1/2))/(2*a^2*d) + (b^(1/2)*log(2*a*b + a^2 + a^2*exp(2*c + 2*d*x) + 8*b
^2*exp(2*c + 2*d*x) + 2*a*b^(1/2)*(a + b)^(1/2) + 8*b^(3/2)*exp(2*c + 2*d*x
)*(a + b)^(1/2) + 8*a*b*exp(2*c + 2*d*x) + 4*a*b^(1/2)*exp(2*c + 2*d*x)*(a
+ b)^(1/2))*(a + b)^(1/2))/(2*a^2*d)
```

3.28 $\int \frac{\sinh(c+dx)}{a+b\,\mathbf{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}d}+\frac{\cosh(c+dx)}{ad}$$

```
[Out] cosh(d*x+c)/a/d-arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4218, 327, 211}

$$\frac{\cosh(c+dx)}{ad}-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(a^(3/2)*d)) + Cosh[c +
 d*x]/(a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\cosh(c+dx)}{ad} - \frac{b\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{ad} \\
&= -\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{a^{3/2}d} + \frac{\cosh(c+dx)}{ad}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.69, size = 328, normalized size = 6.98

$$
\frac{\left(-\frac{(a+4b)\left(\text{ArcTan}\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\,\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\,\sqrt{(\cosh(c)-\sinh(c))^2}\,\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)+\text{ArcTan}\left(\frac{\left(\sqrt{a}+i\sqrt{a+b}\,\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}+i\sqrt{a+b}\,\sqrt{(\cosh(c)-\sinh(c))^2}\,\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)\right)}{\sqrt{b}}+\frac{a\left(\text{ArcTan}\left(\frac{\sqrt{a}-i\sqrt{a+b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\text{ArcTan}\left(\frac{\sqrt{a}+i\sqrt{a+b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{\sqrt{b}}+4\sqrt{a}\,\cosh(c+dx)\right)(a+2b+a\cosh(2(c+dx)))\text{sech}^2(c+dx)}{8a^{3/2}d\,(a+b\text{sech}^2(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((-(((a + 4*b)*(ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2
])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] -
 Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sq
rt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqr
t[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]))/Sqrt[b]) +
(a*(ArcTan[(Sqrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[(S
qrt[a] + I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]]))/Sqrt[b] + 4*Sqrt[a]*Co
sh[c + d*x])*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2)/(8*a^(3/2)*d*
(a + b*Sech[c + d*x]^2))
```

Maple [A]
time = 0.32, size = 45, normalized size = 0.96

method	result
derivativedivides	$-\frac{-\frac{1}{a\,\text{sech}(dx+c)}-\frac{b\arctan\left(\frac{b\,\text{sech}(dx+c)}{\sqrt{ab}}\right)}{a\sqrt{ab}}}{d}$
default	$-\frac{-\frac{1}{a\,\text{sech}(dx+c)}-\frac{b\arctan\left(\frac{b\,\text{sech}(dx+c)}{\sqrt{ab}}\right)}{a\sqrt{ab}}}{d}$

risch	$\frac{e^{dx+c}}{2ad}+\frac{e^{-dx-c}}{2ad}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}\,e^{dx+c}}{a}+1\right)}{2a^2d}-\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}\,e^{dx+c}}{a}+1\right)}{2a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-1/a/sech(d*x+c)-1/a*b/(a*b)^(1/2)*arctan(b*sech(d*x+c)/(a*b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(a*d) - 1/2*integrate(4*(b*e^(3*d*x
 + 3*c) - b*e^(d*x + c))/(a^2*e^(4*d*x + 4*c) + a^2 + 2*(a^2*e^(2*c) + 2*a*b
*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(39) = 78$.
time = 0.36, size = 595, normalized size = 12.66

$$\left[\frac{\sqrt{-\frac{b}{a}}\left(\cosh(dx+c)+\sinh(dx+c)\right)\log\left(\frac{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2(a-2b)\cosh(dx+c)^2+2\left(3a\cosh(dx+c)^2+a-2b\right)\sinh(dx+c)^2+4\left(a\cosh(dx+c)^3+(a-2b)\cosh(dx+c)\right)\sinh(dx+c)-4\left(a\cosh(dx+c)^3+3a\cosh(dx+c)\sinh(dx+c)^2+a\sinh(dx+c)^3+a\cosh(dx+c)+\left(3a\cosh(dx+c)^2+a\right)\sinh(dx+c)\right)\sqrt{-\frac{b}{a}}+a}{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2(a+2b)\cosh(dx+c)^2+2\left(3a\cosh(dx+c)^2+a+2b\right)\sinh(dx+c)^2+4\left(a\cosh(dx+c)^3+(a+2b)\cosh(dx+c)\right)\sinh(dx+c)+a}\right)+\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2+1}{2\left(ad\cosh(dx+c)+ad\sinh(dx+c)\right)},\ \frac{2\sqrt{\frac{b}{a}}\left(\cosh(dx+c)+\sinh(dx+c)\right)\arctan\left(\frac{\left(a\cosh(dx+c)^3+3a\cosh(dx+c)\sinh(dx+c)^2+a\sinh(dx+c)^3+(a+4b)\cosh(dx+c)+\left(3a\cosh(dx+c)^2+a+4b\right)\sinh(dx+c)\right)\sqrt{\frac{b}{a}}}{2b}\right)-2\sqrt{\frac{b}{a}}\left(\cosh(dx+c)+\sinh(dx+c)\right)\arctan\left(\frac{\left(a\cosh(dx+c)+a\sinh(dx+c)\right)\sqrt{\frac{b}{a}}}{2b}\right)+\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2+1}{2\left(ad\cosh(dx+c)+ad\sinh(dx+c)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*(cosh(d*x + c) + sinh(d*x + c))*log((a*cosh(d*x + c)^4 + 4
*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x
 + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x
 + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^3 + 3
*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3
*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 +
 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d
*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d
*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + cosh(d*x + c)^2
+ 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)/(a*d*cosh(d*x + c) +
 a*d*sinh(d*x + c)), 1/2*(2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))*arcta
n(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x +
 c)^3 + (a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x
+ c))*sqrt(b/a)/b) - 2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))*arctan(1/2
*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + cosh(d*x + c)^2 + 2*cos
```

```
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)/(a*d*cosh(d*x + c) + a*d*si
nh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sinh(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 0.14, size = 42, normalized size = 0.89

$$\frac{\cosh(c+d\,x)}{a\,d}-\frac{b\,\mathrm{atan}\left(\frac{a\cosh(c+d\,x)}{\sqrt{a\,b}}\right)}{a\,d\,\sqrt{a\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] cosh(c + d*x)/(a*d) - (b*atan((a*cosh(c + d*x))/(a*b)^(1/2)))/(a*d*(a*b)^(1
/2))
```

3.29 $\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}\,(a+b)d}-\frac{\tanh^{-1}(\cosh(c+dx))}{(a+b)d}$$

```
[Out] -arctanh(cosh(d*x+c))/(a+b)/d+arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/(
a+b)/d/a^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4218, 492, 212, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}\,d(a+b)}-\frac{\tanh^{-1}(\cosh(c+dx))}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(Sqrt[a]*(a + b)*d) - Arc
Tanh[Cosh[c + d*x]]/((a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)(b+ax^2)}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{(a+b)d} + \frac{b\operatorname{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{(a+b)d}\\ &= \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}\,(a+b)d} - \frac{\tanh^{-1}(\cosh(c+dx))}{(a+b)d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.64, size = 232, normalized size = 4.22

$$\frac{\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)}{\sqrt{a}}+\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)}{\sqrt{a}}-\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((Sqrt[b]*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sin
h[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[
c])^2]*Tanh[(d*x)/2]))/Sqrt[b]])/Sqrt[a] + (Sqrt[b]*ArcTan[((Sqrt[a] + I*Sq
rt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqr
t[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]])/
Sqrt[a] - Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]])/((a + b)*d)
```

Maple [A]
time = 1.68, size = 65, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a+b}+\frac{b\arctan\left(\frac{2(a+b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a-2b}{4\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}}{d}$

default	$\dfrac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a+b}+\frac{b\arctan\left(\frac{2(a+b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a-2b}{4\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}}{d}$
risch	$-\frac{\ln(e^{dx+c}+1)}{d(a+b)}+\frac{\ln(e^{dx+c}-1)}{d(a+b)}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}\,e^{dx+c}}{a}+1\right)}{2a(a+b)d}-\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-a\ldots}}{}\right.}{2a(a+b)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a+b)*ln(tanh(1/2*d*x+1/2*c))+b/(a+b)/(a*b)^(1/2)*arctan(1/4*(2*(a+b
)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d + b*d) + log((e^(d*x + c) - 1)*e^(-c))/
(a*d + b*d) + 2*integrate((b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + a*b +
(a^2*e^(4*c) + a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) + 3*a*b*e^(2*c) + 2*
b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(47) = 94$.
time = 0.42, size = 533, normalized size = 9.69

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2(a-2b)\cosh(dx+c)^2+2\left(3a\cosh(dx+c)^2+a-2b\right)\sinh(dx+c)^2+4\left(a\cosh(dx+c)^3+(a-2b)\cosh(dx+c)\right)\sinh(dx+c)+4\left(a\cosh(dx+c)^3+3a\cosh(dx+c)\sinh(dx+c)^2+a\sinh(dx+c)^3+a\cosh(dx+c)+\left(3a\cosh(dx+c)^2+a\right)\sinh(dx+c)\right)\sqrt{-\frac{b}{a}}+a}{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2(a+2b)\cosh(dx+c)^2+2\left(3a\cosh(dx+c)^2+a+2b\right)\sinh(dx+c)^2+4\left(a\cosh(dx+c)^3+(a+2b)\cosh(dx+c)\right)\sinh(dx+c)+a}\right)-2\log\left(\cosh(dx+c)+\sinh(dx+c)+1\right)+2\log\left(\cosh(dx+c)+\sinh(dx+c)-1\right)}{2(a+b)d}, -\frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(a\cosh(dx+c)^3+3a\cosh(dx+c)\sinh(dx+c)^2+a\sinh(dx+c)^3+(a+4b)\cosh(dx+c)+\left(3a\cosh(dx+c)^2+a+4b\right)\sinh(dx+c)\right)\sqrt{\frac{b}{a}}}{2b}\right)-\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(a\cosh(dx+c)+a\sinh(dx+c)\right)\sqrt{\frac{b}{a}}}{2b}\right)+\log\left(\cosh(dx+c)+\sinh(dx+c)+1\right)-\log\left(\cosh(dx+c)+\sinh(dx+c)-1\right)}{(a+b)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2
 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c
))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2
 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x
 + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)
^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)
^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x +
 c))*sinh(d*x + c) + a)) - 2*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*log
```

```
(cosh(d*x + c) + sinh(d*x + c) - 1))/((a + b)*d), -(sqrt(b/a)*arctan(1/2*(a
*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 +
(a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sq
rt(b/a)/b) - sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(
b/a)/b) + log(cosh(d*x + c) + sinh(d*x + c) + 1) - log(cosh(d*x + c) + sinh
(d*x + c) - 1))/((a + b)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 2.18, size = 616, normalized size = 11.20

$$-\frac{2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^c\,\left(b^4\,\sqrt{-a^2\,d^2-2\,a\,b\,d^2-b^2\,d^2}+16\,a^2\,b^2\,\sqrt{-a^2\,d^2-2\,a\,b\,d^2-b^2\,d^2}+8\,a\,b^3\,\sqrt{-a^2\,d^2-2\,a\,b\,d^2-b^2\,d^2}\right)}{16\,d\,a^3\,b^2+24\,d\,a^2\,b^3+9\,d\,a\,b^4+d\,b^5}\right)}{\sqrt{-a^2\,d^2-2\,a\,b\,d^2-b^2\,d^2}}-\frac{\sqrt{b}\,\left(2\,\mathrm{atan}\left(\frac{\left(e^{d\,x}\,e^c\,\left(\frac{64\,\left(2\,b^{7/2}\,d+8\,a^2\,b^{3/2}\,d+10\,a\,b^{5/2}\,d\right)}{a^5\,\left(a+b\right)\,\sqrt{a\,d^2\,{\left(a+b\right)}^2}\,\sqrt{a^3\,d^2+2\,a^2\,b\,d^2+a\,b^2\,d^2}}+\frac{32\,\left(b^2\,\sqrt{a^3\,d^2+2\,a^2\,b\,d^2+a\,b^2\,d^2}+4\,a\,b\,\sqrt{a^3\,d^2+2\,a^2\,b\,d^2+a\,b^2\,d^2}\right)}{a^5\,\sqrt{b}\,d\,{\left(a+b\right)}^2\,\sqrt{a^3\,d^2+2\,a^2\,b\,d^2+a\,b^2\,d^2}}\right)+\frac{32\,e^{3\,c}\,e^{3\,d\,x}\,\left(b^2\,\sqrt{a^3\,d^2+2\,a^2\,b\,d^2+a\,b^2\,d^2}+4\,a\,b\,\sqrt{a^3\,d^2+2\,a^2\,b\,d^2+a\,b^2\,d^2}\right)}{a^5\,\sqrt{b}\,d\,{\left(a+b\right)}^2\,\sqrt{a^3\,d^2+2\,a^2\,b\,d^2+a\,b^2\,d^2}}\right)\,\left(a^6\,\sqrt{a^3\,d^2+2\,a^2\,b\,d^2+a\,b^2\,d^2}+a^5\,b\,\sqrt{a^3\,d^2+2\,a^2\,b\,d^2+a\,b^2\,d^2}\right)}{64\,b^2+256\,a\,b}\right)-2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^c\,\sqrt{a\,d^2\,{\left(a+b\right)}^2}}{2\,\sqrt{b}\,d\,\left(a+b\right)}\right)\right)}{2\,\sqrt{a^3\,d^2+2\,a^2\,b\,d^2+a\,b^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] - (2*atan((exp(d*x)*exp(c)*(b^4*(- a^2*d^2 - b^2*d^2 - 2*a*b*d^2)^(1/2) + 1
6*a^2*b^2*(- a^2*d^2 - b^2*d^2 - 2*a*b*d^2)^(1/2) + 8*a*b^3*(- a^2*d^2 - b^
2*d^2 - 2*a*b*d^2)^(1/2)))/(b^5*d + 24*a^2*b^3*d + 16*a^3*b^2*d + 9*a*b^4*d
)))/(- a^2*d^2 - b^2*d^2 - 2*a*b*d^2)^(1/2) - (b^(1/2)*(2*atan(((exp(d*x)*e
xp(c)*((64*(2*b^(7/2)*d + 8*a^2*b^(3/2)*d + 10*a*b^(5/2)*d))/(a^5*(a + b)*(
a*d^2*(a + b)^2)^(1/2)*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^(1/2)) + (32*(b^
```

```
2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^(1/2) + 4*a*b*(a^3*d^2 + a*b^2*d^2 +
2*a^2*b*d^2)^(1/2)))/(a^5*b^(1/2)*d*(a + b)^2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*
b*d^2)^(1/2))) + (32*exp(3*c)*exp(3*d*x)*(b^2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*
b*d^2)^(1/2) + 4*a*b*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^(1/2)))/(a^5*b^(1/
2)*d*(a + b)^2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^(1/2)))*(a^6*(a^3*d^2 +
a*b^2*d^2 + 2*a^2*b*d^2)^(1/2) + a^5*b*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^
(1/2)))/(256*a*b + 64*b^2)) - 2*atan((exp(d*x)*exp(c)*(a*d^2*(a + b)^2)^(1/
2))/(2*b^(1/2)*d*(a + b)))))/(2*(a^3*d^2 + a*b^2*d^2 + 2*a^2*b*d^2)^(1/2))
```

3.30 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d}-\frac{\coth(c+dx)}{(a+b)d}$$

```
[Out] -coth(d*x+c)/(a+b)/d+arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/(a+b)
^(3/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4217, 331, 214}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}-\frac{\coth(c+dx)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/((a + b)^(3/2)*d) -
Coth[c + d*x]/((a + b)*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
```

```
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{\coth(c+dx)}{(a+b)d} + \frac{b\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)d} \\
&= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} - \frac{\coth(c+dx)}{(a+b)d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(53) = 106.
time = 0.53, size = 179, normalized size = 3.38

$$
\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^2(c+dx)\left(b\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(\cosh(2c)-\sinh(2c))+\sqrt{a+b}\operatorname{csch}(c)\operatorname{csch}(c+dx)\sqrt{b(\cosh(c)-\sinh(c))^4}\sinh(dx)\right)}{2(a+b)^{3/2}d\left(a+b\operatorname{sech}^2(c+dx)\right)\sqrt{b(\cosh(c)-\sinh(c))^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(b*ArcTanh[(Sech[d*x]*(Cos
h[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a +
b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]) + Sqrt[a + b]*Cs
ch[c]*Csch[c + d*x]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*Sinh[d*x]))/(2*(a + b)^(3
/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(45) = 90.
time = 1.75, size = 145, normalized size = 2.74

method	result
risch	$-\frac{2}{d(a+b)\left(e^{2dx+2c}-1\right)}+\frac{\sqrt{b(a+b)}\ln\left(e^{2dx+2c}-\frac{2\sqrt{b(a+b)}-a-2b}{a}\right)}{2(a+b)^2 d}-\frac{\sqrt{b(a+b)}\ln\left(e^{2dx+2c}+\frac{2 \cdots}{\cdots}\right)}{2(a+b)^2 \cdots}$
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}-\frac{1}{2(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\left(-\frac{\ln\left(\sqrt{a+b}\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\sqrt{a+b}}+\frac{\ln\left(\sqrt{a \cdots}\right.}{\cdots}\right.}{a+b}}{d}$

default	$\dfrac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}-\frac{1}{2(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\left(-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a\right.}}{}\right.}{a+b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)*tanh(1/2*d*x+1/2*c)-1/2/(a+b)/tanh(1/2*d*x+1/2*c)-2*b/(a+b)
*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*
d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh
(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(45) = 90$.
time = 0.49, size = 100, normalized size = 1.89

$$-\frac{b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,\sqrt{(a+b)b}\,(a+b)d}+\frac{2}{((a+b)e^{(-2\,dx-2\,c)}-a-b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a + b)*d) + 2/(((a
 + b)*e^(-2*d*x - 2*c) - a - b)*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(45) = 90$.
time = 0.38, size = 588, normalized size = 11.09

$$\left[\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1\right)\sqrt{\frac{b}{a+b}}\log\left(\frac{a^2\cosh(dx+c)^4+4a^2\cosh(dx+c)\sinh(dx+c)^3+a^2\sinh(dx+c)^4+2(a^2+2ab)\cosh(dx+c)^2+2\left(3a^2\cosh(dx+c)^2+a^2+2ab\right)\sinh(dx+c)^2+a^2+8ab+8b^2+4\left(a^2\cosh(dx+c)^3+(a^2+2ab)\cosh(dx+c)\right)\sinh(dx+c)-4\left((a^2+ab)\cosh(dx+c)^2+2(a^2+ab)\cosh(dx+c)\sinh(dx+c)+(a^2+ab)\sinh(dx+c)^2+a^2+3ab+2b^2\right)\sqrt{\frac{b}{a+b}}}{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2(a+2b)\cosh(dx+c)^2+2\left(3a\cosh(dx+c)^2+a+2b\right)\sinh(dx+c)^2+4\left(a\cosh(dx+c)^3+(a+2b)\cosh(dx+c)\right)\sinh(dx+c)+a}\right)-4}{2\left((a+b)d\cosh(dx+c)^2+2(a+b)d\cosh(dx+c)\sinh(dx+c)+(a+b)d\sinh(dx+c)^2-(a+b)d\right)},\ \frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1\right)\sqrt{-\frac{b}{a+b}}\arctan\left(\frac{\left(a\cosh(dx+c)^2+2a\cosh(dx+c)\sinh(dx+c)+a\sinh(dx+c)^2+a+2b\right)\sqrt{-\frac{b}{a+b}}}{2b}\right)-2}{(a+b)d\cosh(dx+c)^2+2(a+b)d\cosh(dx+c)\sinh(dx+c)+(a+b)d\sinh(dx+c)^2-(a+b)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
1)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*c
osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^
2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 +
a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a
*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c
```

```
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*c
osh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4)/((a + b)
*d*cosh(d*x + c)^2 + 2*(a + b)*d*cosh(d*x + c)*sinh(d*x + c) + (a + b)*d*si
nh(d*x + c)^2 - (a + b)*d), ((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 - 1)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 +
2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a +
 b))/b) - 2)/((a + b)*d*cosh(d*x + c)^2 + 2*(a + b)*d*cosh(d*x + c)*sinh(d*
x + c) + (a + b)*d*sinh(d*x + c)^2 - (a + b)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

Giac [A]
time = 0.61, size = 75, normalized size = 1.42

$$\frac{\frac{b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,(a+b)}-\frac{2}{(a+b)\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b -
b^2)*(a + b)) - 2/((a + b)*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [B]
time = 2.47, size = 847, normalized size = 15.98

$$-\frac{2}{\left(e^{2c+2dx}-1\right)(ad+bd)}-\frac{\sqrt{b}\,\mathrm{atan}\left([illegible]\right)}{\sqrt{-a^3d^2-3a^2bd^2-3ab^2d^2-b^3d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] - 2/((exp(2*c + 2*d*x) - 1)*(a*d + b*d)) - (b^(1/2)*atan(((exp(2*c)*exp(2*d
*x)*((2*(8*b^(5/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)
+ 8*a*b^(3/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + a^2
```

```
*b^(1/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))*(8*a*b +
a^2 + 8*b^2))/(a^5*d*(a + b)^3*(2*a*b + a^2 + b^2)*(- a^3*d^2 - b^3*d^2 - 3
*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a + 4*b)*(8*b^4*d + 16*a^2
*b^2*d + 20*a*b^3*d + 4*a^3*b*d))/(a^5*(a + b)*(-d^2*(a + b)^3)^(1/2)*(2*a*
b + a^2 + b^2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))) +
(2*(2*a*b^(3/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + a
^2*b^(1/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))*(8*a*b
+ a^2 + 8*b^2))/(a^5*d*(a + b)^3*(2*a*b + a^2 + b^2)*(- a^3*d^2 - b^3*d^2 -
 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a + 4*b)*(4*a^2*b^2*d +
2*a*b^3*d + 2*a^3*b*d))/(a^5*(a + b)*(-d^2*(a + b)^3)^(1/2)*(2*a*b + a^2 +
b^2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)))*(a^5*(- a^3*
d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 3*a^4*b*(- a^3*d^2 - b^3
*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + a^2*b^3*(- a^3*d^2 - b^3*d^2 - 3*
a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 3*a^3*b^2*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d
^2 - 3*a^2*b*d^2)^(1/2)))/(4*b)))/(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^
2*b*d^2)^(1/2)
```

3.31 $\int \frac{\text{csch}^3(c+dx)}{a+b\text{sech}^2(c+dx)} \, dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{a}\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{(a+b)^2 d}+\frac{(a-b)\tanh^{-1}(\cosh(c+dx))}{2(a+b)^2 d}-\frac{\coth(c+dx)\text{csch}(c+dx)}{2(a+b)d}$$

```
[Out] 1/2*(a-b)*arctanh(cosh(d*x+c))/(a+b)^2/d-1/2*coth(d*x+c)*csch(d*x+c)/(a+b)/
d-arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*a^(1/2)*b^(1/2)/(a+b)^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4218, 482, 536, 212, 211}

$$-\frac{\sqrt{a}\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{d(a+b)^2}+\frac{(a-b)\tanh^{-1}(\cosh(c+dx))}{2d(a+b)^2}-\frac{\coth(c+dx)\text{csch}(c+dx)}{2d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/((a + b)^2*d))
+ ((a - b)*ArcTanh[Cosh[c + d*x]])/(2*(a + b)^2*d) - (Coth[c + d*x]*Csch[c
+ d*x])/(2*(a + b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2\left(b+ax^2\right)}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2(a+b)d} - \frac{\text{Subst}\left(\int \frac{b-ax^2}{\left(1-x^2\right)\left(b+ax^2\right)}\,dx, x, \cosh(c+dx)\right)}{2(a+b)d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2(a+b)d} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{2(a+b)^2d} - \frac{(ab)}{}$$

$$= -\frac{\sqrt{a}\,\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{(a+b)^2d} + \frac{(a-b)\tanh^{-1}(\cosh(c+dx))}{2(a+b)^2d} - \frac{\coth(c+}{2(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.33, size = 338, normalized size = 3.89

$$-\frac{(a+2b+a\cosh(2(c+dx)))\left(8\sqrt{a}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\,\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\,\sqrt{(\cosh(c)-\sinh(c))^2}\,\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)+8\sqrt{a}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\left(\sqrt{a}+i\sqrt{a+b}\,\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}+i\sqrt{a+b}\,\sqrt{(\cosh(c)-\sinh(c))^2}\,\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)+(a+b)\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)-4a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+4b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+4a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)-4b\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+(a+b)\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)\right)\operatorname{sech}^2(c+dx)}{16(a+b)^2d\left(a+b\operatorname{sech}^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/16*((a + 2*b + a*Cosh[2*(c + d*x)])*(8*Sqrt[a]*Sqrt[b]*ArcTan[((Sqrt[a]
- I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c
```

```
]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt
[b]] + 8*Sqrt[a]*Sqrt[b]*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - S
inh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(
Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + (a + b)*Csch[(c + d*x)/2]^
2 - 4*a*Log[Cosh[(c + d*x)/2]] + 4*b*Log[Cosh[(c + d*x)/2]] + 4*a*Log[Sinh[
(c + d*x)/2]] - 4*b*Log[Sinh[(c + d*x)/2]] + (a + b)*Sech[(c + d*x)/2]^2)*S
ech[c + d*x]^2)/((a + b)^2*d*(a + b*Sech[c + d*x]^2))
```

Maple [A]
time = 2.17, size = 111, normalized size = 1.28

method	result
derivativedivides	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a+8b}-\frac{1}{8(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a+2b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2}-\frac{ab\arctan\left(\frac{2(a+b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a-2b}{4\sqrt{ab}}\right)}{(a+b)^2\sqrt{ab}}}{d}$
default	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a+8b}-\frac{1}{8(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a+2b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2}-\frac{ab\arctan\left(\frac{2(a+b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a-2b}{4\sqrt{ab}}\right)}{(a+b)^2\sqrt{ab}}}{d}$
risch	$-\frac{e^{dx+c}\left(1+e^{2dx+2c}\right)}{d(a+b)\left(e^{2dx+2c}-1\right)^2}+\frac{\ln\left(e^{dx+c}+1\right)a}{2d(a^2+2ab+b^2)}-\frac{\ln\left(e^{dx+c}+1\right)b}{2d(a^2+2ab+b^2)}-\frac{\ln\left(e^{dx+c}-1\right)a}{2d(a^2+2ab+b^2)}+\frac{\ln\left(e^{dx+c}-1\right)b}{2d(a^2+2ab+b^2)}+\frac{\sqrt{-ab}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/(a+b)-1/8/(a+b)/tanh(1/2*d*x+1/2*c)^2+1/4/(a
+b)^2*(-2*a+2*b)*ln(tanh(1/2*d*x+1/2*c))-a*b/(a+b)^2/(a*b)^(1/2)*arctan(1/4
*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(a - b)*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d + 2*a*b*d + b^2*d) - 1/2*(
a - b)*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d + 2*a*b*d + b^2*d) - (e^(3*d*x
+ 3*c) + e^(d*x + c))/(a*d + b*d + (a*d*e^(4*c) + b*d*e^(4*c))*e^(4*d*x) -
2*(a*d*e^(2*c) + b*d*e^(2*c))*e^(2*d*x)) - 8*integrate(1/4*(a*b*e^(3*d*x +
3*c) - a*b*e^(d*x + c))/(a^3 + 2*a^2*b + a*b^2 + (a^3*e^(4*c) + 2*a^2*b*e^(
4*c) + a*b^2*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) + 4*a^2*b*e^(2*c) + 5*a*b^
2*e^(2*c) + 2*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(75) = 150$.
time = 0.41, size = 1881, normalized size = 21.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a + b)*cosh(d*x + c)^3 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2
+ 2*(a + b)*sinh(d*x + c)^3 - (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x +
 c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cos
h(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(
-a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d
*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)
*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x
 + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c
)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b) + a
)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^
4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*
x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) +
a)) + 2*(a + b)*cosh(d*x + c) - ((a - b)*cosh(d*x + c)^4 + 4*(a - b)*cosh(d
*x + c)*sinh(d*x + c)^3 + (a - b)*sinh(d*x + c)^4 - 2*(a - b)*cosh(d*x + c)
^2 + 2*(3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 + 4*((a - b)*cos
h(d*x + c)^3 - (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*log(cosh(d*x +
 c) + sinh(d*x + c) + 1) + ((a - b)*cosh(d*x + c)^4 + 4*(a - b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a - b)*sinh(d*x + c)^4 - 2*(a - b)*cosh(d*x + c)^2 +
2*(3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 + 4*((a - b)*cosh(d*x
 + c)^3 - (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*log(cosh(d*x + c) +
 sinh(d*x + c) - 1) + 2*(3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))/
((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^4 - 2*(a^2 + 2*a*
b + b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 - (
a^2 + 2*a*b + b^2)*d)*sinh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d + 4*((a^2 + 2
*a*b + b^2)*d*cosh(d*x + c)^3 - (a^2 + 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d
*x + c)), -1/2*(2*(a + b)*cosh(d*x + c)^3 + 6*(a + b)*cosh(d*x + c)*sinh(d*
x + c)^2 + 2*(a + b)*sinh(d*x + c)^3 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)
*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c
)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c)
 + 1)*sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + sinh(d*x + c))/b) - 2
*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(
3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x +
c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(a*b)*arctan(1/2*(a*cosh(d*x +
 c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*b)*c
osh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(a*b)/(a*
```

```
b)) + 2*(a + b)*cosh(d*x + c) - ((a - b)*cosh(d*x + c)^4 + 4*(a - b)*cosh(d
*x + c)*sinh(d*x + c)^3 + (a - b)*sinh(d*x + c)^4 - 2*(a - b)*cosh(d*x + c)
^2 + 2*(3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 + 4*((a - b)*cos
h(d*x + c)^3 - (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*log(cosh(d*x +
 c) + sinh(d*x + c) + 1) + ((a - b)*cosh(d*x + c)^4 + 4*(a - b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a - b)*sinh(d*x + c)^4 - 2*(a - b)*cosh(d*x + c)^2 +
2*(3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 + 4*((a - b)*cosh(d*x
 + c)^3 - (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*log(cosh(d*x + c) +
 sinh(d*x + c) - 1) + 2*(3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))/
((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^4 - 2*(a^2 + 2*a*
b + b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 - (
a^2 + 2*a*b + b^2)*d)*sinh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d + 4*((a^2 + 2
*a*b + b^2)*d*cosh(d*x + c)^3 - (a^2 + 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d
*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 3.07, size = 1586, normalized size = 18.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] ((a*b)^(1/2)*(2*atan(((exp(d*x)*exp(c)*((64*(2*b^5*d*(a*b)^(1/2) + 2*a*b^4*
d*(a*b)^(1/2) + 2*a^4*b*d*(a*b)^(1/2) + 2*a^3*b^2*d*(a*b)^(1/2)))/(a^4*(a +
 b)^3*(d^2*(a + b)^4)^(1/2)*(2*a*b + a^2 + b^2)*(a^4*d^2 + b^4*d^2 + 4*a*b^
3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)) + (32*(a*b^3*(a^4*d^2 + b^4*d^2
 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + a^3*b*(a^4*d^2 + b^4*
d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) - a^2*b^2*(a^4*d^2 +
 b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)))/(a^3*d*(a*b)^
(1/2)*(a + b)^5*(2*a*b + a^2 + b^2)*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^
3*b*d^2 + 6*a^2*b^2*d^2)^(1/2))) + (32*exp(3*c)*exp(3*d*x)*(a*b^3*(a^4*d^2
+ b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + a^3*b*(a^4*d
^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) - a^2*b^2*(
a^4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)))/(a^3
*d*(a*b)^(1/2)*(a + b)^5*(2*a*b + a^2 + b^2)*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d
^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)))*(a^8*(a^4*d^2 + b^4*d^2 + 4*a*b^3
*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + 5*a^7*b*(a^4*d^2 + b^4*d^2 + 4*
a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + a^3*b^5*(a^4*d^2 + b^4*d^2
 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + 5*a^4*b^4*(a^4*d^2 +
b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + 10*a^5*b^3*(a^
4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + 10*a^6
*b^2*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d^2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2))
)/(64*a^2*b - 64*a*b^2 + 64*b^3)) - 2*atan((a*exp(d*x)*exp(c)*(d^2*(a + b)^
4)^(1/2))/(2*d*(a*b)^(1/2)*(a + b)^2))))/(2*(a^4*d^2 + b^4*d^2 + 4*a*b^3*d^
2 + 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)) - exp(c + d*x)/((exp(2*c + 2*d*x) -
 1)*(a*d + b*d)) - (atan((exp(d*x)*exp(c)*(b^7*(- a^4*d^2 - b^4*d^2 - 4*a*b
^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/2) - 3*a*b^6*(- a^4*d^2 - b^4*d^2
- 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/2) + 5*a^2*b^5*(- a^4*d^2 -
 b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/2) - 5*a^3*b^4*(-
a^4*d^2 - b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/2) + 3*a^
4*b^3*(- a^4*d^2 - b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/
2) - a^5*b^2*(- a^4*d^2 - b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d
^2)^(1/2)))/(b^8*d*(a^2 - 2*a*b + b^2)^(1/2) + 2*a^3*b^5*d*(a^2 - 2*a*b + b
^2)^(1/2) + a^6*b^2*d*(a^2 - 2*a*b + b^2)^(1/2)))*(a^2 - 2*a*b + b^2)^(1/2)
)/(- a^4*d^2 - b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 - 6*a^2*b^2*d^2)^(1/2) -
 (2*exp(c + d*x))/((a*d + b*d)*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.32 $\int \frac{\mathrm{csch}^4(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=75

$$-\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d}+\frac{a\coth(c+dx)}{(a+b)^2d}-\frac{\coth^3(c+dx)}{3(a+b)d}$$

```
[Out] a*coth(d*x+c)/(a+b)^2/d-1/3*coth(d*x+c)^3/(a+b)/d-a*arctanh(b^(1/2)*tanh(d*
x+c)/(a+b)^(1/2))*b^(1/2)/(a+b)^(5/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4217, 464, 331, 214}

$$-\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}-\frac{\coth^3(c+dx)}{3d(a+b)}+\frac{a\coth(c+dx)}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/((a + b)^(5/2)*d
)) + (a*Coth[c + d*x])/((a + b)^2*d) - Coth[c + d*x]^3/(3*(a + b)*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= -\frac{\coth^3(c+dx)}{3(a+b)d} - \frac{a\operatorname{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{(a+b)d}\\ &= \frac{a\coth(c+dx)}{(a+b)^2d} - \frac{\coth^3(c+dx)}{3(a+b)d} - \frac{(ab)\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{(a+b)^2d}\\ &= -\frac{a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d} + \frac{a\coth(c+dx)}{(a+b)^2d} - \frac{\coth^3(c+dx)}{3(a+b)d}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 216 vs. 2(75) = 150.
time = 1.40, size = 216, normalized size = 2.88

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^2(c+dx)\left(3ab\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(-\cosh(2c)+\sinh(2c))+\frac{1}{4}\sqrt{a+b}\operatorname{csch}(c)\operatorname{csch}^3(c+dx)\sqrt{b(\cosh(c)-\sinh(c))^4}\,(6a\sinh(dx)-3b\sinh(2c+dx)+(-2a+b)\sinh(2c+3dx))\right)}{6(a+b)^{5/2}d\left(a+b\operatorname{sech}^2(c+dx)\right)\sqrt{b(\cosh(c)-\sinh(c))^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(3*a*b*ArcTanh[(Sech[d*x]*
(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + (Sqrt[a +
 b]*Csch[c]*Csch[c + d*x]^3*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(6*a*Sinh[d*x] -
3*b*Sinh[2*c + d*x] + (-2*a + b)*Sinh[2*c + 3*d*x]))/4))/(6*(a + b)^(5/2)*d
*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(65) = 130.
time = 1.86, size = 215, normalized size = 2.87

method	result
risch	$-\frac{2(3b\,\mathrm{e}^{4dx+4c}+6a\,\mathrm{e}^{2dx+2c}-2a+b)}{3d(a+b)^2(\mathrm{e}^{2dx+2c}-1)^3}+\frac{\sqrt{b(a+b)}\,a\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{b(a+b)}+a+2b}{a}\right)}{2(a+b)^3 d}-\frac{\sqrt{b(a+b)}\,a}{}$
derivativedivides	$\frac{-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)^2}+2ab\left(\frac{\ln\left(-\sqrt{a+b}\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}\right.\right.}{4\sqrt{b}\sqrt{a+b}}\right.}{}$
default	$\frac{-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)^2}+2ab\left(\frac{\ln\left(-\sqrt{a+b}\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}\right.\right.}{4\sqrt{b}\sqrt{a+b}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/(a+b)^2*(1/3*a*tanh(1/2*d*x+1/2*c)^3+1/3*b*tanh(1/2*d*x+1/2*c)^3-
3*a*tanh(1/2*d*x+1/2*c)+b*tanh(1/2*d*x+1/2*c))+2*a*b/(a+b)^2*(1/4/b^(1/2)/(
a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1
/2)-(a+b)^(1/2))-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)
^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)))-1/24/(a+b)/tanh(1/2*d*x+1/2*
c)^3-1/8/(a+b)^2*(-3*a+b)/tanh(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(65) = 130$.
time = 0.51, size = 195, normalized size = 2.60

$$\frac{ab\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,(a^2+2\,ab+b^2)\sqrt{(a+b)b}\,d}-\frac{2\,\left(6\,ae^{(-2\,dx-2\,c)}+3\,be^{(-4\,dx-4\,c)}-2\,a+b\right)}{3\,(a^2+2\,ab+b^2-3\,(a^2+2\,ab+b^2)e^{(-2\,dx-2\,c)}+3\,(a^2+2\,ab+b^2)e^{(-4\,dx-4\,c)}-(a^2+2\,ab+b^2)e^{(-6\,dx-6\,c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
 - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b
)*d) - 2/3*(6*a*e^(-2*d*x - 2*c) + 3*b*e^(-4*d*x - 4*c) - 2*a + b)/((a^2 +
2*a*b + b^2 - 3*(a^2 + 2*a*b + b^2)*e^(-2*d*x - 2*c) + 3*(a^2 + 2*a*b + b^2
)*e^(-4*d*x - 4*c) - (a^2 + 2*a*b + b^2)*e^(-6*d*x - 6*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(65) = 130$.
time = 0.37, size = 1753, normalized size = 23.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")

[Out] [-1/6*(12*b*cosh(d*x + c)^4 + 48*b*cosh(d*x + c)*sinh(d*x + c)^3 + 12*b*sinh(d*x + c)^4 + 24*a*cosh(d*x + c)^2 + 24*(3*b*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 - 3*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x + c)^6 - 3*a*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*a*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 6*(a*cosh(d*x + c)^5 - 2*a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) - a)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 48*(b*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) - 8*a + 4*b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^6 - 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d)*sinh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + 4*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 - 6*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d)*sinh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d + 6*((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^5 - 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d*x + c)), -1/3*(6*b*cosh(d*x + c)^4 + 24*b*cosh(d*x + c)*sinh(d*x + c)^3 + 6*b*sinh(d*x + c)^4 + 12*a*cosh(d*x + c)^2 + 12*(3*b*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 3*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x + c)^6 - 3*a*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 - 3*a*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 - 6*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 6*(a*cosh(d*x + c)^5 - 2*a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) - a)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) + 24*(b*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) - 4*a + 2*b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^6 - 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d)*sinh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + 4*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 - 3*(a^2 + 2*a*b +

```
 b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^2 + 2*a*b + b^2)*d*cosh(d*
x + c)^4 - 6*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d)
*sinh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d + 6*((a^2 + 2*a*b + b^2)*d*cosh(d*
x + c)^5 - 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*
cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

Giac [A]
time = 0.57, size = 123, normalized size = 1.64

$$-\frac{\frac{3\,ab\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{(a^2+2\,ab+b^2)\sqrt{-ab-b^2}}+\frac{2\,\left(3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}-2\,a+b\right)}{(a^2+2\,ab+b^2)\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^
2 + 2*a*b + b^2)*sqrt(-a*b - b^2)) + 2*(3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x
+ 2*c) - 2*a + b)/((a^2 + 2*a*b + b^2)*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [B]
time = 2.17, size = 248, normalized size = 3.31

$$\frac{a\,\sqrt{b}\,\ln\left(\frac{4\,b\,e^{2\,c+2\,d\,x}}{(a+b)^2}-\frac{2\,\sqrt{b}\,\left(a+a\,e^{2\,c+2\,d\,x}+2\,b\,e^{2\,c+2\,d\,x}\right)}{(a+b)^{5/2}}\right)}{2\,d\,(a+b)^{5/2}}-\frac{8}{3\,(a\,d+b\,d)\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}-\frac{2\,b}{\left(e^{2\,c+2\,d\,x}-1\right)\,(a+b)\,(a\,d+b\,d)}-\frac{4}{(a\,d+b\,d)\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{a\,\sqrt{b}\,\ln\left(\frac{4\,b\,e^{2\,c+2\,d\,x}}{(a+b)^2}+\frac{2\,\sqrt{b}\,\left(a+a\,e^{2\,c+2\,d\,x}+2\,b\,e^{2\,c+2\,d\,x}\right)}{(a+b)^{5/2}}\right)}{2\,d\,(a+b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] (a*b^(1/2)*log((4*b*exp(2*c + 2*d*x))/(a + b)^2 - (2*b^(1/2)*(a + a*exp(2*c
 + 2*d*x) + 2*b*exp(2*c + 2*d*x)))/(a + b)^(5/2)))/(2*d*(a + b)^(5/2)) - 8/
(3*(a*d + b*d)*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x)
- 1)) - (2*b)/((exp(2*c + 2*d*x) - 1)*(a + b)*(a*d + b*d)) - 4/((a*d + b*d)
*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (a*b^(1/2)*log((4*b*exp(2*c
 + 2*d*x))/(a + b)^2 + (2*b^(1/2)*(a + a*exp(2*c + 2*d*x) + 2*b*exp(2*c + 2
*d*x)))/(a + b)^(5/2)))/(2*d*(a + b)^(5/2))
```

3.33 $\int \frac{\sinh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=194

$$\frac{3(a^2+8ab+8b^2)\,x}{8a^4}-\frac{3\sqrt{b}\,\sqrt{a+b}\,(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^4d}-\frac{(5a+6b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)}+$$

```
[Out] 3/8*(a^2+8*a*b+8*b^2)*x/a^4-3/2*(a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(
1/2))*b^(1/2)*(a+b)^(1/2)/a^4/d-1/8*(5*a+6*b)*cosh(d*x+c)*sinh(d*x+c)/a^2/d
/(a+b-b*tanh(d*x+c)^2)+1/4*cosh(d*x+c)^3*sinh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)
^2)-3/8*b*(3*a+4*b)*tanh(d*x+c)/a^3/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.20, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4217, 481, 541, 536, 212, 214}

$$-\frac{3\sqrt{b}\,\sqrt{a+b}\,(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^4d}-\frac{3b(3a+4b)\tanh(c+dx)}{8a^3d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{(5a+6b)\sinh(c+dx)\cosh(c+dx)}{8a^2d\left(a-b\tanh^2(c+dx)+b\right)}+\frac{3x(a^2+8ab+8b^2)}{8a^4}+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4ad\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (3*(a^2 + 8*a*b + 8*b^2)*x)/(8*a^4) - (3*Sqrt[b]*Sqrt[a + b]*(a + 2*b)*ArcT
anh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*a^4*d) - ((5*a + 6*b)*Cosh[c +
 d*x]*Sinh[c + d*x])/(8*a^2*d*(a + b - b*Tanh[c + d*x]^2)) + (Cosh[c + d*x]
^3*Sinh[c + d*x])/(4*a*d*(a + b - b*Tanh[c + d*x]^2)) - (3*b*(3*a + 4*b)*Ta
nh[c + d*x])/(8*a^3*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
```

```
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+b+(4a+5b)x^2}{(1-x^2)^2(a+b-bx^2)^2}\,dx, x, \tanh(c+dx\right.}{4ad}$$

$$= -\frac{(5a+6b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Su}}{}$$

$$= -\frac{(5a+6b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)} - \frac{}{8a}$$

$$= -\frac{(5a+6b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)} - \frac{}{8a}$$

$$= \frac{3(a^2+8ab+8b^2)\,x}{8a^4} - \frac{3\sqrt{b}\,\sqrt{a+b}\,(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^4d} - \frac{(5a}{8}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1080 vs. $2(194) = 388$.
time = 11.81, size = 1080, normalized size = 5.57

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/256*((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(16*x + ((a^3 - 6
*a^2*b - 24*a*b^2 - 16*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a
+ 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sin
h[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(b*(a + b)^(3/2)*d*Sqrt[b*(Cosh[c] - Si
nh[c])^4]) + ((a^2 + 8*a*b + 8*b^2)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh
[2*d*x]))/(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(a^2*(a + b*Sech[
c + d*x]^2)^2) + (3*(a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(((a
+ 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b^(3/2)*(a + b)^(3/
2)*d) - (a*Sinh[2*(c + d*x)])/(8*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]
))))/(128*(a + b*Sech[c + d*x]^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*S
ech[c + d*x]^4*(((a^5 - 30*a^4*b - 480*a^3*b^2 - 1600*a^2*b^3 - 1920*a*b^4
- 768*b^5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x]
- a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[
2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (Sech[2*c]
```

```
*(32*b*(5*a^4 + 39*a^3*b + 106*a^2*b^2 + 120*a*b^3 + 48*b^4)*d*x*Cosh[2*c]
+ 16*a*b*(5*a^3 + 29*a^2*b + 48*a*b^2 + 24*b^3)*d*x*Cosh[2*d*x] + 80*a^4*b*
d*x*Cosh[4*c + 2*d*x] + 464*a^3*b^2*d*x*Cosh[4*c + 2*d*x] + 768*a^2*b^3*d*x
*Cosh[4*c + 2*d*x] + 384*a*b^4*d*x*Cosh[4*c + 2*d*x] + a^5*Sinh[2*c] + 34*a
^4*b*Sinh[2*c] + 224*a^3*b^2*Sinh[2*c] + 576*a^2*b^3*Sinh[2*c] + 640*a*b^4*
Sinh[2*c] + 256*b^5*Sinh[2*c] - a^5*Sinh[2*d*x] - 62*a^4*b*Sinh[2*d*x] - 31
8*a^3*b^2*Sinh[2*d*x] - 512*a^2*b^3*Sinh[2*d*x] - 256*a*b^4*Sinh[2*d*x] - 1
2*a^4*b*Sinh[2*(c + 2*d*x)] - 36*a^3*b^2*Sinh[2*(c + 2*d*x)] - 24*a^2*b^3*S
inh[2*(c + 2*d*x)] - 30*a^4*b*Sinh[4*c + 2*d*x] - 158*a^3*b^2*Sinh[4*c + 2*
d*x] - 256*a^2*b^3*Sinh[4*c + 2*d*x] - 128*a*b^4*Sinh[4*c + 2*d*x] - 12*a^4
*b*Sinh[6*c + 4*d*x] - 36*a^3*b^2*Sinh[6*c + 4*d*x] - 24*a^2*b^3*Sinh[6*c +
 4*d*x] + 2*a^4*b*Sinh[4*c + 6*d*x] + 2*a^3*b^2*Sinh[4*c + 6*d*x] + 2*a^4*b
*Sinh[8*c + 6*d*x] + 2*a^3*b^2*Sinh[8*c + 6*d*x]))/(a + 2*b + a*Cosh[2*(c +
 d*x)])))/(1024*a^4*b*(a + b)*d*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(176) = 352$.
time = 2.60, size = 459, normalized size = 2.37

method	result
risch	$\frac{3x}{8a^2}+\frac{3xb}{a^3}+\frac{3x\,b^2}{a^4}+\frac{e^{4dx+4c}}{64a^2d}-\frac{e^{2dx+2c}b}{4a^3d}-\frac{e^{2dx+2c}}{8a^2d}+\frac{e^{-2dx-2c}b}{4a^3d}+\frac{e^{-2dx-2c}}{8a^2d}-\frac{e^{-4dx-4c}}{64a^2d}+\frac{b(a+b)(}{a^4d(a\,e^{4dx+}}$
derivativedivides	$\frac{\frac{1}{4a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+8b}{8a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3a+8b}{8a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-3a^2-24ab-24b^2\right)\ln\left(\tan\right.}{8a^4}}{}$
default	$\frac{\frac{1}{4a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+8b}{8a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3a+8b}{8a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-3a^2-24ab-24b^2\right)\ln\left(\tan\right.}{8a^4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^2/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/a^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/
8*(a+8*b)/a^3/(tanh(1/2*d*x+1/2*c)-1)^2-1/8*(3*a+8*b)/a^3/(tanh(1/2*d*x+1/2
*c)-1)+1/8/a^4*(-3*a^2-24*a*b-24*b^2)*ln(tanh(1/2*d*x+1/2*c)-1)+2*b/a^4*(((
-1/2*a^2-1/2*a*b)*tanh(1/2*d*x+1/2*c)^3-1/2*a*(a+b)*tanh(1/2*d*x+1/2*c))/(a
```

```
*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*
b*tanh(1/2*d*x+1/2*c)^2+a+b)+1/2*(3*a^2+9*a*b+6*b^2)*(-1/4/b^(1/2)/(a+b)^(1
/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b
)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tan
h(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))-1/4/a^2/(tanh(1/2*d*x+1/2*c)+1)^4+1
/2/a^2/(tanh(1/2*d*x+1/2*c)+1)^3-1/8*(-a-8*b)/a^3/(tanh(1/2*d*x+1/2*c)+1)^2
-1/8*(3*a+8*b)/a^3/(tanh(1/2*d*x+1/2*c)+1)+1/8/a^4*(3*a^2+24*a*b+24*b^2)*ln
(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. $2(185) = 370$.
time = 0.55, size = 1299, normalized size = 6.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/64*(3*a^3*b + 42*a^2*b^2 + 88*a*b^3 + 48*b^4)*log((a*e^(2*d*x + 2*c) + a
 + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b
)))/((a^5 + a^4*b)*sqrt((a + b)*b)*d) - 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*l
og((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a
 + 2*b + 2*sqrt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)*d) + 1/64*(3*a^
3*b + 42*a^2*b^2 + 88*a*b^3 + 48*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2
*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^5
 + a^4*b)*sqrt((a + b)*b)*d) + 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(
-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b
+ 2*sqrt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)*d) + 3/32*(3*a*b + 2*b
^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2
*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((a + b)*b)*d) + 1/1
6*(a^3*b + 8*a^2*b^2 + 8*a*b^3 + (a^3*b + 18*a^2*b^2 + 48*a*b^3 + 32*b^4)*e
^(2*d*x + 2*c))/((a^6 + a^5*b + (a^6 + a^5*b)*e^(4*d*x + 4*c) + 2*(a^6 + 3*
a^5*b + 2*a^4*b^2)*e^(2*d*x + 2*c))*d) - 1/16*(a^3*b + 8*a^2*b^2 + 8*a*b^3
+ (a^3*b + 18*a^2*b^2 + 48*a*b^3 + 32*b^4)*e^(-2*d*x - 2*c))/((a^6 + a^5*b
+ 2*(a^6 + 3*a^5*b + 2*a^4*b^2)*e^(-2*d*x - 2*c) + (a^6 + a^5*b)*e^(-4*d*x
- 4*c))*d) + 1/4*(a^2*b + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(2*d*x + 2*
c))/((a^5 + a^4*b + (a^5 + a^4*b)*e^(4*d*x + 4*c) + 2*(a^5 + 3*a^4*b + 2*a^
3*b^2)*e^(2*d*x + 2*c))*d) - 1/4*(a^2*b + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^
3)*e^(-2*d*x - 2*c))/((a^5 + a^4*b + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(-2*d*
x - 2*c) + (a^5 + a^4*b)*e^(-4*d*x - 4*c))*d) - 3/8*(a*b + (a*b + 2*b^2)*e^
(-2*d*x - 2*c))/((a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2
*c) + (a^4 + a^3*b)*e^(-4*d*x - 4*c))*d) + 3/8*(d*x + c)/(a^2*d) - 1/8*e^(2
*d*x + 2*c)/(a^2*d) + 1/8*e^(-2*d*x - 2*c)/(a^2*d) + 1/2*b*log(a*e^(4*d*x +
 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^3*d) - 1/2*b*log(2*(a + 2*b)*e^
(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d) + 1/64*(a*e^(4*d*x + 4*c)
```

```
- 16*b*e^(2*d*x + 2*c))/(a^3*d) + 1/64*(16*b*e^(-2*d*x - 2*c) - a*e^(-4*d*x
 - 4*c))/(a^3*d) + 1/4*(a*b + 3*b^2)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^
(2*d*x + 2*c) + a)/(a^4*d) - 1/4*(a*b + 3*b^2)*log(2*(a + 2*b)*e^(-2*d*x -
2*c) + a*e^(-4*d*x - 4*c) + a)/(a^4*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2464 vs. $2(185) = 370$.
time = 0.44, size = 5169, normalized size = 26.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/64*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(d*x + c)^11 + a^3*s
inh(d*x + c)^12 - 6*(a^3 + 2*a^2*b)*cosh(d*x + c)^10 + 6*(11*a^3*cosh(d*x +
 c)^2 - a^3 - 2*a^2*b)*sinh(d*x + c)^10 + 20*(11*a^3*cosh(d*x + c)^3 - 3*(a
^3 + 2*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^9 - (15*a^3 + 64*a^2*b + 64*a*b^
2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^8 + (495*a^3*cosh(d*x +
 c)^4 - 15*a^3 - 64*a^2*b - 64*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x - 2
70*(a^3 + 2*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(99*a^3*cosh(d*x +
c)^5 - 90*(a^3 + 2*a^2*b)*cosh(d*x + c)^3 - (15*a^3 + 64*a^2*b + 64*a*b^2 -
 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 + 16*(4*a
^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(
d*x + c)^6 + 4*(231*a^3*cosh(d*x + c)^6 - 315*(a^3 + 2*a^2*b)*cosh(d*x + c)
^4 + 16*a^2*b + 48*a*b^2 + 32*b^3 + 12*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)
*d*x - 7*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*
cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(99*a^3*cosh(d*x + c)^7 - 189*(a^3 + 2
*a^2*b)*cosh(d*x + c)^5 - 7*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2
*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 12*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^
3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + (15
*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x +
 c)^4 + (495*a^3*cosh(d*x + c)^8 - 1260*(a^3 + 2*a^2*b)*cosh(d*x + c)^6 - 7
0*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*
x + c)^4 + 15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*
x + 240*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*a^3*cosh(d*x + c)^9 - 180*(
a^3 + 2*a^2*b)*cosh(d*x + c)^7 - 14*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3
 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^5 + 80*(4*a^2*b + 12*a*b^2 + 8*b^3
 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x + c)^3 + (15*a^3 +
128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c))*si
nh(d*x + c)^3 - a^3 + 6*(a^3 + 2*a^2*b)*cosh(d*x + c)^2 + 2*(33*a^3*cosh(d*
x + c)^10 - 135*(a^3 + 2*a^2*b)*cosh(d*x + c)^8 - 14*(15*a^3 + 64*a^2*b + 6
4*a*b^2 - 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^6 + 120*(4*a^2*b
+ 12*a*b^2 + 8*b^3 + 3*(a^3 + 10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x +
```

```
c)^4 + 3*a^3 + 6*a^2*b + 3*(15*a^3 + 128*a^2*b + 128*a*b^2 + 24*(a^3 + 8*a
^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 48*((a^2 + 2*a*b)*c
osh(d*x + c)^8 + 8*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a
*b)*sinh(d*x + c)^8 + 2*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^6 + 2*(14*(a^2
+ 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*sinh(d*x + c)^6 + 4*(14*(a^
2 + 2*a*b)*cosh(d*x + c)^3 + 3*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*
x + c)^5 + (a^2 + 2*a*b)*cosh(d*x + c)^4 + (70*(a^2 + 2*a*b)*cosh(d*x + c)^
4 + 30*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^4
 + 4*(14*(a^2 + 2*a*b)*cosh(d*x + c)^5 + 10*(a^2 + 4*a*b + 4*b^2)*cosh(d*x
+ c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(14*(a^2 + 2*a*b)
*cosh(d*x + c)^6 + 15*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^4 + 3*(a^2 + 2*a*
b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*(a^2 + 2*a*b)*cosh(d*x + c)^7 +
3*(a^2 + 4*a*b + 4*b^2)*cosh(d*x + c)^5 + (a^2 + 2*a*b)*cosh(d*x + c)^3)*si
nh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)
*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 +
2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b
^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) +
4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2
+ a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
 + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x
 + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(
d*x + c))*sinh(d*x + c) + a)) + 4*(3*a^3*cosh(d*x + c)^11 - 15*(a^3 + 2*a^2
*b)*cosh(d*x + c)^9 - 2*(15*a^3 + 64*a^2*b + 64*a*b^2 - 24*(a^3 + 8*a^2*b +
 8*a*b^2)*d*x)*cosh(d*x + c)^7 + 24*(4*a^2*b + 12*a*b^2 + 8*b^3 + 3*(a^3 +
10*a^2*b + 24*a*b^2 + 16*b^3)*d*x)*cosh(d*x + c)^5 + (15*a^3 + 128*a^2*b +
128*a*b^2 + 24*(a^3 + 8*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c)^3 + 3*(a^3 + 2*
a^2*b)*cosh(d*x + c))*sinh(d*x + c))/(a^5*d*cosh(d*x + c)^8 + 8*a^5*d*cosh(
d*x + c)*sinh(d*x + c)^7 + a^5*d*sinh(d*x + c)^8 + a^5*d*cosh(d*x + c)^4 +
2*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + 2*(14*a^5*d*cosh(d*x + c)^2 + (a^5 +
2*a^4*b)*d)*sinh(d*x + c)^6 + 4*(14*a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*
b)*d*cosh(d*x + c))*sinh(d*x + c)^5 + (70*a^5*d*cosh(d*x + c)^4 + a^5*d + 3
0*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(14*a^5*d*cosh(d*x
 + c)^5 + a^5*d*cosh(d*x + c) + 10*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3)*sinh(
d*x + c)^3 + 2*(14*a^5*d*cosh(d*x + c)^6 + 3*a^5*d*cosh(d*x + c)^2 + 15*(a^
5 + 2*a^4*b)*d*cosh(d*x + c)^4)*sinh(d*x + c)^2 + 4*(2*a^5*d*cosh(d*x + c)^
7 + a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(c+dx)}{(a+b\,\mathrm{sech}^2(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sinh(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [A]
time = 1.27, size = 323, normalized size = 1.66

$$\frac{\frac{24\,(a^2+8\,ab+8\,b^2)(dx+c)}{a^4}-\frac{\left(18\,a^2e^{(4\,dx+4\,c)}+144\,abe^{(4\,dx+4\,c)}+144\,b^2e^{(4\,dx+4\,c)}-8\,a^2e^{(2\,dx+2\,c)}-16\,abe^{(2\,dx+2\,c)}+a^2\right)e^{(-4\,dx-4\,c)}}{a^4}-\frac{96\,(a^2b+3\,ab^2+2\,b^3)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^4}+\frac{a^2e^{(4\,dx+4\,c)}-8\,a^2e^{(2\,dx+2\,c)}-16\,abe^{(2\,dx+2\,c)}}{a^4}+\frac{64\,\left(a^2be^{(2\,dx+2\,c)}+3\,ab^2e^{(2\,dx+2\,c)}+2\,b^3e^{(2\,dx+2\,c)}+a^2b+ab^2\right)}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)a^4}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(24*(a^2 + 8*a*b + 8*b^2)*(d*x + c)/a^4 - (18*a^2*e^(4*d*x + 4*c) + 14
4*a*b*e^(4*d*x + 4*c) + 144*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) - 1
6*a*b*e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c)/a^4 - 96*(a^2*b + 3*a*b^2 + 2
*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b
 - b^2)*a^4) + (a^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) - 16*a*b*e^(2*d
*x + 2*c))/a^4 + 64*(a^2*b*e^(2*d*x + 2*c) + 3*a*b^2*e^(2*d*x + 2*c) + 2*b^
3*e^(2*d*x + 2*c) + a^2*b + a*b^2)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c
) + 4*b*e^(2*d*x + 2*c) + a)*a^4))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^4\,\sinh(c+d\,x)^4}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^4*sinh(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.34 $\int \frac{\sinh^3(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{b}\,(3a+5b)\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{7/2}d}-\frac{(a+2b)\cosh(c+dx)}{a^3d}+\frac{\cosh^3(c+dx)}{3a^2d}-\frac{b(a+b)\cosh(c+dx)}{2a^3d\left(b+a\cosh^2(c+dx)\right)}$$

```
[Out] -(a+2*b)*cosh(d*x+c)/a^3/d+1/3*cosh(d*x+c)^3/a^2/d-1/2*b*(a+b)*cosh(d*x+c)/
a^3/d/(b+a*cosh(d*x+c)^2)+1/2*(3*a+5*b)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))
*b^(1/2)/a^(7/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4218, 466, 1167, 211}

$$\frac{\sqrt{b}\,(3a+5b)\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{7/2}d}-\frac{b(a+b)\cosh(c+dx)}{2a^3d\left(a\cosh^2(c+dx)+b\right)}-\frac{(a+2b)\cosh(c+dx)}{a^3d}+\frac{\cosh^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*a^(7/2)*d)
 - ((a + 2*b)*Cosh[c + d*x])/(a^3*d) + Cosh[c + d*x]^3/(3*a^2*d) - (b*(a +
b)*Cosh[c + d*x])/(2*a^3*d*(b + a*Cosh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4(1-x^2)}{(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{b(a+b)\cosh(c+dx)}{2a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{b(a+b)-2a(a+b)x^2+2a^2x^4}{b+ax^2}\,dx, x, \cosh(c+\right.}{2a^3d}$$

$$= -\frac{b(a+b)\cosh(c+dx)}{2a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \left(-2(a+2b)+2ax^2+\frac{3ab+5b^2}{b+ax^2}\right)\,dx,\right.}{2a^3d}$$

$$= -\frac{(a+2b)\cosh(c+dx)}{a^3d} + \frac{\cosh^3(c+dx)}{3a^2d} - \frac{b(a+b)\cosh(c+dx)}{2a^3d\left(b+a\cosh^2(c+dx)\right)} + \frac{(b(3}{}$$

$$= \frac{\sqrt{b}\,(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{7/2}d} - \frac{(a+2b)\cosh(c+dx)}{a^3d} + \frac{\cosh^3(c+dx}{3a^2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.36, size = 861, normalized size = 7.55

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*((9*a^3*ArcTan[((Sqrt[a]
 - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[
c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqr
```

```
t[b]])/b^(3/2) + 576*a*Sqrt[b]*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[
c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*
Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + 960*b^(3/2)*ArcTan[(
(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2]
 + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/
2]))/Sqrt[b]] + (9*a^3*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sin
h[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Co
sh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]])/b^(3/2) + 576*a*Sqrt[b]*ArcTa
n[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)
/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*
x)/2]))/Sqrt[b]] + 960*b^(3/2)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[
c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*
Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - (9*a^3*ArcTan[(Sqrt[
a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sq
rt[a] + I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]])/b^(3/2) - 96*Sqrt[a]*(3*
a + 8*b)*Cosh[c]*Cosh[d*x] + 32*a^(3/2)*Cosh[3*c]*Cosh[3*d*x] - (384*a^(3/2
)*b*Cosh[c + d*x])/(a + 2*b + a*Cosh[2*(c + d*x)]) - (384*Sqrt[a]*b^2*Cosh[
c + d*x])/(a + 2*b + a*Cosh[2*(c + d*x)]) - 288*a^(3/2)*Sinh[c]*Sinh[d*x] -
 768*Sqrt[a]*b*Sinh[c]*Sinh[d*x] + 32*a^(3/2)*Sinh[3*c]*Sinh[3*d*x]))/(1536
*a^(7/2)*d*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(100) = 200$.

time = 2.25, size = 264, normalized size = 2.32

method	result
derivativedivides	$-\frac{1}{3a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-a-4b}{2a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4b\left(\frac{\left(-\frac{a}{4}+\frac{b}{4}\right)\left(\tanh^2\left(\right.\right.}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(t\right.}\right.}{}$
default	$-\frac{1}{3a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-a-4b}{2a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4b\left(\frac{\left(-\frac{a}{4}+\frac{b}{4}\right)\left(\tanh^2\left(\right.\right.}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(t\right.}\right.}{}$
risch	$\frac{e^{3dx+3c}}{24a^2d}-\frac{3\,e^{dx+c}}{8a^2d}-\frac{e^{dx+c}b}{a^3d}-\frac{3\,e^{-dx-c}}{8a^2d}-\frac{e^{-dx-c}b}{a^3d}+\frac{e^{-3dx-3c}}{24a^2d}-\frac{b(a+b)e^{dx+c}\left(1+e^{2dx+2c}\right)}{a^3d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/a^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/a^2/(tanh(1/2*d*x+1/2*c)-1)^2-1
/2/a^3*(-a-4*b)/(tanh(1/2*d*x+1/2*c)-1)+4/a^3*b*(((-1/4*a+1/4*b)*tanh(1/2*d
```

```
*x+1/2*c)^2-1/4*a-1/4*b)/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2
*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)+1/8*(3*a+5*b)/(a*b)
^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2)))+1/3
/a^2/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/a^2/(tanh(1/2*d*x+1/2*c)+1)^2-1/2*(a+4*b
)/a^3/(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(a^2*e^(10*d*x + 10*c) + a^2 - (7*a^2*e^(8*c) + 20*a*b*e^(8*c))*e^(8*d
*x) - 2*(13*a^2*e^(6*c) + 66*a*b*e^(6*c) + 60*b^2*e^(6*c))*e^(6*d*x) - 2*(1
3*a^2*e^(4*c) + 66*a*b*e^(4*c) + 60*b^2*e^(4*c))*e^(4*d*x) - (7*a^2*e^(2*c)
 + 20*a*b*e^(2*c))*e^(2*d*x))/(a^4*d*e^(7*d*x + 7*c) + a^4*d*e^(3*d*x + 3*c
) + 2*(a^4*d*e^(5*c) + 2*a^3*b*d*e^(5*c))*e^(5*d*x)) + 1/8*integrate(8*((3*
a*b*e^(3*c) + 5*b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + 5*b^2*e^c)*e^(d*x))/(
a^4*e^(4*d*x + 4*c) + a^4 + 2*(a^4*e^(2*c) + 2*a^3*b*e^(2*c))*e^(2*d*x)), x
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2021 vs. 2(100) = 200.
time = 0.39, size = 3804, normalized size = 33.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(a^2*cosh(d*x + c)^10 + 10*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + a^2*si
nh(d*x + c)^10 - (7*a^2 + 20*a*b)*cosh(d*x + c)^8 + (45*a^2*cosh(d*x + c)^2
 - 7*a^2 - 20*a*b)*sinh(d*x + c)^8 + 8*(15*a^2*cosh(d*x + c)^3 - (7*a^2 + 2
0*a*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d
*x + c)^6 + 2*(105*a^2*cosh(d*x + c)^4 - 14*(7*a^2 + 20*a*b)*cosh(d*x + c)^
2 - 13*a^2 - 66*a*b - 60*b^2)*sinh(d*x + c)^6 + 4*(63*a^2*cosh(d*x + c)^5 -
 14*(7*a^2 + 20*a*b)*cosh(d*x + c)^3 - 3*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*
x + c))*sinh(d*x + c)^5 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^4 + 2*
(105*a^2*cosh(d*x + c)^6 - 35*(7*a^2 + 20*a*b)*cosh(d*x + c)^4 - 15*(13*a^2
 + 66*a*b + 60*b^2)*cosh(d*x + c)^2 - 13*a^2 - 66*a*b - 60*b^2)*sinh(d*x +
c)^4 + 8*(15*a^2*cosh(d*x + c)^7 - 7*(7*a^2 + 20*a*b)*cosh(d*x + c)^5 - 5*(
13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^3 - (13*a^2 + 66*a*b + 60*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c)^2 + (45*a^2*cos
h(d*x + c)^8 - 28*(7*a^2 + 20*a*b)*cosh(d*x + c)^6 - 30*(13*a^2 + 66*a*b +
```

60*b^2)*cosh(d*x + c)^4 - 12*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^2 - 7
*a^2 - 20*a*b)*sinh(d*x + c)^2 + 6*((3*a^2 + 5*a*b)*cosh(d*x + c)^7 + 7*(3*
a^2 + 5*a*b)*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 + 5*a*b)*sinh(d*x + c)^
7 + 2*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + 5*a*b)*cosh(
d*x + c)^2 + 6*a^2 + 22*a*b + 20*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + 5*a*b
)*cosh(d*x + c)^3 + 2*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c
)^4 + (3*a^2 + 5*a*b)*cosh(d*x + c)^3 + (35*(3*a^2 + 5*a*b)*cosh(d*x + c)^4
 + 20*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^2 + 3*a^2 + 5*a*b)*sinh(d*x +
 c)^3 + (21*(3*a^2 + 5*a*b)*cosh(d*x + c)^5 + 20*(3*a^2 + 11*a*b + 10*b^2)*
cosh(d*x + c)^3 + 3*(3*a^2 + 5*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(3*
a^2 + 5*a*b)*cosh(d*x + c)^6 + 10*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^4
 + 3*(3*a^2 + 5*a*b)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-b/a)*log((a*cosh
(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a
- 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2
+ 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh
(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cos
h(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*co
sh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(
a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^
2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + a
^2 + 2*(5*a^2*cosh(d*x + c)^9 - 4*(7*a^2 + 20*a*b)*cosh(d*x + c)^7 - 6*(13*
a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^5 - 4*(13*a^2 + 66*a*b + 60*b^2)*cosh(
d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^4*d*cosh(d*x
 + c)^7 + 7*a^4*d*cosh(d*x + c)*sinh(d*x + c)^6 + a^4*d*sinh(d*x + c)^7 + a
^4*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^5 + (21*a^4*d*cosh
(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d)*sinh(d*x + c)^5 + 5*(7*a^4*d*cosh(d*x +
c)^3 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35*a^4*d*cosh(
d*x + c)^4 + a^4*d + 20*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^3
+ (21*a^4*d*cosh(d*x + c)^5 + 3*a^4*d*cosh(d*x + c) + 20*(a^4 + 2*a^3*b)*d*
cosh(d*x + c)^3)*sinh(d*x + c)^2 + (7*a^4*d*cosh(d*x + c)^6 + 3*a^4*d*cosh(
d*x + c)^2 + 10*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^4)*sinh(d*x + c)), 1/24*(a^
2*cosh(d*x + c)^10 + 10*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + a^2*sinh(d*x +
c)^10 - (7*a^2 + 20*a*b)*cosh(d*x + c)^8 + (45*a^2*cosh(d*x + c)^2 - 7*a^2
- 20*a*b)*sinh(d*x + c)^8 + 8*(15*a^2*cosh(d*x + c)^3 - (7*a^2 + 20*a*b)*co
sh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^6
 + 2*(105*a^2*cosh(d*x + c)^4 - 14*(7*a^2 + 20*a*b)*cosh(d*x + c)^2 - 13*a^
2 - 66*a*b - 60*b^2)*sinh(d*x + c)^6 + 4*(63*a^2*cosh(d*x + c)^5 - 14*(7*a^
2 + 20*a*b)*cosh(d*x + c)^3 - 3*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c))*s
inh(d*x + c)^5 - 2*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^4 + 2*(105*a^2*
cosh(d*x + c)^6 - 35*(7*a^2 + 20*a*b)*cosh(d*x + c)^4 - 15*(13*a^2 + 66*a*b
 + 60*b^2)*cosh(d*x + c)^2 - 13*a^2 - 66*a*b - 60*b^2)*sinh(d*x + c)^4 + 8*
(15*a^2*cosh(d*x + c)^7 - 7*(7*a^2 + 20*a*b)*cosh(d*x + c)^5 - 5*(13*a^2 +
66*a*b + 60*b^2)*cosh(d*x + c)^3 - (13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^3 - (7*a^2 + 20*a*b)*cosh(d*x + c)^2 + (45*a^2*cosh(d*x + c
)^8 - 28*(7*a^2 + 20*a*b)*cosh(d*x + c)^6 - 30*(13*a^2 + 66*a*b + 60*b^2)*c

```
osh(d*x + c)^4 - 12*(13*a^2 + 66*a*b + 60*b^2)*cosh(d*x + c)^2 - 7*a^2 - 20
*a*b)*sinh(d*x + c)^2 - 12*((3*a^2 + 5*a*b)*cosh(d*x + c)^7 + 7*(3*a^2 + 5*
a*b)*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 + 5*a*b)*sinh(d*x + c)^7 + 2*(3
*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + 5*a*b)*cosh(d*x + c)
^2 + 6*a^2 + 22*a*b + 20*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + 5*a*b)*cosh(d
*x + c)^3 + 2*(3*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (3
*a^2 + 5*a*b)*cosh(d*x + c)^3 + (35*(3*a^2 + 5*a*b)*cosh(d*x + c)^4 + 20*(3
*a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^2 + 3*a^2...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sinh(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^4\,\sinh(c+d\,x)^3}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^4*sinh(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.35 $\int \frac{\sinh^2(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=131

$$-\frac{(a+4b)x}{2a^3}+\frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}\,d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)}+\frac{b\tanh(c+dx}{a^2d\left(a+b-b\tanh^2(\right.}$$

```
[Out] -1/2*(a+4*b)*x/a^3+1/2*(3*a+4*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b
^(1/2)/a^3/d/(a+b)^(1/2)+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)
^2)+b*tanh(d*x+c)/a^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.13, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4217, 482, 541, 536, 212, 214}

$$\frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3d\sqrt{a+b}}-\frac{x(a+4b)}{2a^3}+\frac{b\tanh(c+dx)}{a^2d\left(a-b\tanh^2(c+dx)+b\right)}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((a + 4*b)*x)/a^3 + (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x
])/Sqrt[a + b]])/(2*a^3*Sqrt[a + b]*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*a
*d*(a + b - b*Tanh[c + d*x]^2)) + (b*Tanh[c + d*x])/(a^2*d*(a + b - b*Tanh[
c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
```

```
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\,(a+b-b\tanh^2(c+dx))} - \frac{\text{Subst}\left(\int \frac{a+b+3bx^2}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+dx\right.}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\,(a+b-b\tanh^2(c+dx))} + \frac{b\tanh(c+dx)}{a^2d\,(a+b-b\tanh^2(c+dx))} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\,(a+b-b\tanh^2(c+dx))} + \frac{b\tanh(c+dx)}{a^2d\,(a+b-b\tanh^2(c+dx))} - \frac{(a+4b)\text{S}}{}$$

$$= -\frac{(a+4b)x}{2a^3} + \frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}\,d} + \frac{\cosh(c+dx)\sinh}{2ad\,(a+b-b\tanh^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 791 vs. $2(131) = 262$.

time = 9.54, size = 791, normalized size = 6.04

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(16*x + ((a^3 - 6*a^2*b
- 24*a*b^2 - 16*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*
Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4
])]*(Cosh[2*c] - Sinh[2*c]))/(b*(a + b)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^
4]) + ((a^2 + 8*a*b + 8*b^2)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]
))/(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(128*a^2*(a + b*Sech[c +
 d*x]^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(-64*(a +
2*b)*x + ((-a^4 + 16*a^3*b + 144*a^2*b^2 + 256*a*b^3 + 128*b^4)*ArcTanh[(Se
ch[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/
(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(b*
(a + b)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (16*a*Cosh[2*d*x]*Sinh[2*c
])/d + (16*a*Cosh[2*c]*Sinh[2*d*x])/d - ((a^3 + 18*a^2*b + 48*a*b^2 + 32*b^
3)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(b*(a + b)*d*(a + 2*b +
 a*Cosh[2*(c + d*x)]))))/(256*a^3*(a + b*Sech[c + d*x]^2)^2) - ((a + 2*b +
a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(-((a*ArcTanh[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a + b]])/(a + b)^(3/2)) + (Sqrt[b]*(a + 2*b)*Sinh[2*(c + d*x)])/((a
+ b)*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(256*b^(3/2)*d*(a + b*Sech[c + d*x]
```

```
^2)^2) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(-1/8*((a + 2*b
)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(3/2)*(a + b)^(3/2)*d) +
 (a*Sinh[2*(c + d*x)])/(8*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(1
6*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(117) = 234$.
time = 2.25, size = 324, normalized size = 2.47

method	result
derivativedivides	$-\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^3}+\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$
default	$-\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^3}+\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$
risch	$-\frac{x}{2a^2}-\frac{2xb}{a^3}+\frac{e^{2dx+2c}}{8a^2d}-\frac{e^{-2dx-2c}}{8a^2d}-\frac{b\left(a\,e^{2dx+2c}+2b\,e^{2dx+2c}+a\right)}{a^3d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)}+\frac{3\sqrt{b(a+b)}\ \ln\left(e^{2dx}\right.}{4(a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^2/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/a^2/(tanh(1/2*d*x+1/2*c)+1)+1/2
/a^3*(-a-4*b)*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/a^2/(tanh(1/2*d*x+1/2*c)-1)^2+1
/2/a^2/(tanh(1/2*d*x+1/2*c)-1)+1/2*(a+4*b)/a^3*ln(tanh(1/2*d*x+1/2*c)-1)-4/
a^3*b*((-1/4*a*tanh(1/2*d*x+1/2*c)^3-1/4*a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2
*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/
2*d*x+1/2*c)^2+a+b)+1/4*(3*a+4*b)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*
tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2
)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^
(1/2)+(a+b)^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(124) = 248$.
time = 0.53, size = 696, normalized size = 5.31

$$\frac{(3a^2b+12ab^2+8b^3)\log\left(\frac{ae^{(2dx+2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(2dx+2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{16(a^4+a^3b)\sqrt{(a+b)b}\,d}-\frac{(3a^2b+12ab^2+8b^3)\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{16(a^4+a^3b)\sqrt{(a+b)b}\,d}-\frac{(3ab+2b^2)\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{8(a^3+a^2b)\sqrt{(a+b)b}\,d}-\frac{a^2b+2ab^2+(a^2b+8ab^2+8b^3)e^{(2dx+2c)}}{4(a^5+a^4b+(a^5+a^4b)e^{(4dx+4c)}+2(a^5+3a^4b+2a^3b^2)e^{(2dx+2c)})d}+\frac{a^2b+2ab^2+(a^2b+8ab^2+8b^3)e^{(-2dx-2c)}}{4(a^5+a^4b+2(a^5+3a^4b+2a^3b^2)e^{(-2dx-2c)}+(a^5+a^4b)e^{(-4dx-4c)})d}+\frac{ab+(ab+2b^2)e^{(-2dx-2c)}}{2(a^4+a^3b+2(a^4+3a^3b+2a^2b^2)e^{(-2dx-2c)}+(a^4+a^3b)e^{(-4dx-4c)})d}-\frac{dx+c}{2a^2d}+\frac{e^{(2dx+2c)}}{8a^2d}-\frac{e^{(-2dx-2c)}}{8a^2d}-\frac{b\log\left(ae^{(4dx+4c)}+2(a+2b)e^{(2dx+2c)}+a\right)}{2a^3d}+\frac{b\log\left(2(a+2b)e^{(-2dx-2c)}+ae^{(-4dx-4c)}+a\right)}{2a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt
((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + a^3
*b)*sqrt((a + b)*b)*d) - 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(-2*d*x
 - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sq
rt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)*d) - 1/8*(3*a*b + 2*b^2)*log
((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a
 + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((a + b)*b)*d) - 1/4*(a^2*b
 + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(2*d*x + 2*c))/((a^5 + a^4*b + (a^
5 + a^4*b)*e^(4*d*x + 4*c) + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(2*d*x + 2*c))
*d) + 1/4*(a^2*b + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(-2*d*x - 2*c))/((
a^5 + a^4*b + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(-2*d*x - 2*c) + (a^5 + a^4*b
)*e^(-4*d*x - 4*c))*d) + 1/2*(a*b + (a*b + 2*b^2)*e^(-2*d*x - 2*c))/((a^4 +
 a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2*c) + (a^4 + a^3*b)*e^(
-4*d*x - 4*c))*d) - 1/2*(d*x + c)/(a^2*d) + 1/8*e^(2*d*x + 2*c)/(a^2*d) - 1
/8*e^(-2*d*x - 2*c)/(a^2*d) - 1/2*b*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(
2*d*x + 2*c) + a)/(a^3*d) + 1/2*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-
4*d*x - 4*c) + a)/(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. $2(124) = 248$.
time = 0.38, size = 2925, normalized size = 22.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(
d*x + c)^8 - 2*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^6 + 2*(14*
a^2*cosh(d*x + c)^2 - 2*(a^2 + 4*a*b)*d*x + a^2 + 2*a*b)*sinh(d*x + c)^6 +
4*(14*a^2*cosh(d*x + c)^3 - 3*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x
+ c))*sinh(d*x + c)^5 - 8*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*
x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 - 4*(a^2 + 6*a*b + 8*b^2)*d*x - 15*(2*
(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^2 - 4*a*b - 8*b^2)*sinh(d*x
+ c)^4 + 8*(7*a^2*cosh(d*x + c)^5 - 5*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*c
osh(d*x + c)^3 - 4*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c))
*sinh(d*x + c)^3 - 2*(2*(a^2 + 4*a*b)*d*x + a^2 + 6*a*b)*cosh(d*x + c)^2 +
2*(14*a^2*cosh(d*x + c)^6 - 15*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x
 + c)^4 - 2*(a^2 + 4*a*b)*d*x - 24*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2
)*cosh(d*x + c)^2 - a^2 - 6*a*b)*sinh(d*x + c)^2 + 2*((3*a^2 + 4*a*b)*cosh(
d*x + c)^6 + 6*(3*a^2 + 4*a*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^2 + 4*a
*b)*sinh(d*x + c)^6 + 2*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c)^4 + (15*(3*a
```

```
^2 + 4*a*b)*cosh(d*x + c)^2 + 6*a^2 + 20*a*b + 16*b^2)*sinh(d*x + c)^4 + 4*
(5*(3*a^2 + 4*a*b)*cosh(d*x + c)^3 + 2*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x +
c))*sinh(d*x + c)^3 + (3*a^2 + 4*a*b)*cosh(d*x + c)^2 + (15*(3*a^2 + 4*a*b)
*cosh(d*x + c)^4 + 12*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c)^2 + 3*a^2 + 4*
a*b)*sinh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b)*cosh(d*x + c)^5 + 4*(3*a^2 + 10
*a*b + 8*b^2)*cosh(d*x + c)^3 + (3*a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c
))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*c
osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^
2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 +
a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a
*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*c
osh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - a^2 + 4*(2*
a^2*cosh(d*x + c)^7 - 3*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^5
 - 8*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^3 - (2*(a^2 +
4*a*b)*d*x + a^2 + 6*a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^4*d*cosh(d*x + c
)^6 + 6*a^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a^4*d*sinh(d*x + c)^6 + a^4*d
*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^4 + (15*a^4*d*cosh(d*x
 + c)^2 + 2*(a^4 + 2*a^3*b)*d)*sinh(d*x + c)^4 + 4*(5*a^4*d*cosh(d*x + c)^3
 + 2*(a^4 + 2*a^3*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*a^4*d*cosh(d*x
+ c)^4 + a^4*d + 12*(a^4 + 2*a^3*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*
(3*a^4*d*cosh(d*x + c)^5 + a^4*d*cosh(d*x + c) + 4*(a^4 + 2*a^3*b)*d*cosh(d
*x + c)^3)*sinh(d*x + c)), 1/8*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*s
inh(d*x + c)^7 + a^2*sinh(d*x + c)^8 - 2*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b
)*cosh(d*x + c)^6 + 2*(14*a^2*cosh(d*x + c)^2 - 2*(a^2 + 4*a*b)*d*x + a^2 +
 2*a*b)*sinh(d*x + c)^6 + 4*(14*a^2*cosh(d*x + c)^3 - 3*(2*(a^2 + 4*a*b)*d*
x - a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*((a^2 + 6*a*b + 8*b^2)*
d*x + a*b + 2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 - 4*(a^2 + 6
*a*b + 8*b^2)*d*x - 15*(2*(a^2 + 4*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^2
- 4*a*b - 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 - 5*(2*(a^2 + 4
*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^3 - 4*((a^2 + 6*a*b + 8*b^2)*d*x + a
*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*(a^2 + 4*a*b)*d*x + a^2 +
 6*a*b)*cosh(d*x + c)^2 + 2*(14*a^2*cosh(d*x + c)^6 - 15*(2*(a^2 + 4*a*b)*d
*x - a^2 - 2*a*b)*cosh(d*x + c)^4 - 2*(a^2 + 4*a*b)*d*x - 24*((a^2 + 6*a*b
+ 8*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^2 - a^2 - 6*a*b)*sinh(d*x + c)^2
+ 4*((3*a^2 + 4*a*b)*cosh(d*x + c)^6 + 6*(3*a^2 + 4*a*b)*cosh(d*x + c)*sinh
(d*x + c)^5 + (3*a^2 + 4*a*b)*sinh(d*x + c)^6 + 2*(3*a^2 + 10*a*b + 8*b^2)*
cosh(d*x + c)^4 + (15*(3*a^2 + 4*a*b)*cosh(d*x + c)^2 + 6*a^2 + 20*a*b + 16
*b^2)*sinh(d*x + c)^4 + 4*(5*(3*a^2 + 4*a*b)*cosh(d*x + c)^3 + 2*(3*a^2 + 1
0*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 + 4*a*b)*cosh(d*x +
c)^2 + (15*(3*a^2 + 4*a*b)*cosh(d*x + c)^4 + 12*(3*a^2 + 10*a*b + 8*b^2)*co
sh(d*x + c)^2 + 3*a^2 + 4*a*b)*sinh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b)*cosh(
d*x + c)^5 + 4*(3*a^2 + 10*a*b + 8*b^2)*cosh(d*x + c)^3 + (3*a^2 + 4*a*b)*c
```

```
osh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2
 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(
a + b))/b) - a^2 + 4*(2*a^2*cosh(d*x + c)^7 - 3*(2*(a^2 + 4*a*b)*d*x - a^2
- 2*a*b)*cosh(d*x + c)^5 - 8*((a^2 + 6*a*b + 8*b^2)*d*x + a*b + 2*b^2)*cosh
(d*x + c)^3 - (2*(a^2 + 4*a*b)*d*x + a^2 + 6*a*b)*cosh(d*x + c))*sinh(d*x +
 c))/(a^4*d*cosh(d*x + c)^6 + 6*a^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a^4*d
*sinh(d*x + c)^6 + a^4*d*cosh(d*x + c)^2 + 2*(a...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sinh(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [**A**]
time = 1.06, size = 234, normalized size = 1.79

$$-\frac{\frac{12\,(dx+c)(a+4\,b)}{a^3}-\frac{3\,e^{(2\,dx+2\,c)}}{a^2}-\frac{12\,(3\,ab+4\,b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^3}-\frac{2\,a^2e^{(6\,dx+6\,c)}+8\,abe^{(6\,dx+6\,c)}+a^2e^{(4\,dx+4\,c)}-16\,b^2e^{(4\,dx+4\,c)}-4\,a^2e^{(2\,dx+2\,c)}-28\,abe^{(2\,dx+2\,c)}-3\,a^2}{\left(ae^{(6\,dx+6\,c)}+2\,ae^{(4\,dx+4\,c)}+4\,be^{(4\,dx+4\,c)}+ae^{(2\,dx+2\,c)}\right)a^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(d*x + c)*(a + 4*b)/a^3 - 3*e^(2*d*x + 2*c)/a^2 - 12*(3*a*b + 4*b
^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b -
 b^2)*a^3) - (2*a^2*e^(6*d*x + 6*c) + 8*a*b*e^(6*d*x + 6*c) + a^2*e^(4*d*x
+ 4*c) - 16*b^2*e^(4*d*x + 4*c) - 4*a^2*e^(2*d*x + 2*c) - 28*a*b*e^(2*d*x +
 2*c) - 3*a^2)/((a*e^(6*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) + 4*b*e^(4*d*x + 4
*c) + a*e^(2*d*x + 2*c))*a^3))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^4\,\sinh(c+d\,x)^2}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^4*sinh(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.36 $\int \frac{\sinh(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{3\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{5/2}d}+\frac{3\cosh(c+dx)}{2a^2d}-\frac{\cosh^3(c+dx)}{2ad\left(b+a\cosh^2(c+dx)\right)}$$

```
[Out] 3/2*cosh(d*x+c)/a^2/d-1/2*cosh(d*x+c)^3/a/d/(b+a*cosh(d*x+c)^2)-3/2*arctan(
cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(5/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4218, 294, 327, 211}

$$-\frac{3\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{5/2}d}+\frac{3\cosh(c+dx)}{2a^2d}-\frac{\cosh^3(c+dx)}{2ad\left(a\cosh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*a^(5/2)*d) + (3*Cos
h[c + d*x])/(2*a^2*d) - Cosh[c + d*x]^3/(2*a*d*(b + a*Cosh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^4}{(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\cosh^3(c+dx)}{2ad\left(b+a\cosh^2(c+dx)\right)} + \frac{3\operatorname{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{2ad} \\
&= \frac{3\cosh(c+dx)}{2a^2d} - \frac{\cosh^3(c+dx)}{2ad\left(b+a\cosh^2(c+dx)\right)} - \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cosh\right.}{2a^2d} \\
&= -\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{5/2}d} + \frac{3\cosh(c+dx)}{2a^2d} - \frac{\cosh^3(c+dx)}{2ad\left(b+a\cosh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.87, size = 479, normalized size = 5.70

$$
\frac{(a+2b+a\cosh(2(c+dx)))^2\operatorname{sech}^4(c+dx)\left(\frac{32\cosh(c)\cosh(dx)}{a^2}+\frac{32b\cosh(c+dx)}{a^2(a+2b+a\cosh(2(c+dx)))}+\frac{2\left(-\left((a^2+24b^2)\operatorname{ArcTan}\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)\right)-a^2\operatorname{ArcTan}\left(\frac{\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)-24b^2\operatorname{ArcTan}\left(\frac{\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)+a^2\operatorname{ArcTan}\left(\frac{\sqrt{a}-i\sqrt{a+b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+a^2\operatorname{ArcTan}\left(\frac{\sqrt{a}+i\sqrt{a+b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+16\sqrt{a}\,b^{3/2}\sinh(c)\sinh(dx)\right)}{a^{5/2}b^{3/2}}\right)}{128d\left(a+b\operatorname{sech}^2(c+dx)\right)^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*((32*Cosh[c]*Cosh[d*x])/
a^2 + (32*b*Cosh[c + d*x])/(a^2*(a + 2*b + a*Cosh[2*(c + d*x)])) + (2*(-((a
^2 + 24*b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*
Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Si
nh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]) - a^2*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*
Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*S
qrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - 24*b^2*Ar
cTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d
*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[
```

```
(d*x)/2]))/Sqrt[b]] + a^2*ArcTan[(Sqrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2]
)/Sqrt[b]] + a^2*ArcTan[(Sqrt[a] + I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]
] + 16*Sqrt[a]*b^(3/2)*Sinh[c]*Sinh[d*x]))/(a^(5/2)*b^(3/2))))/(128*d*(a +
b*Sech[c + d*x]^2)^2)
```

Maple [A]
time = 0.94, size = 70, normalized size = 0.83

method	result
derivativedivides	$-\frac{-\frac{1}{a^{2}\operatorname{sech}(dx+c)}-\frac{b\left(\frac{\operatorname{sech}(dx+c)}{2a+2b\operatorname{sech}(dx+c)^{2}}+\frac{3\arctan\left(\frac{b\operatorname{sech}(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}}{d}$
default	$-\frac{-\frac{1}{a^{2}\operatorname{sech}(dx+c)}-\frac{b\left(\frac{\operatorname{sech}(dx+c)}{2a+2b\operatorname{sech}(dx+c)^{2}}+\frac{3\arctan\left(\frac{b\operatorname{sech}(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}}{d}$
risch	$\frac{e^{dx+c}}{2a^{2}d}+\frac{e^{-dx-c}}{2a^{2}d}+\frac{e^{dx+c}b\left(1+e^{2dx+2c}\right)}{a^{2}d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)}+\frac{3\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}\,e^{dx+c}}{a}+1\right)}{4a^{3}d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-1/a^2/sech(d*x+c)-b/a^2*(1/2*sech(d*x+c)/(a+b*sech(d*x+c)^2)+3/2/(a*
b)^(1/2)*arctan(b*sech(d*x+c)/(a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(3*(a*e^(4*c) + 2*b*e^(4*c))*e^(4*d*x) + 3*(a*e^(2*c) + 2*b*e^(2*c))*e^
(2*d*x) + a*e^(6*d*x + 6*c) + a)/(a^3*d*e^(5*d*x + 5*c) + a^3*d*e^(d*x + c)
 + 2*(a^3*d*e^(3*c) + 2*a^2*b*d*e^(3*c))*e^(3*d*x)) - 1/2*integrate(6*(b*e^
(3*d*x + 3*c) - b*e^(d*x + c))/(a^3*e^(4*d*x + 4*c) + a^3 + 2*(a^3*e^(2*c)
+ 2*a^2*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(70) = 140$.
time = 0.40, size = 1780, normalized size = 21.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(2*a*cosh(d*x + c)^6 + 12*a*cosh(d*x + c)*sinh(d*x + c)^5 + 2*a*sinh(d*x + c)^6 + 6*(a + 2*b)*cosh(d*x + c)^4 + 6*(5*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^4 + 8*(5*a*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(a + 2*b)*cosh(d*x + c)^2 + 6*(5*a*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 3*(a*cosh(d*x + c)^5 + 5*a*cosh(d*x + c)*sinh(d*x + c)^4 + a*sinh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + a*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 12*(a*cosh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + 2*a)/(a^3*d*cosh(d*x + c)^5 + 5*a^3*d*cosh(d*x + c)*sinh(d*x + c)^4 + a^3*d*sinh(d*x + c)^5 + a^3*d*cosh(d*x + c) + 2*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^2 + (a^3 + 2*a^2*b)*d)*sinh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*a^3*d*cosh(d*x + c)^4 + a^3*d + 6*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/2*(a*cosh(d*x + c)^6 + 6*a*cosh(d*x + c)*sinh(d*x + c)^5 + a*sinh(d*x + c)^6 + 3*(a + 2*b)*cosh(d*x + c)^4 + 3*(5*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^4 + 4*(5*a*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c)^2 + 3*(5*a*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 3*(a*cosh(d*x + c)^5 + 5*a*cosh(d*x + c)*sinh(d*x + c)^4 + a*sinh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + a*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x + c))*sqrt(b/a)/b) - 3*(a*cosh(d*x + c)^5 + 5*a*cosh(d*x + c)*sinh(d*x + c)^4 + a*sinh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + a*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + 6*(a*cosh(d*x + c)^5 + 2*(a + 2*b)*cosh(d*x + c)^3 + (a

```
 + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)/(a^3*d*cosh(d*x + c)^5 + 5*a^3*d*
cosh(d*x + c)*sinh(d*x + c)^4 + a^3*d*sinh(d*x + c)^5 + a^3*d*cosh(d*x + c)
 + 2*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^2 + (a^3
+ 2*a^2*b)*d)*sinh(d*x + c)^3 + 2*(5*a^3*d*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2
*b)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*a^3*d*cosh(d*x + c)^4 + a^3*d + 6
*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^2)*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sinh(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.65, size = 71, normalized size = 0.85

$$\frac{b\cosh(c+d\,x)}{2\,\left(d\,a^3\cosh\left(c+d\,x\right)^2+b\,d\,a^2\right)}+\frac{\cosh(c+d\,x)}{a^2\,d}-\frac{3\,\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{a}\,\cosh(c+d\,x)}{\sqrt{b}}\right)}{2\,a^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (b*cosh(c + d*x))/(2*(a^3*d*cosh(c + d*x)^2 + a^2*b*d)) + cosh(c + d*x)/(a^
2*d) - (3*b^(1/2)*atan((a^(1/2)*cosh(c + d*x))/b^(1/2)))/(2*a^(5/2)*d)
```

3.37 $\int \frac{\text{csch}(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=99

$$\frac{\sqrt{b}\,(3a+b)\text{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{3/2}(a+b)^2d}-\frac{\tanh^{-1}(\cosh(c+dx))}{(a+b)^2d}-\frac{b\cosh(c+dx)}{2a(a+b)d\left(b+a\cosh^2(c+dx)\right)}$$

```
[Out] -arctanh(cosh(d*x+c))/(a+b)^2/d-1/2*b*cosh(d*x+c)/a/(a+b)/d/(b+a*cosh(d*x+c
)^2)+1/2*(3*a+b)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/(a+b)^
2/d
```

Rubi [A]
time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4218, 481, 536, 212, 211}

$$\frac{\sqrt{b}\,(3a+b)\text{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{3/2}d(a+b)^2}-\frac{b\cosh(c+dx)}{2ad(a+b)\left(a\cosh^2(c+dx)+b\right)}-\frac{\tanh^{-1}(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(2*a^(3/2)*(a +
 b)^2*d) - ArcTanh[Cosh[c + d*x]]/((a + b)^2*d) - (b*Cosh[c + d*x])/(2*a*(a
 + b)*d*(b + a*Cosh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
```

```
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{b\cosh(c+dx)}{2a(a+b)d\left(b+a\cosh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{b+(-2a-b)x^2}{(1-x^2)(b+ax^2)}\,dx, x, \cosh(c+dx\right.}{2a(a+b)d} \\
&= -\frac{b\cosh(c+dx)}{2a(a+b)d\left(b+a\cosh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{(a+b)^2 d} + \frac{(b(}{} \\
&= \frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{2a^{3/2}(a+b)^2 d} - \frac{\tanh^{-1}(\cosh(c+dx))}{(a+b)^2 d} - \frac{b\cos}{2a(a+b)d\,(b}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.79, size = 377, normalized size = 3.81

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^3(c+dx)\left(-\frac{2b(a+b)}{a}+\frac{\sqrt{b}\,(3a+b)\operatorname{ArcTan}\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\,\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\,\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)(a+2b+a\cosh(2(c+dx)))\operatorname{sech}(c+dx)}{a^{3/2}}+\frac{\sqrt{b}\,(3a+b)\operatorname{ArcTan}\left(\frac{\left(\sqrt{a}+i\sqrt{a+b}\,\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}+i\sqrt{a+b}\,\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)(a+2b+a\cosh(2(c+dx)))\operatorname{sech}(c+dx)}{a^{3/2}}-2(a+2b+a\cosh(2(c+dx)))\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)\operatorname{sech}(c+dx)+2(a+2b+a\cosh(2(c+dx)))\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\operatorname{sech}(c+dx)\right)}{8(a+b)^2 d\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*((-2*b*(a + b))/a + (Sqrt[
b]*(3*a + b)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*
Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Si
nh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c +
 d*x])/a^(3/2) + (Sqrt[b]*(3*a + b)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(
Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a
+ b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh
[2*(c + d*x)])*Sech[c + d*x])/a^(3/2) - 2*(a + 2*b + a*Cosh[2*(c + d*x)])*L
og[Cosh[(c + d*x)/2]]*Sech[c + d*x] + 2*(a + 2*b + a*Cosh[2*(c + d*x)])*Log
[Sinh[(c + d*x)/2]]*Sech[c + d*x]))/(8*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^
2)
```

Maple [A]
time = 2.32, size = 167, normalized size = 1.69

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a+b)^2}+\frac{4b\left(\frac{-\frac{(a-b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}-\frac{a+b}{4a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(3a+b)\arctan\left(\right.}{}\right.}{(a+b)^2}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a+b)^2}+\frac{4b\left(\frac{-\frac{(a-b)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}-\frac{a+b}{4a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(3a+b)\arctan\left(\right.}{}\right.}{(a+b)^2}}{d}$
risch	$-\frac{e^{dx+c}b\left(1+e^{2dx+2c}\right)}{ad(a+b)\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)}+\frac{\ln\left(e^{dx+c}-1\right)}{d\left(a^2+2ab+b^2\right)}-\frac{\ln\left(e^{dx+c}+1\right)}{d\left(a^2+2ab+b^2\right)}+\frac{3\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\right.}{4a(a+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a+b)^2*ln(tanh(1/2*d*x+1/2*c))+4*b/(a+b)^2*((-1/4*(a-b)/a*tanh(1/2*
d*x+1/2*c)^2-1/4/a*(a+b))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+
2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)+1/8*(3*a+b)/a/(a*b
)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^3*d + a^2*b*d + (a^3*d*e^(4*c) + a^
2*b*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c) + 3*a^2*b*d*e^(2*c) + 2*a*b^2*d
*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d + 2*a*b*d + b^2
*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^2*d + 2*a*b*d + b^2*d) + 2*integrate
(1/2*((3*a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + b^2*e^c)*e^(d*
x))/(a^4 + 2*a^3*b + a^2*b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(
4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 4*a^3*b*e^(2*c) + 5*a^2*b^2*e^(2*c) + 2*
a*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(87) = 174$.
time = 0.40, size = 2376, normalized size = 24.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a*b + b^2)*cosh(d*x + c)^3 + 12*(a*b + b^2)*cosh(d*x + c)*sinh(d*
x + c)^2 + 4*(a*b + b^2)*sinh(d*x + c)^3 - ((3*a^2 + a*b)*cosh(d*x + c)^4 +
 4*(3*a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + a*b)*sinh(d*x + c
)^4 + 2*(3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + a*b)*cosh(d
*x + c)^2 + 3*a^2 + 7*a*b + 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + a*b + 4*((3*a^
2 + a*b)*cosh(d*x + c)^3 + (3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2
 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c
))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2
 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x
 + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)
^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)
^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x +
 c))*sinh(d*x + c) + a)) + 4*(a*b + b^2)*cosh(d*x + c) + 4*(a^2*cosh(d*x +
c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 +
 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x
+ c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d
*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 4*(a^2*cosh(d*x + c)^4 +
4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)
*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2
+ a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)
)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(3*(a*b + b^2)*cosh(d*x + c)^2
 + a*b + b^2)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 +
 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3
*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d
*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4
```

```
+ 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*
b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b +
5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(a*b + b^2)*c
osh(d*x + c)^3 + 6*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a*b + b^2
)*sinh(d*x + c)^3 + ((3*a^2 + a*b)*cosh(d*x + c)^4 + 4*(3*a^2 + a*b)*cosh(d
*x + c)*sinh(d*x + c)^3 + (3*a^2 + a*b)*sinh(d*x + c)^4 + 2*(3*a^2 + 7*a*b
+ 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + a*b)*cosh(d*x + c)^2 + 3*a^2 + 7*a
*b + 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + a*b + 4*((3*a^2 + a*b)*cosh(d*x + c)^
3 + (3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(
1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c
)^3 + (a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a + 4*b)*sinh(d*x +
c))*sqrt(b/a)/b) - ((3*a^2 + a*b)*cosh(d*x + c)^4 + 4*(3*a^2 + a*b)*cosh(d*
x + c)*sinh(d*x + c)^3 + (3*a^2 + a*b)*sinh(d*x + c)^4 + 2*(3*a^2 + 7*a*b +
 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + a*b)*cosh(d*x + c)^2 + 3*a^2 + 7*a*
b + 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + a*b + 4*((3*a^2 + a*b)*cosh(d*x + c)^3
 + (3*a^2 + 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1
/2*(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt(b/a)/b) + 2*(a*b + b^2)*cosh(d*
x + c) + 2*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2
*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)
^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2
*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1)
- 2*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d
*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^
2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*c
osh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*
(a*b + b^2)*cosh(d*x + c)^2 + a*b + b^2)*sinh(d*x + c))/((a^4 + 2*a^3*b + a
^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sin
h(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 + 4*a^3
*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2
)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d)*sinh(d*x + c
)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x
 + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c))*sinh(d*x +
 c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^4}{\sinh\left(c+d\,x\right)\,\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(cosh(c + d*x)^4/(sinh(c + d*x)*(b + a*cosh(c + d*x)^2)^2), x)
```

3.38 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=92

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d}-\frac{3\coth(c+dx)}{2(a+b)^2d}+\frac{\coth(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

```
[Out] -3/2*coth(d*x+c)/(a+b)^2/d+3/2*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(
1/2)/(a+b)^(5/2)/d+1/2*coth(d*x+c)/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4217, 296, 331, 214}

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}}-\frac{3\coth(c+dx)}{2d(a+b)^2}+\frac{\coth(c+dx)}{2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*(a + b)^(5/2)*d
) - (3*Coth[c + d*x])/(2*(a + b)^2*d) + Coth[c + d*x]/(2*(a + b)*d*(a + b -
 b*Tanh[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\coth(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{3\operatorname{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)}\,dx, x, \tanh(c+d$$

$$= -\frac{3\coth(c+dx)}{2(a+b)^2d} + \frac{\coth(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{a+b-ba}$$

$$= \frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d} - \frac{3\coth(c+dx)}{2(a+b)^2d} + \frac{\coth(c+dx)}{2(a+b)d\left(a+b-b\tanh^2\right.}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 220 vs. 2(92) = 184.
time = 1.76, size = 220, normalized size = 2.39

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^4(c+dx)\left(\frac{3b\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(a+2b+a\cosh(2(c+dx)))(\cosh(2c)-\sinh(2c))}{\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}+2(a+2b+a\cosh(2(c+dx)))\operatorname{csch}(c)\operatorname{csch}(c+dx)\sinh(dx)+b\operatorname{sech}(2c)\sinh(2dx)-\frac{b(a+2b)\tanh(2c)}{a}\right)}{8(a+b)^2d\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((3*b*ArcTanh[(Sech[d*x]*(
Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a
 + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh
[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + 2*(a + 2*
b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x] + b*Sech[2*c]*Sinh
```

```
[2*d*x] - (b*(a + 2*b)*Tanh[2*c])/a))/(8*(a + b)^2*d*(a + b*Sech[c + d*x]^2
)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(78) = 156$.
time = 2.29, size = 238, normalized size = 2.59

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2+2ab+b^2\right)}-\frac{1}{2(a+b)^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4b\left(\frac{-\frac{\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{}}{}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2+2ab+b^2\right)}-\frac{1}{2(a+b)^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4b\left(\frac{-\frac{\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{}}{}$
risch	$-\frac{2a^2e^{4dx+4c}+ab\,e^{4dx+4c}+2b^2e^{4dx+4c}+4a^2e^{2dx+2c}+8ab\,e^{2dx+2c}-2b^2e^{2dx+2c}+2a^2-ab}{ad(a+b)^2\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)\left(e^{2dx+2c}-1\right)}+\frac{3\sqrt{b\,(a+b)}\,\ln\left(e^{2d}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-1/2/(a+b)^2/tanh(1/2*d*x+1/2*
c)-4*b/(a+b)^2*((-1/4*tanh(1/2*d*x+1/2*c)^3-1/4*tanh(1/2*d*x+1/2*c))/(a*tan
h(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*ta
nh(1/2*d*x+1/2*c)^2+a+b)+3/16/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d
*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))-3/16/b^(1/2)/(a+b)^(
1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+
b)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(81) = 162$.
time = 0.52, size = 262, normalized size = 2.85

$$-\frac{3\,b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{(a+b)b}\,d}-\frac{2\,a^2-ab+2\,(2\,a^2+4\,ab-b^2)e^{(-2\,dx-2\,c)}+(2\,a^2+ab+2\,b^2)e^{(-4\,dx-4\,c)}}{(a^4+2\,a^3b+a^2b^2+(a^4+6\,a^3b+9\,a^2b^2+4\,ab^3)e^{(-2\,dx-2\,c)}-(a^4+6\,a^3b+9\,a^2b^2+4\,ab^3)e^{(-4\,dx-4\,c)}-(a^4+2\,a^3b+a^2b^2)e^{(-6\,dx-6\,c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -3/4*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b)
*d) - (2*a^2 - a*b + 2*(2*a^2 + 4*a*b - b^2)*e^(-2*d*x - 2*c) + (2*a^2 + a*
b + 2*b^2)*e^(-4*d*x - 4*c))/((a^4 + 2*a^3*b + a^2*b^2 + (a^4 + 6*a^3*b + 9
```

```
*a^2*b^2 + 4*a*b^3)*e^(-2*d*x - 2*c) - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3
)*e^(-4*d*x - 4*c) - (a^4 + 2*a^3*b + a^2*b^2)*e^(-6*d*x - 6*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. $2(81) = 162$.
time = 0.40, size = 2407, normalized size = 26.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^4 + 16*(2*a^2 + a*b + 2*b^2)*c
osh(d*x + c)*sinh(d*x + c)^3 + 4*(2*a^2 + a*b + 2*b^2)*sinh(d*x + c)^4 + 8*
(2*a^2 + 4*a*b - b^2)*cosh(d*x + c)^2 + 8*(3*(2*a^2 + a*b + 2*b^2)*cosh(d*x
 + c)^2 + 2*a^2 + 4*a*b - b^2)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^6 + 6
*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + (a^2 + 4*a*b)*co
sh(d*x + c)^4 + (15*a^2*cosh(d*x + c)^2 + a^2 + 4*a*b)*sinh(d*x + c)^4 + 4*
(5*a^2*cosh(d*x + c)^3 + (a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^
2 + 4*a*b)*cosh(d*x + c)^2 + (15*a^2*cosh(d*x + c)^4 + 6*(a^2 + 4*a*b)*cosh
(d*x + c)^2 - a^2 - 4*a*b)*sinh(d*x + c)^2 - a^2 + 2*(3*a^2*cosh(d*x + c)^5
 + 2*(a^2 + 4*a*b)*cosh(d*x + c)^3 - (a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d
*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^
2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*
(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2
 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2
+ a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x
+ c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b
)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(
a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*a^2 -
4*a*b + 16*((2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^3 + (2*a^2 + 4*a*b - b^2)*c
osh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^6 +
 6*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 2*a^3
*b + a^2*b^2)*d*sinh(d*x + c)^6 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*c
osh(d*x + c)^4 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 + 6
*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d)*sinh(d*x + c)^4 - (a^4 + 6*a^3*b + 9*a^2*b
^2 + 4*a*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x
 + c)^3 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c))*sinh(d*x +
 c)^3 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 6*(a^4 + 6*a^3*b
+ 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c)^2 - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a
*b^3)*d)*sinh(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d + 2*(3*(a^4 + 2*a^3*
b + a^2*b^2)*d*cosh(d*x + c)^5 + 2*(a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*
cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c))*si
nh(d*x + c)), -1/2*(2*(2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^4 + 8*(2*a^2 + a*
```

```
b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(2*a^2 + a*b + 2*b^2)*sinh(d*x
 + c)^4 + 4*(2*a^2 + 4*a*b - b^2)*cosh(d*x + c)^2 + 4*(3*(2*a^2 + a*b + 2*b
^2)*cosh(d*x + c)^2 + 2*a^2 + 4*a*b - b^2)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*
x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + (a^2
 + 4*a*b)*cosh(d*x + c)^4 + (15*a^2*cosh(d*x + c)^2 + a^2 + 4*a*b)*sinh(d*x
 + c)^4 + 4*(5*a^2*cosh(d*x + c)^3 + (a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x
+ c)^3 - (a^2 + 4*a*b)*cosh(d*x + c)^2 + (15*a^2*cosh(d*x + c)^4 + 6*(a^2 +
 4*a*b)*cosh(d*x + c)^2 - a^2 - 4*a*b)*sinh(d*x + c)^2 - a^2 + 2*(3*a^2*cos
h(d*x + c)^5 + 2*(a^2 + 4*a*b)*cosh(d*x + c)^3 - (a^2 + 4*a*b)*cosh(d*x + c
))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh
(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b)
+ 4*a^2 - 2*a*b + 8*((2*a^2 + a*b + 2*b^2)*cosh(d*x + c)^3 + (2*a^2 + 4*a*b
 - b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x
 + c)^6 + 6*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^
4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^6 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a
*b^3)*d*cosh(d*x + c)^4 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 +
 (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d)*sinh(d*x + c)^4 - (a^4 + 6*a^3*b
+ 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 2*a^3*b + a^2*b^2)*d
*cosh(d*x + c)^3 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c))*s
inh(d*x + c)^3 + (15*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 6*(a^4 +
 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x + c)^2 - (a^4 + 6*a^3*b + 9*a^2*
b^2 + 4*a*b^3)*d)*sinh(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d + 2*(3*(a^4
 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^5 + 2*(a^4 + 6*a^3*b + 9*a^2*b^2 + 4*
a*b^3)*d*cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*d*cosh(d*x
 + c))*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(81) = 162.
time = 0.62, size = 239, normalized size = 2.60

$$\frac{\frac{3\,b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{(a^2+2\,ab+b^2)\sqrt{-ab-b^2}}-\frac{2\,\left(2\,a^2e^{(4\,dx+4\,c)}+abe^{(4\,dx+4\,c)}+2\,b^2e^{(4\,dx+4\,c)}+4\,a^2e^{(2\,dx+2\,c)}+8\,abe^{(2\,dx+2\,c)}-2\,b^2e^{(2\,dx+2\,c)}+2\,a^2-ab\right)}{(a^3+2\,a^2b+ab^2)\left(ae^{(6\,dx+6\,c)}+ae^{(4\,dx+4\,c)}+4\,be^{(4\,dx+4\,c)}-ae^{(2\,dx+2\,c)}-4\,be^{(2\,dx+2\,c)}-a\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^2 +
 2*a*b + b^2)*sqrt(-a*b - b^2)) - 2*(2*a^2*e^(4*d*x + 4*c) + a*b*e^(4*d*x +
 4*c) + 2*b^2*e^(4*d*x + 4*c) + 4*a^2*e^(2*d*x + 2*c) + 8*a*b*e^(2*d*x + 2*
c) - 2*b^2*e^(2*d*x + 2*c) + 2*a^2 - a*b)/((a^3 + 2*a^2*b + a*b^2)*(a*e^(6*
d*x + 6*c) + a*e^(4*d*x + 4*c) + 4*b*e^(4*d*x + 4*c) - a*e^(2*d*x + 2*c) -
4*b*e^(2*d*x + 2*c) - a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^4}{\sinh\left(c+d\,x\right)^2\,\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(cosh(c + d*x)^4/(sinh(c + d*x)^2*(b + a*cosh(c + d*x)^2)^2), x)
```

3.39 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{(3a-b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2\sqrt{a}\,(a+b)^3d}+\frac{(a-3b)\tanh^{-1}(\cosh(c+dx))}{2(a+b)^3d}-\frac{(a-b)\cosh(c+dx)}{2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)}-$$

```
[Out] 1/2*(a-3*b)*arctanh(cosh(d*x+c))/(a+b)^3/d-1/2*(a-b)*cosh(d*x+c)/(a+b)^2/d/
(b+a*cosh(d*x+c)^2)-1/2*coth(d*x+c)*csch(d*x+c)/(a+b)/d/(b+a*cosh(d*x+c)^2)
-1/2*(3*a-b)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b)^3/d/a^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4218, 481, 541, 536, 212, 211}

$$-\frac{\sqrt{b}\,(3a-b)\text{ArcTan}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{2\sqrt{a}\,d(a+b)^3}-\frac{(a-b)\cosh(c+dx)}{2d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}+\frac{(a-3b)\tanh^{-1}(\cosh(c+dx))}{2d(a+b)^3}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2d(a+b)\left(a\cosh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(Sqrt[a]*(
a + b)^3*d) + ((a - 3*b)*ArcTanh[Cosh[c + d*x]])/(2*(a + b)^3*d) - ((a - b)
*Cosh[c + d*x])/(2*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)) - (Coth[c + d*x]*Cs
ch[c + d*x])/(2*(a + b)*d*(b + a*Cosh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
```

```
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^2(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{b+(-a+2b)x^2}{(1-x^2)(b+ax^2)^2}\,dx, x, \cosh(c+d\right.}{2(a+b)d}$$

$$= -\frac{(a-b)\cosh(c+dx)}{2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)} + \frac{\operatorname{Sub}}{}$$

$$= -\frac{(a-b)\cosh(c+dx)}{2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)} + \frac{(a-}{}$$

$$= -\frac{(3a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{2\sqrt{a}\,(a+b)^3d} + \frac{(a-3b)\tanh^{-1}(\cosh(c+dx))}{2(a+b)^3d} - \frac{}{2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.70, size = 462, normalized size = 3.14

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*(8*b*(a + b) + (4*Sqrt[b]*
(-3*a + b)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Si
nh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh
[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d
*x])/Sqrt[a] + (4*Sqrt[b]*(-3*a + b)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[
(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a
 + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos
h[2*(c + d*x)])*Sech[c + d*x])/Sqrt[a] - (a + b)*(a + 2*b + a*Cosh[2*(c + d
*x)])*Csch[(c + d*x)/2]^2*Sech[c + d*x] + 4*(a - 3*b)*(a + 2*b + a*Cosh[2*(
c + d*x)])*Log[Cosh[(c + d*x)/2]]*Sech[c + d*x] - 4*(a - 3*b)*(a + 2*b + a*
Cosh[2*(c + d*x)])*Log[Sinh[(c + d*x)/2]]*Sech[c + d*x] - (a + b)*(a + 2*b
+ a*Cosh[2*(c + d*x)])*Sech[(c + d*x)/2]^2*Sech[c + d*x]))/(32*(a + b)^3*d*
(a + b*Sech[c + d*x]^2)^2)
```

Maple [A]
time = 2.74, size = 214, normalized size = 1.46

method	result
derivativedivides	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2+16ab+8b^2}-\frac{1}{8(a+b)^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a+6b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^3}-\frac{2b\left(\frac{\left(\frac{b}{2}-\frac{a}{2}\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{d}{2}\cdots\right.\right.}\cdots\right.}{\cdots}}{d}$ …
default	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2+16ab+8b^2}-\frac{1}{8(a+b)^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a+6b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^3}-\frac{2b\left(\frac{\left(\frac{b}{2}-\frac{a}{2}\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{d}{2}\cdots\right.\right.}\cdots\right.}{\cdots}}{d}$ …
risch	$-\frac{e^{dx+c}\left(a\,e^{6dx+6c}-b\,e^{6dx+6c}+3a\,e^{4dx+4c}+5b\,e^{4dx+4c}+3a\,e^{2dx+2c}+5b\,e^{2dx+2c}+a-b\right)}{d(a+b)^2\left(e^{2dx+2c}-1\right)^2\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)}+\frac{\ln\left(e^{dx+c}+1\right)a}{2d\left(a^3+3a^2b+3a\,b^2+b^3\right)}-\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/(a^2+2*a*b+b^2)-1/8/(a+b)^2/tanh(1/2*d*x+1/2
*c)^2+1/4/(a+b)^3*(-2*a+6*b)*ln(tanh(1/2*d*x+1/2*c))-2*b/(a+b)^3*((((1/2*b-1
/2*a)*tanh(1/2*d*x+1/2*c)^2-1/2*a-1/2*b)/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/
2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)+1/4
*(3*a-b)/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*
b)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a - 3*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^3*d + 3*a^2*b*d + 3*a*b^2*d
+ b^3*d) - 1/2*(a - 3*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^3*d + 3*a^2*b*d +
 3*a*b^2*d + b^3*d) - ((a*e^(7*c) - b*e^(7*c))*e^(7*d*x) + (3*a*e^(5*c) + 5
*b*e^(5*c))*e^(5*d*x) + (3*a*e^(3*c) + 5*b*e^(3*c))*e^(3*d*x) + (a*e^c - b*
e^c)*e^(d*x))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*e^(8*c) + 2*a^2*b*d*e^(
8*c) + a*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^2*b*d*e^(6*c) + 2*a*b^2*d*e^(6*c)
+ b^3*d*e^(6*c))*e^(6*d*x) - 2*(a^3*d*e^(4*c) + 6*a^2*b*d*e^(4*c) + 9*a*b^2
*d*e^(4*c) + 4*b^3*d*e^(4*c))*e^(4*d*x) + 4*(a^2*b*d*e^(2*c) + 2*a*b^2*d*e^
(2*c) + b^3*d*e^(2*c))*e^(2*d*x)) - 8*integrate(1/8*((3*a*b*e^(3*c) - b^2*e
^(3*c))*e^(3*d*x) - (3*a*b*e^c - b^2*e^c)*e^(d*x))/(a^4 + 3*a^3*b + 3*a^2*b
```

```
^2 + a*b^3 + (a^4*e^(4*c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c) + a*b^3*e^(
4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 5*a^3*b*e^(2*c) + 9*a^2*b^2*e^(2*c) + 7*
a*b^3*e^(2*c) + 2*b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3620 vs. $2(131) = 262$.
time = 0.53, size = 6878, normalized size = 46.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2 - b^2)*cosh(d*x + c)^7 + 28*(a^2 - b^2)*cosh(d*x + c)*sinh(d*
x + c)^6 + 4*(a^2 - b^2)*sinh(d*x + c)^7 + 4*(3*a^2 + 8*a*b + 5*b^2)*cosh(d
*x + c)^5 + 4*(21*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 5*b^2)*sinh
(d*x + c)^5 + 20*(7*(a^2 - b^2)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 5*b^2)*c
osh(d*x + c))*sinh(d*x + c)^4 + 4*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^3 +
 4*(35*(a^2 - b^2)*cosh(d*x + c)^4 + 10*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x +
c)^2 + 3*a^2 + 8*a*b + 5*b^2)*sinh(d*x + c)^3 + 4*(21*(a^2 - b^2)*cosh(d*x
+ c)^5 + 10*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 8*a*b + 5*
b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a^2 - a*b)*cosh(d*x + c)^8 + 8*(3
*a^2 - a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 - a*b)*sinh(d*x + c)^8 +
 4*(3*a*b - b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 - a*b)*cosh(d*x + c)^2 + 3*a
*b - b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 - a*b)*cosh(d*x + c)^3 + 3*(3*a*b -
 b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(3*a^2 + 11*a*b - 4*b^2)*cosh(d*x
+ c)^4 + 2*(35*(3*a^2 - a*b)*cosh(d*x + c)^4 + 30*(3*a*b - b^2)*cosh(d*x +
c)^2 - 3*a^2 - 11*a*b + 4*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 - a*b)*cosh(d*
x + c)^5 + 10*(3*a*b - b^2)*cosh(d*x + c)^3 - (3*a^2 + 11*a*b - 4*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 + 4*(3*a*b - b^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2
- a*b)*cosh(d*x + c)^6 + 15*(3*a*b - b^2)*cosh(d*x + c)^4 - 3*(3*a^2 + 11*a
*b - 4*b^2)*cosh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x + c)^2 + 3*a^2 - a*b +
8*((3*a^2 - a*b)*cosh(d*x + c)^7 + 3*(3*a*b - b^2)*cosh(d*x + c)^5 - (3*a^2
 + 11*a*b - 4*b^2)*cosh(d*x + c)^3 + (3*a*b - b^2)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2
 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c
))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2
 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a)*sinh(d*x
 + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)
^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)
^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x +
 c))*sinh(d*x + c) + a)) + 4*(a^2 - b^2)*cosh(d*x + c) - 2*((a^2 - 3*a*b)*c
osh(d*x + c)^8 + 8*(a^2 - 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 - 3*a
*b)*sinh(d*x + c)^8 + 4*(a*b - 3*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 - 3*a*b)*
```

```
cosh(d*x + c)^2 + a*b - 3*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 - 3*a*b)*cosh(d*
x + c)^3 + 3*(a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + a*b -
12*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 - 3*a*b)*cosh(d*x + c)^4 + 30*(a*b - 3
*b^2)*cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 - 3
*a*b)*cosh(d*x + c)^5 + 10*(a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 + a*b - 12*
b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a*b - 3*b^2)*cosh(d*x + c)^2 + 4*(
7*(a^2 - 3*a*b)*cosh(d*x + c)^6 + 15*(a*b - 3*b^2)*cosh(d*x + c)^4 - 3*(a^2
 + a*b - 12*b^2)*cosh(d*x + c)^2 + a*b - 3*b^2)*sinh(d*x + c)^2 + a^2 - 3*a
*b + 8*((a^2 - 3*a*b)*cosh(d*x + c)^7 + 3*(a*b - 3*b^2)*cosh(d*x + c)^5 - (
a^2 + a*b - 12*b^2)*cosh(d*x + c)^3 + (a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x
 + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*((a^2 - 3*a*b)*cosh(d*x +
 c)^8 + 8*(a^2 - 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 - 3*a*b)*sinh(
d*x + c)^8 + 4*(a*b - 3*b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 - 3*a*b)*cosh(d*x
+ c)^2 + a*b - 3*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 - 3*a*b)*cosh(d*x + c)^3
+ 3*(a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 + a*b - 12*b^2)*c
osh(d*x + c)^4 + 2*(35*(a^2 - 3*a*b)*cosh(d*x + c)^4 + 30*(a*b - 3*b^2)*cos
h(d*x + c)^2 - a^2 - a*b + 12*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 - 3*a*b)*cos
h(d*x + c)^5 + 10*(a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 + a*b - 12*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 + 4*(a*b - 3*b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 -
3*a*b)*cosh(d*x + c)^6 + 15*(a*b - 3*b^2)*cosh(d*x + c)^4 - 3*(a^2 + a*b -
12*b^2)*cosh(d*x + c)^2 + a*b - 3*b^2)*sinh(d*x + c)^2 + a^2 - 3*a*b + 8*((
a^2 - 3*a*b)*cosh(d*x + c)^7 + 3*(a*b - 3*b^2)*cosh(d*x + c)^5 - (a^2 + a*b
 - 12*b^2)*cosh(d*x + c)^3 + (a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*lo
g(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(7*(a^2 - b^2)*cosh(d*x + c)^6 + 5
*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 5*b^2)*cosh(d
*x + c)^2 + a^2 - b^2)*sinh(d*x + c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*
d*cosh(d*x + c)^8 + 8*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*s
inh(d*x + c)^7 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^8 + 4*
(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^4 + 3*a^3*b
 + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^
4)*d)*sinh(d*x + c)^6 - 2*(a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d
*cosh(d*x + c)^4 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)
^3 + 3*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(35*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 + 30*(a^3*b
+ 3*a^2*b^2 + 3*a*b^3 + b^4)*d*cosh(d*x + c)^2 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^4}{\sinh\left(c+d\,x\right)^3\,{\left(a\cosh\left(c+d\,x\right)^2+b\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(cosh(c + d*x)^4/(sinh(c + d*x)^3*(b + a*cosh(c + d*x)^2)^2), x)
```

3.40 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=123

$$-\frac{(3a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}d}+\frac{(a-b)\coth(c+dx)}{(a+b)^3d}-\frac{\coth^3(c+dx)}{3(a+b)^2d}-\frac{ab\tanh(c+dx)}{2(a+b)^3d\left(a+b-b\tanh^2(\right.}$$

```
[Out] (a-b)*coth(d*x+c)/(a+b)^3/d-1/3*coth(d*x+c)^3/(a+b)^2/d-1/2*(3*a-2*b)*arcta
nh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/d-1/2*a*b*tanh(d*x+
c)/(a+b)^3/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4217, 467, 1275, 214}

$$-\frac{\sqrt{b}\,(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{7/2}}-\frac{ab\tanh(c+dx)}{2d(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\coth^3(c+dx)}{3d(a+b)^2}+\frac{(a-b)\coth(c+dx)}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/((a
 + b)^(7/2)*d) + ((a - b)*Coth[c + d*x])/((a + b)^3*d) - Coth[c + d*x]^3/(3
*(a + b)^2*d) - (a*b*Tanh[c + d*x])/(2*(a + b)^3*d*(a + b - b*Tanh[c + d*x]
^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx=\frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4\left(a+b-bx^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=-\frac{ab\tanh(c+dx)}{2(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right)}+\frac{b\operatorname{Subst}\left(\int \frac{\frac{2}{b(a+b)}-\frac{2ax^2}{b(a+b)^2}-\frac{ax^4}{(a+b)^3}}{x^4(a+b-bx^2)}\,dx,x\right.}{2d}$$

$$=-\frac{ab\tanh(c+dx)}{2(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right)}+\frac{b\operatorname{Subst}\left(\int\left(\frac{2}{b(a+b)^2x^4}-\frac{2(a-b)}{b(a+b)^3x^2}+\right.\right.}{}$$

$$=\frac{(a-b)\coth(c+dx)}{(a+b)^3d}-\frac{\coth^3(c+dx)}{3(a+b)^2d}-\frac{ab\tanh(c+dx)}{2(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right.}$$

$$=-\frac{(3a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}d}+\frac{(a-b)\coth(c+dx)}{(a+b)^3d}-\frac{\coth^3(c+}{3(a+b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 295 vs. 2(123) = 246.
time = 3.90, size = 295, normalized size = 2.40

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^4(c+dx)\left(-2(a+b)(a+2b+a\cosh(2(c+dx)))\coth(c)\operatorname{csch}^2(c+dx)-\frac{3(3a-2b)b\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(a+2b+a\cosh(2(c+dx)))(\cosh(2c)-\sinh(2c))}{\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}-4(a-2b)(a+2b+a\cosh(2(c+dx)))\operatorname{csch}(c)\operatorname{csch}(c+dx)\sinh(dx)+2(a+b)(a+2b+a\cosh(2(c+dx)))\operatorname{csch}(c)\operatorname{csch}^3(c+dx)\sinh(dx)-3ab\operatorname{sech}(2c)\sinh(2dx)+3b(a+2b)\tanh(2c)\right)}{24(a+b)^3d\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(-2*(a + b)*(a + 2*b + a*C
osh[2*(c + d*x)])*Coth[c]*Csch[c + d*x]^2 - (3*(3*a - 2*b)*b*ArcTanh[(Sech[
```

```
d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*
Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])
*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) - 4*(
a - 2*b)*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x] +
2*(a + b)*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]^3*Sinh[d*x]
 - 3*a*b*Sech[2*c]*Sinh[2*d*x] + 3*b*(a + 2*b)*Tanh[2*c]))/(24*(a + b)^3*d*
(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(109) = 218$.
time = 2.62, size = 324, normalized size = 2.63

method	result
derivativedivides	$-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+5b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^2+2ab+b^2\right)(a+b)}+\frac{2b\left(\frac{-\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\cdots}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tan\cdots\right.}\cdots\right.}{\cdots}$
default	$-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+5b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^2+2ab+b^2\right)(a+b)}+\frac{2b\left(\frac{-\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\cdots}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tan\cdots\right.}\cdots\right.}{\cdots}$
risch	$-\frac{9ab\,e^{8dx+8c}-6b^2e^{8dx+8c}+12a^2e^{6dx+6c}+18ab\,e^{6dx+6c}+66b^2e^{6dx+6c}+20a^2e^{4dx+4c}+44ab\,e^{4dx+4c}-66b^2e^{4dx+4c}+4a^2e\cdots}{3d(a+b)^3\left(e^{2dx+2c}-1\right)^3\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/(a^2+2*a*b+b^2)/(a+b)*(1/3*a*tanh(1/2*d*x+1/2*c)^3+1/3*b*tanh(1/2
*d*x+1/2*c)^3-3*a*tanh(1/2*d*x+1/2*c)+5*b*tanh(1/2*d*x+1/2*c))+2*b/(a+b)^3*
((-1/2*a*tanh(1/2*d*x+1/2*c)^3-1/2*a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1
/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+
1/2*c)^2+a+b)+1/2*(3*a-2*b)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1
/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b
)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+
(a+b)^(1/2))))-1/24/(a+b)^2/tanh(1/2*d*x+1/2*c)^3-1/8/(a+b)^3*(-3*a+5*b)/ta
nh(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(112) = 224$.

time = 0.57, size = 430, normalized size = 3.50

$$\frac{(3\,ab - 2\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)b}\,d} + \frac{4\,a^2 - 11\,ab - 2\,(2\,a^2 - 9\,ab + 19\,b^2)e^{(-2\,dx-2\,c)} - 2\,(10\,a^2 + 22\,ab - 33\,b^2)e^{(-4\,dx-4\,c)} - 6\,(2\,a^2 + 3\,ab + 11\,b^2)e^{(-6\,dx-6\,c)} - 3\,(3\,ab - 2\,b^2)e^{(-8\,dx-8\,c)}}{3\,(a^4 + 3\,a^3b + 3\,a^2b^2 + ab^3 - (a^4 - a^3b - 9\,a^2b^2 - 11\,ab^3 - 4\,b^4)e^{(-2\,dx-2\,c)} - 2\,(a^4 + 9\,a^3b + 21\,a^2b^2 + 19\,ab^3 + 6\,b^4)e^{(-4\,dx-4\,c)} + 2\,(a^4 + 9\,a^3b + 21\,a^2b^2 + 19\,ab^3 + 6\,b^4)e^{(-6\,dx-6\,c)} + (a^4 - a^3b - 9\,a^2b^2 - 11\,ab^3 - 4\,b^4)e^{(-8\,dx-8\,c)} - (a^4 + 3\,a^3b + 3\,a^2b^2 + ab^3)e^{(-10\,dx-10\,c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(3*a*b - 2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/
(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*sqrt((a + b)*b)*d) + 1/3*(4*a^2 - 11*a*b - 2*(2*a^2 - 9*a*b + 19*
b^2)*e^(-2*d*x - 2*c) - 2*(10*a^2 + 22*a*b - 33*b^2)*e^(-4*d*x - 4*c) - 6*(
2*a^2 + 3*a*b + 11*b^2)*e^(-6*d*x - 6*c) - 3*(3*a*b - 2*b^2)*e^(-8*d*x - 8*
c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b
^3 - 4*b^4)*e^(-2*d*x - 2*c) - 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6
*b^4)*e^(-4*d*x - 4*c) + 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*
e^(-6*d*x - 6*c) + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*e^(-8*d*x -
 8*c) - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-10*d*x - 10*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2933 vs. $2(112) = 224$.
time = 0.43, size = 6143, normalized size = 49.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(12*(3*a*b - 2*b^2)*cosh(d*x + c)^8 + 96*(3*a*b - 2*b^2)*cosh(d*x +
c)*sinh(d*x + c)^7 + 12*(3*a*b - 2*b^2)*sinh(d*x + c)^8 + 24*(2*a^2 + 3*a*b
 + 11*b^2)*cosh(d*x + c)^6 + 24*(14*(3*a*b - 2*b^2)*cosh(d*x + c)^2 + 2*a^2
 + 3*a*b + 11*b^2)*sinh(d*x + c)^6 + 48*(14*(3*a*b - 2*b^2)*cosh(d*x + c)^3
 + 3*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(10*a^2 +
22*a*b - 33*b^2)*cosh(d*x + c)^4 + 8*(105*(3*a*b - 2*b^2)*cosh(d*x + c)^4 +
 45*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c)^2 + 10*a^2 + 22*a*b - 33*b^2)*si
nh(d*x + c)^4 + 32*(21*(3*a*b - 2*b^2)*cosh(d*x + c)^5 + 15*(2*a^2 + 3*a*b
+ 11*b^2)*cosh(d*x + c)^3 + (10*a^2 + 22*a*b - 33*b^2)*cosh(d*x + c))*sinh(
d*x + c)^3 + 8*(2*a^2 - 9*a*b + 19*b^2)*cosh(d*x + c)^2 + 8*(42*(3*a*b - 2*
b^2)*cosh(d*x + c)^6 + 45*(2*a^2 + 3*a*b + 11*b^2)*cosh(d*x + c)^4 + 6*(10*
a^2 + 22*a*b - 33*b^2)*cosh(d*x + c)^2 + 2*a^2 - 9*a*b + 19*b^2)*sinh(d*x +
 c)^2 + 3*((3*a^2 - 2*a*b)*cosh(d*x + c)^10 + 10*(3*a^2 - 2*a*b)*cosh(d*x +
 c)*sinh(d*x + c)^9 + (3*a^2 - 2*a*b)*sinh(d*x + c)^10 - (3*a^2 - 14*a*b +
8*b^2)*cosh(d*x + c)^8 + (45*(3*a^2 - 2*a*b)*cosh(d*x + c)^2 - 3*a^2 + 14*a
*b - 8*b^2)*sinh(d*x + c)^8 + 8*(15*(3*a^2 - 2*a*b)*cosh(d*x + c)^3 - (3*a^
2 - 14*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(3*a^2 + 16*a*b - 12
*b^2)*cosh(d*x + c)^6 + 2*(105*(3*a^2 - 2*a*b)*cosh(d*x + c)^4 - 14*(3*a^2
```

- 14*a*b + 8*b^2)*cosh(d*x + c)^2 - 3*a^2 - 16*a*b + 12*b^2)*sinh(d*x + c)^
6 + 4*(63*(3*a^2 - 2*a*b)*cosh(d*x + c)^5 - 14*(3*a^2 - 14*a*b + 8*b^2)*cos
h(d*x + c)^3 - 3*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 +
2*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^4 + 2*(105*(3*a^2 - 2*a*b)*cosh(
d*x + c)^6 - 35*(3*a^2 - 14*a*b + 8*b^2)*cosh(d*x + c)^4 - 15*(3*a^2 + 16*a
*b - 12*b^2)*cosh(d*x + c)^2 + 3*a^2 + 16*a*b - 12*b^2)*sinh(d*x + c)^4 + 8
(15(3*a^2 - 2*a*b)*cosh(d*x + c)^7 - 7*(3*a^2 - 14*a*b + 8*b^2)*cosh(d*x
+ c)^5 - 5*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^3 + (3*a^2 + 16*a*b - 12
*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 - 14*a*b + 8*b^2)*cosh(d*x +
c)^2 + (45*(3*a^2 - 2*a*b)*cosh(d*x + c)^8 - 28*(3*a^2 - 14*a*b + 8*b^2)*co
sh(d*x + c)^6 - 30*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^4 + 12*(3*a^2 +
16*a*b - 12*b^2)*cosh(d*x + c)^2 + 3*a^2 - 14*a*b + 8*b^2)*sinh(d*x + c)^2
- 3*a^2 + 2*a*b + 2*(5*(3*a^2 - 2*a*b)*cosh(d*x + c)^9 - 4*(3*a^2 - 14*a*b
+ 8*b^2)*cosh(d*x + c)^7 - 6*(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^5 + 4*
(3*a^2 + 16*a*b - 12*b^2)*cosh(d*x + c)^3 + (3*a^2 - 14*a*b + 8*b^2)*cosh(d
*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*co
sh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*
x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 +
8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(
d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sin
h(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a +
b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x +
c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sin
h(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c
) + a)) - 16*a^2 + 44*a*b + 16*(6*(3*a*b - 2*b^2)*cosh(d*x + c)^7 + 9*(2*a^
2 + 3*a*b + 11*b^2)*cosh(d*x + c)^5 + 2*(10*a^2 + 22*a*b - 33*b^2)*cosh(d*x
+ c)^3 + (2*a^2 - 9*a*b + 19*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^10 + 10*(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*d*sinh(d*x + c)^10 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*
cosh(d*x + c)^8 + (45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2
- (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d)*sinh(d*x + c)^8 - 2*(a^4
+ 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^6 + 8*(15*(a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 - (a^4 - a^3*b - 9*a^2*b^2
- 11*a*b^3 - 4*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^4 + 3*a^3
*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 - 14*(a^4 - a^3*b - 9*a^2*b^2 - 1
1*a*b^3 - 4*b^4)*d*cosh(d*x + c)^2 - (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3
+ 6*b^4)*d)*sinh(d*x + c)^6 + 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6
*b^4)*d*cosh(d*x + c)^4 + 4*(63*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(
d*x + c)^5 - 14*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c
)^3 - 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(105*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^
6 - 35*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c)^4 - 15*
(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*cosh(d*x + c)^2 + (a^4 +
9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d)*sinh(d*x + c)^4 + (a^4 - a^3*b

```
- 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*cosh(d*x + c)^2 + 8*(15*(a^4 + 3*a^3*b +
3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^7 - 7*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^
3 - 4*b^4)*d*cosh(d*x + c)^5 - 5*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6
*b^4)*d*cosh(d*x + c)^3 + (a^4 + 9*a^3*b + 21*a...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(112) = 224.
time = 0.63, size = 253, normalized size = 2.06

$$-\frac{\frac{3\,(3\,ab-2\,b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{-ab-b^2}}-\frac{6\,(abe^{(2\,dx+2\,c)}+2\,b^2e^{(2\,dx+2\,c)}+ab)}{(a^3+3\,a^2b+3\,ab^2+b^3)(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a)}+\frac{8\,(3\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}-3\,be^{(2\,dx+2\,c)}-a+2\,b)}{(a^3+3\,a^2b+3\,ab^2+b^3)(e^{(2\,dx+2\,c)}-1)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*(3*a*b - 2*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b
- b^2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a*b - b^2)) - 6*(a*b*e^(2*d*
x + 2*c) + 2*b^2*e^(2*d*x + 2*c) + a*b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(a
*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)) + 8*(3*b
*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) - a + 2*b)/((a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^4}{\sinh\left(c+d\,x\right)^4\,\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(cosh(c + d*x)^4/(sinh(c + d*x)^4*(b + a*cosh(c + d*x)^2)^2), x)
```

3.41 $\int \frac{\sinh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=242

$$\frac{3(a^2+12ab+16b^2)\,x}{8a^5}-\frac{3\sqrt{b}\,(5a^2+20ab+16b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^5\sqrt{a+b}\,d}-\frac{(5a+8b)\cosh(c+dx)\sinh(c+}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

```
[Out] 3/8*(a^2+12*a*b+16*b^2)*x/a^5-3/8*(5*a^2+20*a*b+16*b^2)*arctanh(b^(1/2)*tan
h(d*x+c)/(a+b)^(1/2))*b^(1/2)/a^5/d/(a+b)^(1/2)-1/8*(5*a+8*b)*cosh(d*x+c)*s
inh(d*x+c)/a^2/d/(a+b-b*tanh(d*x+c)^2)^2+1/4*cosh(d*x+c)^3*sinh(d*x+c)/a/d/
(a+b-b*tanh(d*x+c)^2)^2-1/8*b*(7*a+12*b)*tanh(d*x+c)/a^3/d/(a+b-b*tanh(d*x+
c)^2)^2-3/2*b*(a+2*b)*tanh(d*x+c)/a^4/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.28, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4217, 481, 541, 536, 212, 214}

$$-\frac{3b(a+2b)\tanh(c+dx)}{2a^4d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{b(7a+12b)\tanh(c+dx)}{8a^3d\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{(5a+8b)\sinh(c+dx)\cosh(c+dx)}{8a^2d\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{3\sqrt{b}\,(5a^2+20ab+16b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^5d\sqrt{a+b}}+\frac{3x(a^2+12ab+16b^2)}{8a^5}+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4ad\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (3*(a^2 + 12*a*b + 16*b^2)*x)/(8*a^5) - (3*Sqrt[b]*(5*a^2 + 20*a*b + 16*b^2
)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^5*Sqrt[a + b]*d) - ((5
*a + 8*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*a^2*d*(a + b - b*Tanh[c + d*x]^2)
^2) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*a*d*(a + b - b*Tanh[c + d*x]^2)^2)
 - (b*(7*a + 12*b)*Tanh[c + d*x])/(8*a^3*d*(a + b - b*Tanh[c + d*x]^2)^2) -
 (3*b*(a + 2*b)*Tanh[c + d*x])/(2*a^4*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+b+(4a+7b)x^2}{(1-x^2)^2(a+b-bx^2)^3}\,dx, x, \tanh(c+dx\right.}{4ad}$$

$$= -\frac{(5a+8b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Su}}{}$$

$$= -\frac{(5a+8b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{}{8a}$$

$$= -\frac{(5a+8b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{}{8a}$$

$$= -\frac{(5a+8b)\cosh(c+dx)\sinh(c+dx)}{8a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{}{8a}$$

$$= \frac{3(a^2+12ab+16b^2)\,x}{8a^5} - \frac{3\sqrt{b}\,(5a^2+20ab+16b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^5\sqrt{a+b}\,d} -$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3080 vs. $2(242) = 484$.
time = 20.18, size = 3080, normalized size = 12.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (3*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*(((3*a^2 + 8*a*b + 8*b
^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b
]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d
*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(16384*b^(5/2)*d*(a +
 b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6
*((-3*a*(a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/
2) + (Sqrt[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*
b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(
c + d*x)])^2)))/(16384*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3) - (3*(a + 2*b +
 a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2
```

+ 480*a^2*b^3 + 640*a*b^4 + 256*b^5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[
2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos
h[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c]
- Sinh[c])^4]) + (Sech[2*c]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*
Cosh[2*c] + 512*a*b^2*(a + b)^2*(a + 2*b)*d*x*Cosh[2*d*x] + 128*a^4*b^2*d*x
Cosh[2(c + 2*d*x)] + 256*a^3*b^3*d*x*Cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*
x*Cosh[2*(c + 2*d*x)] + 512*a^4*b^2*d*x*Cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*
x*Cosh[4*c + 2*d*x] + 2560*a^2*b^4*d*x*Cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*C
osh[4*c + 2*d*x] + 128*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*Cosh
[6*c + 4*d*x] + 128*a^2*b^4*d*x*Cosh[6*c + 4*d*x] - 9*a^6*Sinh[2*c] + 12*a^
5*b*Sinh[2*c] + 684*a^4*b^2*Sinh[2*c] + 2880*a^3*b^3*Sinh[2*c] + 5280*a^2*b
^4*Sinh[2*c] + 4608*a*b^5*Sinh[2*c] + 1536*b^6*Sinh[2*c] + 9*a^6*Sinh[2*d*x
] - 14*a^5*b*Sinh[2*d*x] - 608*a^4*b^2*Sinh[2*d*x] - 2112*a^3*b^3*Sinh[2*d*
x] - 2560*a^2*b^4*Sinh[2*d*x] - 1024*a*b^5*Sinh[2*d*x] + 3*a^6*Sinh[2*(c +
2*d*x)] - 12*a^5*b*Sinh[2*(c + 2*d*x)] - 204*a^4*b^2*Sinh[2*(c + 2*d*x)] -
384*a^3*b^3*Sinh[2*(c + 2*d*x)] - 192*a^2*b^4*Sinh[2*(c + 2*d*x)] - 3*a^6*S
inh[4*c + 2*d*x] + 10*a^5*b*Sinh[4*c + 2*d*x] + 304*a^4*b^2*Sinh[4*c + 2*d*
x] + 1056*a^3*b^3*Sinh[4*c + 2*d*x] + 1280*a^2*b^4*Sinh[4*c + 2*d*x] + 512*
a*b^5*Sinh[4*c + 2*d*x]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(65536*a^3*b^
2*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])
^3*Sech[c + d*x]^6*((6*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a
^2*b^4 + 3072*a*b^5 + 1024*b^6)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*
((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] -
Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh
[c])^4]) + (Sech[2*c]*(-1536*b^2*(a + b)^2*(3*a^3 + 14*a^2*b + 24*a*b^2 + 1
6*b^3)*d*x*Cosh[2*c] - 3072*a*b^2*(a^2 + 3*a*b + 2*b^2)^2*d*x*Cosh[2*d*x] -
768*a^5*b^2*d*x*Cosh[2*(c + 2*d*x)] - 3072*a^4*b^3*d*x*Cosh[2*(c + 2*d*x)]
- 3840*a^3*b^4*d*x*Cosh[2*(c + 2*d*x)] - 1536*a^2*b^5*d*x*Cosh[2*(c + 2*d*
x)] - 3072*a^5*b^2*d*x*Cosh[4*c + 2*d*x] - 18432*a^4*b^3*d*x*Cosh[4*c + 2*d
*x] - 39936*a^3*b^4*d*x*Cosh[4*c + 2*d*x] - 36864*a^2*b^5*d*x*Cosh[4*c + 2*
d*x] - 12288*a*b^6*d*x*Cosh[4*c + 2*d*x] - 768*a^5*b^2*d*x*Cosh[6*c + 4*d*x
] - 3072*a^4*b^3*d*x*Cosh[6*c + 4*d*x] - 3840*a^3*b^4*d*x*Cosh[6*c + 4*d*x]
- 1536*a^2*b^5*d*x*Cosh[6*c + 4*d*x] + 9*a^7*Sinh[2*c] - 54*a^6*b*Sinh[2*c
] - 2392*a^5*b^2*Sinh[2*c] - 13968*a^4*b^3*Sinh[2*c] - 36480*a^3*b^4*Sinh[2
*c] - 50432*a^2*b^5*Sinh[2*c] - 35840*a*b^6*Sinh[2*c] - 10240*b^7*Sinh[2*c]
- 9*a^7*Sinh[2*d*x] + 56*a^6*b*Sinh[2*d*x] + 2552*a^5*b^2*Sinh[2*d*x] + 13
184*a^4*b^3*Sinh[2*d*x] + 27072*a^3*b^4*Sinh[2*d*x] + 24576*a^2*b^5*Sinh[2*
d*x] + 8192*a*b^6*Sinh[2*d*x] - 3*a^7*Sinh[2*(c + 2*d*x)] + 26*a^6*b*Sinh[2
*(c + 2*d*x)] + 992*a^5*b^2*Sinh[2*(c + 2*d*x)] + 3648*a^4*b^3*Sinh[2*(c +
2*d*x)] + 4480*a^3*b^4*Sinh[2*(c + 2*d*x)] + 1792*a^2*b^5*Sinh[2*(c + 2*d*x
)] + 3*a^7*Sinh[4*c + 2*d*x] - 24*a^6*b*Sinh[4*c + 2*d*x] - 600*a^5*b^2*Sin
h[4*c + 2*d*x] - 3200*a^4*b^3*Sinh[4*c + 2*d*x] - 6720*a^3*b^4*Sinh[4*c + 2
*d*x] - 6144*a^2*b^5*Sinh[4*c + 2*d*x] - 2048*a*b^6*Sinh[4*c + 2*d*x] + 256
*a^5*b^2*Sinh[6*c + 4*d*x] + 1024*a^4*b^3*Sinh[6*c + 4*d*x] + 1280*a^3*b^4*
Sinh[6*c + 4*d*x] + 512*a^2*b^5*Sinh[6*c + 4*d*x] + 64*a^5*b^2*Sinh[4*c + 6

```
*d*x] + 128*a^4*b^3*Sinh[4*c + 6*d*x] + 64*a^3*b^4*Sinh[4*c + 6*d*x] + 64*a
^5*b^2*Sinh[8*c + 6*d*x] + 128*a^4*b^3*Sinh[8*c + 6*d*x] + 64*a^3*b^4*Sinh[
8*c + 6*d*x]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(32768*a^4*b^2*(a + b)^2
*d*(a + b*Sech[c + d*x]^2)^3) - ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c +
 d*x]^6*((6*a^2*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[
d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(
Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (a*Se
ch[2*c]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sinh[2*d*x]
+ a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sinh[2*(c + 2*d*x)] + (3*a^4 - 6
4*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sinh[4*c + 2*d*x]) + (9*a^5 + 18*a^4*b - 64
*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tanh[2*c])/(a^2*(a + 2*b + a*
Cosh[2*(c + d*x)])^2)))/(8192*b^2*(a + b)^2*d*(...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(222) = 444$.
time = 3.27, size = 538, normalized size = 2.22 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^3/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/a^3/(tanh(1/2*d*x+1/2*c)-1)^3-1/
8*(a+12*b)/a^4/(tanh(1/2*d*x+1/2*c)-1)^2-1/8*(3*a+12*b)/a^4/(tanh(1/2*d*x+1
/2*c)-1)+1/8/a^5*(-3*a^2-36*a*b-48*b^2)*ln(tanh(1/2*d*x+1/2*c)-1)+2*b/a^5*(
((-9/8*a^3-21/8*a^2*b-3/2*a*b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(27*a^2+35*a*b-1
2*b^2)*a*tanh(1/2*d*x+1/2*c)^5-1/8*(27*a^2+35*a*b-12*b^2)*a*tanh(1/2*d*x+1/
2*c)^3+(-9/8*a^3-21/8*a^2*b-3/2*a*b^2)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x
+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*
x+1/2*c)^2+a+b)^2+1/8*(15*a^2+60*a*b+48*b^2)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((
a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))
+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*
x+1/2*c)*b^(1/2)+(a+b)^(1/2))))-1/4/a^3/(tanh(1/2*d*x+1/2*c)+1)^4+1/2/a^3/(
tanh(1/2*d*x+1/2*c)+1)^3-1/8*(-a-12*b)/a^4/(tanh(1/2*d*x+1/2*c)+1)^2-1/8*(3
*a+12*b)/a^4/(tanh(1/2*d*x+1/2*c)+1)+1/8/a^5*(3*a^2+36*a*b+48*b^2)*ln(tanh(
1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2468 vs. $2(234) = 468$.
time = 0.62, size = 2468, normalized size = 10.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -3/256*(5*a^4*b + 100*a^3*b^2 + 320*a^2*b^3 + 352*a*b^4 + 128*b^5)*log((a*e
^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b
```

```
+ 2*sqrt((a + b)*b)))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt((a + b)*b)*d) - 3/64*
(5*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a + 2*b
 - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((
a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)*d) + 3/256*(5*a^4*b + 100*a^3*b^2
+ 320*a^2*b^3 + 352*a*b^4 + 128*b^5)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*
sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^7
+ 2*a^6*b + a^5*b^2)*sqrt((a + b)*b)*d) + 3/64*(5*a^3*b + 30*a^2*b^2 + 40*a
*b^3 + 16*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^
(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 2*a^5*b + a^4*b^2)*s
qrt((a + b)*b)*d) + 3/128*(15*a^2*b + 20*a*b^2 + 8*b^3)*log((a*e^(-2*d*x -
2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt(
(a + b)*b)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d) + 1/64*(9*a^5*b
+ 110*a^4*b^2 + 216*a^3*b^3 + 112*a^2*b^4 + (9*a^5*b + 228*a^4*b^2 + 920*a^
3*b^3 + 1216*a^2*b^4 + 512*a*b^5)*e^(6*d*x + 6*c) + (27*a^5*b + 594*a^4*b^2
 + 2816*a^3*b^3 + 5696*a^2*b^4 + 5248*a*b^5 + 1792*b^6)*e^(4*d*x + 4*c) + (
27*a^5*b + 476*a^4*b^2 + 1720*a^3*b^3 + 2176*a^2*b^4 + 896*a*b^5)*e^(2*d*x
+ 2*c))/((a^9 + 2*a^8*b + a^7*b^2 + (a^9 + 2*a^8*b + a^7*b^2)*e^(8*d*x + 8*
c) + 4*(a^9 + 4*a^8*b + 5*a^7*b^2 + 2*a^6*b^3)*e^(6*d*x + 6*c) + 2*(3*a^9 +
 14*a^8*b + 27*a^7*b^2 + 24*a^6*b^3 + 8*a^5*b^4)*e^(4*d*x + 4*c) + 4*(a^9 +
 4*a^8*b + 5*a^7*b^2 + 2*a^6*b^3)*e^(2*d*x + 2*c))*d) - 1/64*(9*a^5*b + 110
*a^4*b^2 + 216*a^3*b^3 + 112*a^2*b^4 + (27*a^5*b + 476*a^4*b^2 + 1720*a^3*b
^3 + 2176*a^2*b^4 + 896*a*b^5)*e^(-2*d*x - 2*c) + (27*a^5*b + 594*a^4*b^2 +
 2816*a^3*b^3 + 5696*a^2*b^4 + 5248*a*b^5 + 1792*b^6)*e^(-4*d*x - 4*c) + (9
*a^5*b + 228*a^4*b^2 + 920*a^3*b^3 + 1216*a^2*b^4 + 512*a*b^5)*e^(-6*d*x -
6*c))/((a^9 + 2*a^8*b + a^7*b^2 + 4*(a^9 + 4*a^8*b + 5*a^7*b^2 + 2*a^6*b^3)
*e^(-2*d*x - 2*c) + 2*(3*a^9 + 14*a^8*b + 27*a^7*b^2 + 24*a^6*b^3 + 8*a^5*b
^4)*e^(-4*d*x - 4*c) + 4*(a^9 + 4*a^8*b + 5*a^7*b^2 + 2*a^6*b^3)*e^(-6*d*x
- 6*c) + (a^9 + 2*a^8*b + a^7*b^2)*e^(-8*d*x - 8*c))*d) + 1/16*(9*a^4*b + 3
2*a^3*b^2 + 20*a^2*b^3 + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b^3 + 32*a*b^4)*e
^(6*d*x + 6*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 + 832*a*b^4 + 320*b^
5)*e^(4*d*x + 4*c) + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b^3 + 160*a*b^4)*e^(
2*d*x + 2*c))/((a^8 + 2*a^7*b + a^6*b^2 + (a^8 + 2*a^7*b + a^6*b^2)*e^(8*d*
x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(6*d*x + 6*c) + 2*(3
*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*e^(4*d*x + 4*c) + 4*
(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(2*d*x + 2*c))*d) - 1/16*(9*a^4*b
 + 32*a^3*b^2 + 20*a^2*b^3 + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b^3 + 160*a*
b^4)*e^(-2*d*x - 2*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 + 832*a*b^4 +
 320*b^5)*e^(-4*d*x - 4*c) + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b^3 + 32*a*b^
4)*e^(-6*d*x - 6*c))/((a^8 + 2*a^7*b + a^6*b^2 + 4*(a^8 + 4*a^7*b + 5*a^6*b
^2 + 2*a^5*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^
5*b^3 + 8*a^4*b^4)*e^(-4*d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*
b^3)*e^(-6*d*x - 6*c) + (a^8 + 2*a^7*b + a^6*b^2)*e^(-8*d*x - 8*c))*d) - 3/
32*(9*a^3*b + 6*a^2*b^2 + (27*a^3*b + 68*a^2*b^2 + 32*a*b^3)*e^(-2*d*x - 2*
c) + 3*(9*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*e^(-4*d*x - 4*c) + (9*a^3
*b + 28*a^2*b^2 + 16*a*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a^5*b^2 + 4
```

```
*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 14*a
^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*(a^7 + 4*a
^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b + a^5*b^2)*
e^(-8*d*x - 8*c))*d) + 3/8*(d*x + c)/(a^3*d) - 1/8*e^(2*d*x + 2*c)/(a^3*d)
+ 1/8*e^(-2*d*x - 2*c)/(a^3*d) + 3/4*b*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*
e^(2*d*x + 2*c) + a)/(a^4*d) - 3/4*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e
^(-4*d*x - 4*c) + a)/(a^4*d) + 1/64*(a*e^(4*d*x + 4*c) - 24*b*e^(2*d*x + 2*
c))/(a^4*d) + 1/64*(24*b*e^(-2*d*x - 2*c) - a*e^(-4*d*x - 4*c))/(a^4*d) + 3
/8*(a*b + 4*b^2)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(
a^5*d) - 3/8*(a*b + 4*b^2)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x -
 4*c) + a)/(a^5*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6038 vs. $2(234) = 468$.
time = 0.58, size = 12353, normalized size = 51.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/64*(a^4*cosh(d*x + c)^16 + 16*a^4*cosh(d*x + c)*sinh(d*x + c)^15 + a^4*s
inh(d*x + c)^16 - 4*(a^4 + 4*a^3*b)*cosh(d*x + c)^14 + 4*(30*a^4*cosh(d*x +
 c)^2 - a^4 - 4*a^3*b)*sinh(d*x + c)^14 + 56*(10*a^4*cosh(d*x + c)^3 - (a^4
 + 4*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^13 - 2*(13*a^4 + 72*a^3*b + 88*a^2
*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^12 + 2*(910*a^4*
cosh(d*x + c)^4 - 13*a^4 - 72*a^3*b - 88*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*
a^2*b^2)*d*x - 182*(a^4 + 4*a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 8*(5
46*a^4*cosh(d*x + c)^5 - 182*(a^4 + 4*a^3*b)*cosh(d*x + c)^3 - 3*(13*a^4 +
72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)
)*sinh(d*x + c)^11 - 4*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4
+ 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^10 + 4*(2002*a^4*cos
h(d*x + c)^6 - 1001*(a^4 + 4*a^3*b)*cosh(d*x + c)^4 - 9*a^4 - 24*a^3*b + 16
*a^2*b^2 + 32*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x - 33*
(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cos
h(d*x + c)^2)*sinh(d*x + c)^10 + 8*(1430*a^4*cosh(d*x + c)^7 - 1001*(a^4 +
4*a^3*b)*cosh(d*x + c)^5 - 55*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 1
2*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^3 - 5*(9*a^4 + 24*a^3*b - 16*a^2*b
^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x +
 c))*sinh(d*x + c)^9 + 16*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3
*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^
8 + 2*(6435*a^4*cosh(d*x + c)^8 - 6006*(a^4 + 4*a^3*b)*cosh(d*x + c)^6 - 49
5*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*c
osh(d*x + c)^4 + 216*a^3*b + 912*a^2*b^2 + 1472*a*b^3 + 896*b^4 + 24*(3*a^4
 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x - 90*(9*a^4 + 24*a^3*b
```

```
 - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)
*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(715*a^4*cosh(d*x + c)^9 - 858*(a^4
+ 4*a^3*b)*cosh(d*x + c)^7 - 99*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 +
 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^5 - 30*(9*a^4 + 24*a^3*b - 16*a^
2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*
x + c)^3 + 8*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*
a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c
)^7 + 4*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b +
 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^6 + 4*(2002*a^4*cosh(d*x + c)^10
 - 3003*(a^4 + 4*a^3*b)*cosh(d*x + c)^8 - 462*(13*a^4 + 72*a^3*b + 88*a^2*b
^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^6 - 210*(9*a^4 + 2
4*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^
3)*d*x)*cosh(d*x + c)^4 + 9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*
(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x + 112*(27*a^3*b + 114*a^2*b^2
+ 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128
*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(546*a^4*cosh(d*x + c)^11 -
 1001*(a^4 + 4*a^3*b)*cosh(d*x + c)^9 - 198*(13*a^4 + 72*a^3*b + 88*a^2*b^2
 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^7 - 126*(9*a^4 + 24*
a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)
*d*x)*cosh(d*x + c)^5 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 +
 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c
)^3 + 3*(9*a^4 + 168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b +
 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(13*a^4 + 1
44*a^3*b + 200*a^2*b^2 + 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c
)^4 + 2*(910*a^4*cosh(d*x + c)^12 - 2002*(a^4 + 4*a^3*b)*cosh(d*x + c)^10 -
 495*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x
)*cosh(d*x + c)^8 - 420*(9*a^4 + 24*a^3*b - 16*a^2*b^2 - 32*a*b^3 - 24*(a^4
 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c)^6 + 560*(27*a^3*b +
 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3*b + 152*a^2*b^2 + 22
4*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^4 + 13*a^4 + 144*a^3*b + 200*a^2*b^2
+ 12*(a^4 + 12*a^3*b + 16*a^2*b^2)*d*x + 30*(9*a^4 + 168*a^3*b + 496*a^2*b^
2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x +
 c)^2)*sinh(d*x + c)^4 - a^4 + 8*(70*a^4*cosh(d*x + c)^13 - 182*(a^4 + 4*a^
3*b)*cosh(d*x + c)^11 - 55*(13*a^4 + 72*a^3*b + 88*a^2*b^2 - 12*(a^4 + 12*a
^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c)^9 - 60*(9*a^4 + 24*a^3*b - 16*a^2*b^2
 - 32*a*b^3 - 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*a*b^3)*d*x)*cosh(d*x + c
)^7 + 112*(27*a^3*b + 114*a^2*b^2 + 184*a*b^3 + 112*b^4 + 3*(3*a^4 + 44*a^3
*b + 152*a^2*b^2 + 224*a*b^3 + 128*b^4)*d*x)*cosh(d*x + c)^5 + 10*(9*a^4 +
168*a^3*b + 496*a^2*b^2 + 416*a*b^3 + 24*(a^4 + 14*a^3*b + 40*a^2*b^2 + 32*
a*b^3)*d*x)*cosh(d*x + c)^3 + (13*a^4 + 144*a^3*b + 200*a^2*b^2 + 12*(a^4 +
 12*a^3*b + 16*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 4*a^
3*b)*cosh(d*x + c)^2 + 4*(30*a^4*cosh(d*x + c)^14 - 91*(a^4 + 4*a^3*b)*cosh
(d*x + c)^12 - 33*(13*a^4 + 72*a^3*b + 88*a^2*b...
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(234) = 468.
time = 1.75, size = 518, normalized size = 2.14

$$\frac{\frac{24\left(a^{2}+12\,ab+16\,b^{2}\right)(dx+c)}{a^{5}}-\frac{24\left(5\,a^{2}b+20\,ab^{2}+16\,b^{3}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\sqrt{-ab-b^{2}}}\right)}{\sqrt{-ab-b^{2}}\,a^{5}}+\frac{a^{3}e^{(4\,dx+4\,c)}-8\,a^{3}e^{(2\,dx+2\,c)}-24\,a^{2}be^{(2\,dx+2\,c)}}{a^{6}}-\frac{6\,a^{4}e^{(12\,dx+12\,c)}+72\,a^{3}be^{(12\,dx+12\,c)}+96\,a^{2}b^{2}e^{(12\,dx+12\,c)}+16\,a^{4}e^{(10\,dx+10\,c)}+168\,a^{3}be^{(10\,dx+10\,c)}+384\,a^{2}b^{2}e^{(10\,dx+10\,c)}+256\,ab^{3}e^{(10\,dx+10\,c)}+5\,a^{4}e^{(8\,dx+8\,c)}-64\,a^{3}be^{(8\,dx+8\,c)}-192\,a^{2}b^{2}e^{(8\,dx+8\,c)}-256\,ab^{3}e^{(8\,dx+8\,c)}-256\,b^{4}e^{(8\,dx+8\,c)}-20\,a^{4}e^{(6\,dx+6\,c)}-360\,a^{3}be^{(6\,dx+6\,c)}-1024\,a^{2}b^{2}e^{(6\,dx+6\,c)}-896\,ab^{3}e^{(6\,dx+6\,c)}-20\,a^{4}e^{(4\,dx+4\,c)}-216\,a^{3}be^{(4\,dx+4\,c)}-304\,a^{2}b^{2}e^{(4\,dx+4\,c)}-4\,a^{4}e^{(2\,dx+2\,c)}-16\,a^{3}be^{(2\,dx+2\,c)}+a^{4}}{\left(ae^{(6\,dx+6\,c)}+2\,ae^{(4\,dx+4\,c)}+4\,be^{(4\,dx+4\,c)}+ae^{(2\,dx+2\,c)}\right)^{2}a^{5}}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(24*(a^2 + 12*a*b + 16*b^2)*(d*x + c)/a^5 - 24*(5*a^2*b + 20*a*b^2 + 1
6*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*
b - b^2)*a^5) + (a^3*e^(4*d*x + 4*c) - 8*a^3*e^(2*d*x + 2*c) - 24*a^2*b*e^(
2*d*x + 2*c))/a^6 - (6*a^4*e^(12*d*x + 12*c) + 72*a^3*b*e^(12*d*x + 12*c) +
 96*a^2*b^2*e^(12*d*x + 12*c) + 16*a^4*e^(10*d*x + 10*c) + 168*a^3*b*e^(10*
d*x + 10*c) + 384*a^2*b^2*e^(10*d*x + 10*c) + 256*a*b^3*e^(10*d*x + 10*c) +
 5*a^4*e^(8*d*x + 8*c) - 64*a^3*b*e^(8*d*x + 8*c) - 192*a^2*b^2*e^(8*d*x +
8*c) - 256*a*b^3*e^(8*d*x + 8*c) - 256*b^4*e^(8*d*x + 8*c) - 20*a^4*e^(6*d*
x + 6*c) - 360*a^3*b*e^(6*d*x + 6*c) - 1024*a^2*b^2*e^(6*d*x + 6*c) - 896*a
*b^3*e^(6*d*x + 6*c) - 20*a^4*e^(4*d*x + 4*c) - 216*a^3*b*e^(4*d*x + 4*c) -
 304*a^2*b^2*e^(4*d*x + 4*c) - 4*a^4*e^(2*d*x + 2*c) - 16*a^3*b*e^(2*d*x +
2*c) + a^4)/((a*e^(6*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) + 4*b*e^(4*d*x + 4*c)
 + a*e^(2*d*x + 2*c))^2*a^5))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^6\,{\mathrm{sinh}\left(c+d\,x\right)}^4}{{\left(a\,{\mathrm{cosh}\left(c+d\,x\right)}^2+b\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*sinh(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.42 $\int \frac{\sinh^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=154

$$\frac{5\sqrt{b}\,(3a+7b)\text{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{9/2}d}-\frac{(a+3b)\cosh(c+dx)}{a^4d}+\frac{\cosh^3(c+dx)}{3a^3d}+\frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(b+a\cosh^2(c+dx)\right)}$$

```
[Out] -(a+3*b)*cosh(d*x+c)/a^4/d+1/3*cosh(d*x+c)^3/a^3/d+1/4*b^2*(a+b)*cosh(d*x+c
)/a^4/d/(b+a*cosh(d*x+c)^2)^2-1/8*b*(9*a+13*b)*cosh(d*x+c)/a^4/d/(b+a*cosh(
d*x+c)^2)+5/8*(3*a+7*b)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(9/2)
/d
```

Rubi [A]

time = 0.16, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4218, 466, 1828, 1167, 211}

$$\frac{5\sqrt{b}\,(3a+7b)\text{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{9/2}d}+\frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(a\cosh^2(c+dx)+b\right)^2}-\frac{b(9a+13b)\cosh(c+dx)}{8a^4d\left(a\cosh^2(c+dx)+b\right)}-\frac{(a+3b)\cosh(c+dx)}{a^4d}+\frac{\cosh^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (5*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(8*a^(9/2)*
d) - ((a + 3*b)*Cosh[c + d*x])/(a^4*d) + Cosh[c + d*x]^3/(3*a^3*d) + (b^2*(
a + b)*Cosh[c + d*x])/(4*a^4*d*(b + a*Cosh[c + d*x]^2)^2) - (b*(9*a + 13*b)
*Cosh[c + d*x])/(8*a^4*d*(b + a*Cosh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^6(1-x^2)}{(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{-b^2(a+b)+4ab(a+b)x^2-4a^2(a+b)x^4+4a^3x^6}{(b+ax^2)^2}\,dx,\right.}{4a^4d}$$

$$= \frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(9a+13b)\cosh(c+dx)}{8a^4d\left(b+a\cosh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{-b^2(7a+}{}\right.}{}$$

$$= \frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(9a+13b)\cosh(c+dx)}{8a^4d\left(b+a\cosh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \left(8b(a+\right.\right.}{}$$

$$= -\frac{(a+3b)\cosh(c+dx)}{a^4d} + \frac{\cosh^3(c+dx)}{3a^3d} + \frac{b^2(a+b)\cosh(c+dx)}{4a^4d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(}{8a}$$

$$= \frac{5\sqrt{b}\,(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{9/2}d} - \frac{(a+3b)\cosh(c+dx)}{a^4d} + \frac{\cosh^3(c+d}{3a^3d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.06, size = 1217, normalized size = 7.90

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((24*(3*a - 4*b)*(ArcTan
[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/
2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x
)/2]))/Sqrt[b]] + ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])
^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c]
 - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]))/(a^(3/2)*b^(5/2)) - (54*(ArcTan[(
Sqrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[(Sqrt[a] + I*S
qrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]]))/(Sqrt[a]*b^(5/2)) - (36*Cosh[c + d
*x]*(3*a + 10*b + 3*a*Cosh[2*(c + d*x)]))/(b^2*(a + 2*b + a*Cosh[2*(c + d*x
)])^2) + (48*Cosh[c + d*x]*(3*a^2 + 6*a*b + 8*b^2 + a*(3*a - 4*b)*Cosh[2*(c
 + d*x)]))/(a*b^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) + (3*(3*a^4 - 40*a^3*b
 + 720*a^2*b^2 + 6720*a*b^3 + 8960*b^4)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sq
rt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqr
t[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + 3*(3*a^4 -
40*a^3*b + 720*a^2*b^2 + 6720*a*b^3 + 8960*b^4)*ArcTan[((Sqrt[a] + I*Sqrt[a
 + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a]
 + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] + (2*
Sqrt[a]*Sqrt[b]*Cosh[c + d*x]*(9*a^5 - 90*a^4*b - 10144*a^3*b^2 - 48672*a^2
*b^3 - 85120*a*b^4 - 53760*b^5 + a*(9*a^4 - 120*a^3*b - 12432*a^2*b^2 - 479
36*a*b^3 - 44800*b^4)*Cosh[2*(c + d*x)] - 128*a^2*b^2*(15*a + 28*b)*Cosh[4*
(c + d*x)] + 128*a^3*b^2*Cosh[6*(c + d*x)]))/(a + 2*b + a*Cosh[2*(c + d*x)]
)^2)/(a^(9/2)*b^(5/2)) + (9*((-3*(a^3 - 8*a^2*b + 80*a*b^2 + 320*b^3)*ArcTa
n[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)
/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*
x)/2]))/Sqrt[b]])/b^(5/2) - (3*(a^3 - 8*a^2*b + 80*a*b^2 + 320*b^3)*ArcTan[
((Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2
] + Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)
/2]))/Sqrt[b]])/b^(5/2) + 512*Sqrt[a]*Cosh[c]*Cosh[d*x] - (8*Sqrt[a]*(a^3 +
 24*a^2*b + 80*a*b^2 + 64*b^3)*Cosh[c + d*x])/(b*(a + 2*b + a*Cosh[2*(c + d
*x)])^2) - (2*Sqrt[a]*(3*a^3 - 24*a^2*b - 400*a*b^2 - 576*b^3)*Cosh[c + d*x
])/(b^2*(a + 2*b + a*Cosh[2*(c + d*x)])) + 512*Sqrt[a]*Sinh[c]*Sinh[d*x]))/
a^(7/2)))/(49152*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(138) = 276$.
time = 2.34, size = 348, normalized size = 2.26

method	result
derivativedivides	$\frac{2b\left(\frac{\left(-\frac{9}{8}a^2+\frac{1}{4}ab+\frac{11}{8}b^2\right)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\left(27a^3+15a^2b+5a\,b^2+33b^3\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}+\left(-\frac{27}{8}a^2-\frac{5}{4}ab+\frac{33}{8}b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}\right.}{a^4}$
default	$\frac{2b\left(\frac{\left(-\frac{9}{8}a^2+\frac{1}{4}ab+\frac{11}{8}b^2\right)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\left(27a^3+15a^2b+5a\,b^2+33b^3\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}+\left(-\frac{27}{8}a^2-\frac{5}{4}ab+\frac{33}{8}b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}\right.}{a^4}$
risch	$\frac{e^{3dx+3c}}{24a^3d}-\frac{3\,e^{dx+c}}{8a^3d}-\frac{3\,e^{dx+c}b}{2a^4d}-\frac{3\,e^{-dx-c}}{8a^3d}-\frac{3\,e^{-dx-c}b}{2a^4d}+\frac{e^{-3dx-3c}}{24a^3d}-\frac{e^{dx+c}b\left(9a^2e^{6dx+6c}+13ab\,e^{6dx+6c}+27a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/a^4*(((-9/8*a^2+1/4*a*b+11/8*b^2)*tanh(1/2*d*x+1/2*c)^6-1/8*(27*a^
3+15*a^2*b+5*a*b^2+33*b^3)/(a+b)*tanh(1/2*d*x+1/2*c)^4+(-27/8*a^2-5/4*a*b+3
3/8*b^2)*tanh(1/2*d*x+1/2*c)^2-9/8*a^2-5/2*a*b-11/8*b^2)/(a*tanh(1/2*d*x+1/
2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1
/2*c)^2+a+b)^2+5/16*(3*a+7*b)/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+
1/2*c)^2+2*a-2*b)/(a*b)^(1/2)))-1/3/a^3/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/a^3/(
tanh(1/2*d*x+1/2*c)-1)^2-1/2/a^4*(-a-6*b)/(tanh(1/2*d*x+1/2*c)-1)+1/3/a^3/(
tanh(1/2*d*x+1/2*c)+1)^3-1/2/a^3/(tanh(1/2*d*x+1/2*c)+1)^2-1/2*(a+6*b)/a^4/
(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4793 vs. 2(138) = 276.
time = 0.45, size = 8667, normalized size = 56.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/48*(2*a^3*cosh(d*x + c)^14 + 28*a^3*cosh(d*x + c)*sinh(d*x + c)^13 + 2*a^3*sinh(d*x + c)^14 - 2*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^12 + 2*(91*a^3*cosh(d*x + c)^2 - 5*a^3 - 28*a^2*b)*sinh(d*x + c)^12 + 8*(91*a^3*cosh(d*x + c)^3 - 3*(5*a^3 + 28*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^11 - 2*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^10 + 2*(1001*a^3*cosh(d*x + c)^4 - 39*a^3 - 290*a^2*b - 350*a*b^2 - 66*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(1001*a^3*cosh(d*x + c)^5 - 110*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^3 - 5*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 10*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^8 + 2*(3003*a^3*cosh(d*x + c)^6 - 495*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^4 - 85*a^3 - 730*a^2*b - 1410*a*b^2 - 840*b^3 - 45*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(429*a^3*cosh(d*x + c)^7 - 99*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^5 - 15*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^3 - 5*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 10*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^6 + 2*(3003*a^3*cosh(d*x + c)^8 - 924*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^6 - 210*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^4 - 85*a^3 - 730*a^2*b - 1410*a*b^2 - 840*b^3 - 140*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(1001*a^3*cosh(d*x + c)^9 - 396*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^7 - 126*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^5 - 140*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^3 - 15*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^4 + 2*(1001*a^3*cosh(d*x + c)^10 - 495*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^8 - 210*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^6 - 350*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^4 - 39*a^3 - 290*a^2*b - 350*a*b^2 - 75*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(91*a^3*cosh(d*x + c)^11 - 55*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^9 - 30*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^7 - 70*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^5 - 25*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^3 - (39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*a^3 - 2*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^2 + 2*(91*a^3*cosh(d*x + c)^12 - 66*(5*a^3 + 28*a^2*b)*cosh(d*x + c)^10 - 45*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^8 - 140*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^6 - 75*(17*a^3 + 146*a^2*b + 282*a*b^2 + 168*b^3)*cosh(d*x + c)^4 - 5*a^3 - 28*a^2*b - 6*(39*a^3 + 290*a^2*b + 350*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 15*((3*a^3 + 7*a^2*b)*cosh(d*x + c)^11 + 11*(3*a^3 + 7*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^10 + (3*a^3 + 7*a^2*b)*sinh(d*x + c)^11 + 4*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^9 + (12*a^3 + 52*a^2*b + 56*a*b^2 + 55*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 3*(55*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^3 + 12*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c))*sinh(d*x + c)

```
^8 + 2*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^7 + 2*(165*(3*a
^3 + 7*a^2*b)*cosh(d*x + c)^4 + 9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3 + 72*(
3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 14*(33*(3*a
^3 + 7*a^2*b)*cosh(d*x + c)^5 + 24*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x +
 c)^3 + (9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c))*sinh(d*x + c)
^6 + 4*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^5 + 2*(231*(3*a^3 + 7*a^
2*b)*cosh(d*x + c)^6 + 252*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^4 +
6*a^3 + 26*a^2*b + 28*a*b^2 + 21*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^5 + 2*(165*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^7 +
252*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^5 + 35*(9*a^3 + 45*a^2*b +
80*a*b^2 + 56*b^3)*cosh(d*x + c)^3 + 10*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(
d*x + c))*sinh(d*x + c)^4 + (3*a^3 + 7*a^2*b)*cosh(d*x + c)^3 + (165*(3*a^3
 + 7*a^2*b)*cosh(d*x + c)^8 + 336*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x +
c)^6 + 70*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)^4 + 3*a^3 +
7*a^2*b + 40*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3
 + (55*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^9 + 144*(3*a^3 + 13*a^2*b + 14*a*b^2
)*cosh(d*x + c)^7 + 42*(9*a^3 + 45*a^2*b + 80*a*b^2 + 56*b^3)*cosh(d*x + c)
^5 + 40*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^3 + 3*(3*a^3 + 7*a^2*b)
*cosh(d*x + c))*sinh(d*x + c)^2 + (11*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^10 +
36*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d*x + c)^8 + 14*(9*a^3 + 45*a^2*b + 8
0*a*b^2 + 56*b^3)*cosh(d*x + c)^6 + 20*(3*a^3 + 13*a^2*b + 14*a*b^2)*cosh(d
*x + c)^4 + 3*(3*a^3 + 7*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-b/a)*
log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c
)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(
d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c)
+ 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^6\sinh(c+d\,x)^3}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*sinh(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.43 $\int \frac{\sinh^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=187

$$-\frac{(a+6b)x}{2a^4}+\frac{\sqrt{b}\,(15a^2+40ab+24b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2}+\frac{3b}{4a^2d\,(a+}$$

```
[Out] -1/2*(a+6*b)*x/a^4+1/8*(15*a^2+40*a*b+24*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(
a+b)^(1/2))*b^(1/2)/a^4/(a+b)^(3/2)/d+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d/(a+b-
b*tanh(d*x+c)^2)^2+3/4*b*tanh(d*x+c)/a^2/d/(a+b-b*tanh(d*x+c)^2)^2+1/8*b*(1
1*a+12*b)*tanh(d*x+c)/a^3/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.21, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$,
Rules used = {4217, 482, 541, 536, 212, 214}

$$-\frac{x(a+6b)}{2a^4}+\frac{b(11a+12b)\tanh(c+dx)}{8a^3d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}+\frac{3b\tanh(c+dx)}{4a^2d\left(a-b\tanh^2(c+dx)+b\right)^2}+\frac{\sqrt{b}\,(15a^2+40ab+24b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4d(a+b)^{3/2}}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/2*((a + 6*b)*x)/a^4 + (Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTanh[(Sqrt[
b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(3/2)*d) + (Cosh[c + d*x]*Si
nh[c + d*x])/(2*a*d*(a + b - b*Tanh[c + d*x]^2)^2) + (3*b*Tanh[c + d*x])/(4
*a^2*d*(a + b - b*Tanh[c + d*x]^2)^2) + (b*(11*a + 12*b)*Tanh[c + d*x])/(8*
a^3*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+b+5bx^2}{(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right.}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3b\tanh(c+dx)}{4a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3b\tanh(c+dx)}{4a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(1}{8a^3(a+}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3b\tanh(c+dx)}{4a^2d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(1}{8a^3(a+}$$

$$= -\frac{(a+6b)x}{2a^4} + \frac{\sqrt{b}\,(15a^2+40ab+24b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}d} + \frac{\cosh(c}{2ad\,(a+}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2544 vs. $2(187) = 374$.
time = 15.67, size = 2544, normalized size = 13.60

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-5*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*(((3*a^2 + 8*a*b + 8*
b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[
b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*(c +
d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(8192*b^(5/2)*d*(a +
 b*Sech[c + d*x]^2)^3) - ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6
*((-3*a*(a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/
2) + (Sqrt[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*
b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(
c + d*x)])^2)))/(2048*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*
Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^6*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 +
480*a^2*b^3 + 640*a*b^4 + 256*b^5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c
])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c
] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - S
```

inh[c])^4]) + (Sech[2*c]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*Cos
h[2*c] + 512*a*b^2*(a + b)^2*(a + 2*b)*d*x*Cosh[2*d*x] + 128*a^4*b^2*d*x*Co
sh[2*(c + 2*d*x)] + 256*a^3*b^3*d*x*Cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*x*C
osh[2*(c + 2*d*x)] + 512*a^4*b^2*d*x*Cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*x*C
osh[4*c + 2*d*x] + 2560*a^2*b^4*d*x*Cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*Cosh
[4*c + 2*d*x] + 128*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*Cosh[6*
c + 4*d*x] + 128*a^2*b^4*d*x*Cosh[6*c + 4*d*x] - 9*a^6*Sinh[2*c] + 12*a^5*b
*Sinh[2*c] + 684*a^4*b^2*Sinh[2*c] + 2880*a^3*b^3*Sinh[2*c] + 5280*a^2*b^4*
Sinh[2*c] + 4608*a*b^5*Sinh[2*c] + 1536*b^6*Sinh[2*c] + 9*a^6*Sinh[2*d*x] -
14*a^5*b*Sinh[2*d*x] - 608*a^4*b^2*Sinh[2*d*x] - 2112*a^3*b^3*Sinh[2*d*x]
- 2560*a^2*b^4*Sinh[2*d*x] - 1024*a*b^5*Sinh[2*d*x] + 3*a^6*Sinh[2*(c + 2*d
*x)] - 12*a^5*b*Sinh[2*(c + 2*d*x)] - 204*a^4*b^2*Sinh[2*(c + 2*d*x)] - 384
*a^3*b^3*Sinh[2*(c + 2*d*x)] - 192*a^2*b^4*Sinh[2*(c + 2*d*x)] - 3*a^6*Sinh
[4*c + 2*d*x] + 10*a^5*b*Sinh[4*c + 2*d*x] + 304*a^4*b^2*Sinh[4*c + 2*d*x]
+ 1056*a^3*b^3*Sinh[4*c + 2*d*x] + 1280*a^2*b^4*Sinh[4*c + 2*d*x] + 512*a*b
^5*Sinh[4*c + 2*d*x]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(4096*a^3*b^2*(a
+ b)^2*d*(a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*S
ech[c + d*x]^6*((6*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b
^4 + 3072*a*b^5 + 1024*b^6)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a
+ 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sin
h[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])
^4]) + (Sech[2*c]*(-1536*b^2*(a + b)^2*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^
3)*d*x*Cosh[2*c] - 3072*a*b^2*(a^2 + 3*a*b + 2*b^2)^2*d*x*Cosh[2*d*x] - 768
*a^5*b^2*d*x*Cosh[2*(c + 2*d*x)] - 3072*a^4*b^3*d*x*Cosh[2*(c + 2*d*x)] - 3
840*a^3*b^4*d*x*Cosh[2*(c + 2*d*x)] - 1536*a^2*b^5*d*x*Cosh[2*(c + 2*d*x)]
- 3072*a^5*b^2*d*x*Cosh[4*c + 2*d*x] - 18432*a^4*b^3*d*x*Cosh[4*c + 2*d*x]
- 39936*a^3*b^4*d*x*Cosh[4*c + 2*d*x] - 36864*a^2*b^5*d*x*Cosh[4*c + 2*d*x]
- 12288*a*b^6*d*x*Cosh[4*c + 2*d*x] - 768*a^5*b^2*d*x*Cosh[6*c + 4*d*x] -
3072*a^4*b^3*d*x*Cosh[6*c + 4*d*x] - 3840*a^3*b^4*d*x*Cosh[6*c + 4*d*x] - 1
536*a^2*b^5*d*x*Cosh[6*c + 4*d*x] + 9*a^7*Sinh[2*c] - 54*a^6*b*Sinh[2*c] -
2392*a^5*b^2*Sinh[2*c] - 13968*a^4*b^3*Sinh[2*c] - 36480*a^3*b^4*Sinh[2*c]
- 50432*a^2*b^5*Sinh[2*c] - 35840*a*b^6*Sinh[2*c] - 10240*b^7*Sinh[2*c] - 9
*a^7*Sinh[2*d*x] + 56*a^6*b*Sinh[2*d*x] + 2552*a^5*b^2*Sinh[2*d*x] + 13184*
a^4*b^3*Sinh[2*d*x] + 27072*a^3*b^4*Sinh[2*d*x] + 24576*a^2*b^5*Sinh[2*d*x]
+ 8192*a*b^6*Sinh[2*d*x] - 3*a^7*Sinh[2*(c + 2*d*x)] + 26*a^6*b*Sinh[2*(c
+ 2*d*x)] + 992*a^5*b^2*Sinh[2*(c + 2*d*x)] + 3648*a^4*b^3*Sinh[2*(c + 2*d*
x)] + 4480*a^3*b^4*Sinh[2*(c + 2*d*x)] + 1792*a^2*b^5*Sinh[2*(c + 2*d*x)] +
3*a^7*Sinh[4*c + 2*d*x] - 24*a^6*b*Sinh[4*c + 2*d*x] - 600*a^5*b^2*Sinh[4*
c + 2*d*x] - 3200*a^4*b^3*Sinh[4*c + 2*d*x] - 6720*a^3*b^4*Sinh[4*c + 2*d*x
] - 6144*a^2*b^5*Sinh[4*c + 2*d*x] - 2048*a*b^6*Sinh[4*c + 2*d*x] + 256*a^5
*b^2*Sinh[6*c + 4*d*x] + 1024*a^4*b^3*Sinh[6*c + 4*d*x] + 1280*a^3*b^4*Sinh
[6*c + 4*d*x] + 512*a^2*b^5*Sinh[6*c + 4*d*x] + 64*a^5*b^2*Sinh[4*c + 6*d*x
] + 128*a^4*b^3*Sinh[4*c + 6*d*x] + 64*a^3*b^4*Sinh[4*c + 6*d*x] + 64*a^5*b
^2*Sinh[8*c + 6*d*x] + 128*a^4*b^3*Sinh[8*c + 6*d*x] + 64*a^3*b^4*Sinh[8*c
+ 6*d*x]))/(a + 2*b + a*Cosh[2*(c + d*x)])^2))/(16384*a^4*b^2*(a + b)^2*d*(

```
a + b*Sech[c + d*x]^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x
]^6*((6*a^2*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x]
 - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh
[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (a*Sech[2
*c]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sinh[2*d*x] + a*
(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sinh[2*(c + 2*d*x)] + (3*a^4 - 64*a^
2*b^2 - 128*a*b^3 - 64*b^4)*Sinh[4*c + 2*d*x]) + (9*a^5 + 18*a^4*b - 64*a^3
*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tanh[2*c])/(a^2*(a + 2*b + a*Cosh
[2*(c + d*x)])^2)))/(4096*b^2*(a + b)^2*d*(a + ...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(169) = 338$.
time = 2.35, size = 421, normalized size = 2.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^3/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/a^3/(tanh(1/2*d*x+1/2*c)+1)+1/2
/a^4*(-a-6*b)*ln(tanh(1/2*d*x+1/2*c)+1)-2*b/a^4*(((-9/8*a^2-a*b)*tanh(1/2*d
*x+1/2*c)^7-1/8*a*(27*a^2+23*a*b-8*b^2)/(a+b)*tanh(1/2*d*x+1/2*c)^5-1/8*a*(
27*a^2+23*a*b-8*b^2)/(a+b)*tanh(1/2*d*x+1/2*c)^3+(-9/8*a^2-a*b)*tanh(1/2*d*
x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x
+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)^2+1/8*(15*a^2+40*a*b+24*b^2)/(a+b)
*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*
d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh
(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))+1/2/a^3/(tan
h(1/2*d*x+1/2*c)-1)^2+1/2/a^3/(tanh(1/2*d*x+1/2*c)-1)+1/2*(a+6*b)/a^4*ln(ta
nh(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. $2(178) = 356$.
time = 0.58, size = 1373, normalized size = 7.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3/64*(5*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a
+ 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)
))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)*d) - 3/64*(5*a^3*b + 30*a^2*b
^2 + 40*a*b^3 + 16*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*
b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 2*a^5*b + a
^4*b^2)*sqrt((a + b)*b)*d) - 1/32*(15*a^2*b + 20*a*b^2 + 8*b^3)*log((a*e^(-
2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b +
 2*sqrt((a + b)*b)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d) - 1/16*(
```

```
9*a^4*b + 32*a^3*b^2 + 20*a^2*b^3 + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b^3 +
32*a*b^4)*e^(6*d*x + 6*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 + 832*a*b
^4 + 320*b^5)*e^(4*d*x + 4*c) + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b^3 + 160
*a*b^4)*e^(2*d*x + 2*c))/((a^8 + 2*a^7*b + a^6*b^2 + (a^8 + 2*a^7*b + a^6*b
^2)*e^(8*d*x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(6*d*x +
6*c) + 2*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*e^(4*d*x
+ 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(2*d*x + 2*c))*d) + 1/
16*(9*a^4*b + 32*a^3*b^2 + 20*a^2*b^3 + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b
^3 + 160*a*b^4)*e^(-2*d*x - 2*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 +
832*a*b^4 + 320*b^5)*e^(-4*d*x - 4*c) + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b^
3 + 32*a*b^4)*e^(-6*d*x - 6*c))/((a^8 + 2*a^7*b + a^6*b^2 + 4*(a^8 + 4*a^7*
b + 5*a^6*b^2 + 2*a^5*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^8 + 14*a^7*b + 27*a^6*
b^2 + 24*a^5*b^3 + 8*a^4*b^4)*e^(-4*d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b
^2 + 2*a^5*b^3)*e^(-6*d*x - 6*c) + (a^8 + 2*a^7*b + a^6*b^2)*e^(-8*d*x - 8*
c))*d) + 1/8*(9*a^3*b + 6*a^2*b^2 + (27*a^3*b + 68*a^2*b^2 + 32*a*b^3)*e^(-
2*d*x - 2*c) + 3*(9*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*e^(-4*d*x - 4*c
) + (9*a^3*b + 28*a^2*b^2 + 16*a*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a
^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*
a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*
(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b +
 a^5*b^2)*e^(-8*d*x - 8*c))*d) - 1/2*(d*x + c)/(a^3*d) + 1/8*e^(2*d*x + 2*c
)/(a^3*d) - 1/8*e^(-2*d*x - 2*c)/(a^3*d) - 3/4*b*log(a*e^(4*d*x + 4*c) + 2*
(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^4*d) + 3/4*b*log(2*(a + 2*b)*e^(-2*d*x -
2*c) + a*e^(-4*d*x - 4*c) + a)/(a^4*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4724 vs. $2(178) = 356$.
time = 0.49, size = 9730, normalized size = 52.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(a^4 + a^3*b)*cosh(d*x + c)^12 + 24*(a^4 + a^3*b)*cosh(d*x + c)*si
nh(d*x + c)^11 + 2*(a^4 + a^3*b)*sinh(d*x + c)^12 + 8*(a^4 + 3*a^3*b + 2*a^
2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^10 + 4*(2*a^4 + 6*a^
3*b + 4*a^2*b^2 - 2*(a^4 + 7*a^3*b + 6*a^2*b^2)*d*x + 33*(a^4 + a^3*b)*cosh
(d*x + c)^2)*sinh(d*x + c)^10 + 40*(11*(a^4 + a^3*b)*cosh(d*x + c)^3 + 2*(a
^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c))*
sinh(d*x + c)^9 + 2*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*
a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^8 + 2*(495*(a^4 + a^3*b)*
cosh(d*x + c)^4 + 5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3
*b + 20*a^2*b^2 + 12*a*b^3)*d*x + 180*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7
*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(99*(a^4 + a
```

^3*b)*cosh(d*x + c)^5 + 60*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*
a^2*b^2)*d*x)*cosh(d*x + c)^3 + (5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 -
16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c
)^7 - 4*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b
+ 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh(d*x + c)^6 + 4*(462*(a^4 + a^3
*b)*cosh(d*x + c)^6 + 420*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a
^2*b^2)*d*x)*cosh(d*x + c)^4 - 27*a^3*b - 102*a^2*b^2 - 152*a*b^3 - 80*b^4
- 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x + 14*(5*a^4 +
3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(198*(a^4 + a^3*b)*cosh(d*x + c
)^7 + 252*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cos
h(d*x + c)^5 + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^
3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^3 - 3*(27*a^3*b + 102*a^2*b
^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48
*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(5*a^4 + 75*a^3*b + 192*a^2*b
^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x +
c)^4 + 2*(495*(a^4 + a^3*b)*cosh(d*x + c)^8 + 840*(a^4 + 3*a^3*b + 2*a^2*b
^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^6 + 70*(5*a^4 + 3*a^3*b
- 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*
cosh(d*x + c)^4 - 5*a^4 - 75*a^3*b - 192*a^2*b^2 - 128*a*b^3 - 16*(a^4 + 9*
a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x - 30*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3
+ 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cos
h(d*x + c)^2)*sinh(d*x + c)^4 - 2*a^4 - 2*a^3*b + 8*(55*(a^4 + a^3*b)*cosh(
d*x + c)^9 + 120*(a^4 + 3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d
*x)*cosh(d*x + c)^7 + 14*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4
+ 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^5 - 10*(27*a^3*b + 1
02*a^2*b^2 + 152*a*b^3 + 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*
b^3 + 48*b^4)*d*x)*cosh(d*x + c)^3 - (5*a^4 + 75*a^3*b + 192*a^2*b^2 + 128*
a*b^3 + 16*(a^4 + 9*a^3*b + 20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c))*sinh
(d*x + c)^3 - 4*(2*a^4 + 15*a^3*b + 14*a^2*b^2 + 2*(a^4 + 7*a^3*b + 6*a^2*b
^2)*d*x)*cosh(d*x + c)^2 + 4*(33*(a^4 + a^3*b)*cosh(d*x + c)^10 + 90*(a^4 +
3*a^3*b + 2*a^2*b^2 - (a^4 + 7*a^3*b + 6*a^2*b^2)*d*x)*cosh(d*x + c)^8 + 1
4*(5*a^4 + 3*a^3*b - 32*a^2*b^2 - 32*a*b^3 - 16*(a^4 + 9*a^3*b + 20*a^2*b^2
+ 12*a*b^3)*d*x)*cosh(d*x + c)^6 - 15*(27*a^3*b + 102*a^2*b^2 + 152*a*b^3
+ 80*b^4 + 4*(3*a^4 + 29*a^3*b + 82*a^2*b^2 + 104*a*b^3 + 48*b^4)*d*x)*cosh
(d*x + c)^4 - 2*a^4 - 15*a^3*b - 14*a^2*b^2 - 2*(a^4 + 7*a^3*b + 6*a^2*b^2)
*d*x - 3*(5*a^4 + 75*a^3*b + 192*a^2*b^2 + 128*a*b^3 + 16*(a^4 + 9*a^3*b +
20*a^2*b^2 + 12*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((15*a^4 + 4
0*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^10 + 10*(15*a^4 + 40*a^3*b + 24*a^2*b^2
)*cosh(d*x + c)*sinh(d*x + c)^9 + (15*a^4 + 40*a^3*b + 24*a^2*b^2)*sinh(d*x
+ c)^10 + 4*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^8 +
(60*a^4 + 280*a^3*b + 416*a^2*b^2 + 192*a*b^3 + 45*(15*a^4 + 40*a^3*b + 24
*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(15*a^4 + 40*a^3*b + 24*
a^2*b^2)*cosh(d*x + c)^3 + 4*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*c
osh(d*x + c))*sinh(d*x + c)^7 + 2*(45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a

```
*b^3 + 192*b^4)*cosh(d*x + c)^6 + 2*(105*(15*a^4 + 40*a^3*b + 24*a^2*b^2)*c
osh(d*x + c)^4 + 45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4 + 5
6*(15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^6 + 4*(63*(15*a^4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^5 + 56*(15*a^4
+ 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^3 + 3*(45*a^4 + 240*a^3*
b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(
15*a^4 + 70*a^3*b + 104*a^2*b^2 + 48*a*b^3)*cosh(d*x + c)^4 + 2*(105*(15*a^
4 + 40*a^3*b + 24*a^2*b^2)*cosh(d*x + c)^6 + 140*(15*a^4 + 70*a^3*b + 104*a
^2*b^2 + 48*a*b^3)*cosh(d*x + c)^4 + 30*a^4 + 140*a^3*b + 208*a^2*b^2 + 96*
a*b^3 + 15*(45*a^4 + 240*a^3*b + 512*a^2*b^2 + 512*a*b^3 + 192*b^4)*cosh(d*
x + c)^2)*sinh(d*x + c)^4 + 8*(15*(15*a^4 + 40*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(178) = 356.
time = 1.65, size = 370, normalized size = 1.98

$$\frac{\frac{\left(15\,a^{2}b+40\,ab^{2}+24\,b^{3}\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^{2}}}\right)}{\left(a^{5}+a^{4}b\right)\sqrt{-ab-b^{2}}}-\frac{2\,\left(9\,a^{3}be^{(6\,dx+6\,c)}+32\,a^{2}b^{2}e^{(6\,dx+6\,c)}+24\,ab^{3}e^{(6\,dx+6\,c)}+27\,a^{3}be^{(4\,dx+4\,c)}+102\,a^{2}b^{2}e^{(4\,dx+4\,c)}+152\,ab^{3}e^{(4\,dx+4\,c)}+80\,b^{4}e^{(4\,dx+4\,c)}+27\,a^{3}be^{(2\,dx+2\,c)}+80\,a^{2}b^{2}e^{(2\,dx+2\,c)}+56\,ab^{3}e^{(2\,dx+2\,c)}+9\,a^{3}b+10\,a^{2}b^{2}\right)}{\left(a^{5}+a^{4}b\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)^{2}}-\frac{4\,(dx+c)(a+6\,b)}{a^{4}}+\frac{e^{(2\,dx+2\,c)}}{a^{3}}+\frac{\left(2\,ae^{(2\,dx+2\,c)}+12\,be^{(2\,dx+2\,c)}-a\right)e^{(-2\,dx-2\,c)}}{a^{4}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((15*a^2*b + 40*a*b^2 + 24*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b
)/sqrt(-a*b - b^2))/((a^5 + a^4*b)*sqrt(-a*b - b^2)) - 2*(9*a^3*b*e^(6*d*x
+ 6*c) + 32*a^2*b^2*e^(6*d*x + 6*c) + 24*a*b^3*e^(6*d*x + 6*c) + 27*a^3*b*e
^(4*d*x + 4*c) + 102*a^2*b^2*e^(4*d*x + 4*c) + 152*a*b^3*e^(4*d*x + 4*c) +
80*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 80*a^2*b^2*e^(2*d*x + 2
*c) + 56*a*b^3*e^(2*d*x + 2*c) + 9*a^3*b + 10*a^2*b^2)/((a^5 + a^4*b)*(a*e^
(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2) - 4*(d*x
+ c)*(a + 6*b)/a^4 + e^(2*d*x + 2*c)/a^3 + (2*a*e^(2*d*x + 2*c) + 12*b*e^(2
*d*x + 2*c) - a)*e^(-2*d*x - 2*c)/a^4)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^6\,{\mathrm{sinh}\left(c+d\,x\right)}^2}{{\left(a\,{\mathrm{cosh}\left(c+d\,x\right)}^2+b\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3,x)
[Out] int((cosh(c + d*x)^6*sinh(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.44 $\int \frac{\sinh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=116

$$-\frac{15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{7/2}d}+\frac{15\cosh(c+dx)}{8a^3d}-\frac{\cosh^5(c+dx)}{4ad\left(b+a\cosh^2(c+dx)\right)^2}-\frac{5\cosh^3(c+dx)}{8a^2d\left(b+a\cosh^2(c+dx)\right)}$$

```
[Out] 15/8*cosh(d*x+c)/a^3/d-1/4*cosh(d*x+c)^5/a/d/(b+a*cosh(d*x+c)^2)^2-5/8*cosh
(d*x+c)^3/a^2/d/(b+a*cosh(d*x+c)^2)-15/8*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2)
)*b^(1/2)/a^(7/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4218, 294, 327, 211}

$$-\frac{15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{7/2}d}+\frac{15\cosh(c+dx)}{8a^3d}-\frac{5\cosh^3(c+dx)}{8a^2d\left(a\cosh^2(c+dx)+b\right)}-\frac{\cosh^5(c+dx)}{4ad\left(a\cosh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])/(8*a^(7/2)*d) + (15*C
osh[c + d*x])/(8*a^3*d) - Cosh[c + d*x]^5/(4*a*d*(b + a*Cosh[c + d*x]^2)^2)
 - (5*Cosh[c + d*x]^3)/(8*a^2*d*(b + a*Cosh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\cosh^5(c+dx)}{4ad\left(b+a\cosh^2(c+dx)\right)^2} + \frac{5\text{Subst}\left(\int \frac{x^4}{(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{4ad}$$

$$= -\frac{\cosh^5(c+dx)}{4ad\left(b+a\cosh^2(c+dx)\right)^2} - \frac{5\cosh^3(c+dx)}{8a^2d\left(b+a\cosh^2(c+dx)\right)} + \frac{15\text{Subst}\left(\int \frac{x^2}{b+ax}\right.}{}$$

$$= \frac{15\cosh(c+dx)}{8a^3d} - \frac{\cosh^5(c+dx)}{4ad\left(b+a\cosh^2(c+dx)\right)^2} - \frac{5\cosh^3(c+dx)}{8a^2d\left(b+a\cosh^2(c+dx)\right)} -$$

$$= -\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{7/2}d} + \frac{15\cosh(c+dx)}{8a^3d} - \frac{\cosh^5(c+dx)}{4ad\left(b+a\cosh^2(c+dx\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.16, size = 453, normalized size = 3.91

$$\frac{(a+2b+a\cosh(2(c+dx)))^3\text{sech}^6(c+dx)\left(-\frac{15\left((a^3+64b^3)\text{ArcTan}\left(\frac{\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)+(a^3+64b^3)\text{ArcTan}\left(\frac{\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right)-a^3\left(\text{ArcTan}\left(\frac{\sqrt{a}-i\sqrt{a+b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\text{ArcTan}\left(\frac{\sqrt{a}+i\sqrt{a+b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)\right)}{a^{7/2}b^{5/2}}+\frac{512\cosh(c)\cosh(dx)}{a^3}+\frac{8\cosh(c+dx)(16b^3(9a+14b)+3(a^4+48ab^3)\cosh(2(c+dx)))}{a^3b^2(a+2b+a\cosh(2(c+dx)))^2}+\frac{512\sinh(c)\sinh(dx)}{a^3}-\frac{6a\,\text{csch}(c+dx)\sinh(4(c+dx))}{b^2(a+2b+a\cosh(2(c+dx)))^2}\right)}{4096d\left(a+b\text{sech}^2(c+dx)\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((-15*((a^3 + 64*b^3)*Ar
cTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d
*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[
```

```
(d*x)/2]))/Sqrt[b]] + (a^3 + 64*b^3)*ArcTan[((Sqrt[a] + I*Sqrt[a + b]*Sqrt[
(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] + I*Sqrt[a
 + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]] - a^3*(ArcTan[(S
qrt[a] - I*Sqrt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[(Sqrt[a] + I*Sq
rt[a + b]*Tanh[(c + d*x)/2])/Sqrt[b]])))/(a^(7/2)*b^(5/2)) + (512*Cosh[c]*C
osh[d*x])/a^3 + (8*Cosh[c + d*x]*(16*b^3*(9*a + 14*b) + 3*(a^4 + 48*a*b^3)*
Cosh[2*(c + d*x)]))/(a^3*b^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) + (512*Sinh
[c]*Sinh[d*x])/a^3 - (6*a*Csch[c + d*x]*Sinh[4*(c + d*x)])/(b^2*(a + 2*b +
a*Cosh[2*(c + d*x)])^2)))/(4096*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [A]
time = 0.95, size = 84, normalized size = 0.72

method	result
derivativedivides	$-\frac{-\frac{1}{a^{3}\operatorname{sech}(dx+c)}-\frac{b\left(\frac{\frac{7b\operatorname{sech}(dx+c)^{3}}{8}+\frac{9a\operatorname{sech}(dx+c)}{8}}{\left(a+b\operatorname{sech}(dx+c)^{2}\right)^{2}}+\frac{15\arctan\left(\frac{b\operatorname{sech}(dx+c)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}}{d}$
default	$-\frac{-\frac{1}{a^{3}\operatorname{sech}(dx+c)}-\frac{b\left(\frac{\frac{7b\operatorname{sech}(dx+c)^{3}}{8}+\frac{9a\operatorname{sech}(dx+c)}{8}}{\left(a+b\operatorname{sech}(dx+c)^{2}\right)^{2}}+\frac{15\arctan\left(\frac{b\operatorname{sech}(dx+c)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}}{d}$
risch	$\frac{e^{dx+c}}{2a^{3}d}+\frac{e^{-dx-c}}{2a^{3}d}+\frac{\left(9a\,e^{6dx+6c}+27a\,e^{4dx+4c}+28b\,e^{4dx+4c}+27a\,e^{2dx+2c}+28b\,e^{2dx+2c}+9a\right)b\,e^{dx+c}}{4a^{3}\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)^{2}d}+15\sqrt{-ab}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-1/a^3/sech(d*x+c)-1/a^3*b*((7/8*b*sech(d*x+c)^3+9/8*a*sech(d*x+c))/(
a+b*sech(d*x+c)^2)^2+15/8/(a*b)^(1/2)*arctan(b*sech(d*x+c)/(a*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a^2*e^(10*d*x + 10*c) + 2*a^2 + 5*(2*a^2*e^(8*c) + 5*a*b*e^(8*c))*e^
(8*d*x) + 5*(4*a^2*e^(6*c) + 15*a*b*e^(6*c) + 12*b^2*e^(6*c))*e^(6*d*x) + 5
*(4*a^2*e^(4*c) + 15*a*b*e^(4*c) + 12*b^2*e^(4*c))*e^(4*d*x) + 5*(2*a^2*e^(
```

```
2*c) + 5*a*b*e^(2*c))*e^(2*d*x))/(a^5*d*e^(9*d*x + 9*c) + a^5*d*e^(d*x + c)
 + 4*(a^5*d*e^(7*c) + 2*a^4*b*d*e^(7*c))*e^(7*d*x) + 2*(3*a^5*d*e^(5*c) + 8
*a^4*b*d*e^(5*c) + 8*a^3*b^2*d*e^(5*c))*e^(5*d*x) + 4*(a^5*d*e^(3*c) + 2*a^
4*b*d*e^(3*c))*e^(3*d*x)) - 1/2*integrate(15/2*(b*e^(3*d*x + 3*c) - b*e^(d*
x + c))/(a^4*e^(4*d*x + 4*c) + a^4 + 2*(a^4*e^(2*c) + 2*a^3*b*e^(2*c))*e^(2
*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2599 vs. $2(100) = 200$.
time = 0.48, size = 4829, normalized size = 41.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*a^2*cosh(d*x + c)^10 + 80*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + 8*a^
2*sinh(d*x + c)^10 + 20*(2*a^2 + 5*a*b)*cosh(d*x + c)^8 + 20*(18*a^2*cosh(d
*x + c)^2 + 2*a^2 + 5*a*b)*sinh(d*x + c)^8 + 160*(6*a^2*cosh(d*x + c)^3 + (
2*a^2 + 5*a*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(4*a^2 + 15*a*b + 12*b^2
)*cosh(d*x + c)^6 + 20*(84*a^2*cosh(d*x + c)^4 + 28*(2*a^2 + 5*a*b)*cosh(d*
x + c)^2 + 4*a^2 + 15*a*b + 12*b^2)*sinh(d*x + c)^6 + 8*(252*a^2*cosh(d*x +
 c)^5 + 140*(2*a^2 + 5*a*b)*cosh(d*x + c)^3 + 15*(4*a^2 + 15*a*b + 12*b^2)*
cosh(d*x + c))*sinh(d*x + c)^5 + 20*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)
^4 + 20*(84*a^2*cosh(d*x + c)^6 + 70*(2*a^2 + 5*a*b)*cosh(d*x + c)^4 + 15*(
4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^2 + 4*a^2 + 15*a*b + 12*b^2)*sinh(d*
x + c)^4 + 80*(12*a^2*cosh(d*x + c)^7 + 14*(2*a^2 + 5*a*b)*cosh(d*x + c)^5
 + 5*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^3 + (4*a^2 + 15*a*b + 12*b^2)*c
osh(d*x + c))*sinh(d*x + c)^3 + 20*(2*a^2 + 5*a*b)*cosh(d*x + c)^2 + 20*(18
*a^2*cosh(d*x + c)^8 + 28*(2*a^2 + 5*a*b)*cosh(d*x + c)^6 + 15*(4*a^2 + 15*
a*b + 12*b^2)*cosh(d*x + c)^4 + 6*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^2
 + 2*a^2 + 5*a*b)*sinh(d*x + c)^2 + 15*(a^2*cosh(d*x + c)^9 + 9*a^2*cosh(d*
x + c)*sinh(d*x + c)^8 + a^2*sinh(d*x + c)^9 + 4*(a^2 + 2*a*b)*cosh(d*x + c
)^7 + 4*(9*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^7 + 28*(3*a^2*c
osh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(3*a^2 +
8*a*b + 8*b^2)*cosh(d*x + c)^5 + 2*(63*a^2*cosh(d*x + c)^4 + 42*(a^2 + 2*a*
b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^5 + 2*(63*a^2*cos
h(d*x + c)^5 + 70*(a^2 + 2*a*b)*cosh(d*x + c)^3 + 5*(3*a^2 + 8*a*b + 8*b^2)
*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^3 + 4*(21*a
^2*cosh(d*x + c)^6 + 35*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 5*(3*a^2 + 8*a*b +
8*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^3 + a^2*cosh(d*x + c) +
 4*(9*a^2*cosh(d*x + c)^7 + 21*(a^2 + 2*a*b)*cosh(d*x + c)^5 + 5*(3*a^2 + 8
*a*b + 8*b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c
)^2 + (9*a^2*cosh(d*x + c)^8 + 28*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 10*(3*a^2
 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 12*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2)
```

```
*sinh(d*x + c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(
d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(
d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a - 2*b)*co
sh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(
d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + a
)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sin
h(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cos
h(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*
cosh(d*x + c))*sinh(d*x + c) + a)) + 8*a^2 + 40*(2*a^2*cosh(d*x + c)^9 + 4*
(2*a^2 + 5*a*b)*cosh(d*x + c)^7 + 3*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)
^5 + 2*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^3 + (2*a^2 + 5*a*b)*cosh(d*x
 + c))*sinh(d*x + c))/(a^5*d*cosh(d*x + c)^9 + 9*a^5*d*cosh(d*x + c)*sinh(d
*x + c)^8 + a^5*d*sinh(d*x + c)^9 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^7 + 4
*(9*a^5*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^7 + a^5*d*cosh
(d*x + c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^5 + 28*(3*a^5*d
*cosh(d*x + c)^3 + (a^5 + 2*a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63
*a^5*d*cosh(d*x + c)^4 + 42*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + (3*a^5 + 8*
a^4*b + 8*a^3*b^2)*d)*sinh(d*x + c)^5 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3
 + 2*(63*a^5*d*cosh(d*x + c)^5 + 70*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3 + 5*(
3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*a^5*d
*cosh(d*x + c)^6 + 35*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^4 + 5*(3*a^5 + 8*a^4*
b + 8*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^3 + 4*(
9*a^5*d*cosh(d*x + c)^7 + 21*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^5 + 5*(3*a^5 +
 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c)
)*sinh(d*x + c)^2 + (9*a^5*d*cosh(d*x + c)^8 + 28*(a^5 + 2*a^4*b)*d*cosh(d*
x + c)^6 + a^5*d + 10*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^4 + 12*
(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/8*(4*a^2*cosh(d*x + c)
^10 + 40*a^2*cosh(d*x + c)*sinh(d*x + c)^9 + 4*a^2*sinh(d*x + c)^10 + 10*(2
*a^2 + 5*a*b)*cosh(d*x + c)^8 + 10*(18*a^2*cosh(d*x + c)^2 + 2*a^2 + 5*a*b)
*sinh(d*x + c)^8 + 80*(6*a^2*cosh(d*x + c)^3 + (2*a^2 + 5*a*b)*cosh(d*x + c
))*sinh(d*x + c)^7 + 10*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^6 + 10*(84*
a^2*cosh(d*x + c)^4 + 28*(2*a^2 + 5*a*b)*cosh(d*x + c)^2 + 4*a^2 + 15*a*b +
 12*b^2)*sinh(d*x + c)^6 + 4*(252*a^2*cosh(d*x + c)^5 + 140*(2*a^2 + 5*a*b)
*cosh(d*x + c)^3 + 15*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c))*sinh(d*x + c
)^5 + 10*(4*a^2 + 15*a*b + 12*b^2)*cosh(d*x + c)^4 + 10*(84*a^2*cosh(d*x +
c)^6 + 70*(2*a^2 + 5*a*b)*cosh(d*x + c)^4 + 15*(4*a^2 + 15*a*b + 12*b^2)*co
sh(d*x + c)^2 + 4*a^2 + 15*a*b + 12*b^2)*sinh(d*x + c)^4 + 40*(12*a^2*cosh(
d*x + c)^7 + 14*(2*a^2 + 5*a*b)*cosh(d*x + c)^5...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.61, size = 103, normalized size = 0.89

$$\frac{\frac{7\,b^2\cosh(c+d\,x)}{8}+\frac{9\,a\,b\cosh(c+d\,x)^3}{8}}{d\,a^5\cosh\left(c+d\,x\right)^4+2\,d\,a^4\,b\cosh\left(c+d\,x\right)^2+d\,a^3\,b^2}+\frac{\cosh(c+d\,x)}{a^3\,d}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\cosh(c+d\,x)}{\sqrt{b}}\right)}{8\,a^{7/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] ((7*b^2*cosh(c + d*x))/8 + (9*a*b*cosh(c + d*x)^3)/8)/(a^5*d*cosh(c + d*x)^
4 + a^3*b^2*d + 2*a^4*b*d*cosh(c + d*x)^2) + cosh(c + d*x)/(a^3*d) - (15*b^
(1/2)*atan((a^(1/2)*cosh(c + d*x))/b^(1/2)))/(8*a^(7/2)*d)
```

3.45 $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=154

$$\frac{\sqrt{b}\,(15a^2+10ab+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{5/2}(a+b)^3d}-\frac{\tanh^{-1}(\cosh(c+dx))}{(a+b)^3d}-\frac{b\cosh^3(c+dx)}{4a(a+b)d\left(b+a\cosh^2(c+dx)\right)^2}$$

```
[Out] -arctanh(cosh(d*x+c))/(a+b)^3/d-1/4*b*cosh(d*x+c)^3/a/(a+b)/d/(b+a*cosh(d*x
+c)^2)^2-1/8*b*(7*a+3*b)*cosh(d*x+c)/a^2/(a+b)^2/d/(b+a*cosh(d*x+c)^2)+1/8*
(15*a^2+10*a*b+3*b^2)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(5/2)/(
a+b)^3/d
```

Rubi [A]
time = 0.16, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4218, 481, 592, 536, 212, 211}

$$-\frac{b(7a+3b)\cosh(c+dx)}{8a^2d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}+\frac{\sqrt{b}\,(15a^2+10ab+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{5/2}d(a+b)^3}-\frac{b\cosh^3(c+dx)}{4ad(a+b)\left(a\cosh^2(c+dx)+b\right)^2}-\frac{\tanh^{-1}(\cosh(c+dx))}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b]])
/(8*a^(5/2)*(a + b)^3*d) - ArcTanh[Cosh[c + d*x]]/((a + b)^3*d) - (b*Cosh[c
 + d*x]^3)/(4*a*(a + b)*d*(b + a*Cosh[c + d*x]^2)^2) - (b*(7*a + 3*b)*Cosh[
c + d*x])/(8*a^2*(a + b)^2*d*(b + a*Cosh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
```

```
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^6}{(1-x^2)(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{b\cosh^3(c+dx)}{4a(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{x^2(3b+(-4a-3b)x^2)}{(1-x^2)(b+ax^2)^2}\,dx, x, \cosh(\right.}{4a(a+b)d}$$

$$= -\frac{b\cosh^3(c+dx)}{4a(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(7a+3b)\cosh(c+dx)}{8a^2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} +$$

$$= -\frac{b\cosh^3(c+dx)}{4a(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{b(7a+3b)\cosh(c+dx)}{8a^2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} -$$

$$= \frac{\sqrt{b}\,(15a^2+10ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{5/2}(a+b)^3d} - \frac{\tanh^{-1}(\cosh(c+dx))}{(a+b)^3d} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.65, size = 440, normalized size = 2.86

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^5(c+dx)\left(\frac{8b^2(a+b)^2}{a^2}-\frac{2b(a+b)(9a+5b)(a+2b+a\cosh(2(c+dx)))}{a^2}+\frac{\sqrt{b}(15a^2+10ab+3b^2)\operatorname{ArcTan}\left[\frac{\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}-i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right](a+2b+a\cosh(2(c+dx)))^2\operatorname{sech}(c+dx)}{a^{5/2}}+\frac{\sqrt{b}(15a^2+10ab+3b^2)\operatorname{ArcTan}\left[\frac{\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\right)\sinh(c)\tanh\left(\frac{dx}{2}\right)+\cosh(c)\left(\sqrt{a}+i\sqrt{a+b}\sqrt{(\cosh(c)-\sinh(c))^2}\tanh\left(\frac{dx}{2}\right)\right)}{\sqrt{b}}\right](a+2b+a\cosh(2(c+dx)))^2\operatorname{sech}(c+dx)}{a^{5/2}}-8(a+2b+a\cosh(2(c+dx)))^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)\operatorname{sech}(c+dx)+8(a+2b+a\cosh(2(c+dx)))^2\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\operatorname{sech}(c+dx)\right)}{64(a+b)^3d\left(a+b\operatorname{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*((8*b^2*(a + b)^2)/a^2 - (
2*b*(a + b)*(9*a + 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)]))/a^2 + (Sqrt[b]*(15
*a^2 + 10*a*b + 3*b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sin
h[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Co
sh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)]
)^2*Sech[c + d*x])/a^(5/2) + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[((Sq
rt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] +
Cosh[c]*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2])
)/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x])/a^(5/2) - 8*(a
+ 2*b + a*Cosh[2*(c + d*x)])^2*Log[Cosh[(c + d*x)/2]]*Sech[c + d*x] + 8*(a
+ 2*b + a*Cosh[2*(c + d*x)])^2*Log[Sinh[(c + d*x)/2]]*Sech[c + d*x]))/(64*(
a + b)^3*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(140) = 280$.
time = 2.50, size = 291, normalized size = 1.89

method	result
derivativedivides	$\frac{2b\left(\frac{-\frac{\left(9a^3-a^2b-13a\,b^2-3b^3\right)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}-\frac{3\left(9a^3-3a^2b+7a\,b^2+3b^3\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}-\frac{\left(27a^3+13a^2b-23a\,b^2-9b^3\right)\left(\tanh^2\left(\right.\right.}{8a^2}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}}{(a+b)^3}}{d}$
default	$\frac{2b\left(\frac{-\frac{\left(9a^3-a^2b-13a\,b^2-3b^3\right)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}-\frac{3\left(9a^3-3a^2b+7a\,b^2+3b^3\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2}-\frac{\left(27a^3+13a^2b-23a\,b^2-9b^3\right)\left(\tanh^2\left(\right.\right.}{8a^2}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}}{(a+b)^3}}{d}$
risch	$-\frac{b\,e^{dx+c}\left(9a^2e^{6dx+6c}+5ab\,e^{6dx+6c}+27a^2e^{4dx+4c}+43ab\,e^{4dx+4c}+12b^2e^{4dx+4c}+27a^2e^{2dx+2c}+43ab\,e^{2dx+2c}+12b^2e^{2dx+}\right.}{4a^2(a+b)^2d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/(a+b)^3*((-1/8*(9*a^3-a^2*b-13*a*b^2-3*b^3)/a^2*tanh(1/2*d*x+1/2*c
)^6-3/8*(9*a^3-3*a^2*b+7*a*b^2+3*b^3)/a^2*tanh(1/2*d*x+1/2*c)^4-1/8*(27*a^3
+13*a^2*b-23*a*b^2-9*b^3)/a^2*tanh(1/2*d*x+1/2*c)^2-3/8*(3*a^3+7*a^2*b+5*a*
b^2+b^3)/a^2)/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2
*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)^2+1/16*(15*a^2+10*a*b+3*b^2)/a
^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/
2)))+1/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((9*a^2*b*e^(7*c) + 5*a*b^2*e^(7*c))*e^(7*d*x) + (27*a^2*b*e^(5*c) + 4
3*a*b^2*e^(5*c) + 12*b^3*e^(5*c))*e^(5*d*x) + (27*a^2*b*e^(3*c) + 43*a*b^2*
e^(3*c) + 12*b^3*e^(3*c))*e^(3*d*x) + (9*a^2*b*e^c + 5*a*b^2*e^c)*e^(d*x))/
(a^6*d + 2*a^5*b*d + a^4*b^2*d + (a^6*d*e^(8*c) + 2*a^5*b*d*e^(8*c) + a^4*b
^2*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 4*a^5*b*d*e^(6*c) + 5*a^4*b^2*
d*e^(6*c) + 2*a^3*b^3*d*e^(6*c))*e^(6*d*x) + 2*(3*a^6*d*e^(4*c) + 14*a^5*b*
d*e^(4*c) + 27*a^4*b^2*d*e^(4*c) + 24*a^3*b^3*d*e^(4*c) + 8*a^2*b^4*d*e^(4*
c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 4*a^5*b*d*e^(2*c) + 5*a^4*b^2*d*e^(2*c)
+ 2*a^3*b^3*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^3*d +
3*a^2*b*d + 3*a*b^2*d + b^3*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^3*d + 3*a
```

```
^2*b*d + 3*a*b^2*d + b^3*d) + 2*integrate(1/8*((15*a^2*b*e^(3*c) + 10*a*b^2
*e^(3*c) + 3*b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^c + 10*a*b^2*e^c + 3*b^3*
e^c)*e^(d*x))/(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 + (a^6*e^(4*c) + 3*a^5*b
*e^(4*c) + 3*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c))*e^(4*d*x) + 2*(a^6*e^(2*c)
+ 5*a^5*b*e^(2*c) + 9*a^4*b^2*e^(2*c) + 7*a^3*b^3*e^(2*c) + 2*a^2*b^4*e^(2*
c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4721 vs. $2(140) = 280$.
time = 0.50, size = 8742, normalized size = 56.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^7 + 28*(9*a^3*b +
14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(9*a^3*b + 14*a^2*b
^2 + 5*a*b^3)*sinh(d*x + c)^7 + 4*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^
4)*cosh(d*x + c)^5 + 4*(27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4 + 21*(9*a
^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(9*a^
3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^3 + (27*a^3*b + 70*a^2*b^2 + 55*a
*b^3 + 12*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(27*a^3*b + 70*a^2*b^2 +
55*a*b^3 + 12*b^4)*cosh(d*x + c)^3 + 4*(35*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)
*cosh(d*x + c)^4 + 27*a^3*b + 70*a^2*b^2 + 55*a*b^3 + 12*b^4 + 10*(27*a^3*b
 + 70*a^2*b^2 + 55*a*b^3 + 12*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21
*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^5 + 10*(27*a^3*b + 70*a^2*b
^2 + 55*a*b^3 + 12*b^4)*cosh(d*x + c)^3 + 3*(27*a^3*b + 70*a^2*b^2 + 55*a*b
^3 + 12*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 - ((15*a^4 + 10*a^3*b + 3*a^2*b
^2)*cosh(d*x + c)^8 + 8*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)*sinh(
d*x + c)^7 + (15*a^4 + 10*a^3*b + 3*a^2*b^2)*sinh(d*x + c)^8 + 4*(15*a^4 +
40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^6 + 4*(15*a^4 + 40*a^3*b + 2
3*a^2*b^2 + 6*a*b^3 + 7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^2)*si
nh(d*x + c)^6 + 8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 + 3*(1
5*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2
*(45*a^4 + 150*a^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)^4 +
2*(35*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^4 + 45*a^4 + 150*a^3*b
+ 209*a^2*b^2 + 104*a*b^3 + 24*b^4 + 30*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6
*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 10*a^3*b + 3*a^2*b^2 +
8*(7*(15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^5 + 10*(15*a^4 + 40*a^3*
b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^3 + (45*a^4 + 150*a^3*b + 209*a^2*b
^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 + 40*a^
3*b + 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2 + 4*(7*(15*a^4 + 10*a^3*b + 3*a
^2*b^2)*cosh(d*x + c)^6 + 15*(15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3)*cos
h(d*x + c)^4 + 15*a^4 + 40*a^3*b + 23*a^2*b^2 + 6*a*b^3 + 3*(45*a^4 + 150*a
```

```
^3*b + 209*a^2*b^2 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
 8*((15*a^4 + 10*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^7 + 3*(15*a^4 + 40*a^3*b
+ 23*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^5 + (45*a^4 + 150*a^3*b + 209*a^2*b^2
 + 104*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (15*a^4 + 40*a^3*b + 23*a^2*b^2 +
6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log((a*cosh(d*x + c)^4 +
4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a - 2*b)*cosh(d*
x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*
x + c)^3 + (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 +
3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + a*cosh(d*x + c) + (
3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*sqrt(-b/a) + a)/(a*cosh(d*x + c)^4
+ 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(
d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(
d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*(9*a^3*b + 14
*a^2*b^2 + 5*a*b^3)*cosh(d*x + c) + 16*(a^4*cosh(d*x + c)^8 + 8*a^4*cosh(d*
x + c)*sinh(d*x + c)^7 + a^4*sinh(d*x + c)^8 + 4*(a^4 + 2*a^3*b)*cosh(d*x +
 c)^6 + 4*(7*a^4*cosh(d*x + c)^2 + a^4 + 2*a^3*b)*sinh(d*x + c)^6 + 8*(7*a^
4*cosh(d*x + c)^3 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3
*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*a^4*cosh(d*x + c)^4 + 3
*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(a^4 + 2*a^3*b)*cosh(d*x + c)^2)*sinh(d*x +
 c)^4 + a^4 + 8*(7*a^4*cosh(d*x + c)^5 + 10*(a^4 + 2*a^3*b)*cosh(d*x + c)^3
 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 +
2*a^3*b)*cosh(d*x + c)^2 + 4*(7*a^4*cosh(d*x + c)^6 + 15*(a^4 + 2*a^3*b)*co
sh(d*x + c)^4 + a^4 + 2*a^3*b + 3*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 8*(a^4*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b)*cosh(d*x
 + c)^5 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*a^3*b)*c
osh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 16*(a
^4*cosh(d*x + c)^8 + 8*a^4*cosh(d*x + c)*sinh(d*x + c)^7 + a^4*sinh(d*x + c
)^8 + 4*(a^4 + 2*a^3*b)*cosh(d*x + c)^6 + 4*(7*a^4*cosh(d*x + c)^2 + a^4 +
2*a^3*b)*sinh(d*x + c)^6 + 8*(7*a^4*cosh(d*x + c)^3 + 3*(a^4 + 2*a^3*b)*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)
^4 + 2*(35*a^4*cosh(d*x + c)^4 + 3*a^4 + 8*a^3*b + 8*a^2*b^2 + 30*(a^4 + 2*
a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 8*(7*a^4*cosh(d*x + c)^5 +
10*(a^4 + 2*a^3*b)*cosh(d*x + c)^3 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b)*cosh(d*x + c)^2 + 4*(7*a^4*cosh(
d*x + c)^6 + 15*(a^4 + 2*a^3*b)*cosh(d*x + c)^4 + a^4 + 2*a^3*b + 3*(3*a^4
+ 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a^4*cosh(d*x +
 c)^7 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c)^5 + (3*a^4 + 8*a^3*b + 8*a^2*b^2)*c
osh(d*x + c)^3 + (a^4 + 2*a^3*b)*cosh(d*x + c))...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^6}{\sinh\left(c+d\,x\right)\,\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(cosh(c + d*x)^6/(sinh(c + d*x)*(b + a*cosh(c + d*x)^2)^3), x)
```

3.46 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=126

$$\frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}d}-\frac{15\coth(c+dx)}{8(a+b)^3d}+\frac{\coth(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}+\frac{5\coth(c}{8(a+b)^2d\left(a+b-\right.}$$

```
[Out] -15/8*coth(d*x+c)/(a+b)^3/d+15/8*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b
^(1/2)/(a+b)^(7/2)/d+1/4*coth(d*x+c)/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2+5/8*co
th(d*x+c)/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4217, 296, 331, 214}

$$\frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{7/2}}-\frac{15\coth(c+dx)}{8d(a+b)^3}+\frac{5\coth(c+dx)}{8d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{\coth(c+dx)}{4d(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (15*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*(a + b)^(7/2)*
d) - (15*Coth[c + d*x])/(8*(a + b)^3*d) + Coth[c + d*x]/(4*(a + b)*d*(a + b
 - b*Tanh[c + d*x]^2)^2) + (5*Coth[c + d*x])/(8*(a + b)^2*d*(a + b - b*Tanh
[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\coth(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{5\operatorname{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)^2}\,dx, x, \tanh(c-\right.}{4(a+b)d}$$

$$= \frac{\coth(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{5\coth(c+dx)}{8(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

$$= -\frac{15\coth(c+dx)}{8(a+b)^3d} + \frac{\coth(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{5\operatorname{cot}}{8(a+b)^2d\,(a+}$$

$$= \frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}d} - \frac{15\coth(c+dx)}{8(a+b)^3d} + \frac{\coth(c+d}{4(a+b)d\left(a+b-b\operatorname{ta}\right.}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 741 vs. 2(126) = 252.
time = 5.35, size = 741, normalized size = 5.88

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*((120*b*ArcTanh[(Sech[d*x]
*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt
[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*(
```

```
Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (Csch
[c]*Csch[c + d*x]*Sech[2*c]*((-32*a^4 - 64*a^3*b + 22*a^2*b^2 + 80*a*b^3 +
16*b^4)*Sinh[d*x] + 2*a*(16*a^3 + 23*a^2*b - 27*a*b^2 - 4*b^3)*Sinh[3*d*x]
- 48*a^4*Sinh[2*c - d*x] - 128*a^3*b*Sinh[2*c - d*x] - 106*a^2*b^2*Sinh[2*c
 - d*x] + 80*a*b^3*Sinh[2*c - d*x] + 16*b^4*Sinh[2*c - d*x] + 48*a^4*Sinh[2
*c + d*x] + 146*a^3*b*Sinh[2*c + d*x] + 182*a^2*b^2*Sinh[2*c + d*x] + 80*a*
b^3*Sinh[2*c + d*x] + 16*b^4*Sinh[2*c + d*x] - 32*a^4*Sinh[4*c + d*x] - 82*
a^3*b*Sinh[4*c + d*x] - 54*a^2*b^2*Sinh[4*c + d*x] - 80*a*b^3*Sinh[4*c + d*
x] - 16*b^4*Sinh[4*c + d*x] - 8*a^4*Sinh[2*c + 3*d*x] + 18*a^3*b*Sinh[2*c +
 3*d*x] + 54*a^2*b^2*Sinh[2*c + 3*d*x] + 8*a*b^3*Sinh[2*c + 3*d*x] + 32*a^4
*Sinh[4*c + 3*d*x] + 73*a^3*b*Sinh[4*c + 3*d*x] + 24*a^2*b^2*Sinh[4*c + 3*d
*x] + 8*a*b^3*Sinh[4*c + 3*d*x] - 8*a^4*Sinh[6*c + 3*d*x] - 9*a^3*b*Sinh[6*
c + 3*d*x] - 24*a^2*b^2*Sinh[6*c + 3*d*x] - 8*a*b^3*Sinh[6*c + 3*d*x] + 8*a
^4*Sinh[2*c + 5*d*x] - 9*a^3*b*Sinh[2*c + 5*d*x] - 2*a^2*b^2*Sinh[2*c + 5*d
*x] + 9*a^3*b*Sinh[4*c + 5*d*x] + 2*a^2*b^2*Sinh[4*c + 5*d*x] + 8*a^4*Sinh[
6*c + 5*d*x]))/a^2))/(512*(a + b)^3*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(110) = 220$.
time = 2.42, size = 299, normalized size = 2.37

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^3+3a^2b+3a\,b^2+b^3\right)}-\frac{1}{2(a+b)^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\left(\frac{\left(-\frac{9a}{8}-\frac{9b}{8}\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{27a}{8}+\frac{b}{8}\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{27a}{8}+\frac{b}{8}\right)(}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{}}{}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^3+3a^2b+3a\,b^2+b^3\right)}-\frac{1}{2(a+b)^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\left(\frac{\left(-\frac{9a}{8}-\frac{9b}{8}\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{27a}{8}+\frac{b}{8}\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{27a}{8}+\frac{b}{8}\right)(}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{}}{}$
risch	$-\frac{8a^4e^{8dx+8c}+9a^3b\,e^{8dx+8c}+24a^2b^2e^{8dx+8c}+8a\,b^3e^{8dx+8c}+32a^4e^{6dx+6c}+82a^3b\,e^{6dx+6c}+54a^2b^2e^{6dx+6c}+80a\,b^3e^{6dx+}}{4a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)-1/2/(a+b)^3/tanh(1/
2*d*x+1/2*c)-2*b/(a+b)^3*(((-9/8*a-9/8*b)*tanh(1/2*d*x+1/2*c)^7+(-27/8*a+1/
8*b)*tanh(1/2*d*x+1/2*c)^5+(-27/8*a+1/8*b)*tanh(1/2*d*x+1/2*c)^3+(-9/8*a-9/
8*b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+
2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)^2-15/32/b^(1/2)/(a
+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2
)+(a+b)^(1/2))+15/32/b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x+1/2*c
)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(116) = 232$.
time = 0.56, size = 533, normalized size = 4.23

$$-\frac{15\,b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)b}\,d}-\frac{8\,a^4-9\,a^3b-2\,a^2b^2+2\,(16\,a^4+23\,a^3b-27\,a^2b^2-4\,ab^3)e^{(-2\,dx-2\,c)}+2\,(24\,a^4+64\,a^3b+53\,a^2b^2-40\,ab^3-8\,b^4)e^{(-4\,dx-4\,c)}+2\,(16\,a^4+41\,a^3b+27\,a^2b^2+40\,ab^3+8\,b^4)e^{(-6\,dx-6\,c)}+(8\,a^4+9\,a^3b+24\,a^2b^2+8\,ab^3)e^{(-8\,dx-8\,c)}}{4\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3+(3\,a^7+17\,a^6b+33\,a^5b^2+27\,a^4b^3+8\,a^3b^4)e^{(-2\,dx-2\,c)}+2\,(a^7+7\,a^6b+23\,a^5b^2+37\,a^4b^3+28\,a^3b^4+8\,a^2b^5)e^{(-4\,dx-4\,c)}-2\,(a^7+7\,a^6b+23\,a^5b^2+37\,a^4b^3+28\,a^3b^4+8\,a^2b^5)e^{(-6\,dx-6\,c)}-(3\,a^7+17\,a^6b+33\,a^5b^2+27\,a^4b^3+8\,a^3b^4)e^{(-8\,dx-8\,c)}-(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)e^{(-10\,dx-10\,c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -15/16*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*
x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*s
qrt((a + b)*b)*d) - 1/4*(8*a^4 - 9*a^3*b - 2*a^2*b^2 + 2*(16*a^4 + 23*a^3*b
 - 27*a^2*b^2 - 4*a*b^3)*e^(-2*d*x - 2*c) + 2*(24*a^4 + 64*a^3*b + 53*a^2*b
^2 - 40*a*b^3 - 8*b^4)*e^(-4*d*x - 4*c) + 2*(16*a^4 + 41*a^3*b + 27*a^2*b^2
 + 40*a*b^3 + 8*b^4)*e^(-6*d*x - 6*c) + (8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a
*b^3)*e^(-8*d*x - 8*c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3 + (3*a^7 + 17
*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*e^(-2*d*x - 2*c) + 2*(a^7 + 7
*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*e^(-4*d*x - 4*c)
 - 2*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*e^(
-6*d*x - 6*c) - (3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*e^
(-8*d*x - 8*c) - (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*e^(-10*d*x - 10*c))*
d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3499 vs. $2(116) = 232$.
time = 0.44, size = 7275, normalized size = 57.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^8 + 32*(8*
a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + 4*(8*
a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*sinh(d*x + c)^8 + 8*(16*a^4 + 41*a^3*
b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*x + c)^6 + 8*(16*a^4 + 41*a^3*b +
 27*a^2*b^2 + 40*a*b^3 + 8*b^4 + 14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 +
 8*a*b^3)*cosh(d*x + c)^3 + 3*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 +
8*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(24*a^4 + 64*a^3*b + 53*a^2*b^2 -
 40*a*b^3 - 8*b^4)*cosh(d*x + c)^4 + 8*(35*(8*a^4 + 9*a^3*b + 24*a^2*b^2 +
8*a*b^3)*cosh(d*x + c)^4 + 24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^
4 + 15*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)*cosh(d*x + c)^2)
*sinh(d*x + c)^4 + 32*a^4 - 36*a^3*b - 8*a^2*b^2 + 32*(7*(8*a^4 + 9*a^3*b +
 24*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)^5 + 5*(16*a^4 + 41*a^3*b + 27*a^2*b^2
```

+ 40*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + (24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*
a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(16*a^4 + 23*a^3*b - 27*a
^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2 + 8*(14*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8
*a*b^3)*cosh(d*x + c)^6 + 15*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8
*b^4)*cosh(d*x + c)^4 + 16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3 + 6*(24*a^
4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 - 15*(a^4*cosh(d*x + c)^10 + 10*a^4*cosh(d*x + c)*sinh(d*x + c)^9 + a^4
*sinh(d*x + c)^10 + (3*a^4 + 8*a^3*b)*cosh(d*x + c)^8 + (45*a^4*cosh(d*x +
c)^2 + 3*a^4 + 8*a^3*b)*sinh(d*x + c)^8 + 8*(15*a^4*cosh(d*x + c)^3 + (3*a^
4 + 8*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(a^4 + 4*a^3*b + 8*a^2*b^2)
*cosh(d*x + c)^6 + 2*(105*a^4*cosh(d*x + c)^4 + a^4 + 4*a^3*b + 8*a^2*b^2 +
14*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*a^4*cosh(d*x
+ c)^5 + 14*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^3 + 3*(a^4 + 4*a^3*b + 8*a^2*b
^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x
+ c)^4 + 2*(105*a^4*cosh(d*x + c)^6 + 35*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^4
- a^4 - 4*a^3*b - 8*a^2*b^2 + 15*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 - a^4 + 8*(15*a^4*cosh(d*x + c)^7 + 7*(3*a^4 + 8*a^3*b)
*cosh(d*x + c)^5 + 5*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 - (a^4 + 4
*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (3*a^4 + 8*a^3*b)*cosh
(d*x + c)^2 + (45*a^4*cosh(d*x + c)^8 + 28*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^
6 + 30*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 - 3*a^4 - 8*a^3*b - 12*(
a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*a^4*cosh
(d*x + c)^9 + 4*(3*a^4 + 8*a^3*b)*cosh(d*x + c)^7 + 6*(a^4 + 4*a^3*b + 8*a^
2*b^2)*cosh(d*x + c)^5 - 4*(a^4 + 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 - (3
*a^4 + 8*a^3*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh
(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2
*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*si
nh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b
)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 +
a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*
b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
+ c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x
+ c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(
d*x + c))*sinh(d*x + c) + a)) + 16*(2*(8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b
^3)*cosh(d*x + c)^7 + 3*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b^4)
*cosh(d*x + c)^5 + 2*(24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8*b^4)*co
sh(d*x + c)^3 + (16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*s
inh(d*x + c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^10 + 1
0*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (
a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*sinh(d*x + c)^10 + (3*a^7 + 17*a^6*b
+ 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^8 + (45*(a^7 + 3*a^
6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (3*a^7 + 17*a^6*b + 33*a^5*b
^2 + 27*a^4*b^3 + 8*a^3*b^4)*d)*sinh(d*x + c)^8 + 2*(a^7 + 7*a^6*b + 23*a^5
*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^6 + 8*(15*(a^7
+ 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^3 + (3*a^7 + 17*a^6*b + 33

```
*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(10
5*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^4 + 14*(3*a^7 + 17*
a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 7*a
^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d)*sinh(d*x + c)^6
 - 2*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*d*c
osh(d*x + c)^4 + 4*(63*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c
)^5 + 14*(3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*
x + c)^3 + 3*(a^7 + 7*a^6*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*
b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
[Out] Integral(csch(c + d*x)**2/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(116) = 232.
time = 0.86, size = 347, normalized size = 2.75

$$\frac{\frac{15\,b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{-ab-b^2}}-\frac{2\,(9\,a^3be^{(6\,dx+6\,c)}+24\,a^2b^2e^{(6\,dx+6\,c)}+8\,ab^3e^{(6\,dx+6\,c)}+27\,a^3be^{(4\,dx+4\,c)}+78\,a^2b^2e^{(4\,dx+4\,c)}+88\,ab^3e^{(4\,dx+4\,c)}+16\,b^4e^{(4\,dx+4\,c)}+27\,a^3be^{(2\,dx+2\,c)}+56\,a^2b^2e^{(2\,dx+2\,c)}+8\,ab^3e^{(2\,dx+2\,c)}+9\,a^3b+2\,a^2b^2)}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)^2}-\frac{16}{(a^3+3\,a^2b+3\,ab^2+b^3)\left(e^{(2\,dx+2\,c)}-1\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
[Out] 1/8*(15*b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a*b - b^2)) - 2*(9*a^3*b*e^(6*d*x + 6*c) +
 24*a^2*b^2*e^(6*d*x + 6*c) + 8*a*b^3*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x +
 4*c) + 78*a^2*b^2*e^(4*d*x + 4*c) + 88*a*b^3*e^(4*d*x + 4*c) + 16*b^4*e^(4
*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 56*a^2*b^2*e^(2*d*x + 2*c) + 8*a*b
^3*e^(2*d*x + 2*c) + 9*a^3*b + 2*a^2*b^2)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2
*b^3)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2
) - 16/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^6}{\sinh\left(c+d\,x\right)^2\,\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3),x)
[Out] int(cosh(c + d*x)^6/(sinh(c + d*x)^2*(b + a*cosh(c + d*x)^2)^3), x)
```

3.47 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=213

$$-\frac{\sqrt{b}\,(15a^2-10ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{3/2}(a+b)^4d}+\frac{(a-5b)\tanh^{-1}(\cosh(c+dx))}{2(a+b)^4d}+\frac{(2a-b)b\cosh(c+}{4a(a+b)^2d\left(b+a\cosh^2\right.}$$

```
[Out] 1/2*(a-5*b)*arctanh(cosh(d*x+c))/(a+b)^4/d+1/4*(2*a-b)*b*cosh(d*x+c)/a/(a+b
)^2/d/(b+a*cosh(d*x+c)^2)^2-1/8*(4*a^2-9*a*b-b^2)*cosh(d*x+c)/a/(a+b)^3/d/(
b+a*cosh(d*x+c)^2)-1/2*cosh(d*x+c)*coth(d*x+c)^2/(a+b)/d/(b+a*cosh(d*x+c)^2
)^2-1/8*(15*a^2-10*a*b-b^2)*arctan(cosh(d*x+c)*a^(1/2)/b^(1/2))*b^(1/2)/a^(
3/2)/(a+b)^4/d
```

Rubi [A]
time = 0.25, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4218, 481, 592, 541, 536, 212, 211}

$$-\frac{(4a^2-9ab-b^2)\cosh(c+dx)}{8ad(a+b)^3\left(a\cosh^2(c+dx)+b\right)}-\frac{\sqrt{b}\,(15a^2-10ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{3/2}d(a+b)^4}+\frac{b(2a-b)\cosh(c+dx)}{4ad(a+b)^2\left(a\cosh^2(c+dx)+b\right)^2}-\frac{\cosh(c+dx)\coth^2(c+dx)}{2d(a+b)\left(a\cosh^2(c+dx)+b\right)^2}+\frac{(a-5b)\tanh^{-1}(\cosh(c+dx))}{2d(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/8*(Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqrt[a]*Cosh[c + d*x])/Sqrt[b
]])/(a^(3/2)*(a + b)^4*d) + ((a - 5*b)*ArcTanh[Cosh[c + d*x]])/(2*(a + b)^4
*d) + ((2*a - b)*b*Cosh[c + d*x])/(4*a*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)^
2) - ((4*a^2 - 9*a*b - b^2)*Cosh[c + d*x])/(8*a*(a + b)^3*d*(b + a*Cosh[c +
 d*x]^2)) - (Cosh[c + d*x]*Coth[c + d*x]^2)/(2*(a + b)*d*(b + a*Cosh[c + d*
x]^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^6}{(1-x^2)^2(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\cosh(c+dx)\coth^2(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{x^2(3b+(-a+2b)x^2)}{(1-x^2)(b+ax^2)^3}\,dx, x, \cosh(c+\right.}{2(a+b)d}$$

$$= \frac{(2a-b)b\cosh(c+dx)}{4a(a+b)^2d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{\cosh(c+dx)\coth^2(c+dx)}{2(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{\text{Sub}}{}$$

$$= \frac{(2a-b)b\cosh(c+dx)}{4a(a+b)^2d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{(4a^2-9ab-b^2)\cosh(c+dx)}{8a(a+b)^3d\left(b+a\cosh^2(c+dx)\right)} - \frac{}{2(}$$

$$= \frac{(2a-b)b\cosh(c+dx)}{4a(a+b)^2d\left(b+a\cosh^2(c+dx)\right)^2} - \frac{(4a^2-9ab-b^2)\cosh(c+dx)}{8a(a+b)^3d\left(b+a\cosh^2(c+dx)\right)} - \frac{}{2(}$$

$$= -\frac{\sqrt{b}\,(15a^2-10ab-b^2)\tan^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{b}}\right)}{8a^{3/2}(a+b)^4d} + \frac{(a-5b)\tanh^{-1}(\cosh(c+}{2(a+b)^4d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.79, size = 524, normalized size = 2.46

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*((-8*b^2*(a + b)^2)/a + (2
*b*(a + b)*(9*a + b)*(a + 2*b + a*Cosh[2*(c + d*x)]))/a + (Sqrt[b]*(-15*a^2
 + 10*a*b + b^2)*ArcTan[((Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^
2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]*(Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cosh[c]
- Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Se
ch[c + d*x])/a^(3/2) + (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[((Sqrt[a] +
 I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2])*Sinh[c]*Tanh[(d*x)/2] + Cosh[c]
*(Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[(d*x)/2]))/Sqrt[
b]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x])/a^(3/2) - (a + b)*(a +
 2*b + a*Cosh[2*(c + d*x)])^2*Csch[(c + d*x)/2]^2*Sech[c + d*x] + 4*(a - 5*
b)*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Log[Cosh[(c + d*x)/2]]*Sech[c + d*x] -
 4*(a - 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Log[Sinh[(c + d*x)/2]]*Sech[
c + d*x] - (a + b)*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[(c + d*x)/2]^2*Se
ch[c + d*x]))/(64*(a + b)^4*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [A]
time = 2.85, size = 350, normalized size = 1.64

method	result
derivativedivides	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3+24a^2b+24a\,b^2+8b^3}-\frac{2b\left(\frac{-\frac{\left(9a^3-5a^2b-13a\,b^2+b^3\right)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{\left(27a^3-21a^2b+29a\,b^2-3b^3\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{(27a}{}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}\right.}{}}{}$
default	$\frac{\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3+24a^2b+24a\,b^2+8b^3}-\frac{2b\left(\frac{-\frac{\left(9a^3-5a^2b-13a\,b^2+b^3\right)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{\left(27a^3-21a^2b+29a\,b^2-3b^3\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{(27a}{}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}\right.}{}}{}$
risch	$-\frac{e^{dx+c}\left(4a^3e^{10dx+10c}-9a^2b\,e^{10dx+10c}-a\,b^2e^{10dx+10c}+20a^3e^{8dx+8c}+23a^2b\,e^{8dx+8c}-29a\,b^2e^{8dx+8c}+4b^3e^{8dx+8c}+40\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/(a^3+3*a^2*b+3*a*b^2+b^3)-2*b/(a+b)^4*((-1/8
*(9*a^3-5*a^2*b-13*a*b^2+b^3)/a*tanh(1/2*d*x+1/2*c)^6-1/8*(27*a^3-21*a^2*b+
29*a*b^2-3*b^3)/a*tanh(1/2*d*x+1/2*c)^4-1/8*(27*a^3+a^2*b-23*a*b^2+3*b^3)/a
*tanh(1/2*d*x+1/2*c)^2-1/8*(9*a^3+17*a^2*b+7*a*b^2-b^3)/a)/(a*tanh(1/2*d*x+
1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x
+1/2*c)^2+a+b)^2+1/16*(15*a^2-10*a*b-b^2)/a/(a*b)^(1/2)*arctan(1/4*(2*(a+b)
*tanh(1/2*d*x+1/2*c)^2+2*a-2*b)/(a*b)^(1/2)))-1/8/(a+b)^3/tanh(1/2*d*x+1/2*
c)^2+1/4/(a+b)^4*(-2*a+10*b)*ln(tanh(1/2*d*x+1/2*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a - 5*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^4*d + 4*a^3*b*d + 6*a^2*b^2*
d + 4*a*b^3*d + b^4*d) - 1/2*(a - 5*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^4*d
 + 4*a^3*b*d + 6*a^2*b^2*d + 4*a*b^3*d + b^4*d) - 1/4*((4*a^3*e^(11*c) - 9*
a^2*b*e^(11*c) - a*b^2*e^(11*c))*e^(11*d*x) + (20*a^3*e^(9*c) + 23*a^2*b*e^
(9*c) - 29*a*b^2*e^(9*c) + 4*b^3*e^(9*c))*e^(9*d*x) + 2*(20*a^3*e^(7*c) + 5
7*a^2*b*e^(7*c) + 47*a*b^2*e^(7*c) - 2*b^3*e^(7*c))*e^(7*d*x) + 2*(20*a^3*e
```

```
^(5*c) + 57*a^2*b*e^(5*c) + 47*a*b^2*e^(5*c) - 2*b^3*e^(5*c))*e^(5*d*x) + (
20*a^3*e^(3*c) + 23*a^2*b*e^(3*c) - 29*a*b^2*e^(3*c) + 4*b^3*e^(3*c))*e^(3*
d*x) + (4*a^3*e^c - 9*a^2*b*e^c - a*b^2*e^c)*e^(d*x))/(a^6*d + 3*a^5*b*d +
3*a^4*b^2*d + a^3*b^3*d + (a^6*d*e^(12*c) + 3*a^5*b*d*e^(12*c) + 3*a^4*b^2*
d*e^(12*c) + a^3*b^3*d*e^(12*c))*e^(12*d*x) + 2*(a^6*d*e^(10*c) + 7*a^5*b*d
*e^(10*c) + 15*a^4*b^2*d*e^(10*c) + 13*a^3*b^3*d*e^(10*c) + 4*a^2*b^4*d*e^(
10*c))*e^(10*d*x) - (a^6*d*e^(8*c) + 3*a^5*b*d*e^(8*c) - 13*a^4*b^2*d*e^(8*
c) - 47*a^3*b^3*d*e^(8*c) - 48*a^2*b^4*d*e^(8*c) - 16*a*b^5*d*e^(8*c))*e^(8
*d*x) - 4*(a^6*d*e^(6*c) + 7*a^5*b*d*e^(6*c) + 23*a^4*b^2*d*e^(6*c) + 37*a^
3*b^3*d*e^(6*c) + 28*a^2*b^4*d*e^(6*c) + 8*a*b^5*d*e^(6*c))*e^(6*d*x) - (a^
6*d*e^(4*c) + 3*a^5*b*d*e^(4*c) - 13*a^4*b^2*d*e^(4*c) - 47*a^3*b^3*d*e^(4*
c) - 48*a^2*b^4*d*e^(4*c) - 16*a*b^5*d*e^(4*c))*e^(4*d*x) + 2*(a^6*d*e^(2*c
) + 7*a^5*b*d*e^(2*c) + 15*a^4*b^2*d*e^(2*c) + 13*a^3*b^3*d*e^(2*c) + 4*a^2
*b^4*d*e^(2*c))*e^(2*d*x)) - 8*integrate(1/32*((15*a^2*b*e^(3*c) - 10*a*b^2
*e^(3*c) - b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^c - 10*a*b^2*e^c - b^3*e^c)
*e^(d*x))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + (a^6*e^(4*c) +
 4*a^5*b*e^(4*c) + 6*a^4*b^2*e^(4*c) + 4*a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c))
*e^(4*d*x) + 2*(a^6*e^(2*c) + 6*a^5*b*e^(2*c) + 14*a^4*b^2*e^(2*c) + 16*a^3
*b^3*e^(2*c) + 9*a^2*b^4*e^(2*c) + 2*a*b^5*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 10990 vs. $2(195) = 390$.
time = 0.58, size = 20341, normalized size = 95.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(4*a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cosh(d*x + c)^11 + 44*(4*a
^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^10 + 4*(4*a^
4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*sinh(d*x + c)^11 + 4*(20*a^4 + 43*a^3*b -
 6*a^2*b^2 - 25*a*b^3 + 4*b^4)*cosh(d*x + c)^9 + 4*(20*a^4 + 43*a^3*b - 6*a
^2*b^2 - 25*a*b^3 + 4*b^4 + 55*(4*a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^9 + 12*(55*(4*a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)
*cosh(d*x + c)^3 + 3*(20*a^4 + 43*a^3*b - 6*a^2*b^2 - 25*a*b^3 + 4*b^4)*cos
h(d*x + c))*sinh(d*x + c)^8 + 8*(20*a^4 + 77*a^3*b + 104*a^2*b^2 + 45*a*b^3
 - 2*b^4)*cosh(d*x + c)^7 + 8*(165*(4*a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*c
osh(d*x + c)^4 + 20*a^4 + 77*a^3*b + 104*a^2*b^2 + 45*a*b^3 - 2*b^4 + 18*(2
0*a^4 + 43*a^3*b - 6*a^2*b^2 - 25*a*b^3 + 4*b^4)*cosh(d*x + c)^2)*sinh(d*x
+ c)^7 + 56*(33*(4*a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cosh(d*x + c)^5 + 6*
(20*a^4 + 43*a^3*b - 6*a^2*b^2 - 25*a*b^3 + 4*b^4)*cosh(d*x + c)^3 + (20*a^
4 + 77*a^3*b + 104*a^2*b^2 + 45*a*b^3 - 2*b^4)*cosh(d*x + c))*sinh(d*x + c)
^6 + 8*(20*a^4 + 77*a^3*b + 104*a^2*b^2 + 45*a*b^3 - 2*b^4)*cosh(d*x + c)^5
 + 8*(231*(4*a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cosh(d*x + c)^6 + 63*(20*a
```

```
^4 + 43*a^3*b - 6*a^2*b^2 - 25*a*b^3 + 4*b^4)*cosh(d*x + c)^4 + 20*a^4 + 77
*a^3*b + 104*a^2*b^2 + 45*a*b^3 - 2*b^4 + 21*(20*a^4 + 77*a^3*b + 104*a^2*b
^2 + 45*a*b^3 - 2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 8*(165*(4*a^4 - 5
*a^3*b - 10*a^2*b^2 - a*b^3)*cosh(d*x + c)^7 + 63*(20*a^4 + 43*a^3*b - 6*a^
2*b^2 - 25*a*b^3 + 4*b^4)*cosh(d*x + c)^5 + 35*(20*a^4 + 77*a^3*b + 104*a^2
*b^2 + 45*a*b^3 - 2*b^4)*cosh(d*x + c)^3 + 5*(20*a^4 + 77*a^3*b + 104*a^2*b
^2 + 45*a*b^3 - 2*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(20*a^4 + 43*a^3*
b - 6*a^2*b^2 - 25*a*b^3 + 4*b^4)*cosh(d*x + c)^3 + 4*(165*(4*a^4 - 5*a^3*b
 - 10*a^2*b^2 - a*b^3)*cosh(d*x + c)^8 + 84*(20*a^4 + 43*a^3*b - 6*a^2*b^2
- 25*a*b^3 + 4*b^4)*cosh(d*x + c)^6 + 70*(20*a^4 + 77*a^3*b + 104*a^2*b^2 +
 45*a*b^3 - 2*b^4)*cosh(d*x + c)^4 + 20*a^4 + 43*a^3*b - 6*a^2*b^2 - 25*a*b
^3 + 4*b^4 + 20*(20*a^4 + 77*a^3*b + 104*a^2*b^2 + 45*a*b^3 - 2*b^4)*cosh(d
*x + c)^2)*sinh(d*x + c)^3 + 4*(55*(4*a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*c
osh(d*x + c)^9 + 36*(20*a^4 + 43*a^3*b - 6*a^2*b^2 - 25*a*b^3 + 4*b^4)*cosh
(d*x + c)^7 + 42*(20*a^4 + 77*a^3*b + 104*a^2*b^2 + 45*a*b^3 - 2*b^4)*cosh(
d*x + c)^5 + 20*(20*a^4 + 77*a^3*b + 104*a^2*b^2 + 45*a*b^3 - 2*b^4)*cosh(d
*x + c)^3 + 3*(20*a^4 + 43*a^3*b - 6*a^2*b^2 - 25*a*b^3 + 4*b^4)*cosh(d*x +
 c))*sinh(d*x + c)^2 + ((15*a^4 - 10*a^3*b - a^2*b^2)*cosh(d*x + c)^12 + 12
*(15*a^4 - 10*a^3*b - a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (15*a^4 - 1
0*a^3*b - a^2*b^2)*sinh(d*x + c)^12 + 2*(15*a^4 + 50*a^3*b - 41*a^2*b^2 - 4
*a*b^3)*cosh(d*x + c)^10 + 2*(15*a^4 + 50*a^3*b - 41*a^2*b^2 - 4*a*b^3 + 33
*(15*a^4 - 10*a^3*b - a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(
15*a^4 - 10*a^3*b - a^2*b^2)*cosh(d*x + c)^3 + (15*a^4 + 50*a^3*b - 41*a^2*
b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 - (15*a^4 - 10*a^3*b - 241*a^
2*b^2 + 160*a*b^3 + 16*b^4)*cosh(d*x + c)^8 + (495*(15*a^4 - 10*a^3*b - a^2
*b^2)*cosh(d*x + c)^4 - 15*a^4 + 10*a^3*b + 241*a^2*b^2 - 160*a*b^3 - 16*b^
4 + 90*(15*a^4 + 50*a^3*b - 41*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^8 + 8*(99*(15*a^4 - 10*a^3*b - a^2*b^2)*cosh(d*x + c)^5 + 30*(15*a^4
+ 50*a^3*b - 41*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 - (15*a^4 - 10*a^3*b - 2
41*a^2*b^2 + 160*a*b^3 + 16*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*a^4
 + 50*a^3*b + 79*a^2*b^2 - 84*a*b^3 - 8*b^4)*cosh(d*x + c)^6 + 4*(231*(15*a
^4 - 10*a^3*b - a^2*b^2)*cosh(d*x + c)^6 + 105*(15*a^4 + 50*a^3*b - 41*a^2*
b^2 - 4*a*b^3)*cosh(d*x + c)^4 - 15*a^4 - 50*a^3*b - 79*a^2*b^2 + 84*a*b^3
+ 8*b^4 - 7*(15*a^4 - 10*a^3*b - 241*a^2*b^2 + 160*a*b^3 + 16*b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c)^6 + 8*(99*(15*a^4 - 10*a^3*b - a^2*b^2)*cosh(d*x + c
)^7 + 63*(15*a^4 + 50*a^3*b - 41*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^5 - 7*(15
*a^4 - 10*a^3*b - 241*a^2*b^2 + 160*a*b^3 + 16*b^4)*cosh(d*x + c)^3 - 3*(15
*a^4 + 50*a^3*b + 79*a^2*b^2 - 84*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x +
c)^5 - (15*a^4 - 10*a^3*b - 241*a^2*b^2 + 160*a*b^3 + 16*b^4)*cosh(d*x + c)
^4 + (495*(15*a^4 - 10*a^3*b - a^2*b^2)*cosh(d*x + c)^8 + 420*(15*a^4 + 50*
a^3*b - 41*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^6 - 70*(15*a^4 - 10*a^3*b - 241
*a^2*b^2 + 160*a*b^3 + 16*b^4)*cosh(d*x + c)^4 - 15*a^4 + 10*a^3*b + 241*a^
2*b^2 - 160*a*b^3 - 16*b^4 - 60*(15*a^4 + 50*a^3*b + 79*a^2*b^2 - 84*a*b^3
- 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 - 10*a^3*b - a^2*b^2 + 4
*(55*(15*a^4 - 10*a^3*b - a^2*b^2)*cosh(d*x + c)^9 + 60*(15*a^4 + 50*a^3*b
```

```
- 41*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^7 - 14*(15*a^4 - 10*a^3*b - 241*a^2*b
^2 + 160*a*b^3 + 16*b^4)*cosh(d*x + c)^5 - 20*(15*a^4 + 50*a^3*b + 79*a^2*b
^2 - 84*a*b^3 - 8*b^4)*cosh(d*x + c)^3 - (15*a^4 - 10*a^3*b - 241*a^2*b^2 +
 160*a*b^3 + 16*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(15*a^4 + 50*a^3*b
- 41*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2 + 2*(33*(15*a^4 - 10*a^3*b - a^2*b^
2)*cosh(d*x + c)^10 + 45*(15*a^4 + 50*a^3*b - 4...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^6}{\sinh(c+d\,x)^3\,\left(a\cosh(c+d\,x)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(cosh(c + d*x)^6/(sinh(c + d*x)^3*(b + a*cosh(c + d*x)^2)^3), x)
```

3.48 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=165

$$-\frac{5(3a-4b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}d}+\frac{(a-2b)\coth(c+dx)}{(a+b)^4d}-\frac{\coth^3(c+dx)}{3(a+b)^3d}-\frac{ab\tanh(c+a}{4(a+b)^3d\,(a+b-b\tan}$$

[Out] (a-2*b)*coth(d*x+c)/(a+b)^4/d-1/3*coth(d*x+c)^3/(a+b)^3/d-5/8*(3*a-4*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/(a+b)^(9/2)/d-1/4*a*b*tanh(d*x+c)/(a+b)^3/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*(7*a-4*b)*b*tanh(d*x+c)/(a+b)^4/d/(a+b-b*tanh(d*x+c)^2)

Rubi [A]

time = 0.19, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4217, 467, 1273, 1275, 214}

$$-\frac{5\sqrt{b}\,(3a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{8d(a+b)^{9/2}}-\frac{b(7a-4b)\tanh(c+dx)}{8d(a+b)^4\left(a-b\tanh^2(c+dx)+b\right)}-\frac{ab\tanh(c+dx)}{4d(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{\coth^3(c+dx)}{3d(a+b)^3}+\frac{(a-2b)\coth(c+dx)}{d(a+b)^4}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]

[Out] (-5*(3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*(a + b)^(9/2)*d) + ((a - 2*b)*Coth[c + d*x])/((a + b)^4*d) - Coth[c + d*x]^3/(3*(a + b)^3*d) - (a*b*Tanh[c + d*x])/(4*(a + b)^3*d*(a + b - b*Tanh[c + d*x]^2)^2) - ((7*a - 4*b)*b*Tanh[c + d*x])/(8*(a + b)^4*d*(a + b - b*Tanh[c + d*x]^2))

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4217

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + ff
^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{ab\tanh(c+dx)}{4(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b\operatorname{Subst}\left(\int \frac{\frac{4}{b(a+b)}-\frac{4ax^2}{b(a+b)^2}-\frac{3ax^4}{(a+b)^3}}{x^4(a+b-bx^2)^2}\,dx,\right.}{4d}$$

$$= -\frac{ab\tanh(c+dx)}{4(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{(7a-4b)b\tanh(c+dx)}{8(a+b)^4d\left(a+b-b\tanh^2(c+d\right.}$$

$$= -\frac{ab\tanh(c+dx)}{4(a+b)^3d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{(7a-4b)b\tanh(c+dx)}{8(a+b)^4d\left(a+b-b\tanh^2(c+d\right.}$$

$$= \frac{(a-2b)\coth(c+dx)}{(a+b)^4d} - \frac{\coth^3(c+dx)}{3(a+b)^3d} - \frac{ab\tanh(c+dx)}{4(a+b)^3d\left(a+b-b\tanh^2(c+dx\right.}$$

$$= -\frac{5(3a-4b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}d} + \frac{(a-2b)\coth(c+dx)}{(a+b)^4d} - \frac{\coth^3(}{3(a+}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 985 vs. 2(165) = 330.
time = 4.10, size = 985, normalized size = 5.97

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/6144*((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*((480*(3*a - 4*b)*
b*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[
2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Co
sh[2*(c + d*x)])^2*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] -
Sinh[c])^4]) + (Csch[c]*Csch[c + d*x]^3*Sech[2*c]*(4*(44*a^4 + 122*a^3*b +
63*a^2*b^2 + 126*a*b^3 + 36*b^4)*Sinh[d*x] + (-96*a^4 - 71*a^3*b + 344*a^2*
b^2 - 1208*a*b^3 + 48*b^4)*Sinh[3*d*x] + 224*a^4*Sinh[2*c - d*x] + 576*a^3*
b*Sinh[2*c - d*x] + 124*a^2*b^2*Sinh[2*c - d*x] - 2184*a*b^3*Sinh[2*c - d*x
] + 144*b^4*Sinh[2*c - d*x] - 224*a^4*Sinh[2*c + d*x] - 657*a^3*b*Sinh[2*c
+ d*x] - 538*a^2*b^2*Sinh[2*c + d*x] + 984*a*b^3*Sinh[2*c + d*x] + 144*b^4*
Sinh[2*c + d*x] + 176*a^4*Sinh[4*c + d*x] + 569*a^3*b*Sinh[4*c + d*x] + 666
*a^2*b^2*Sinh[4*c + d*x] + 1704*a*b^3*Sinh[4*c + d*x] - 144*b^4*Sinh[4*c +
```

d*x] + 48*a^4*Sinh[2*c + 3*d*x] + 111*a^3*b*Sinh[2*c + 3*d*x] + 360*a^2*b^2
*Sinh[2*c + 3*d*x] + 312*a*b^3*Sinh[2*c + 3*d*x] - 48*b^4*Sinh[2*c + 3*d*x]
 - 96*a^4*Sinh[4*c + 3*d*x] - 152*a^3*b*Sinh[4*c + 3*d*x] + 146*a^2*b^2*Sin
h[4*c + 3*d*x] - 728*a*b^3*Sinh[4*c + 3*d*x] - 48*b^4*Sinh[4*c + 3*d*x] + 4
8*a^4*Sinh[6*c + 3*d*x] + 192*a^3*b*Sinh[6*c + 3*d*x] + 558*a^2*b^2*Sinh[6*
c + 3*d*x] - 168*a*b^3*Sinh[6*c + 3*d*x] + 48*b^4*Sinh[6*c + 3*d*x] + 16*a^
4*Sinh[2*c + 5*d*x] - 598*a^2*b^2*Sinh[2*c + 5*d*x] + 48*a*b^3*Sinh[2*c + 5
*d*x] + 72*a^3*b*Sinh[4*c + 5*d*x] + 150*a^2*b^2*Sinh[4*c + 5*d*x] - 48*a*b
^3*Sinh[4*c + 5*d*x] + 16*a^4*Sinh[6*c + 5*d*x] + 27*a^3*b*Sinh[6*c + 5*d*x
] - 388*a^2*b^2*Sinh[6*c + 5*d*x] + 45*a^3*b*Sinh[8*c + 5*d*x] - 60*a^2*b^2
*Sinh[8*c + 5*d*x] + 16*a^4*Sinh[4*c + 7*d*x] - 83*a^3*b*Sinh[4*c + 7*d*x]
+ 6*a^2*b^2*Sinh[4*c + 7*d*x] + 27*a^3*b*Sinh[6*c + 7*d*x] - 6*a^2*b^2*Sinh
[6*c + 7*d*x] + 16*a^4*Sinh[8*c + 7*d*x] - 56*a^3*b*Sinh[8*c + 7*d*x]))/a))
/((a + b)^4*d*(a + b*Sech[c + d*x]^2)^3)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(149) = 298$.
time = 2.61, size = 412, normalized size = 2.50 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/8/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)*(1/3*a*tanh(1/2*d*x+1/2*c)^3+1/3*
b*tanh(1/2*d*x+1/2*c)^3-3*a*tanh(1/2*d*x+1/2*c)+9*b*tanh(1/2*d*x+1/2*c))-1/
24/(a+b)^3/tanh(1/2*d*x+1/2*c)^3-1/8/(a+b)^4*(-3*a+9*b)/tanh(1/2*d*x+1/2*c)
+2*b/(a+b)^4*(((-9/8*a^2-5/8*a*b+1/2*b^2)*tanh(1/2*d*x+1/2*c)^7+(-27/8*a^2+
13/8*a*b-1/2*b^2)*tanh(1/2*d*x+1/2*c)^5+(-27/8*a^2+13/8*a*b-1/2*b^2)*tanh(1
/2*d*x+1/2*c)^3+(-9/8*a^2-5/8*a*b+1/2*b^2)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2
*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/
2*d*x+1/2*c)^2+a+b)^2+1/8*(15*a-20*b)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1
/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^
(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c
)*b^(1/2)+(a+b)^(1/2)))))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(155) = 310$.
time = 0.66, size = 782, normalized size = 4.74

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 5/16*(3*a*b - 4*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))
/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 1/12*(16*a^4 - 83*a^3*b + 6*a^2

```
*b^2 + 2*(8*a^4 - 299*a^2*b^2 + 24*a*b^3)*e^(-2*d*x - 2*c) - (96*a^4 + 71*a
^3*b - 344*a^2*b^2 + 1208*a*b^3 - 48*b^4)*e^(-4*d*x - 4*c) - 4*(56*a^4 + 14
4*a^3*b + 31*a^2*b^2 - 546*a*b^3 + 36*b^4)*e^(-6*d*x - 6*c) - (176*a^4 + 56
9*a^3*b + 666*a^2*b^2 + 1704*a*b^3 - 144*b^4)*e^(-8*d*x - 8*c) - 6*(8*a^4 +
 32*a^3*b + 93*a^2*b^2 - 28*a*b^3 + 8*b^4)*e^(-10*d*x - 10*c) - 15*(3*a^3*b
 - 4*a^2*b^2)*e^(-12*d*x - 12*c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 +
 a^3*b^4 + (a^7 + 12*a^6*b + 38*a^5*b^2 + 52*a^4*b^3 + 33*a^3*b^4 + 8*a^2*b
^5)*e^(-2*d*x - 2*c) - (3*a^7 + 20*a^6*b + 34*a^5*b^2 - 4*a^4*b^3 - 61*a^3*
b^4 - 56*a^2*b^5 - 16*a*b^6)*e^(-4*d*x - 4*c) - (3*a^7 + 28*a^6*b + 130*a^5
*b^2 + 300*a^4*b^3 + 355*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*e^(-6*d*x - 6*c)
 + (3*a^7 + 28*a^6*b + 130*a^5*b^2 + 300*a^4*b^3 + 355*a^3*b^4 + 208*a^2*b^
5 + 48*a*b^6)*e^(-8*d*x - 8*c) + (3*a^7 + 20*a^6*b + 34*a^5*b^2 - 4*a^4*b^3
 - 61*a^3*b^4 - 56*a^2*b^5 - 16*a*b^6)*e^(-10*d*x - 10*c) - (a^7 + 12*a^6*b
 + 38*a^5*b^2 + 52*a^4*b^3 + 33*a^3*b^4 + 8*a^2*b^5)*e^(-12*d*x - 12*c) - (
a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*e^(-14*d*x - 14*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7442 vs. $2(155) = 310$.
time = 0.47, size = 15161, normalized size = 91.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(60*(3*a^3*b - 4*a^2*b^2)*cosh(d*x + c)^12 + 720*(3*a^3*b - 4*a^2*b^
2)*cosh(d*x + c)*sinh(d*x + c)^11 + 60*(3*a^3*b - 4*a^2*b^2)*sinh(d*x + c)^
12 + 24*(8*a^4 + 32*a^3*b + 93*a^2*b^2 - 28*a*b^3 + 8*b^4)*cosh(d*x + c)^10
 + 24*(8*a^4 + 32*a^3*b + 93*a^2*b^2 - 28*a*b^3 + 8*b^4 + 165*(3*a^3*b - 4*
a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 240*(55*(3*a^3*b - 4*a^2*b^2)*
cosh(d*x + c)^3 + (8*a^4 + 32*a^3*b + 93*a^2*b^2 - 28*a*b^3 + 8*b^4)*cosh(d
*x + c))*sinh(d*x + c)^9 + 4*(176*a^4 + 569*a^3*b + 666*a^2*b^2 + 1704*a*b^
3 - 144*b^4)*cosh(d*x + c)^8 + 4*(7425*(3*a^3*b - 4*a^2*b^2)*cosh(d*x + c)^
4 + 176*a^4 + 569*a^3*b + 666*a^2*b^2 + 1704*a*b^3 - 144*b^4 + 270*(8*a^4 +
 32*a^3*b + 93*a^2*b^2 - 28*a*b^3 + 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8
 + 32*(1485*(3*a^3*b - 4*a^2*b^2)*cosh(d*x + c)^5 + 90*(8*a^4 + 32*a^3*b +
93*a^2*b^2 - 28*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + (176*a^4 + 569*a^3*b + 666
*a^2*b^2 + 1704*a*b^3 - 144*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 16*(56*a^
4 + 144*a^3*b + 31*a^2*b^2 - 546*a*b^3 + 36*b^4)*cosh(d*x + c)^6 + 16*(3465
*(3*a^3*b - 4*a^2*b^2)*cosh(d*x + c)^6 + 315*(8*a^4 + 32*a^3*b + 93*a^2*b^2
 - 28*a*b^3 + 8*b^4)*cosh(d*x + c)^4 + 56*a^4 + 144*a^3*b + 31*a^2*b^2 - 54
6*a*b^3 + 36*b^4 + 7*(176*a^4 + 569*a^3*b + 666*a^2*b^2 + 1704*a*b^3 - 144*
b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 32*(1485*(3*a^3*b - 4*a^2*b^2)*cosh
(d*x + c)^7 + 189*(8*a^4 + 32*a^3*b + 93*a^2*b^2 - 28*a*b^3 + 8*b^4)*cosh(d
*x + c)^5 + 7*(176*a^4 + 569*a^3*b + 666*a^2*b^2 + 1704*a*b^3 - 144*b^4)*co
```

sh(d*x + c)^3 + 3*(56*a^4 + 144*a^3*b + 31*a^2*b^2 - 546*a*b^3 + 36*b^4)*co
sh(d*x + c))*sinh(d*x + c)^5 + 4*(96*a^4 + 71*a^3*b - 344*a^2*b^2 + 1208*a*
b^3 - 48*b^4)*cosh(d*x + c)^4 + 4*(7425*(3*a^3*b - 4*a^2*b^2)*cosh(d*x + c)
^8 + 1260*(8*a^4 + 32*a^3*b + 93*a^2*b^2 - 28*a*b^3 + 8*b^4)*cosh(d*x + c)^
6 + 70*(176*a^4 + 569*a^3*b + 666*a^2*b^2 + 1704*a*b^3 - 144*b^4)*cosh(d*x
+ c)^4 + 96*a^4 + 71*a^3*b - 344*a^2*b^2 + 1208*a*b^3 - 48*b^4 + 60*(56*a^4
 + 144*a^3*b + 31*a^2*b^2 - 546*a*b^3 + 36*b^4)*cosh(d*x + c)^2)*sinh(d*x +
 c)^4 - 64*a^4 + 332*a^3*b - 24*a^2*b^2 + 16*(825*(3*a^3*b - 4*a^2*b^2)*cos
h(d*x + c)^9 + 180*(8*a^4 + 32*a^3*b + 93*a^2*b^2 - 28*a*b^3 + 8*b^4)*cosh(
d*x + c)^7 + 14*(176*a^4 + 569*a^3*b + 666*a^2*b^2 + 1704*a*b^3 - 144*b^4)*
cosh(d*x + c)^5 + 20*(56*a^4 + 144*a^3*b + 31*a^2*b^2 - 546*a*b^3 + 36*b^4)
*cosh(d*x + c)^3 + (96*a^4 + 71*a^3*b - 344*a^2*b^2 + 1208*a*b^3 - 48*b^4)*
cosh(d*x + c))*sinh(d*x + c)^3 - 8*(8*a^4 - 299*a^2*b^2 + 24*a*b^3)*cosh(d*
x + c)^2 + 8*(495*(3*a^3*b - 4*a^2*b^2)*cosh(d*x + c)^10 + 135*(8*a^4 + 32*
a^3*b + 93*a^2*b^2 - 28*a*b^3 + 8*b^4)*cosh(d*x + c)^8 + 14*(176*a^4 + 569*
a^3*b + 666*a^2*b^2 + 1704*a*b^3 - 144*b^4)*cosh(d*x + c)^6 + 30*(56*a^4 +
144*a^3*b + 31*a^2*b^2 - 546*a*b^3 + 36*b^4)*cosh(d*x + c)^4 - 8*a^4 + 299*
a^2*b^2 - 24*a*b^3 + 3*(96*a^4 + 71*a^3*b - 344*a^2*b^2 + 1208*a*b^3 - 48*b
^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 15*((3*a^4 - 4*a^3*b)*cosh(d*x + c)^
14 + 14*(3*a^4 - 4*a^3*b)*cosh(d*x + c)*sinh(d*x + c)^13 + (3*a^4 - 4*a^3*b
)*sinh(d*x + c)^14 + (3*a^4 + 20*a^3*b - 32*a^2*b^2)*cosh(d*x + c)^12 + (3*
a^4 + 20*a^3*b - 32*a^2*b^2 + 91*(3*a^4 - 4*a^3*b)*cosh(d*x + c)^2)*sinh(d*
x + c)^12 + 4*(91*(3*a^4 - 4*a^3*b)*cosh(d*x + c)^3 + 3*(3*a^4 + 20*a^3*b -
 32*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^11 - (9*a^4 + 12*a^3*b - 80*a^2*b
^2 + 64*a*b^3)*cosh(d*x + c)^10 + (1001*(3*a^4 - 4*a^3*b)*cosh(d*x + c)^4 -
 9*a^4 - 12*a^3*b + 80*a^2*b^2 - 64*a*b^3 + 66*(3*a^4 + 20*a^3*b - 32*a^2*b
^2)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(1001*(3*a^4 - 4*a^3*b)*cosh(d*x
+ c)^5 + 110*(3*a^4 + 20*a^3*b - 32*a^2*b^2)*cosh(d*x + c)^3 - 5*(9*a^4 + 1
2*a^3*b - 80*a^2*b^2 + 64*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 - (9*a^4 +
36*a^3*b + 80*a^2*b^2 - 192*a*b^3)*cosh(d*x + c)^8 + (3003*(3*a^4 - 4*a^3*b
)*cosh(d*x + c)^6 + 495*(3*a^4 + 20*a^3*b - 32*a^2*b^2)*cosh(d*x + c)^4 - 9
*a^4 - 36*a^3*b - 80*a^2*b^2 + 192*a*b^3 - 45*(9*a^4 + 12*a^3*b - 80*a^2*b^
2 + 64*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(429*(3*a^4 - 4*a^3*b)*c
osh(d*x + c)^7 + 99*(3*a^4 + 20*a^3*b - 32*a^2*b^2)*cosh(d*x + c)^5 - 15*(9
*a^4 + 12*a^3*b - 80*a^2*b^2 + 64*a*b^3)*cosh(d*x + c)^3 - (9*a^4 + 36*a^3*
b + 80*a^2*b^2 - 192*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (9*a^4 + 36*a^
3*b + 80*a^2*b^2 - 192*a*b^3)*cosh(d*x + c)^6 + (3003*(3*a^4 - 4*a^3*b)*cos
h(d*x + c)^8 + 924*(3*a^4 + 20*a^3*b - 32*a^2*b^2)*cosh(d*x + c)^6 - 210*(9
*a^4 + 12*a^3*b - 80*a^2*b^2 + 64*a*b^3)*cosh(d*x + c)^4 + 9*a^4 + 36*a^3*b
 + 80*a^2*b^2 - 192*a*b^3 - 28*(9*a^4 + 36*a^3*b + 80*a^2*b^2 - 192*a*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c)^6 + 2*(1001*(3*a^4 - 4*a^3*b)*cosh(d*x + c)^
9 + 396*(3*a^4 + 20*a^3*b - 32*a^2*b^2)*cosh(d*x + c)^7 - 126*(9*a^4 + 12*a
^3*b - 80*a^2*b^2 + 64*a*b^3)*cosh(d*x + c)^5 - 28*(9*a^4 + 36*a^3*b + 80*a
^2*b^2 - 192*a*b^3)*cosh(d*x + c)^3 + 3*(9*a^4 + 36*a^3*b + 80*a^2*b^2 - 19
2*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (9*a^4 + 12*a^3*b - 80*a^2*b^2 +

```
64*a*b^3)*cosh(d*x + c)^4 + (1001*(3*a^4 - 4*a^3*b)*cosh(d*x + c)^10 + 495*
(3*a^4 + 20*a^3*b - 32*a^2*b^2)*cosh(d*x + c)^8...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(csch(c + d*x)**4/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(155) = 310.
time = 0.87, size = 406, normalized size = 2.46

$$-\frac{\frac{15\,(3\,ab-4\,b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\sqrt{-ab-b^2}}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{-ab-b^2}}-\frac{6\,\left(9\,a^3be^{(6\,dx+6\,c)}+20\,a^2b^2e^{(6\,dx+6\,c)}+27\,a^3be^{(4\,dx+4\,c)}+66\,a^2b^2e^{(4\,dx+4\,c)}+56\,ab^3e^{(4\,dx+4\,c)}-16\,b^4e^{(4\,dx+4\,c)}+27\,a^3be^{(2\,dx+2\,c)}+44\,a^2b^2e^{(2\,dx+2\,c)}-16\,ab^3e^{(2\,dx+2\,c)}+9\,a^3b-2\,a^2b^2\right)}{(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)^2}+\frac{16\,\left(9\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}-12\,be^{(2\,dx+2\,c)}-2\,a+7\,b\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\left(e^{(2\,dx+2\,c)}-1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(15*(3*a*b - 4*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*
b - b^2))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(-a*b - b^2)) -
6*(9*a^3*b*e^(6*d*x + 6*c) + 20*a^2*b^2*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x
 + 4*c) + 66*a^2*b^2*e^(4*d*x + 4*c) + 56*a*b^3*e^(4*d*x + 4*c) - 16*b^4*e^
(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 44*a^2*b^2*e^(2*d*x + 2*c) - 16*
a*b^3*e^(2*d*x + 2*c) + 9*a^3*b - 2*a^2*b^2)/((a^5 + 4*a^4*b + 6*a^3*b^2 +
4*a^2*b^3 + a*b^4)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x
+ 2*c) + a)^2) + 16*(9*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) - 12*b*e^(2*
d*x + 2*c) - 2*a + 7*b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(e^(2*
d*x + 2*c) - 1)^3))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^6}{\sinh\left(c+d\,x\right)^4\,\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(cosh(c + d*x)^6/(sinh(c + d*x)^4*(b + a*cosh(c + d*x)^2)^3), x)
```

3.49 $\int \cosh^4(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{1}{8}(3a+4b)x+\frac{(3a+4b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

```
[Out] 1/8*(3*a+4*b)*x+1/8*(3*a+4*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a*cosh(d*x+c)^3
*sinh(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4130, 2715, 8}

$$\frac{(3a+4b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x(3a+4b)+\frac{a\sinh(c+dx)\cosh^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((3*a + 4*b)*x)/8 + ((3*a + 4*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (a*Co
sh[c + d*x]^3*Sinh[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = \frac{a\cosh^3(c+dx)\sinh(c+dx)}{4d}+\frac{1}{4}(3a+4b)\int \cosh^2(c+dx)\,dx$$
$$= \frac{(3a+4b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a\cosh^3(c+dx)\sinh(c+$$
$$= \frac{1}{8}(3a+4b)x+\frac{(3a+4b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a\cosh^3(c$$

Mathematica [A]
time = 0.08, size = 45, normalized size = 0.74

$$\frac{4(3a+4b)(c+dx)+8(a+b)\sinh(2(c+dx))+a\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (4*(3*a + 4*b)*(c + d*x) + 8*(a + b)*Sinh[2*(c + d*x)] + a*Sinh[4*(c + d*x)
])/(32*d)
```

Maple [A]
time = 2.68, size = 46, normalized size = 0.75

method	result	size
default	$\frac{\left(\frac{a}{2}+\frac{b}{2}\right)\sinh(2dx+2c)}{2d}+\frac{3ax}{8}+\frac{bx}{2}+\frac{a\sinh(4dx+4c)}{32d}$	46
risch	$\frac{3ax}{8}+\frac{bx}{2}+\frac{a\,\mathrm{e}^{4dx+4c}}{64d}+\frac{\mathrm{e}^{2dx+2c}a}{8d}+\frac{\mathrm{e}^{2dx+2c}b}{8d}-\frac{\mathrm{e}^{-2dx-2c}a}{8d}-\frac{\mathrm{e}^{-2dx-2c}b}{8d}-\frac{a\,\mathrm{e}^{-4dx-4c}}{64d}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1/2*a+1/2*b)/d*sinh(2*d*x+2*c)+3/8*a*x+1/2*b*x+1/32*a/d*sinh(4*d*x+4*c
)
```

Maxima [A]
time = 0.27, size = 97, normalized size = 1.59

$$\frac{1}{64}\,a\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}+\frac{8\,e^{(2\,dx+2\,c)}}{d}-\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{1}{8}\,b\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + 1/8*b*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c
)/d)
```

Fricas [A]
time = 0.36, size = 61, normalized size = 1.00

$$\frac{a \cosh \left(dx+c\right) \sinh \left(dx+c\right)^{3}+\left(3 a+4 b\right) dx+\left(a \cosh \left(dx+c\right)^{3}+4 \left(a+b\right) \cosh \left(dx+c\right)\right) \sinh \left(dx+c\right)}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(a*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a + 4*b)*d*x + (a*cosh(d*x + c)^3
 + 4*(a + b)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b \operatorname{sech}^{2}\left(c+dx\right)\right) \cosh^{4}\left(c+dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*cosh(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(55) = 110.
time = 0.39, size = 116, normalized size = 1.90

$$\frac{8 \left(dx+c\right) \left(3 a+4 b\right)+a e^{\left(4 dx+4 c\right)}+8 a e^{\left(2 dx+2 c\right)}+8 b e^{\left(2 dx+2 c\right)}-\left(18 a e^{\left(4 dx+4 c\right)}+24 b e^{\left(4 dx+4 c\right)}+8 a e^{\left(2 dx+2 c\right)}+8 b e^{\left(2 dx+2 c\right)}+a\right) e^{\left(-4 dx-4 c\right)}}{64 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(8*(d*x + c)*(3*a + 4*b) + a*e^(4*d*x + 4*c) + 8*a*e^(2*d*x + 2*c) + 8
*b*e^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) + 24*b*e^(4*d*x + 4*c) + 8*a*e^(
2*d*x + 2*c) + 8*b*e^(2*d*x + 2*c) + a)*e^(-4*d*x - 4*c))/d
```

Mupad [B]
time = 0.13, size = 50, normalized size = 0.82

$$\frac{\frac{a \sinh(2 c+2 d x)}{4}+\frac{a \sinh(4 c+4 d x)}{32}+\frac{b \sinh(2 c+2 d x)}{4}}{d}+\frac{3 a x}{8}+\frac{b x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] ((a*sinh(2*c + 2*d*x))/4 + (a*sinh(4*c + 4*d*x))/32 + (b*sinh(2*c + 2*d*x))
/4)/d + (3*a*x)/8 + (b*x)/2
```

3.50 $\int \cosh^3(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=30

$$\frac{(a+b)\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}$$

```
[Out] (a+b)*sinh(d*x+c)/d+1/3*a*sinh(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4129, 3092}

$$\frac{(a+b)\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + b)*Sinh[c + d*x])/d + (a*Sinh[c + d*x]^3)/(3*d)
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)\,dx &= \int \cosh(c+dx)\left(b+a\cosh^2(c+dx)\right)\,dx\\ &= \frac{i\mathrm{Subst}\left(\int \left(a+b-ax^2\right)\,dx, x, -i\sinh(c+dx)\right)}{d}\\ &= \frac{(a+b)\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 1.67

$$\frac{b\cosh(dx)\sinh(c)}{d}+\frac{b\cosh(c)\sinh(dx)}{d}+\frac{a\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*x])/d + (a*Sinh[c + d*x])/d + (
a*Sinh[c + d*x]^3)/(3*d)
```

Maple [A]
time = 2.52, size = 32, normalized size = 1.07

method	result	size
default	$\frac{\left(\frac{3a}{4}+b\right)\sinh(dx+c)}{d}+\frac{a\sinh(3dx+3c)}{12d}$	32
risch	$\frac{a\,\mathrm{e}^{3dx+3c}}{24d}+\frac{3a\,\mathrm{e}^{dx+c}}{8d}+\frac{b\,\mathrm{e}^{dx+c}}{2d}-\frac{3\,\mathrm{e}^{-dx-c}a}{8d}-\frac{\mathrm{e}^{-dx-c}b}{2d}-\frac{a\,\mathrm{e}^{-3dx-3c}}{24d}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] (3/4*a+b)/d*sinh(d*x+c)+1/12*a/d*sinh(3*d*x+3*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(28) = 56.
time = 0.27, size = 85, normalized size = 2.83

$$\frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{1}{2}\,b\left(\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/24*a*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x
- 3*c)/d) + 1/2*b*(e^(d*x + c)/d - e^(-d*x - c)/d)
```

Fricas [A]
time = 0.35, size = 41, normalized size = 1.37

$$\frac{a\sinh\left(dx+c\right)^{3}+3\left(a\cosh\left(dx+c\right)^{2}+3\,a+4\,b\right)\sinh\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(a*sinh(d*x + c)^3 + 3*(a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))/
d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2\left(c + dx\right)\right)\cosh^3\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*cosh(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(28) = 56.
time = 0.40, size = 72, normalized size = 2.40

$$\frac{ae^{(3\,dx+3\,c)} + 9\,ae^{(dx+c)} + 12\,be^{(dx+c)} - \left(9\,ae^{(2\,dx+2\,c)} + 12\,be^{(2\,dx+2\,c)} + a\right)e^{(-3\,dx-3\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*(a*e^(3*d*x + 3*c) + 9*a*e^(d*x + c) + 12*b*e^(d*x + c) - (9*a*e^(2*d*
x + 2*c) + 12*b*e^(2*d*x + 2*c) + a)*e^(-3*d*x - 3*c))/d
```

Mupad [B]
time = 0.09, size = 34, normalized size = 1.13

$$\frac{3\,a\sinh(c + d\,x) + 3\,b\sinh(c + d\,x) + a\sinh(c + d\,x)^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (3*a*sinh(c + d*x) + 3*b*sinh(c + d*x) + a*sinh(c + d*x)^3)/(3*d)
```

3.51 $\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}(a+2b)x+\frac{a\cosh(c+dx)\sinh(c+dx)}{2d}$$

```
[Out] 1/2*(a+2*b)*x+1/2*a*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4130, 8}

$$\frac{1}{2}x(a+2b)+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b)*x)/2 + (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx &= \frac{a\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{1}{2}(a+2b)\int 1\,dx\\ &= \frac{1}{2}(a+2b)x+\frac{a\cosh(c+dx)\sinh(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.06

$$bx+\frac{a(c+dx)}{2d}+\frac{a\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
[Out] b*x + (a*(c + d*x))/(2*d) + (a*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 1.77, size = 37, normalized size = 1.19

method	result	size
derivativedivides	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b(dx+c)}{d}$	37
default	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b(dx+c)}{d}$	37
risch	$\frac{ax}{2}+bx+\frac{e^{2dx+2c}a}{8d}-\frac{e^{-2dx-2c}a}{8d}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+b*(d*x+c))
```

Maxima [A]
time = 0.30, size = 38, normalized size = 1.23

$$\frac{1}{8}\,a\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
[Out] 1/8*a*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + b*x
```

Fricas [A]
time = 0.35, size = 28, normalized size = 0.90

$$\frac{(a+2\,b)dx+a\cosh\left(dx+c\right)\sinh\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
[Out] 1/2*((a + 2*b)*d*x + a*cosh(d*x + c)*sinh(d*x + c))/d
```

Sympy [A]
time = 8.08, size = 60, normalized size = 1.94

$$a\left(\begin{cases}-\frac{x\sinh^2(c+dx)}{2}+\frac{x\cosh^2(c+dx)}{2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2d} & \text{for}\ d\neq 0\\ x\cosh^2(c) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}x & \text{for}\ |x|<1\\ G_{2,2}^{1,1}\left(\begin{matrix}1 & 2\\ 1 & 0\end{matrix}\middle|x\right)+G_{2,2}^{0,2}\left(\begin{matrix}2,1 & \\ & 1,0\end{matrix}\middle|x\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sech(d*x+c)**2),x)
```

```
[Out] a*Piecewise((-x*sinh(c + d*x)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*c
osh(c + d*x)/(2*d), Ne(d, 0)), (x*cosh(c)**2, True)) + b*Piecewise((x, Abs(
x) < 1), (meijerg(((1,), (2,)), ((1,), (0,)), x) + meijerg(((2, 1), ()), ((
), (1, 0)), x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(27) = 54$.
time = 0.41, size = 66, normalized size = 2.13

$$\frac{4\,(dx+c)(a+2\,b)+ae^{(2\,dx+2\,c)}-\left(2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)e^{(-2\,dx-2\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*(d*x + c)*(a + 2*b) + a*e^(2*d*x + 2*c) - (2*a*e^(2*d*x + 2*c) + 4*b
*e^(2*d*x + 2*c) + a)*e^(-2*d*x - 2*c))/d
```

Mupad [B]
time = 0.08, size = 23, normalized size = 0.74

$$\frac{a\,x}{2}+b\,x+\frac{a\sinh(2\,c+2\,d\,x)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (a*x)/2 + b*x + (a*sinh(2*c + 2*d*x))/(4*d)
```

3.52 $\int \cosh(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=24

$$\frac{b\text{ArcTan}(\sinh(c+dx))}{d}+\frac{a\sinh(c+dx)}{d}$$

```
[Out] b*arctan(sinh(d*x+c))/d+a*sinh(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {4130, 3855}

$$\frac{a\sinh(c+dx)}{d}+\frac{b\text{ArcTan}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/d + (a*Sinh[c + d*x])/d
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cosh(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx&=\frac{a\sinh(c+dx)}{d}+b\int \text{sech}(c+dx)\,dx\\&=\frac{b\tan^{-1}(\sinh(c+dx))}{d}+\frac{a\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.46

$$\frac{b\text{ArcTan}(\sinh(c+dx))}{d}+\frac{a\cosh(dx)\sinh(c)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/d + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x
])/d
```

Maple [A]
time = 1.50, size = 24, normalized size = 1.00

method	result	size
derivativedivides	$\frac{a\sinh(dx+c)+2b\arctan\left(e^{dx+c}\right)}{d}$	24
default	$\frac{a\sinh(dx+c)+2b\arctan\left(e^{dx+c}\right)}{d}$	24
risch	$\frac{a\,e^{dx+c}}{2d}-\frac{e^{-dx-c}a}{2d}+\frac{ib\ln\left(e^{dx+c}+i\right)}{d}-\frac{ib\ln\left(e^{dx+c}-i\right)}{d}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*sinh(d*x+c)+2*b*arctan(exp(d*x+c)))
```

Maxima [A]
time = 0.52, size = 28, normalized size = 1.17

$$-\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{a\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -2*b*arctan(e^(-d*x - c))/d + a*sinh(d*x + c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(24) = 48.
time = 0.37, size = 93, normalized size = 3.88

$$\frac{a\cosh\left(dx+c\right)^{2}+2\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a\sinh\left(dx+c\right)^{2}+4\left(b\cosh\left(dx+c\right)+b\sinh\left(dx+c\right)\right)\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)-a}{2\left(d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^
2 + 4*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x +
 c)) - a)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)\cosh\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*cosh(c + d*x), x)
```

Giac [A]
time = 0.40, size = 36, normalized size = 1.50

$$\frac{4\,b\arctan\left(e^{(dx+c)}\right)+ae^{(dx+c)}-ae^{(-dx-c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(4*b*arctan(e^(d*x + c)) + a*e^(d*x + c) - a*e^(-d*x - c))/d
```

Mupad [B]
time = 1.44, size = 62, normalized size = 2.58

$$\frac{2\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}}-\frac{a\,\mathrm{e}^{-c-d\,x}}{2\,d}+\frac{a\,\mathrm{e}^{c+d\,x}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (2*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2))/(d^2)
^(1/2) - (a*exp(- c - d*x))/(2*d) + (a*exp(c + d*x))/(2*d)
```

3.53 $\int \text{sech}(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=40

$$\frac{(2a+b)\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{b\,\text{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[Out] 1/2*(2*a+b)*arctan(sinh(d*x+c))/d+1/2*b*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4131, 3855}

$$\frac{(2a+b)\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{b\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((2*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(
2*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \text{sech}(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)\,dx&=\frac{b\,\text{sech}(c+dx)\tanh(c+dx)}{2d}+\frac{1}{2}(2a+b)\int \text{sech}(c+dx)\,dx\\&=\frac{(2a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b\,\text{sech}(c+dx)\tanh(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 1.20

$$\frac{a\text{ArcTan}(\sinh(c+dx))}{d}+\frac{b\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{b\,\text{sech}(c+dx)\tanh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/d + (b*ArcTan[Sinh[c + d*x]])/(2*d) + (b*Sech[c +
 d*x]*Tanh[c + d*x])/(2*d)
```

Maple [C] Result contains complex when optimal does not.
time = 1.17, size = 105, normalized size = 2.62

method	result	size
risch	$\frac{b\,{\mathrm e}^{d x +c} \left({\mathrm e}^{2 d x +2 c}-1\right)}{d \left(1+{\mathrm e}^{2 d x +2 c}\right)^{2}}+\frac{i \ln \left({\mathrm e}^{d x +c}+i\right) a}{d}+\frac{i b \ln \left({\mathrm e}^{d x +c}+i\right)}{2 d}-\frac{i \ln \left({\mathrm e}^{d x +c}-i\right) a}{d}-\frac{i b \ln \left({\mathrm e}^{d x +c}-i\right)}{2 d}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] b*exp(d*x+c)*(exp(2*d*x+2*c)-1)/d/(1+exp(2*d*x+2*c))^2+I/d*ln(exp(d*x+c)+I)
*a+1/2*I*b/d*ln(exp(d*x+c)+I)-I/d*ln(exp(d*x+c)-I)*a-1/2*I*b/d*ln(exp(d*x+c
)-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(36) = 72$.
time = 0.48, size = 81, normalized size = 2.02

$$-b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) + \frac{a\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*arctan(sinh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(36) = 72$.
time = 0.35, size = 321, normalized size = 8.02

$$\frac{b\cosh\left(dx+c\right)^{3} + 3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2} + b\sinh\left(dx+c\right)^{3} + \left((2\,a+b)\cosh\left(dx+c\right)^{4} + 4\,(2\,a+b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3} + (2\,a+b)\sinh\left(dx+c\right)^{4} + 2\,(2\,a+b)\cosh\left(dx+c\right)^{2} + 2\,\left(3\,(2\,a+b)\cosh\left(dx+c\right)^{2} + 2\,a+b\right)\sinh\left(dx+c\right)^{2} + 4\,\left((2\,a+b)\cosh\left(dx+c\right)^{3} + (2\,a+b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right) + 2\,a+b\right)\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right) - b\cosh\left(dx+c\right) + \left(3\,b\cosh\left(dx+c\right)^{2} - b\right)\sinh\left(dx+c\right)}{d\cosh\left(dx+c\right)^{4} + 4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3} + d\sinh\left(dx+c\right)^{4} + 2\,d\cosh\left(dx+c\right)^{2} + 2\,\left(3\,d\cosh\left(dx+c\right)^{2} + d\right)\sinh\left(dx+c\right)^{2} + 4\,\left(d\cosh\left(dx+c\right)^{3} + d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] (b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3
+ ((2*a + b)*cosh(d*x + c)^4 + 4*(2*a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
(2*a + b)*sinh(d*x + c)^4 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*co
```

```
sh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 + 4*((2*a + b)*cosh(d*x + c)^3 + (
2*a + b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + b)*arctan(cosh(d*x + c) + sin
h(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(d
*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 +
2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*co
sh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)\mathrm{sech}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sech(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(36) = 72.
time = 0.40, size = 84, normalized size = 2.10

$$\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(2\,a+b)+\frac{4\,b\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(2*a + b) + 4*
b*(e^(d*x + c) - e^(-d*x - c))/((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

Mupad [B]
time = 0.16, size = 124, normalized size = 3.10

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(2\,a\,\sqrt{d^2}+b\,\sqrt{d^2}\right)}{d\,\sqrt{4\,a^2+4\,a\,b+b^2}}\right)\,\sqrt{4\,a^2+4\,a\,b+b^2}}{\sqrt{d^2}}+\frac{b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)/cosh(c + d*x),x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(2*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(4*a*b + 4*a^2
 + b^2)^(1/2)))*(4*a*b + 4*a^2 + b^2)^(1/2))/(d^2)^(1/2) + (b*exp(c + d*x))
/(d*(exp(2*c + 2*d*x) + 1)) - (2*b*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + e
xp(4*c + 4*d*x) + 1))
```

3.54 $\int \text{sech}^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=30

$$\frac{(a+b)\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}$$

```
[Out] (a+b)*tanh(d*x+c)/d-1/3*b*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.43, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4131, 3852, 8}

$$\frac{(3a+2b)\tanh(c+dx)}{3d}+\frac{b\tanh(c+dx)\text{sech}^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((3*a + 2*b)*Tanh[c + d*x])/(3*d) + (b*Sech[c + d*x]^2*Tanh[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \text{sech}^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx &= \frac{b\text{sech}^2(c+dx)\tanh(c+dx)}{3d}+\frac{1}{3}(3a+2b)\int \text{sech}^2(c+dx)\,dx\\ &= \frac{b\text{sech}^2(c+dx)\tanh(c+dx)}{3d}+\frac{(i(3a+2b))\text{Subst}(\int 1\,dx,x,-i\\ &\quad 3d\\ &= \frac{(3a+2b)\tanh(c+dx)}{3d}+\frac{b\text{sech}^2(c+dx)\tanh(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.30

$$\frac{a\tanh(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}-\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x])/d - (b*Tanh[c + d*x]^3)/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(28) = 56.
time = 1.17, size = 62, normalized size = 2.07

method	result	size
risch	$-\frac{2\left(3a\,\mathrm{e}^{4dx+4c}+6a\,\mathrm{e}^{2dx+2c}+6b\,\mathrm{e}^{2dx+2c}+3a+2b\right)}{3d\left(1+\mathrm{e}^{2dx+2c}\right)^3}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(3*a*exp(4*d*x+4*c)+6*a*exp(2*d*x+2*c)+6*b*exp(2*d*x+2*c)+3*a+2*b)/d/(
1+exp(2*d*x+2*c))^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(28) = 56.
time = 0.28, size = 112, normalized size = 3.73

$$\frac{4}{3}\,b\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)+\frac{2\,a}{d(e^{(-2\,dx-2\,c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/3*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) + 1))) + 2*a/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(28) = 56.
time = 0.33, size = 158, normalized size = 5.27

$$-\frac{4\left((3\,a+b)\cosh\left(dx+c\right)^2-2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(3\,a+b)\sinh\left(dx+c\right)^2+3\,a+3\,b\right)}{3\left(d\cosh\left(dx+c\right)^4+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+d\sinh\left(dx+c\right)^4+4\,d\cosh\left(dx+c\right)^2+2\left(3\,d\cosh\left(dx+c\right)^2+2\,d\right)\sinh\left(dx+c\right)^2+4\left(d\cosh\left(dx+c\right)^3+d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+3\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -4/3*((3*a + b)*cosh(d*x + c)^2 - 2*b*cosh(d*x + c)*sinh(d*x + c) + (3*a +
b)*sinh(d*x + c)^2 + 3*a + 3*b)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh
(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c
)^2 + 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d
*x + c) + 3*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)\mathrm{sech}^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sech(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(28) = 56.
time = 0.40, size = 61, normalized size = 2.03

$$-\frac{2\left(3\,ae^{(4\,dx+4\,c)} + 6\,ae^{(2\,dx+2\,c)} + 6\,be^{(2\,dx+2\,c)} + 3\,a + 2\,b\right)}{3\,d(e^{(2\,dx+2\,c)} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/3*(3*a*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + 3*a
 + 2*b)/(d*(e^(2*d*x + 2*c) + 1)^3)
```

Mupad [**B**]
time = 1.38, size = 61, normalized size = 2.03

$$-\frac{2\left(3\,a + 2\,b + 6\,a\,\mathrm{e}^{2\,c+2\,d\,x} + 3\,a\,\mathrm{e}^{4\,c+4\,d\,x} + 6\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{3\,d\left(\mathrm{e}^{2\,c+2\,d\,x} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)/cosh(c + d*x)^2,x)
```

```
[Out] -(2*(3*a + 2*b + 6*a*exp(2*c + 2*d*x) + 3*a*exp(4*c + 4*d*x) + 6*b*exp(2*c
+ 2*d*x)))/(3*d*(exp(2*c + 2*d*x) + 1)^3)
```

3.55 $\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=70

$$\frac{(4a+3b)\operatorname{ArcTan}(\sinh(c+dx))}{8d}+\frac{(4a+3b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d}+\frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

```
[Out] 1/8*(4*a+3*b)*arctan(sinh(d*x+c))/d+1/8*(4*a+3*b)*sech(d*x+c)*tanh(d*x+c)/d
+1/4*b*sech(d*x+c)^3*tanh(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4131, 3853, 3855}

$$\frac{(4a+3b)\operatorname{ArcTan}(\sinh(c+dx))}{8d}+\frac{(4a+3b)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d}+\frac{b\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((4*a + 3*b)*ArcTan[Sinh[c + d*x]])/(8*d) + ((4*a + 3*b)*Sech[c + d*x]*Tanh
[c + d*x])/(8*d) + (b*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = \frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d} + \frac{1}{4}(4a+3b)\int \operatorname{sech}^3(c+dx)\,dx$$
$$= \frac{(4a+3b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{b\operatorname{sech}^3(c+dx)\tanh(c+$$
$$= \frac{(4a+3b)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{(4a+3b)\operatorname{sech}(c+dx)\tanh(c$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 0.86

$$\frac{(4a+3b)\operatorname{ArcTan}(\sinh(c+dx)) + (4a+3b)\operatorname{sech}(c+dx)\tanh(c+dx) + 2b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
[Out] ((4*a + 3*b)*ArcTan[Sinh[c + d*x]] + (4*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x
] + 2*b*Sech[c + d*x]^3*Tanh[c + d*x])/(8*d)
```

Maple [C] Result contains complex when optimal does not.
time = 1.36, size = 173, normalized size = 2.47

method	result
risch	$\frac{e^{dx+c}\left(4a\,e^{6dx+6c}+3b\,e^{6dx+6c}+4a\,e^{4dx+4c}+11b\,e^{4dx+4c}-4a\,e^{2dx+2c}-11b\,e^{2dx+2c}-4a-3b\right)}{4d\left(1+e^{2dx+2c}\right)^4} + \frac{i\ln\left(e^{dx+c}+i\right)a}{2d} + \frac{3ib\ln\left(e^{dx+c}+i\right)}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/4*exp(d*x+c)*(4*a*exp(6*d*x+6*c)+3*b*exp(6*d*x+6*c)+4*a*exp(4*d*x+4*c)+11
*b*exp(4*d*x+4*c)-4*a*exp(2*d*x+2*c)-11*b*exp(2*d*x+2*c)-4*a-3*b)/d/(1+exp(
2*d*x+2*c))^4+1/2*I/d*ln(exp(d*x+c)+I)*a+3/8*I*b/d*ln(exp(d*x+c)+I)-1/2*I/d
*ln(exp(d*x+c)-I)*a-3/8*I*b/d*ln(exp(d*x+c)-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(64) = 128.
time = 0.50, size = 184, normalized size = 2.63

$$-\frac{1}{4}b\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{3e^{(-dx-c)}+11e^{(-3dx-3c)}-11e^{(-5dx-5c)}-3e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right) - a\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}-e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) -
11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a*(arctan(e^(-d
*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(
-4*d*x - 4*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(64) = 128$.
time = 0.38, size = 1112, normalized size = 15.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((4*a + 3*b)*cosh(d*x + c)^7 + 7*(4*a + 3*b)*cosh(d*x + c)*sinh(d*x + c
)^6 + (4*a + 3*b)*sinh(d*x + c)^7 + (4*a + 11*b)*cosh(d*x + c)^5 + (21*(4*a
 + 3*b)*cosh(d*x + c)^2 + 4*a + 11*b)*sinh(d*x + c)^5 + 5*(7*(4*a + 3*b)*co
sh(d*x + c)^3 + (4*a + 11*b)*cosh(d*x + c))*sinh(d*x + c)^4 - (4*a + 11*b)*
cosh(d*x + c)^3 + (35*(4*a + 3*b)*cosh(d*x + c)^4 + 10*(4*a + 11*b)*cosh(d*
x + c)^2 - 4*a - 11*b)*sinh(d*x + c)^3 + (21*(4*a + 3*b)*cosh(d*x + c)^5 +
10*(4*a + 11*b)*cosh(d*x + c)^3 - 3*(4*a + 11*b)*cosh(d*x + c))*sinh(d*x +
c)^2 + ((4*a + 3*b)*cosh(d*x + c)^8 + 8*(4*a + 3*b)*cosh(d*x + c)*sinh(d*x
+ c)^7 + (4*a + 3*b)*sinh(d*x + c)^8 + 4*(4*a + 3*b)*cosh(d*x + c)^6 + 4*(7
*(4*a + 3*b)*cosh(d*x + c)^2 + 4*a + 3*b)*sinh(d*x + c)^6 + 8*(7*(4*a + 3*b
)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(4*a +
 3*b)*cosh(d*x + c)^4 + 2*(35*(4*a + 3*b)*cosh(d*x + c)^4 + 30*(4*a + 3*b)*
cosh(d*x + c)^2 + 12*a + 9*b)*sinh(d*x + c)^4 + 8*(7*(4*a + 3*b)*cosh(d*x +
 c)^5 + 10*(4*a + 3*b)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d*x + c))*sinh(
d*x + c)^3 + 4*(4*a + 3*b)*cosh(d*x + c)^2 + 4*(7*(4*a + 3*b)*cosh(d*x + c)
^6 + 15*(4*a + 3*b)*cosh(d*x + c)^4 + 9*(4*a + 3*b)*cosh(d*x + c)^2 + 4*a +
 3*b)*sinh(d*x + c)^2 + 8*((4*a + 3*b)*cosh(d*x + c)^7 + 3*(4*a + 3*b)*cosh
(d*x + c)^5 + 3*(4*a + 3*b)*cosh(d*x + c)^3 + (4*a + 3*b)*cosh(d*x + c))*si
nh(d*x + c) + 4*a + 3*b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - (4*a + 3*b
)*cosh(d*x + c) + (7*(4*a + 3*b)*cosh(d*x + c)^6 + 5*(4*a + 11*b)*cosh(d*x
+ c)^4 - 3*(4*a + 11*b)*cosh(d*x + c)^2 - 4*a - 3*b)*sinh(d*x + c))/(d*cosh
(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*c
osh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(
d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(
35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d
*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9
*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d
*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)\mathrm{sech}^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sech(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(64) = 128.
time = 0.40, size = 156, normalized size = 2.23

$$\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(4\,a+3\,b)+\frac{4\left(4\,a\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+3\,b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+16\,a\left(e^{(dx+c)}-e^{(-dx-c)}\right)+20\,b\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(4*a + 3*b) +
 4*(4*a*(e^(d*x + c) - e^(-d*x - c))^3 + 3*b*(e^(d*x + c) - e^(-d*x - c))^3
 + 16*a*(e^(d*x + c) - e^(-d*x - c)) + 20*b*(e^(d*x + c) - e^(-d*x - c)))/(
(e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

Mupad [B]
time = 1.38, size = 283, normalized size = 4.04

$$\frac{\mathrm{atan}\left(\frac{e^{dx}\,e^{c}\left(4\,a\,\sqrt{d^2}+3\,b\,\sqrt{d^2}\right)}{d\,\sqrt{16\,a^2+24\,a\,b+9\,b^2}}\right)\sqrt{16\,a^2+24\,a\,b+9\,b^2}}{4\,\sqrt{d^2}}-\frac{\frac{a\,e^{5\,c+5\,dx}}{d}+\frac{2\,e^{3\,c+3\,dx}\,(a+2\,b)}{d}+\frac{a\,e^{c+dx}}{d}}{4\,e^{2\,c+2\,dx}+6\,e^{4\,c+4\,dx}+4\,e^{6\,c+6\,dx}+e^{8\,c+8\,dx}+1}-\frac{e^{c+dx}\,(2\,a-b)}{2\,d\,\left(2\,e^{2\,c+2\,dx}+e^{4\,c+4\,dx}+1\right)}-\frac{2\,b\,e^{c+dx}}{d\,\left(3\,e^{2\,c+2\,dx}+3\,e^{4\,c+4\,dx}+e^{6\,c+6\,dx}+1\right)}+\frac{e^{c+dx}\,(4\,a+3\,b)}{4\,d\,\left(e^{2\,c+2\,dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)/cosh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(4*a*(d^2)^(1/2) + 3*b*(d^2)^(1/2)))/(d*(24*a*b + 16
*a^2 + 9*b^2)^(1/2)))*(24*a*b + 16*a^2 + 9*b^2)^(1/2))/(4*(d^2)^(1/2)) - ((
a*exp(5*c + 5*d*x))/d + (2*exp(3*c + 3*d*x)*(a + 2*b))/d + (a*exp(c + d*x))
/d)/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c
 + 8*d*x) + 1) - (exp(c + d*x)*(2*a - b))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*
c + 4*d*x) + 1)) - (2*b*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c +
4*d*x) + exp(6*c + 6*d*x) + 1)) + (exp(c + d*x)*(4*a + 3*b))/(4*d*(exp(2*c
+ 2*d*x) + 1))
```

3.56 $\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=50

$$\frac{(a+b)\tanh(c+dx)}{d}-\frac{(a+2b)\tanh^3(c+dx)}{3d}+\frac{b\tanh^5(c+dx)}{5d}$$

```
[Out] (a+b)*tanh(d*x+c)/d-1/3*(a+2*b)*tanh(d*x+c)^3/d+1/5*b*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.30, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4131, 3852}

$$-\frac{(5a+4b)\tanh^3(c+dx)}{15d}+\frac{(5a+4b)\tanh(c+dx)}{5d}+\frac{b\tanh(c+dx)\operatorname{sech}^4(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((5*a + 4*b)*Tanh[c + d*x])/(5*d) + (b*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)
 - ((5*a + 4*b)*Tanh[c + d*x]^3)/(15*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx &= \frac{b\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}+\frac{1}{5}(5a+4b)\int \operatorname{sech}^4(c+dx)\,dx\\ &= \frac{b\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}+\frac{(i(5a+4b))\operatorname{Subst}\left(\int\left(1+x^2\right)\,dx,\right.}{5d}\\ &= \frac{(5a+4b)\tanh(c+dx)}{5d}+\frac{b\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}-\frac{(5a+}{}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 71, normalized size = 1.42

$$\frac{a\tanh(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}-\frac{a\tanh^3(c+dx)}{3d}-\frac{2b\tanh^3(c+dx)}{3d}+\frac{b\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) - (2*
b*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^5)/(5*d)
```

Maple [A]
time = 1.31, size = 86, normalized size = 1.72

method	result	size
risch	$-\frac{4\left(15a\,\mathrm{e}^{6dx+6c}+35a\,\mathrm{e}^{4dx+4c}+40b\,\mathrm{e}^{4dx+4c}+25a\,\mathrm{e}^{2dx+2c}+20b\,\mathrm{e}^{2dx+2c}+5a+4b\right)}{15d\left(1+\mathrm{e}^{2dx+2c}\right)^{5}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/15*(15*a*exp(6*d*x+6*c)+35*a*exp(4*d*x+4*c)+40*b*exp(4*d*x+4*c)+25*a*exp
(2*d*x+2*c)+20*b*exp(2*d*x+2*c)+5*a+4*b)/d/(1+exp(2*d*x+2*c))^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(46) = 92.
time = 0.28, size = 300, normalized size = 6.00

$$\frac{16}{15}\,b\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{10\,e^{(-4\,dx-4\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{1}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}\right)+\frac{4}{3}\,a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 16/15*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x
- 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) + 1))) + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c)
 + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) +
 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(46) = 92.
time = 0.33, size = 343, normalized size = 6.86

$$-\frac{8\,\left(2\,(5\,a+b)\cosh\left(dx+c\right)^{3}+6\,(5\,a+b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+(5\,a-2\,b)\sinh\left(dx+c\right)^{3}+30\,(a+b)\cosh\left(dx+c\right)+\left(3\,(5\,a-2\,b)\cosh\left(dx+c\right)^{2}+5\,a+10\,b\right)\sinh\left(dx+c\right)\right)}{15\,\left(d\cosh\left(dx+c\right)^{7}+7\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{6}+d\sinh\left(dx+c\right)^{7}+5\,d\cosh\left(dx+c\right)^{5}+\left(21\,d\cosh\left(dx+c\right)^{2}+5\,d\right)\sinh\left(dx+c\right)^{5}+5\,\left(7\,d\cosh\left(dx+c\right)^{3}+5\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{4}+11\,d\cosh\left(dx+c\right)^{3}+\left(35\,d\cosh\left(dx+c\right)^{4}+50\,d\cosh\left(dx+c\right)^{2}+9\,d\right)\sinh\left(dx+c\right)^{3}+\left(21\,d\cosh\left(dx+c\right)^{5}+50\,d\cosh\left(dx+c\right)^{3}+33\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+15\,d\cosh\left(dx+c\right)+\left(7\,d\cosh\left(dx+c\right)^{6}+25\,d\cosh\left(dx+c\right)^{4}+27\,d\cosh\left(dx+c\right)^{2}+5\,d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -8/15*(2*(5*a + b)*cosh(d*x + c)^3 + 6*(5*a + b)*cosh(d*x + c)*sinh(d*x + c
)^2 + (5*a - 2*b)*sinh(d*x + c)^3 + 30*(a + b)*cosh(d*x + c) + (3*(5*a - 2*
b)*cosh(d*x + c)^2 + 5*a + 10*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*co
sh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 5*d*cosh(d*x + c)^5 + (21
*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 5*d*co
sh(d*x + c))*sinh(d*x + c)^4 + 11*d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4
 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5 + 50
*d*cosh(d*x + c)^3 + 33*d*cosh(d*x + c))*sinh(d*x + c)^2 + 15*d*cosh(d*x +
c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2 + 5
*d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)\operatorname{sech}^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*sech(c + d*x)**4, x)
```

Giac [A]
time = 0.40, size = 85, normalized size = 1.70

$$-\frac{4\left(15\,ae^{(6\,dx+6\,c)}+35\,ae^{(4\,dx+4\,c)}+40\,be^{(4\,dx+4\,c)}+25\,ae^{(2\,dx+2\,c)}+20\,be^{(2\,dx+2\,c)}+5\,a+4\,b\right)}{15\,d(e^{(2\,dx+2\,c)}+1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -4/15*(15*a*e^(6*d*x + 6*c) + 35*a*e^(4*d*x + 4*c) + 40*b*e^(4*d*x + 4*c) +
 25*a*e^(2*d*x + 2*c) + 20*b*e^(2*d*x + 2*c) + 5*a + 4*b)/(d*(e^(2*d*x + 2*
c) + 1)^5)
```

Mupad [B]
time = 1.43, size = 292, normalized size = 5.84

$$-\frac{\frac{8\,(a+2\,b)}{15\,d}+\frac{4\,a\,\mathrm{e}^{2\,c+2\,d\,x}}{5\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}-\frac{\frac{8\,a\,\mathrm{e}^{2\,c+2\,d\,x}}{5\,d}+\frac{8\,a\,\mathrm{e}^{6\,c+6\,d\,x}}{5\,d}+\frac{16\,\mathrm{e}^{4\,c+4\,d\,x}\,(a+2\,b)}{5\,d}}{5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1}-\frac{\frac{2\,a}{5\,d}+\frac{6\,a\,\mathrm{e}^{4\,c+4\,d\,x}}{5\,d}+\frac{8\,\mathrm{e}^{2\,c+2\,d\,x}\,(a+2\,b)}{5\,d}}{4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1}-\frac{2\,a}{5\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)/cosh(c + d*x)^4,x)
```

```
[Out] - ((8*(a + 2*b))/(15*d) + (4*a*exp(2*c + 2*d*x))/(5*d))/(3*exp(2*c + 2*d*x)
 + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((8*a*exp(2*c + 2*d*x))/(5*
d) + (8*a*exp(6*c + 6*d*x))/(5*d) + (16*exp(4*c + 4*d*x)*(a + 2*b))/(5*d))/
(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c
 + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*a)/(5*d) + (6*a*exp(4*c + 4*d*x))
/(5*d) + (8*exp(2*c + 2*d*x)*(a + 2*b))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(
4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - (2*a)/(5*d*(2*e
xp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.57 $\int \cosh^4(c+dx)\left(a+b\textbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=82

$$\frac{1}{8}\left(3a^2+8ab+8b^2\right)x+\frac{3a(a+2b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a\cosh^3(c+dx)\sinh(c+dx)\left(a+b-b\tanh^2(\right.}{4d}$$

```
[Out] 1/8*(3*a^2+8*a*b+8*b^2)*x+3/8*a*(a+2*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a*cos
h(d*x+c)^3*sinh(d*x+c)*(a+b-b*tanh(d*x+c)^2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4231, 424, 393, 212}

$$\frac{1}{8}x\left(3a^2+8ab+8b^2\right)+\frac{3a(a+2b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{a\sinh(c+dx)\cosh^3(c+dx)\left(a-b\tanh^2(c+dx)+b\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*x)/8 + (3*a*(a + 2*b)*Cosh[c + d*x]*Sinh[c + d*x])
/(8*d) + (a*Cosh[c + d*x]^3*Sinh[c + d*x]*(a + b - b*Tanh[c + d*x]^2))/(4*d
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{(a+b-bx^2)^2}{(1-x^2)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{a\cosh^3(c+dx)\sinh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)}{4d}-\frac{\text{Sub}}{}$$

$$=\frac{3a(a+2b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a\cosh^3(c+dx)\sinh(c}{}$$

$$=\frac{1}{8}\left(3a^2+8ab+8b^2\right)x+\frac{3a(a+2b)\cosh(c+dx)\sinh(c+dx)}{8d}$$

Mathematica [A]
time = 0.09, size = 58, normalized size = 0.71

$$\frac{4(3a^2+8ab+8b^2)(c+dx)+8a(a+2b)\sinh(2(c+dx))+a^2\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
[Out] (4*(3*a^2 + 8*a*b + 8*b^2)*(c + d*x) + 8*a*(a + 2*b)*Sinh[2*(c + d*x)] + a^
2*Sinh[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 2.15, size = 57, normalized size = 0.70

method	result	size
default	$abx+b^2x+\frac{\left(\frac{1}{2}a^2+ab\right)\sinh(2dx+2c)}{2d}+\frac{3a^2x}{8}+\frac{a^2\sinh(4dx+4c)}{32d}$	57
risch	$\frac{3a^2x}{8}+abx+b^2x+\frac{a^2 e^{4dx+4c}}{64d}+\frac{a\,e^{2dx+2c}b}{4d}+\frac{a^2 e^{2dx+2c}}{8d}-\frac{a\,e^{-2dx-2c}b}{4d}-\frac{a^2 e^{-2dx-2c}}{8d}-\frac{a^2 e^{-4dx-4c}}{64d}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*b*x+b^2*x+1/2*(1/2*a^2+a*b)/d*sinh(2*d*x+2*c)+3/8*a^2*x+1/32*a^2/d*sinh(4
*d*x+4*c)
```

Maxima [A]
time = 0.29, size = 105, normalized size = 1.28

$$\frac{1}{64}a^2\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{1}{4}ab\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)+b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 1/4*a*b*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x -
 2*c)/d) + b^2*x
```

Fricas [A]
time = 0.36, size = 78, normalized size = 0.95

$$\frac{a^2\cosh(dx+c)\sinh(dx+c)^3+(3a^2+8ab+8b^2)dx+\left(a^2\cosh(dx+c)^3+4(a^2+2ab)\cosh(dx+c)\right)\sinh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(a^2*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*d*x + (a^2
*cosh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.40, size = 151, normalized size = 1.84

$$\frac{a^2e^{(4dx+4c)}+8a^2e^{(2dx+2c)}+16abe^{(2dx+2c)}+8(3a^2+8ab+8b^2)(dx+c)-\left(18a^2e^{(4dx+4c)}+48abe^{(4dx+4c)}+48b^2e^{(4dx+4c)}+8a^2e^{(2dx+2c)}+16abe^{(2dx+2c)}+a^2\right)e^{(-4dx-4c)}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(a^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) + 16*a*b*e^(2*d*x + 2*c)
+ 8*(3*a^2 + 8*a*b + 8*b^2)*(d*x + c) - (18*a^2*e^(4*d*x + 4*c) + 48*a*b*e^
```

```
(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) + 16*a*b*e^(
2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c))/d
```

Mupad [B]
time = 1.37, size = 66, normalized size = 0.80

$$\frac{3\,a^2\,x}{8}+b^2\,x+a\,b\,x+\frac{a^2\,\sinh(2\,c+2\,d\,x)}{4\,d}+\frac{a^2\,\sinh(4\,c+4\,d\,x)}{32\,d}+\frac{a\,b\,\sinh(2\,c+2\,d\,x)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (3*a^2*x)/8 + b^2*x + a*b*x + (a^2*sinh(2*c + 2*d*x))/(4*d) + (a^2*sinh(4*c
 + 4*d*x))/(32*d) + (a*b*sinh(2*c + 2*d*x))/(2*d)
```

3.58 $\int \cosh^3(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{b^2\mathrm{ArcTan}(\sinh(c+dx))}{d}+\frac{a(a+2b)\sinh(c+dx)}{d}+\frac{a^2\sinh^3(c+dx)}{3d}$$

```
[Out] b^2*arctan(sinh(d*x+c))/d+a*(a+2*b)*sinh(d*x+c)/d+1/3*a^2*sinh(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4232, 398, 209}

$$\frac{a^2\sinh^3(c+dx)}{3d}+\frac{a(a+2b)\sinh(c+dx)}{d}+\frac{b^2\mathrm{ArcTan}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (b^2*ArcTan[Sinh[c + d*x]])/d + (a*(a + 2*b)*Sinh[c + d*x])/d + (a^2*Sinh[c
 + d*x]^3)/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+ax^2)^2}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(a(a+2b)+a^2x^2+\frac{b^2}{1+x^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{a(a+2b)\sinh(c+dx)}{d} + \frac{a^2\sinh^3(c+dx)}{3d} + \frac{b^2\text{Subst}\left(\int \frac{1}{1+x^2}\,d\right.}{}$$
$$= \frac{b^2\tan^{-1}(\sinh(c+dx))}{d} + \frac{a(a+2b)\sinh(c+dx)}{d} + \frac{a^2\sinh^3(c}{3d}$$

Mathematica [A]
time = 0.02, size = 72, normalized size = 1.47

$$\frac{b^2\text{ArcTan}(\sinh(c+dx))}{d} + \frac{2ab\cosh(dx)\sinh(c)}{d} + \frac{2ab\cosh(c)\sinh(dx)}{d} + \frac{a^2\sinh(c+dx)}{d} + \frac{a^2\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (b^2*ArcTan[Sinh[c + d*x]])/d + (2*a*b*Cosh[d*x]*Sinh[c])/d + (2*a*b*Cosh[c
]*Sinh[d*x]])/d + (a^2*Sinh[c + d*x])/d + (a^2*Sinh[c + d*x]^3)/(3*d)
```

Maple [C] Result contains complex when optimal does not.
time = 1.98, size = 133, normalized size = 2.71

method	result	si
risch	$\frac{a^2 e^{3dx+3c}}{24d}+\frac{3a^2 e^{dx+c}}{8d}+\frac{ab\, e^{dx+c}}{d}-\frac{3a^2 e^{-dx-c}}{8d}-\frac{a\, e^{-dx-c} b}{d}-\frac{a^2 e^{-3dx-3c}}{24d}+\frac{ib^2\ln(e^{dx+c}+i)}{d}-\frac{ib^2\ln(e^{dx+c}-i)}{d}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*a^2/d*exp(3*d*x+3*c)+3/8*a^2/d*exp(d*x+c)+a*b/d*exp(d*x+c)-3/8*a^2/d*e
xp(-d*x-c)-a/d*exp(-d*x-c)*b-1/24*a^2/d*exp(-3*d*x-3*c)+I*b^2/d*ln(exp(d*x+
c)+I)-I*b^2/d*ln(exp(d*x+c)-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(47) = 94.
time = 0.48, size = 105, normalized size = 2.14

$$\frac{1}{24}a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)+ab\left(\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)-\frac{2\,b^2\arctan\left(e^{(-dx-c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24*a^2*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*
x - 3*c)/d) + a*b*(e^(d*x + c)/d - e^(-d*x - c)/d) - 2*b^2*arctan(e^(-d*x -
c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(47) = 94$.
time = 0.39, size = 414, normalized size = 8.45

$$\frac{a^2\cosh(dx+c)^6+6a^2\cosh(dx+c)\sinh(dx+c)^5+a^2\sinh(dx+c)^6+3(3a^2+8ab)\cosh(dx+c)^4+3(5a^2\cosh(dx+c)^2+3a^2+8ab)\sinh(dx+c)^4+4(5a^2\cosh(dx+c)^3+3(3a^2+8ab)\cosh(dx+c))\sinh(dx+c)^3-3(3a^2+8ab)\cosh(dx+c)^2+3(5a^2\cosh(dx+c)^4+6(3a^2+8ab)\cosh(dx+c)^2-3a^2-8ab)\sinh(dx+c)^2-a^2+48(b^2\cosh(dx+c)^3+3b^2\cosh(dx+c)^2\sinh(dx+c)+3b^2\cosh(dx+c)\sinh(dx+c)^2+b^2\sinh(dx+c)^3)\arctan(\cosh(dx+c)+\sinh(dx+c))+6(a^2\cosh(dx+c)^5+2(3a^2+8ab)\cosh(dx+c)^3-(3a^2+8ab)\cosh(dx+c))\sinh(dx+c)}{24(d\cosh(dx+c)^3+3d\cosh(dx+c)^2\sinh(dx+c)+3d\cosh(dx+c)\sinh(dx+c)^2+d\sinh(dx+c)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(a^2*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(
d*x + c)^6 + 3*(3*a^2 + 8*a*b)*cosh(d*x + c)^4 + 3*(5*a^2*cosh(d*x + c)^2 +
3*a^2 + 8*a*b)*sinh(d*x + c)^4 + 4*(5*a^2*cosh(d*x + c)^3 + 3*(3*a^2 + 8*a
*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a^2 + 8*a*b)*cosh(d*x + c)^2 + 3*
(5*a^2*cosh(d*x + c)^4 + 6*(3*a^2 + 8*a*b)*cosh(d*x + c)^2 - 3*a^2 - 8*a*b)
*sinh(d*x + c)^2 - a^2 + 48*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*si
nh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*ar
ctan(cosh(d*x + c) + sinh(d*x + c)) + 6*(a^2*cosh(d*x + c)^5 + 2*(3*a^2 + 8
*a*b)*cosh(d*x + c)^3 - (3*a^2 + 8*a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*co
sh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(d*x + c)*sinh(
d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2\cosh^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*cosh(c + d*x)**3, x)
```

Giac [A]
time = 0.40, size = 94, normalized size = 1.92

$$\frac{48\,b^2\arctan\left(e^{(dx+c)}\right)+a^2e^{(3\,dx+3\,c)}+9\,a^2e^{(dx+c)}+24\,abe^{(dx+c)}-\left(9\,a^2e^{(2\,dx+2\,c)}+24\,abe^{(2\,dx+2\,c)}+a^2\right)e^{(-3\,dx-3\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(48*b^2*arctan(e^(d*x + c)) + a^2*e^(3*d*x + 3*c) + 9*a^2*e^(d*x + c)
+ 24*a*b*e^(d*x + c) - (9*a^2*e^(2*d*x + 2*c) + 24*a*b*e^(2*d*x + 2*c) + a^
2)*e^(-3*d*x - 3*c))/d
```

Mupad [B]
time = 0.17, size = 114, normalized size = 2.33

$$\frac{2\,\mathrm{atan}\left(\frac{b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{d^2}}{d\,\sqrt{b^4}}\right)\,\sqrt{b^4}}{\sqrt{d^2}}-\frac{\mathrm{e}^{-c-d\,x}\,\left(3\,a^2+8\,b\,a\right)}{8\,d}-\frac{a^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}+\frac{a^2\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}+\frac{a\,\mathrm{e}^{c+d\,x}\,\left(3\,a+8\,b\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (2*atan((b^2*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(b^4)^(1/2)))*(b^4)^(1/2))/(d^
2)^(1/2) - (exp(- c - d*x)*(8*a*b + 3*a^2))/(8*d) - (a^2*exp(- 3*c - 3*d*x)
)/(24*d) + (a^2*exp(3*c + 3*d*x))/(24*d) + (a*exp(c + d*x)*(3*a + 8*b))/(8*
d)
```

3.59 $\int \cosh^2(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=47

$$\frac{1}{2}a(a+4b)x+\frac{a^2\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2\tanh(c+dx)}{d}$$

```
[Out] 1/2*a*(a+4*b)*x+1/2*a^2*cosh(d*x+c)*sinh(d*x+c)/d+b^2*tanh(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4231, 398, 393, 212}

$$\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}ax(a+4b)+\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (a*(a + 4*b)*x)/2 + (a^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (b^2*Tanh[c +
 d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
```

```
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^2 dx &= \frac{\mathrm{Subst}\left(\int \frac{(a+b-bx^2)^2}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\mathrm{Subst}\left(\int \left(b^2+\frac{a(a+2b)-2abx^2}{(1-x^2)^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{b^2\tanh(c+dx)}{d}+\frac{\mathrm{Subst}\left(\int \frac{a(a+2b)-2abx^2}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{a^2\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2\tanh(c+dx)}{d}+\frac{(a(a+4b))}{}\\ &= \frac{1}{2}a(a+4b)x+\frac{a^2\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2\tanh(c+dx}{d}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 52, normalized size = 1.11

$$2abx+\frac{a^2(c+dx)}{2d}+\frac{a^2\sinh(2(c+dx))}{4d}+\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] 2*a*b*x + (a^2*(c + d*x))/(2*d) + (a^2*Sinh[2*(c + d*x)])/(4*d) + (b^2*Tanh
[c + d*x])/d
```

Maple [A]
time = 1.96, size = 68, normalized size = 1.45

method	result	size
risch	$\frac{a^2x}{2}+2abx+\frac{a^2e^{2dx+2c}}{8d}-\frac{a^2e^{-2dx-2c}}{8d}-\frac{2b^2}{d(1+e^{2dx+2c})}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x+2*a*b*x+1/8*a^2/d*exp(2*d*x+2*c)-1/8*a^2/d*exp(-2*d*x-2*c)-2*b^2/
d/(1+exp(2*d*x+2*c))
```

Maxima [A]
time = 0.28, size = 63, normalized size = 1.34

$$\frac{1}{8}\,a^{2}\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+2\,abx+\frac{2\,b^{2}}{d(e^{(-2\,dx-2\,c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + 2*a*b*x + 2*b^2/(d
*(e^(-2*d*x - 2*c) + 1))
```

Fricas [A]
time = 0.39, size = 80, normalized size = 1.70

$$\frac{a^{2}\sinh\left(dx+c\right)^{3}+4\left(\left(a^{2}+4\,ab\right)dx-2\,b^{2}\right)\cosh\left(dx+c\right)+\left(3\,a^{2}\cosh\left(dx+c\right)^{2}+a^{2}+8\,b^{2}\right)\sinh\left(dx+c\right)}{8\,d\cosh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(a^2*sinh(d*x + c)^3 + 4*((a^2 + 4*a*b)*d*x - 2*b^2)*cosh(d*x + c) + (3
*a^2*cosh(d*x + c)^2 + a^2 + 8*b^2)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^{2}\left(c+dx\right)\right)^{2}\cosh^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*cosh(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(43) = 86.
time = 0.40, size = 128, normalized size = 2.72

$$\frac{a^{2}e^{(2\,dx+2\,c)}+4\left(a^{2}+4\,ab\right)(dx+c)-\frac{a^{2}e^{(4\,dx+4\,c)}+4\,abe^{(4\,dx+4\,c)}+2\,a^{2}e^{(2\,dx+2\,c)}+4\,abe^{(2\,dx+2\,c)}+16\,b^{2}e^{(2\,dx+2\,c)}+a^{2}}{e^{(4\,dx+4\,c)}+e^{(2\,dx+2\,c)}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*e^(2*d*x + 2*c) + 4*(a^2 + 4*a*b)*(d*x + c) - (a^2*e^(4*d*x + 4*c)
 + 4*a*b*e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) + 4*a*b*e^(2*d*x + 2*c) +
16*b^2*e^(2*d*x + 2*c) + a^2)/(e^(4*d*x + 4*c) + e^(2*d*x + 2*c)))/d
```

Mupad [B]
time = 0.16, size = 65, normalized size = 1.38

$$\frac{a^2 e^{2c+2dx}}{8d} - \frac{a^2 e^{-2c-2dx}}{8d} - \frac{2b^2}{d\,(e^{2c+2dx}+1)} + \frac{a\,x\,(a+4\,b)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (a^2*exp(2*c + 2*d*x))/(8*d) - (a^2*exp(- 2*c - 2*d*x))/(8*d) - (2*b^2)/(d*
(exp(2*c + 2*d*x) + 1)) + (a*x*(a + 4*b))/2
```

3.60 $\int \cosh(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=56

$$\frac{b(4a+b)\mathrm{ArcTan}(\sinh(c+dx))}{2d}+\frac{a^2\sinh(c+dx)}{d}+\frac{b^2\mathrm{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[Out] 1/2*b*(4*a+b)*arctan(sinh(d*x+c))/d+a^2*sinh(d*x+c)/d+1/2*b^2*sech(d*x+c)*t
anh(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4232, 398, 393, 209}

$$\frac{a^2\sinh(c+dx)}{d}+\frac{b(4a+b)\mathrm{ArcTan}(\sinh(c+dx))}{2d}+\frac{b^2\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (b*(4*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + (a^2*Sinh[c + d*x])/d + (b^2*Se
ch[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2 dx = \frac{\mathrm{Subst}\left(\int \frac{(a+b+ax^2)^2}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\mathrm{Subst}\left(\int \left(a^2+\frac{b(2a+b)+2abx^2}{(1+x^2)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{a^2\sinh(c+dx)}{d}+\frac{\mathrm{Subst}\left(\int \frac{b(2a+b)+2abx^2}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{a^2\sinh(c+dx)}{d}+\frac{b^2\mathrm{sech}(c+dx)\tanh(c+dx)}{2d}+\frac{(b(4a+b))\mathrm{Su}}{}$$
$$= \frac{b(4a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^2\sinh(c+dx)}{d}+\frac{b^2\mathrm{sech}(c+}{}$$

Mathematica [A]
time = 0.03, size = 80, normalized size = 1.43

$$\frac{2ab\mathrm{ArcTan}(\sinh(c+dx))}{d}+\frac{b^2\mathrm{ArcTan}(\sinh(c+dx))}{2d}+\frac{a^2\cosh(dx)\sinh(c)}{d}+\frac{a^2\cosh(c)\sinh(dx)}{d}+\frac{b^2\mathrm{sech}(c+dx)\tanh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (2*a*b*ArcTan[Sinh[c + d*x]])/d + (b^2*ArcTan[Sinh[c + d*x]])/(2*d) + (a^2*
Cosh[d*x]*Sinh[c])/d + (a^2*Cosh[c]*Sinh[d*x])/d + (b^2*Sech[c + d*x]*Tanh[
c + d*x])/(2*d)
```

Maple [C] Result contains complex when optimal does not.
time = 1.96, size = 144, normalized size = 2.57

method	result
risch	$\frac{a^2 e^{dx+c}}{2d}-\frac{a^2 e^{-dx-c}}{2d}+\frac{b^2 e^{dx+c}\left(e^{2dx+2c}-1\right)}{d\left(1+e^{2dx+2c}\right)^2}+\frac{2iba\ln\left(e^{dx+c}+i\right)}{d}+\frac{ib^2\ln\left(e^{dx+c}+i\right)}{2d}-\frac{2iba\ln\left(e^{dx+c}-i\right)}{d}-\frac{ib^2\ln\left(e^{dx+c}-\right.}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2/d*exp(d*x+c)-1/2*a^2/d*exp(-d*x-c)+b^2*exp(d*x+c)*(exp(2*d*x+2*c)-1
)/d/(1+exp(2*d*x+2*c))^2+2*I*b*a/d*ln(exp(d*x+c)+I)+1/2*I*b^2/d*ln(exp(d*x+
c)+I)-2*I*b*a/d*ln(exp(d*x+c)-I)-1/2*I*b^2/d*ln(exp(d*x+c)-I)
```

Maxima [A]
time = 0.50, size = 101, normalized size = 1.80

$$-b^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)-\frac{4\,ab\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{a^2\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -b^2*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-
2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 4*a*b*arctan(e^(-d*x - c))/d + a^2
*sinh(d*x + c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. 2(52) = 104.
time = 0.43, size = 653, normalized size = 11.66

$$\frac{a^2\cosh\left(dx+c\right)^6+6\,a^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+a^2\sinh\left(dx+c\right)^6+(a^2+2\,b^2)\cosh\left(dx+c\right)^4+(15\,a^2\cosh\left(dx+c\right)^2+a^2+2\,b^2)\sinh\left(dx+c\right)^4+4\,(5\,a^2\cosh\left(dx+c\right)^3+(a^2+2\,b^2)\cosh\left(dx+c\right))\sinh\left(dx+c\right)^3-(a^2+2\,b^2)\cosh\left(dx+c\right)^2+(15\,a^2\cosh\left(dx+c\right)^4+6\,(a^2+2\,b^2)\cosh\left(dx+c\right)^2-a^2-2\,b^2)\sinh\left(dx+c\right)^2-a^2+2\,((4\,ab+b^2)\cosh\left(dx+c\right)^5+5\,(4\,ab+b^2)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+(4\,ab+b^2)\sinh\left(dx+c\right)^5+2\,(4\,ab+b^2)\cosh\left(dx+c\right)^3+2\,(5\,(4\,ab+b^2)\cosh\left(dx+c\right)^2+4\,ab+b^2)\sinh\left(dx+c\right)^3+2\,(5\,(4\,ab+b^2)\cosh\left(dx+c\right)^3+3\,(4\,ab+b^2)\cosh\left(dx+c\right))\sinh\left(dx+c\right)^2+(4\,ab+b^2)\cosh\left(dx+c\right)+(5\,(4\,ab+b^2)\cosh\left(dx+c\right)^4+6\,(4\,ab+b^2)\cosh\left(dx+c\right)^2+4\,ab+b^2)\sinh\left(dx+c\right))\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)+2\,(3\,a^2\cosh\left(dx+c\right)^5+2\,(a^2+2\,b^2)\cosh\left(dx+c\right)^3-(a^2+2\,b^2)\cosh\left(dx+c\right))\sinh\left(dx+c\right)}{2\,(d\cosh\left(dx+c\right)^5+5\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+d\sinh\left(dx+c\right)^5+2\,d\cosh\left(dx+c\right)^3+2\,(5\,d\cosh\left(dx+c\right)^2+d)\sinh\left(dx+c\right)^3+2\,(5\,d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right))\sinh\left(dx+c\right)^2+d\cosh\left(dx+c\right)+(5\,d\cosh\left(dx+c\right)^4+6\,d\cosh\left(dx+c\right)^2+d)\sinh\left(dx+c\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d
*x + c)^6 + (a^2 + 2*b^2)*cosh(d*x + c)^4 + (15*a^2*cosh(d*x + c)^2 + a^2 +
 2*b^2)*sinh(d*x + c)^4 + 4*(5*a^2*cosh(d*x + c)^3 + (a^2 + 2*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c)^2 + (15*a^2*cosh(d*x +
 c)^4 + 6*(a^2 + 2*b^2)*cosh(d*x + c)^2 - a^2 - 2*b^2)*sinh(d*x + c)^2 - a^
2 + 2*((4*a*b + b^2)*cosh(d*x + c)^5 + 5*(4*a*b + b^2)*cosh(d*x + c)*sinh(d
*x + c)^4 + (4*a*b + b^2)*sinh(d*x + c)^5 + 2*(4*a*b + b^2)*cosh(d*x + c)^3
 + 2*(5*(4*a*b + b^2)*cosh(d*x + c)^2 + 4*a*b + b^2)*sinh(d*x + c)^3 + 2*(5
*(4*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + b^2)*cosh(d*x + c))*sinh(d*x +
c)^2 + (4*a*b + b^2)*cosh(d*x + c) + (5*(4*a*b + b^2)*cosh(d*x + c)^4 + 6*(
4*a*b + b^2)*cosh(d*x + c)^2 + 4*a*b + b^2)*sinh(d*x + c))*arctan(cosh(d*x
+ c) + sinh(d*x + c)) + 2*(3*a^2*cosh(d*x + c)^5 + 2*(a^2 + 2*b^2)*cosh(d*x
 + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^5 +
5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^3
 + 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 + 3
*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4
+ 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)^2\cosh\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*cosh(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(52) = 104.
time = 0.39, size = 112, normalized size = 2.00

$$\frac{2\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(4\,ab+b^2)+\frac{4\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*a^2*(e^(d*x + c) - e^(-d*x - c)) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*
c) - 1)*e^(-d*x - c)))*(4*a*b + b^2) + 4*b^2*(e^(d*x + c) - e^(-d*x - c))/(
(e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

Mupad [B]
time = 1.44, size = 172, normalized size = 3.07

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(b^2\,\sqrt{d^2}+4\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{16\,a^2\,b^2+8\,a\,b^3+b^4}}\right)\,\sqrt{16\,a^2\,b^2+8\,a\,b^3+b^4}}{\sqrt{d^2}}+\frac{a^2\,\mathrm{e}^{c+d\,x}}{2\,d}-\frac{a^2\,\mathrm{e}^{-c-d\,x}}{2\,d}+\frac{b^2\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,b^2\,\mathrm{e}^{c+d\,x}}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(b^2*(d^2)^(1/2) + 4*a*b*(d^2)^(1/2)))/(d*(8*a*b^3 +
 b^4 + 16*a^2*b^2)^(1/2)))*(8*a*b^3 + b^4 + 16*a^2*b^2)^(1/2))/(d^2)^(1/2)
+ (a^2*exp(c + d*x))/(2*d) - (a^2*exp(- c - d*x))/(2*d) + (b^2*exp(c + d*x)
)/(d*(exp(2*c + 2*d*x) + 1)) - (2*b^2*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x)
+ exp(4*c + 4*d*x) + 1))
```

3.61 $\int \text{sech}(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=90

$$\frac{(8a^2+8ab+3b^2)\,\text{ArcTan}(\sinh(c+dx))}{8d}+\frac{3b(2a+b)\text{sech}(c+dx)\tanh(c+dx)}{8d}+\frac{b\text{sech}^3(c+dx)\,(a+b+a\sinh$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctan(sinh(d*x+c))/d+3/8*b*(2*a+b)*sech(d*x+c)*tan
h(d*x+c)/d+1/4*b*sech(d*x+c)^3*(a+b+a*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {4232, 424, 393, 209}

$$\frac{(8a^2+8ab+3b^2)\,\text{ArcTan}(\sinh(c+dx))}{8d}+\frac{3b(2a+b)\tanh(c+dx)\text{sech}(c+dx)}{8d}+\frac{b\tanh(c+dx)\text{sech}^3(c+dx)\,(a\sinh^2(c+dx)+a+b)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (3*b*(2*a + b)*Sech
[c + d*x]*Tanh[c + d*x])/(8*d) + (b*Sech[c + d*x]^3*(a + b + a*Sinh[c + d*x
]^2)*Tanh[c + d*x])/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+b+ax^2)^2}{(1+x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\operatorname{sech}^3(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx)}{4d} + \frac{\text{Subst}}{}$$

$$= \frac{3b(2a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{b\operatorname{sech}^3(c+dx)\left(a+b+\right.}{}$$

$$= \frac{(8a^2+8ab+3b^2)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{3b(2a+b)\operatorname{sech}(c+dx}{8d}$$

Mathematica [A]
time = 0.10, size = 71, normalized size = 0.79

$$\frac{(8a^2+8ab+3b^2)\,\text{ArcTan}(\sinh(c+dx))+b(8a+3b)\operatorname{sech}(c+dx)\tanh(c+dx)+2b^2\operatorname{sech}^3(c+dx)\tanh(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]] + b*(8*a + 3*b)*Sech[c + d*x
]*Tanh[c + d*x] + 2*b^2*Sech[c + d*x]^3*Tanh[c + d*x])/(8*d)
```

Maple [C] Result contains complex when optimal does not.
time = 1.62, size = 218, normalized size = 2.42

method	result
risch	$\frac{b\,e^{dx+c}\left(8a\,e^{6dx+6c}+3b\,e^{6dx+6c}+8a\,e^{4dx+4c}+11b\,e^{4dx+4c}-8a\,e^{2dx+2c}-11b\,e^{2dx+2c}-8a-3b\right)}{4d\left(1+e^{2dx+2c}\right)^4}+\frac{i\ln\left(e^{dx+c}+i\right)a^2}{d}+\frac{iba\ln\left(e^{dx+c}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b*exp(d*x+c)*(8*a*exp(6*d*x+6*c)+3*b*exp(6*d*x+6*c)+8*a*exp(4*d*x+4*c)+
11*b*exp(4*d*x+4*c)-8*a*exp(2*d*x+2*c)-11*b*exp(2*d*x+2*c)-8*a-3*b)/d/(1+ex
p(2*d*x+2*c))^4+I/d*ln(exp(d*x+c)+I)*a^2+I*b*a/d*ln(exp(d*x+c)+I)+3/8*I*b^2
/d*ln(exp(d*x+c)+I)-I/d*ln(exp(d*x+c)-I)*a^2-I*b*a/d*ln(exp(d*x+c)-I)-3/8*I
/d*ln(exp(d*x+c)-I)*b^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(84) = 168$.
time = 0.51, size = 201, normalized size = 2.23

$$-\frac{1}{4}b^{2}\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}+11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}-3\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)-2\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+\frac{a^{2}\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c)
- 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-
4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 2*a*b*(arctan
(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c)
 + e^(-4*d*x - 4*c) + 1))) + a^2*arctan(sinh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1372 vs. $2(84) = 168$.
time = 0.36, size = 1372, normalized size = 15.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((8*a*b + 3*b^2)*cosh(d*x + c)^7 + 7*(8*a*b + 3*b^2)*cosh(d*x + c)*sinh
(d*x + c)^6 + (8*a*b + 3*b^2)*sinh(d*x + c)^7 + (8*a*b + 11*b^2)*cosh(d*x +
 c)^5 + (21*(8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a*b + 11*b^2)*sinh(d*x + c)
^5 + 5*(7*(8*a*b + 3*b^2)*cosh(d*x + c)^3 + (8*a*b + 11*b^2)*cosh(d*x + c))
*sinh(d*x + c)^4 - (8*a*b + 11*b^2)*cosh(d*x + c)^3 + (35*(8*a*b + 3*b^2)*c
osh(d*x + c)^4 + 10*(8*a*b + 11*b^2)*cosh(d*x + c)^2 - 8*a*b - 11*b^2)*sinh
(d*x + c)^3 + (21*(8*a*b + 3*b^2)*cosh(d*x + c)^5 + 10*(8*a*b + 11*b^2)*cos
h(d*x + c)^3 - 3*(8*a*b + 11*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + ((8*a^2
+ 8*a*b + 3*b^2)*cosh(d*x + c)^8 + 8*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)*
sinh(d*x + c)^7 + (8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^8 + 4*(8*a^2 + 8*a*
b + 3*b^2)*cosh(d*x + c)^6 + 4*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 +
 8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh
(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*
(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(8*a^2 + 8*a*b + 3*b^2)*cos
h(d*x + c)^4 + 30*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 24*a^2 + 24*a*b
```

```
 + 9*b^2)*sinh(d*x + c)^4 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^5 +
10*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*
(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^6 + 15*(8*a^2 + 8*a*b + 3*b^2)*cos
h(d*x + c)^4 + 9*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b +
3*b^2)*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 8*((8*a^2 + 8*a*b + 3*b^2)
*cosh(d*x + c)^7 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^5 + 3*(8*a^2 + 8
*a*b + 3*b^2)*cosh(d*x + c)^3 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh
(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (8*a*b + 3*b^2)*cosh(d*x
 + c) + (7*(8*a*b + 3*b^2)*cosh(d*x + c)^6 + 5*(8*a*b + 11*b^2)*cosh(d*x +
c)^4 - 3*(8*a*b + 11*b^2)*cosh(d*x + c)^2 - 8*a*b - 3*b^2)*sinh(d*x + c))/(
d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 +
 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d
*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4
 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 +
8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x
 + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)
^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*
cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^2 \mathrm{sech}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(84) = 168.
time = 0.40, size = 170, normalized size = 1.89

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)\left(8a^2 + 8ab + 3b^2\right) + \frac{4\left(8ab\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 3b^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 32ab\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 20b^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 8*a*
b + 3*b^2) + 4*(8*a*b*(e^(d*x + c) - e^(-d*x - c))^3 + 3*b^2*(e^(d*x + c) -
 e^(-d*x - c))^3 + 32*a*b*(e^(d*x + c) - e^(-d*x - c)) + 20*b^2*(e^(d*x + c
) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

Mupad [B]
time = 1.52, size = 303, normalized size = 3.37

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(8\,a^2\,\sqrt{d^2}+3\,b^2\,\sqrt{d^2}+8\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{64\,a^4+128\,a^3\,b+112\,a^2\,b^2+48\,a\,b^3+9\,b^4}}\right)\,\sqrt{64\,a^4+128\,a^3\,b+112\,a^2\,b^2+48\,a\,b^3+9\,b^4}}{4\,\sqrt{d^2}}-\frac{6\,b^2\,\mathrm{e}^{c+d\,x}}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}+\frac{4\,b^2\,\mathrm{e}^{c+d\,x}}{d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}+\frac{\mathrm{e}^{c+d\,x}\,\left(3\,b^2+8\,a\,b\right)}{4\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{\mathrm{e}^{c+d\,x}\,\left(8\,a\,b-b^2\right)}{2\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2/cosh(c + d*x),x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(8*a^2*(d^2)^(1/2) + 3*b^2*(d^2)^(1/2) + 8*a*b*(d^2)
^(1/2)))/(d*(48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2)))*(
48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2))/(4*(d^2)^(1/2))
 - (6*b^2*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6
*c + 6*d*x) + 1)) + (4*b^2*exp(c + d*x))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c
 + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (exp(c + d*x)*(8*
a*b + 3*b^2))/(4*d*(exp(2*c + 2*d*x) + 1)) - (exp(c + d*x)*(8*a*b - b^2))/(
2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.62 $\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{2b(a+b)\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] (a+b)^2*tanh(d*x+c)/d-2/3*b*(a+b)*tanh(d*x+c)^3/d+1/5*b^2*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {4231, 200}

$$-\frac{2b(a+b)\tanh^3(c+dx)}{3d}+\frac{(a+b)^2\tanh(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tanh[c + d*x])/d - (2*b*(a + b)*Tanh[c + d*x]^3)/(3*d) + (b^2*Ta
nh[c + d*x]^5)/(5*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \left(a+b-bx^2\right)^2\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a^2\left(1+\frac{b(2a+b)}{a^2}\right)-2ab\left(1+\frac{b}{a}\right)x^2+b^2x^4\right)\,dx,x,\tan\right.}{d}\\ &= \frac{(a+b)^2\tanh(c+dx)}{d}-\frac{2b(a+b)\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c}{5d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 93, normalized size = 1.75

$$\frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh(c+dx)}{d}-\frac{2ab\tanh^3(c+dx)}{3d}-\frac{2b^2\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x])/d + (b^2*Tanh[c + d*x])/d - (
2*a*b*Tanh[c + d*x]^3)/(3*d) - (2*b^2*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c
+ d*x]^5)/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(49) = 98.
time = 1.59, size = 157, normalized size = 2.96

method	result
risch	$-\frac{2(15a^2e^{8dx+8c}+60a^2e^{6dx+6c}+60ab\,e^{6dx+6c}+90a^2e^{4dx+4c}+140ab\,e^{4dx+4c}+80b^2e^{4dx+4c}+60a^2e^{2dx+2c}+100ab\,e^{2dx+2c}+40b^2e^{2dx+}}{15d(1+e^{2dx+2c})^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(15*a^2*exp(8*d*x+8*c)+60*a^2*exp(6*d*x+6*c)+60*a*b*exp(6*d*x+6*c)+90
*a^2*exp(4*d*x+4*c)+140*a*b*exp(4*d*x+4*c)+80*b^2*exp(4*d*x+4*c)+60*a^2*exp
(2*d*x+2*c)+100*a*b*exp(2*d*x+2*c)+40*b^2*exp(2*d*x+2*c)+15*a^2+20*a*b+8*b^
2)/d/(1+exp(2*d*x+2*c))^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(49) = 98.
time = 0.29, size = 324, normalized size = 6.11

$$\frac{16}{15}b^2\left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{10e^{(-4dx-4c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{1}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)+\frac{8}{3}ab\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)+\frac{2a^2}{d(e^{(-2dx-2c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 16/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*
e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*
x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
 e^(-10*d*x - 10*c) + 1))) + 8/3*a*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x -
2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*
```

```
c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2*a^2/(d*(e^(-2*d*x - 2
*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(49) = 98$.
time = 0.33, size = 404, normalized size = 7.62

$$-\frac{4\left(\left(15a^2+10ab+4b^2\right)\cosh\left(dx+c\right)^4-8\left(5ab+2b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+\left(15a^2+10ab+4b^2\right)\sinh\left(dx+c\right)^4+20\left(3a^2+4ab+b^2\right)\cosh\left(dx+c\right)^2+2\left(3\left(15a^2+10ab+4b^2\right)\cosh\left(dx+c\right)^2+30a^2+40ab+10b^2\right)\sinh\left(dx+c\right)^2+45a^2+70ab+40b^2-8\left(\left(5ab+2b^2\right)\cosh\left(dx+c\right)^3+5\left(ab+b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)}{15\left(d\cosh\left(dx+c\right)^6+6d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+d\sinh\left(dx+c\right)^6+6d\cosh\left(dx+c\right)^4+3\left(5d\cosh\left(dx+c\right)^2+2d\right)\sinh\left(dx+c\right)^4+4\left(5d\cosh\left(dx+c\right)^3+4d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^3+15d\cosh\left(dx+c\right)^2+3\left(5d\cosh\left(dx+c\right)^4+12d\cosh\left(dx+c\right)^2+5d\right)\sinh\left(dx+c\right)^2+2\left(3d\cosh\left(dx+c\right)^5+8d\cosh\left(dx+c\right)^3+5d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+10d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -4/15*((15*a^2 + 10*a*b + 4*b^2)*cosh(d*x + c)^4 - 8*(5*a*b + 2*b^2)*cosh(d
*x + c)*sinh(d*x + c)^3 + (15*a^2 + 10*a*b + 4*b^2)*sinh(d*x + c)^4 + 20*(3
*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(15*a^2 + 10*a*b + 4*b^2)*cosh(d
*x + c)^2 + 30*a^2 + 40*a*b + 10*b^2)*sinh(d*x + c)^2 + 45*a^2 + 70*a*b + 4
0*b^2 - 8*((5*a*b + 2*b^2)*cosh(d*x + c)^3 + 5*(a*b + b^2)*cosh(d*x + c))*s
inh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*si
nh(d*x + c)^6 + 6*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + 2*d)*sinh(d*
x + c)^4 + 4*(5*d*cosh(d*x + c)^3 + 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15
*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + 5*d)*s
inh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 + 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*
x + c))*sinh(d*x + c) + 10*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^2\operatorname{sech}^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(49) = 98$.
time = 0.40, size = 156, normalized size = 2.94

$$-\frac{2\left(15a^2e^{(8dx+8c)}+60a^2e^{(6dx+6c)}+60abe^{(6dx+6c)}+90a^2e^{(4dx+4c)}+140abe^{(4dx+4c)}+80b^2e^{(4dx+4c)}+60a^2e^{(2dx+2c)}+100abe^{(2dx+2c)}+40b^2e^{(2dx+2c)}+15a^2+20ab+8b^2\right)}{15d\left(e^{(2dx+2c)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^2*e^(8*d*x + 8*c) + 60*a^2*e^(6*d*x + 6*c) + 60*a*b*e^(6*d*x +
6*c) + 90*a^2*e^(4*d*x + 4*c) + 140*a*b*e^(4*d*x + 4*c) + 80*b^2*e^(4*d*x +
```

```
 4*c) + 60*a^2*e^(2*d*x + 2*c) + 100*a*b*e^(2*d*x + 2*c) + 40*b^2*e^(2*d*x
+ 2*c) + 15*a^2 + 20*a*b + 8*b^2)/(d*(e^(2*d*x + 2*c) + 1)^5)
```

Mupad [B]
time = 1.46, size = 452, normalized size = 8.53

$$-\frac{\frac{2\,a\,\left(a+2\,b\right)}{5\,d}+\frac{2\,a^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{\frac{2\,a^2}{5\,d}+\frac{2\,a^2\,{\mathrm{e}}^{8\,c+8\,d\,x}}{5\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2+8\,a\,b+8\,b^2\right)}{5\,d}+\frac{8\,a\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+2\,b\right)}{5\,d}+\frac{8\,a\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a+2\,b\right)}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}-\frac{\frac{2\,a\,\left(a+2\,b\right)}{5\,d}+\frac{2\,a^2\,{\mathrm{e}}^{6\,c+6\,d\,x}}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2+8\,a\,b+8\,b^2\right)}{5\,d}+\frac{6\,a\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a+2\,b\right)}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{\frac{2\,\left(3\,a^2+8\,a\,b+8\,b^2\right)}{15\,d}+\frac{2\,a^2\,{\mathrm{e}}^{4\,c+4\,d\,x}}{5\,d}+\frac{4\,a\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+2\,b\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}-\frac{2\,a^2}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2/cosh(c + d*x)^2,x)
```

```
[Out] - ((2*a*(a + 2*b))/(5*d) + (2*a^2*exp(2*c + 2*d*x))/(5*d))/(2*exp(2*c + 2*d
*x) + exp(4*c + 4*d*x) + 1) - ((2*a^2)/(5*d) + (2*a^2*exp(8*c + 8*d*x))/(5*
d) + (4*exp(4*c + 4*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(5*d) + (8*a*exp(2*c + 2*
d*x)*(a + 2*b))/(5*d) + (8*a*exp(6*c + 6*d*x)*(a + 2*b))/(5*d))/(5*exp(2*c
+ 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) +
 exp(10*c + 10*d*x) + 1) - ((2*a*(a + 2*b))/(5*d) + (2*a^2*exp(6*c + 6*d*x)
)/(5*d) + (2*exp(2*c + 2*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(5*d) + (6*a*exp(4*c
 + 4*d*x)*(a + 2*b))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*ex
p(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(8*a*b + 3*a^2 + 8*b^2))/(15*d
) + (2*a^2*exp(4*c + 4*d*x))/(5*d) + (4*a*exp(2*c + 2*d*x)*(a + 2*b))/(5*d)
)/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*a^2
)/(5*d*(exp(2*c + 2*d*x) + 1))
```

3.63 $\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=128

$$\frac{(8a^2+12ab+5b^2)\operatorname{ArcTan}(\sinh(c+dx))}{16d}+\frac{(8a^2+12ab+5b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d}+\frac{b(8a+5b)\operatorname{sech}^3(}{}$$

```
[Out] 1/16*(8*a^2+12*a*b+5*b^2)*arctan(sinh(d*x+c))/d+1/16*(8*a^2+12*a*b+5*b^2)*s
ech(d*x+c)*tanh(d*x+c)/d+1/24*b*(8*a+5*b)*sech(d*x+c)^3*tanh(d*x+c)/d+1/6*b
*sech(d*x+c)^5*(a+b+a*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4232, 424, 393, 205, 209}

$$\frac{(8a^2+12ab+5b^2)\operatorname{ArcTan}(\sinh(c+dx))}{16d}+\frac{(8a^2+12ab+5b^2)\tanh(c+dx)\operatorname{sech}(c+dx)}{16d}+\frac{b(8a+5b)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{24d}+\frac{b\tanh(c+dx)\operatorname{sech}^5(c+dx)\left(a\sinh^2(c+dx)+a+b\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 12*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((8*a^2 + 12*a*b
+ 5*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) + (b*(8*a + 5*b)*Sech[c + d*x]
^3*Tanh[c + d*x])/(24*d) + (b*Sech[c + d*x]^5*(a + b + a*Sinh[c + d*x]^2)*T
anh[c + d*x])/(6*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b+ax^2\right)^2}{\left(1+x^2\right)^4}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\operatorname{sech}^5(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx)}{6d} + \frac{\operatorname{Subst}}{}$$

$$= \frac{b(8a+5b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{24d} + \frac{b\operatorname{sech}^5(c+dx)\left(a+b+\right.}{}$$

$$= \frac{\left(8a^2+12ab+5b^2\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d} + \frac{b(8a+5b)\operatorname{sech}}{}$$

$$= \frac{\left(8a^2+12ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{\left(8a^2+12ab+5b^2\right)\operatorname{se}}{}$$

Mathematica [A]
time = 0.17, size = 104, normalized size = 0.81

$$\frac{3\left(8a^2+12ab+5b^2\right)\operatorname{ArcTan}(\sinh(c+dx))+3\left(8a^2+12ab+5b^2\right)\operatorname{sech}(c+dx)\tanh(c+dx)+2b(12a+5b)\operatorname{sech}^3(c+dx)\tanh(c+dx)+8b^2\operatorname{sech}^5(c+dx)\tanh(c+dx)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (3*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]] + 3*(8*a^2 + 12*a*b + 5*b
^2)*Sech[c + d*x]*Tanh[c + d*x] + 2*b*(12*a + 5*b)*Sech[c + d*x]^3*Tanh[c +
 d*x] + 8*b^2*Sech[c + d*x]^5*Tanh[c + d*x])/(48*d)
```

Maple [C] Result contains complex when optimal does not.
time = 1.75, size = 358, normalized size = 2.80

method	result
risch	$\frac{e^{dx+c}\left(24a^2 e^{10dx+10c}+36ab\, e^{10dx+10c}+15b^2 e^{10dx+10c}+72a^2 e^{8dx+8c}+204ab\, e^{8dx+8c}+85b^2 e^{8dx+8c}+48a^2 e^{6dx+6c}+168ab\, e^{6dx+6c}+\right.}{24d\left(1+e^{2dx}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*exp(d*x+c)*(24*a^2*exp(10*d*x+10*c)+36*a*b*exp(10*d*x+10*c)+15*b^2*exp
(10*d*x+10*c)+72*a^2*exp(8*d*x+8*c)+204*a*b*exp(8*d*x+8*c)+85*b^2*exp(8*d*x
+8*c)+48*a^2*exp(6*d*x+6*c)+168*a*b*exp(6*d*x+6*c)+198*b^2*exp(6*d*x+6*c)-4
8*a^2*exp(4*d*x+4*c)-168*a*b*exp(4*d*x+4*c)-198*b^2*exp(4*d*x+4*c)-72*a^2*e
xp(2*d*x+2*c)-204*a*b*exp(2*d*x+2*c)-85*b^2*exp(2*d*x+2*c)-24*a^2-36*a*b-15
*b^2)/d/(1+exp(2*d*x+2*c))^6+1/2*I/d*ln(exp(d*x+c)+I)*a^2+3/4*I*b*a/d*ln(ex
p(d*x+c)+I)+5/16*I*b^2/d*ln(exp(d*x+c)+I)-1/2*I/d*ln(exp(d*x+c)-I)*a^2-3/4*
I/d*ln(exp(d*x+c)-I)*a*b-5/16*I/d*ln(exp(d*x+c)-I)*b^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(120) = 240.
time = 0.50, size = 348, normalized size = 2.72

$$-\frac{1}{24}\,b^2\left(\frac{15\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{15\,e^{(-dx-c)}+85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}-198\,e^{(-7\,dx-7\,c)}-85\,e^{(-9\,dx-9\,c)}-15\,e^{(-11\,dx-11\,c)}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}\right)-\frac{1}{2}\,ab\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}+11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}-3\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)-a^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*b^2*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3*
c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) - 15
*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-
6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) - 1/2*a*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3
*d*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2
*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a
^2*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2946 vs. 2(120) = 240.
time = 0.39, size = 2946, normalized size = 23.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^11 + 33*(8*a^2 + 12*a*b + 5*
b^2)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(8*a^2 + 12*a*b + 5*b^2)*sinh(d*x +
 c)^11 + (72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^9 + (165*(8*a^2 + 12*a*b
 + 5*b^2)*cosh(d*x + c)^2 + 72*a^2 + 204*a*b + 85*b^2)*sinh(d*x + c)^9 + 9*
(55*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^3 + (72*a^2 + 204*a*b + 85*b^2)*
cosh(d*x + c))*sinh(d*x + c)^8 + 6*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^
7 + 6*(165*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^4 + 6*(72*a^2 + 204*a*b +
 85*b^2)*cosh(d*x + c)^2 + 8*a^2 + 28*a*b + 33*b^2)*sinh(d*x + c)^7 + 42*(3
3*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^5 + 2*(72*a^2 + 204*a*b + 85*b^2)*
cosh(d*x + c)^3 + (8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c))*sinh(d*x + c)^6
- 6*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^5 + 6*(231*(8*a^2 + 12*a*b + 5*
b^2)*cosh(d*x + c)^6 + 21*(72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^4 + 21*
(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^2 - 8*a^2 - 28*a*b - 33*b^2)*sinh(d
*x + c)^5 + 6*(165*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^7 + 21*(72*a^2 +
204*a*b + 85*b^2)*cosh(d*x + c)^5 + 35*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x +
 c)^3 - 5*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - (72*a^
2 + 204*a*b + 85*b^2)*cosh(d*x + c)^3 + (495*(8*a^2 + 12*a*b + 5*b^2)*cosh(
d*x + c)^8 + 84*(72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^6 + 210*(8*a^2 +
28*a*b + 33*b^2)*cosh(d*x + c)^4 - 60*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x +
c)^2 - 72*a^2 - 204*a*b - 85*b^2)*sinh(d*x + c)^3 + 3*(55*(8*a^2 + 12*a*b +
 5*b^2)*cosh(d*x + c)^9 + 12*(72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^7 +
42*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^5 - 20*(8*a^2 + 28*a*b + 33*b^2)
*cosh(d*x + c)^3 - (72*a^2 + 204*a*b + 85*b^2)*cosh(d*x + c))*sinh(d*x + c)
^2 + 3*((8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^12 + 12*(8*a^2 + 12*a*b + 5*
b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (8*a^2 + 12*a*b + 5*b^2)*sinh(d*x + c
)^12 + 6*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^10 + 6*(11*(8*a^2 + 12*a*b
+ 5*b^2)*cosh(d*x + c)^2 + 8*a^2 + 12*a*b + 5*b^2)*sinh(d*x + c)^10 + 20*(1
1*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 12*a*b + 5*b^2)*cos
h(d*x + c))*sinh(d*x + c)^9 + 15*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^8 +
 15*(33*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^4 + 18*(8*a^2 + 12*a*b + 5*b
^2)*cosh(d*x + c)^2 + 8*a^2 + 12*a*b + 5*b^2)*sinh(d*x + c)^8 + 24*(33*(8*a
^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^5 + 30*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x
 + c)^3 + 5*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(8
*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^6 + 4*(231*(8*a^2 + 12*a*b + 5*b^2)*co
sh(d*x + c)^6 + 315*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^4 + 105*(8*a^2 +
 12*a*b + 5*b^2)*cosh(d*x + c)^2 + 40*a^2 + 60*a*b + 25*b^2)*sinh(d*x + c)^
6 + 24*(33*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^7 + 63*(8*a^2 + 12*a*b +
5*b^2)*cosh(d*x + c)^5 + 35*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^3 + 5*(8
*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(8*a^2 + 12*a*b
+ 5*b^2)*cosh(d*x + c)^4 + 15*(33*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^8
+ 84*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^6 + 70*(8*a^2 + 12*a*b + 5*b^2)
```

```
*cosh(d*x + c)^4 + 20*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^2 + 8*a^2 + 12
*a*b + 5*b^2)*sinh(d*x + c)^4 + 20*(11*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x +
c)^9 + 36*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^7 + 42*(8*a^2 + 12*a*b + 5
*b^2)*cosh(d*x + c)^5 + 20*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^3 + 3*(8*
a^2 + 12*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(8*a^2 + 12*a*b +
5*b^2)*cosh(d*x + c)^2 + 6*(11*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^10 +
45*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^8 + 70*(8*a^2 + 12*a*b + 5*b^2)*c
osh(d*x + c)^6 + 50*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^4 + 15*(8*a^2 +
12*a*b + 5*b^2)*cosh(d*x + c)^2 + 8*a^2 + 12*a*b + 5*b^2)*sinh(d*x + c)^2 +
 8*a^2 + 12*a*b + 5*b^2 + 12*((8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^11 + 5
*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^9 + 10*(8*a^2 + 12*a*b + 5*b^2)*cos
h(d*x + c)^7 + 10*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^5 + 5*(8*a^2 + 12*
a*b + 5*b^2)*cosh(d*x + c)^3 + (8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c))*sinh
(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 3*(8*a^2 + 12*a*b + 5*b^
2)*cosh(d*x + c) + 3*(11*(8*a^2 + 12*a*b + 5*b^2)*cosh(d*x + c)^10 + 3*(72*
a^2 + 204*a*b + 85*b^2)*cosh(d*x + c)^8 + 14*(8*a^2 + 28*a*b + 33*b^2)*cosh
(d*x + c)^6 - 10*(8*a^2 + 28*a*b + 33*b^2)*cosh(d*x + c)^4 - (72*a^2 + 204*
a*b + 85*b^2)*cosh(d*x + c)^2 - 8*a^2 - 12*a*b - 5*b^2)*sinh(d*x + c))/(d*c
osh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12
+ 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 20
*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x
 + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c
)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*
sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 + 315*d*c
osh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6 + 24*(33*d*co
sh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cos...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^2 \mathrm{sech}^3\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(120) = 240.
time = 0.41, size = 293, normalized size = 2.29

$$\frac{3\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)\left(8a^2 + 12ab + 5b^2\right) + \frac{4\left(24a^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 + 36ab\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 + 15b^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 + 192a^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 384ab\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 160b^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 384a^2\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 960ab\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 528b^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)^3}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 12
*a*b + 5*b^2) + 4*(24*a^2*(e^(d*x + c) - e^(-d*x - c))^5 + 36*a*b*(e^(d*x +
 c) - e^(-d*x - c))^5 + 15*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 192*a^2*(e^
(d*x + c) - e^(-d*x - c))^3 + 384*a*b*(e^(d*x + c) - e^(-d*x - c))^3 + 160*
b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 384*a^2*(e^(d*x + c) - e^(-d*x - c)) +
 960*a*b*(e^(d*x + c) - e^(-d*x - c)) + 528*b^2*(e^(d*x + c) - e^(-d*x - c)
))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3)/d
```

Mupad [B]
time = 1.57, size = 569, normalized size = 4.45

$$\frac{\mathrm{atan}\left(\frac{e^{dx}\,e^c\,\left(8\,a^2\,\sqrt{d^2}+5\,b^2\,\sqrt{d^2}+12\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{64\,a^4+192\,a^3\,b+224\,a^2\,b^2+120\,a\,b^3+25\,b^4}}\right)\,\sqrt{64\,a^4+192\,a^3\,b+224\,a^2\,b^2+120\,a\,b^3+25\,b^4}}{8\,\sqrt{d^2}}-\frac{\frac{2\,a^2\,e^{c+d\,x}}{3\,d}+\frac{2\,a^2\,e^{9\,c+9\,d\,x}}{3\,d}+\frac{4\,e^{5\,c+5\,d\,x}\,\left(3\,a^2+8\,a\,b+8\,b^2\right)}{3\,d}+\frac{8\,a\,e^{3\,c+3\,d\,x}\,\left(a+2\,b\right)}{3\,d}+\frac{8\,a\,e^{7\,c+7\,d\,x}\,\left(a+2\,b\right)}{3\,d}}{6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1}+\frac{2\,e^{c+d\,x}\,\left(4\,a\,b-11\,b^2\right)}{3\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}+\frac{16\,b^2\,e^{c+d\,x}}{3\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}+\frac{e^{c+d\,x}\,\left(8\,a^2+12\,a\,b+5\,b^2\right)}{8\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}+\frac{e^{c+d\,x}\,\left(-16\,a^2+12\,a\,b+5\,b^2\right)}{12\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}-\frac{e^{c+d\,x}\,\left(20\,a\,b-b^2\right)}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2/cosh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(8*a^2*(d^2)^(1/2) + 5*b^2*(d^2)^(1/2) + 12*a*b*(d^2
)^(1/2)))/(d*(120*a*b^3 + 192*a^3*b + 64*a^4 + 25*b^4 + 224*a^2*b^2)^(1/2))
)*(120*a*b^3 + 192*a^3*b + 64*a^4 + 25*b^4 + 224*a^2*b^2)^(1/2))/(8*(d^2)^(
1/2)) - ((2*a^2*exp(c + d*x))/(3*d) + (2*a^2*exp(9*c + 9*d*x))/(3*d) + (4*e
xp(5*c + 5*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(3*d) + (8*a*exp(3*c + 3*d*x)*(a +
 2*b))/(3*d) + (8*a*exp(7*c + 7*d*x)*(a + 2*b))/(3*d))/(6*exp(2*c + 2*d*x)
+ 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(1
0*c + 10*d*x) + exp(12*c + 12*d*x) + 1) + (2*exp(c + d*x)*(4*a*b - 11*b^2))
/(3*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8
*c + 8*d*x) + 1)) + (16*b^2*exp(c + d*x))/(3*d*(5*exp(2*c + 2*d*x) + 10*exp
(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*
x) + 1)) + (exp(c + d*x)*(12*a*b + 8*a^2 + 5*b^2))/(8*d*(exp(2*c + 2*d*x) +
 1)) + (exp(c + d*x)*(12*a*b - 16*a^2 + 5*b^2))/(12*d*(2*exp(2*c + 2*d*x) +
 exp(4*c + 4*d*x) + 1)) - (exp(c + d*x)*(20*a*b - b^2))/(3*d*(3*exp(2*c + 2
*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
```

3.64 $\int \mathrm{sech}^4(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=80

$$\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)(a+3b)\tanh^3(c+dx)}{3d}+\frac{b(2a+3b)\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

```
[Out] (a+b)^2*tanh(d*x+c)/d-1/3*(a+b)*(a+3*b)*tanh(d*x+c)^3/d+1/5*b*(2*a+3*b)*tan
h(d*x+c)^5/d-1/7*b^2*tanh(d*x+c)^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4231, 380}

$$\frac{b(2a+3b)\tanh^5(c+dx)}{5d}-\frac{(a+b)(a+3b)\tanh^3(c+dx)}{3d}+\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tanh[c + d*x])/d - ((a + b)*(a + 3*b)*Tanh[c + d*x]^3)/(3*d) + (
b*(2*a + 3*b)*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \mathrm{sech}^4(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx &= \frac{\mathrm{Subst}\left(\int \left(1-x^2\right)\left(a+b-bx^2\right)^2\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\mathrm{Subst}\left(\int \left((a+b)^2+(-a-3b)(a+b)x^2+b(2a+3b)x^4-b^2x^6\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)(a+3b)\tanh^3(c+dx)}{3d}+\frac{b(2a+3b)\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 144, normalized size = 1.80

$$\frac{a^2\tanh(c+dx)}{d}+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh(c+dx)}{d}-\frac{a^2\tanh^3(c+dx)}{3d}-\frac{4ab\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^3(c+dx)}{d}+\frac{2ab\tanh^5(c+dx)}{5d}+\frac{3b^2\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x])/d + (b^2*Tanh[c + d*x])/d - (
a^2*Tanh[c + d*x]^3)/(3*d) - (4*a*b*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c +
d*x]^3)/d + (2*a*b*Tanh[c + d*x]^5)/(5*d) + (3*b^2*Tanh[c + d*x]^5)/(5*d) -
 (b^2*Tanh[c + d*x]^7)/(7*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(74) = 148.
time = 1.76, size = 198, normalized size = 2.48

method	result
risch	$-\frac{4\left(105a^2e^{10dx+10c}+455a^2e^{8dx+8c}+560ab\,e^{8dx+8c}+770a^2e^{6dx+6c}+1400ab\,e^{6dx+6c}+840b^2e^{6dx+6c}+630a^2e^{4dx+4c}+1176ab\,e^{4dx+4c}\cdots\right.}{105d\left(1+e^{2dx+2c}\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/105*(105*a^2*exp(10*d*x+10*c)+455*a^2*exp(8*d*x+8*c)+560*a*b*exp(8*d*x+8
*c)+770*a^2*exp(6*d*x+6*c)+1400*a*b*exp(6*d*x+6*c)+840*b^2*exp(6*d*x+6*c)+6
30*a^2*exp(4*d*x+4*c)+1176*a*b*exp(4*d*x+4*c)+504*b^2*exp(4*d*x+4*c)+245*a^
2*exp(2*d*x+2*c)+392*a*b*exp(2*d*x+2*c)+168*b^2*exp(2*d*x+2*c)+35*a^2+56*a*
b+24*b^2)/d/(1+exp(2*d*x+2*c))^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 671 vs. 2(74) = 148.
time = 0.28, size = 671, normalized size = 8.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 32/35*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(
-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-
2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8
*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1
)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*
```

```
e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d
*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4
*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 1
0*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 32/15*a*b*(5*e^(-
2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d*x - 4*c)/
(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-
8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x
- 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(74) = 148$.

time = 0.38, size = 677, normalized size = 8.46

$$-\frac{8\,\left(2\,\left(35\,a^{2}+14\,ab+6\,b^{2}\right)\cosh\left(dx+c\right)^{5}+10\,\left(35\,a^{2}+14\,ab+6\,b^{2}\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}+\left(35\,a^{2}-28\,ab-12\,b^{2}\right)\sinh\left(dx+c\right)^{5}+14\,\left(25\,a^{2}+34\,ab+6\,b^{2}\right)\cosh\left(dx+c\right)^{3}+\left(10\,\left(35\,a^{2}-28\,ab-12\,b^{2}\right)\cosh\left(dx+c\right)^{2}+105\,a^{2}+84\,ab-84\,b^{2}\right)\sinh\left(dx+c\right)^{3}+2\,\left(10\,\left(35\,a^{2}+14\,ab+6\,b^{2}\right)\cosh\left(dx+c\right)^{3}+21\,\left(25\,a^{2}+34\,ab+6\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+28\,\left(25\,a^{2}+46\,ab+24\,b^{2}\right)\cosh\left(dx+c\right)+\left(5\,\left(35\,a^{2}-28\,ab-12\,b^{2}\right)\cosh\left(dx+c\right)^{4}+63\,\left(5\,a^{2}+4\,ab-4\,b^{2}\right)\cosh\left(dx+c\right)^{2}+70\,a^{2}+112\,ab+168\,b^{2}\right)\sinh\left(dx+c\right)\right)}{105\,\left(d\cosh\left(dx+c\right)^{9}+9\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{8}+d\sinh\left(dx+c\right)^{9}+7\,d\cosh\left(dx+c\right)^{7}+\left(36\,d\cosh\left(dx+c\right)^{2}+7\,d\right)\sinh\left(dx+c\right)^{7}+7\,\left(12\,d\cosh\left(dx+c\right)^{3}+7\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{6}+22\,d\cosh\left(dx+c\right)^{5}+\left(126\,d\cosh\left(dx+c\right)^{4}+147\,d\cosh\left(dx+c\right)^{2}+20\,d\right)\sinh\left(dx+c\right)^{5}+\left(126\,d\cosh\left(dx+c\right)^{5}+245\,d\cosh\left(dx+c\right)^{3}+110\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{4}+42\,d\cosh\left(dx+c\right)^{3}+\left(84\,d\cosh\left(dx+c\right)^{6}+245\,d\cosh\left(dx+c\right)^{4}+200\,d\cosh\left(dx+c\right)^{2}+28\,d\right)\sinh\left(dx+c\right)^{3}+\left(36\,d\cosh\left(dx+c\right)^{7}+147\,d\cosh\left(dx+c\right)^{5}+220\,d\cosh\left(dx+c\right)^{3}+126\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+56\,d\cosh\left(dx+c\right)+\left(9\,d\cosh\left(dx+c\right)^{8}+49\,d\cosh\left(dx+c\right)^{6}+100\,d\cosh\left(dx+c\right)^{4}+84\,d\cosh\left(dx+c\right)^{2}+14\,d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -8/105*(2*(35*a^2 + 14*a*b + 6*b^2)*cosh(d*x + c)^5 + 10*(35*a^2 + 14*a*b +
 6*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (35*a^2 - 28*a*b - 12*b^2)*sinh(d*x
 + c)^5 + 14*(25*a^2 + 34*a*b + 6*b^2)*cosh(d*x + c)^3 + (10*(35*a^2 - 28*a
*b - 12*b^2)*cosh(d*x + c)^2 + 105*a^2 + 84*a*b - 84*b^2)*sinh(d*x + c)^3 +
 2*(10*(35*a^2 + 14*a*b + 6*b^2)*cosh(d*x + c)^3 + 21*(25*a^2 + 34*a*b + 6*
b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 28*(25*a^2 + 46*a*b + 24*b^2)*cosh(d*
x + c) + (5*(35*a^2 - 28*a*b - 12*b^2)*cosh(d*x + c)^4 + 63*(5*a^2 + 4*a*b
- 4*b^2)*cosh(d*x + c)^2 + 70*a^2 + 112*a*b + 168*b^2)*sinh(d*x + c))/(d*co
sh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x + c)^9 + 7*d
*cosh(d*x + c)^7 + (36*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^7 + 7*(12*d*c
osh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^6 + 22*d*cosh(d*x + c)^5
+ (126*d*cosh(d*x + c)^4 + 147*d*cosh(d*x + c)^2 + 20*d)*sinh(d*x + c)^5 +
(126*d*cosh(d*x + c)^5 + 245*d*cosh(d*x + c)^3 + 110*d*cosh(d*x + c))*sinh(
d*x + c)^4 + 42*d*cosh(d*x + c)^3 + (84*d*cosh(d*x + c)^6 + 245*d*cosh(d*x
+ c)^4 + 200*d*cosh(d*x + c)^2 + 28*d)*sinh(d*x + c)^3 + (36*d*cosh(d*x + c
)^7 + 147*d*cosh(d*x + c)^5 + 220*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c))*
sinh(d*x + c)^2 + 56*d*cosh(d*x + c) + (9*d*cosh(d*x + c)^8 + 49*d*cosh(d*x
 + c)^6 + 100*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c
))
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{sech}^{2}{\left(c + dx \right)}\right)^{2} \operatorname{sech}^{4}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*sech(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(74) = 148$.
time = 0.41, size = 197, normalized size = 2.46

$$-\frac{4\,\left(105\,a^2 e^{(10\,dx+10\,c)} + 455\,a^2 e^{(8\,dx+8\,c)} + 560\,abe^{(8\,dx+8\,c)} + 770\,a^2 e^{(6\,dx+6\,c)} + 1400\,abe^{(6\,dx+6\,c)} + 840\,b^2 e^{(6\,dx+6\,c)} + 630\,a^2 e^{(4\,dx+4\,c)} + 1176\,abe^{(4\,dx+4\,c)} + 504\,b^2 e^{(4\,dx+4\,c)} + 245\,a^2 e^{(2\,dx+2\,c)} + 392\,abe^{(2\,dx+2\,c)} + 168\,b^2 e^{(2\,dx+2\,c)} + 35\,a^2 + 56\,ab + 24\,b^2\right)}{105\,d\left(e^{(2\,dx+2\,c)} + 1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -4/105*(105*a^2*e^(10*d*x + 10*c) + 455*a^2*e^(8*d*x + 8*c) + 560*a*b*e^(8*
d*x + 8*c) + 770*a^2*e^(6*d*x + 6*c) + 1400*a*b*e^(6*d*x + 6*c) + 840*b^2*e
^(6*d*x + 6*c) + 630*a^2*e^(4*d*x + 4*c) + 1176*a*b*e^(4*d*x + 4*c) + 504*b
^2*e^(4*d*x + 4*c) + 245*a^2*e^(2*d*x + 2*c) + 392*a*b*e^(2*d*x + 2*c) + 16
8*b^2*e^(2*d*x + 2*c) + 35*a^2 + 56*a*b + 24*b^2)/(d*(e^(2*d*x + 2*c) + 1)^
7)
```

Mupad [B]
time = 1.42, size = 692, normalized size = 8.65

$$-\frac{\frac{32\,a\,(a+2\,b)}{105\,d}+\frac{8\,a^2\,\mathrm{e}^{2\,c+2\,d\,x}}{21\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}-\frac{\frac{8\,a^2\,\mathrm{e}^{2\,c+2\,d\,x}}{7\,d}+\frac{8\,a^2\,\mathrm{e}^{10\,c+10\,d\,x}}{7\,d}+\frac{16\,\mathrm{e}^{6\,c+6\,d\,x}\,(3\,a^2+8\,a\,b+8\,b^2)}{7\,d}+\frac{32\,a\,\mathrm{e}^{4\,c+4\,d\,x}\,(a+2\,b)}{7\,d}+\frac{32\,a\,\mathrm{e}^{8\,c+8\,d\,x}\,(a+2\,b)}{7\,d}}{7\,\mathrm{e}^{2\,c+2\,d\,x}+21\,\mathrm{e}^{4\,c+4\,d\,x}+35\,\mathrm{e}^{6\,c+6\,d\,x}+35\,\mathrm{e}^{8\,c+8\,d\,x}+21\,\mathrm{e}^{10\,c+10\,d\,x}+7\,\mathrm{e}^{12\,c+12\,d\,x}+\mathrm{e}^{14\,c+14\,d\,x}+1}-\frac{\frac{4\,a^2}{21\,d}+\frac{20\,a^2\,\mathrm{e}^{8\,c+8\,d\,x}}{21\,d}+\frac{8\,\mathrm{e}^{4\,c+4\,d\,x}\,(3\,a^2+8\,a\,b+8\,b^2)}{7\,d}+\frac{32\,a\,\mathrm{e}^{2\,c+2\,d\,x}\,(a+2\,b)}{21\,d}+\frac{64\,a\,\mathrm{e}^{6\,c+6\,d\,x}\,(a+2\,b)}{21\,d}}{6\,\mathrm{e}^{2\,c+2\,d\,x}+15\,\mathrm{e}^{4\,c+4\,d\,x}+20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}+6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1}-\frac{\frac{32\,a\,(a+2\,b)}{105\,d}+\frac{16\,a^2\,\mathrm{e}^{6\,c+6\,d\,x}}{21\,d}+\frac{16\,\mathrm{e}^{2\,c+2\,d\,x}\,(3\,a^2+8\,a\,b+8\,b^2)}{35\,d}+\frac{64\,a\,\mathrm{e}^{4\,c+4\,d\,x}\,(a+2\,b)}{35\,d}}{5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1}-\frac{\frac{4\,(3\,a^2+8\,a\,b+8\,b^2)}{35\,d}+\frac{4\,a^2\,\mathrm{e}^{4\,c+4\,d\,x}}{7\,d}+\frac{32\,a\,\mathrm{e}^{2\,c+2\,d\,x}\,(a+2\,b)}{35\,d}}{4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1}-\frac{4\,a^2}{21\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2/cosh(c + d*x)^4,x)
```

```
[Out] - ((32*a*(a + 2*b))/(105*d) + (8*a^2*exp(2*c + 2*d*x))/(21*d))/(3*exp(2*c +
 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((8*a^2*exp(2*c + 2*
d*x))/(7*d) + (8*a^2*exp(10*c + 10*d*x))/(7*d) + (16*exp(6*c + 6*d*x)*(8*a*
b + 3*a^2 + 8*b^2))/(7*d) + (32*a*exp(4*c + 4*d*x)*(a + 2*b))/(7*d) + (32*a
*exp(8*c + 8*d*x)*(a + 2*b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*
x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*
exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((4*a^2)/(21*d) + (20*a^2*ex
p(8*c + 8*d*x))/(21*d) + (8*exp(4*c + 4*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(7*d)
 + (32*a*exp(2*c + 2*d*x)*(a + 2*b))/(21*d) + (64*a*exp(6*c + 6*d*x)*(a + 2
*b))/(21*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x
) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) -
((32*a*(a + 2*b))/(105*d) + (16*a^2*exp(6*c + 6*d*x))/(21*d) + (16*exp(2*c
+ 2*d*x)*(8*a*b + 3*a^2 + 8*b^2))/(35*d) + (64*a*exp(4*c + 4*d*x)*(a + 2*b)
)/(35*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) +
 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((4*(8*a*b + 3*a^2 + 8*b^2)
)/(35*d) + (4*a^2*exp(4*c + 4*d*x))/(7*d) + (32*a*exp(2*c + 2*d*x)*(a + 2*b
))/(35*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) +
exp(8*c + 8*d*x) + 1) - (4*a^2)/(21*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x
) + 1))
```

3.65 $\int \cosh^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=84

$$\frac{3}{8}a\left(a^2+4ab+8b^2\right)x+\frac{3a^2(a+4b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a^3\cosh^3(c+dx)\sinh(c+dx)}{4d}+\frac{b^3\tanh(c+dx)}{d}$$

```
[Out] 3/8*a*(a^2+4*a*b+8*b^2)*x+3/8*a^2*(a+4*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a^3
*cosh(d*x+c)^3*sinh(d*x+c)/d+b^3*tanh(d*x+c)/d
```

Rubi [A]

time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4231, 398, 1171, 393, 212}

$$\frac{a^3\sinh(c+dx)\cosh^3(c+dx)}{4d}+\frac{3}{8}ax\left(a^2+4ab+8b^2\right)+\frac{3a^2(a+4b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{b^3\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (3*a*(a^2 + 4*a*b + 8*b^2)*x)/8 + (3*a^2*(a + 4*b)*Cosh[c + d*x]*Sinh[c + d
*x])/(8*d) + (a^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) + (b^3*Tanh[c + d*x]
)/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\frac{\operatorname{Subst}\left(\int\frac{\left(a+b-bx^2\right)^3}{\left(1-x^2\right)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{\operatorname{Subst}\left(\int\left(b^3+\frac{a\left(a^2+3ab+3b^2\right)-3ab(a+2b)x^2+3ab^2x^4}{\left(1-x^2\right)^3}\right)\,dx,x,\tanh(c+$$

$$=\frac{b^3\tanh(c+dx)}{d}+\frac{\operatorname{Subst}\left(\int\frac{a\left(a^2+3ab+3b^2\right)-3ab(a+2b)x^2+3ab^2x^4}{\left(1-x^2\right)^3}\,dx,x,}{d}$$

$$=\frac{a^3\cosh^3(c+dx)\sinh(c+dx)}{4d}+\frac{b^3\tanh(c+dx)}{d}-\frac{\operatorname{Subst}\left(\int\frac{-3}{}\right.}{}$$

$$=\frac{3a^2(a+4b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a^3\cosh^3(c+dx)\sinh(}{4d}$$

$$=\frac{3}{8}a\left(a^2+4ab+8b^2\right)x+\frac{3a^2(a+4b)\cosh(c+dx)\sinh(c+dx)}{8d}$$

Mathematica [A]
time = 0.29, size = 70, normalized size = 0.83

$$\frac{12a(a^2+4ab+8b^2)\,(c+dx)+8a^2(a+3b)\sinh(2(c+dx))+a^3\sinh(4(c+dx))+32b^3\tanh(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (12*a*(a^2 + 4*a*b + 8*b^2)*(c + d*x) + 8*a^2*(a + 3*b)*Sinh[2*(c + d*x)] +
 a^3*Sinh[4*(c + d*x)] + 32*b^3*Tanh[c + d*x])/(32*d)
```

Maple [A]
time = 2.14, size = 147, normalized size = 1.75

method	result
risch	$\frac{3a^3x}{8}+\frac{3a^2bx}{2}+3a\,b^2x+\frac{a^3\mathrm{e}^{4dx+4c}}{64d}+\frac{a^3\mathrm{e}^{2dx+2c}}{8d}+\frac{3a^2\mathrm{e}^{2dx+2c}b}{8d}-\frac{a^3\mathrm{e}^{-2dx-2c}}{8d}-\frac{3a^2\mathrm{e}^{-2dx-2c}b}{8d}-\frac{a^3\mathrm{e}^{-4dx-4c}}{64d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a^3*x+3/2*a^2*b*x+3*a*b^2*x+1/64*a^3/d*exp(4*d*x+4*c)+1/8*a^3/d*exp(2*d
*x+2*c)+3/8*a^2/d*exp(2*d*x+2*c)*b-1/8*a^3/d*exp(-2*d*x-2*c)-3/8*a^2/d*exp(
-2*d*x-2*c)*b-1/64*a^3/d*exp(-4*d*x-4*c)-2*b^3/d/(1+exp(2*d*x+2*c))
```

Maxima [A]
time = 0.28, size = 130, normalized size = 1.55

$$\frac{1}{64}\,a^3\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}+\frac{8\,e^{(2\,dx+2\,c)}}{d}-\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{3}{8}\,a^2b\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+3\,ab^2x+\frac{2\,b^3}{d(e^{(-2\,dx-2\,c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3/8*a^2*b*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x
 - 2*c)/d) + 3*a*b^2*x + 2*b^3/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [A]
time = 0.39, size = 153, normalized size = 1.82

$$\frac{a^3\sinh\left(dx+c\right)^5+\left(10\,a^3\cosh\left(dx+c\right)^2+9\,a^3+24\,a^2b\right)\sinh\left(dx+c\right)^3-8\left(8\,b^3-3\left(a^3+4\,a^2b+8\,ab^2\right)dx\right)\cosh\left(dx+c\right)+\left(5\,a^3\cosh\left(dx+c\right)^4+8\,a^3+24\,a^2b+64\,b^3+9\left(3\,a^3+8\,a^2b\right)\cosh\left(dx+c\right)^2\right)\sinh\left(dx+c\right)}{64\,d\cosh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(a^3*sinh(d*x + c)^5 + (10*a^3*cosh(d*x + c)^2 + 9*a^3 + 24*a^2*b)*sin
h(d*x + c)^3 - 8*(8*b^3 - 3*(a^3 + 4*a^2*b + 8*a*b^2)*d*x)*cosh(d*x + c) +
(5*a^3*cosh(d*x + c)^4 + 8*a^3 + 24*a^2*b + 64*b^3 + 9*(3*a^3 + 8*a^2*b)*co
sh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(78) = 156.
time = 0.43, size = 177, normalized size = 2.11

$$\frac{a^3e^{(4\,dx+4\,c)}+8\,a^3e^{(2\,dx+2\,c)}+24\,a^2be^{(2\,dx+2\,c)}+24\,(a^3+4\,a^2b+8\,ab^2)(dx+c)-\frac{128\,b^3}{e^{(2\,dx+2\,c)}+1}-\left(18\,a^3e^{(4\,dx+4\,c)}+72\,a^2be^{(4\,dx+4\,c)}+144\,ab^2e^{(4\,dx+4\,c)}+8\,a^3e^{(2\,dx+2\,c)}+24\,a^2be^{(2\,dx+2\,c)}+a^3\right)e^{(-4\,dx-4\,c)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*e^(4*d*x + 4*c) + 8*a^3*e^(2*d*x + 2*c) + 24*a^2*b*e^(2*d*x + 2*c
) + 24*(a^3 + 4*a^2*b + 8*a*b^2)*(d*x + c) - 128*b^3/(e^(2*d*x + 2*c) + 1)
- (18*a^3*e^(4*d*x + 4*c) + 72*a^2*b*e^(4*d*x + 4*c) + 144*a*b^2*e^(4*d*x +
 4*c) + 8*a^3*e^(2*d*x + 2*c) + 24*a^2*b*e^(2*d*x + 2*c) + a^3)*e^(-4*d*x -
 4*c))/d
```

Mupad [B]
time = 1.53, size = 117, normalized size = 1.39

$$\frac{3\,a\,x\,\left(a^2+4\,a\,b+8\,b^2\right)}{8}-\frac{2\,b^3}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{a^3\,{\mathrm{e}}^{-4\,c-4\,d\,x}}{64\,d}+\frac{a^3\,{\mathrm{e}}^{4\,c+4\,d\,x}}{64\,d}-\frac{a^2\,{\mathrm{e}}^{-2\,c-2\,d\,x}\,\left(a+3\,b\right)}{8\,d}+\frac{a^2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+3\,b\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (3*a*x*(4*a*b + a^2 + 8*b^2))/8 - (2*b^3)/(d*(exp(2*c + 2*d*x) + 1)) - (a^3
*exp(- 4*c - 4*d*x))/(64*d) + (a^3*exp(4*c + 4*d*x))/(64*d) - (a^2*exp(- 2*
c - 2*d*x)*(a + 3*b))/(8*d) + (a^2*exp(2*c + 2*d*x)*(a + 3*b))/(8*d)
```

3.66 $\int \cosh^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=81

$$\frac{b^2(6a+b)\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{a^3\sinh^3(c+dx)}{3d}+\frac{b^3\text{sech}(c+dx)\tanh(c+dx)}{2d}$$

```
[Out] 1/2*b^2*(6*a+b)*arctan(sinh(d*x+c))/d+a^2*(a+3*b)*sinh(d*x+c)/d+1/3*a^3*sin
h(d*x+c)^3/d+1/2*b^3*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4232, 398, 393, 209}

$$\frac{a^3\sinh^3(c+dx)}{3d}+\frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{b^2(6a+b)\text{ArcTan}(\sinh(c+dx))}{2d}+\frac{b^3\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (b^2*(6*a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + (a^2*(a + 3*b)*Sinh[c + d*x])
/d + (a^3*Sinh[c + d*x]^3)/(3*d) + (b^3*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx=\frac{\operatorname{Subst}\left(\int\frac{\left(a+b+ax^2\right)^3}{\left(1+x^2\right)^2}\,dx,x,\sinh(c+dx)\right)}{d}$$

$$=\frac{\operatorname{Subst}\left(\int\left(a^2(a+3b)+a^3x^2+\frac{b^2(3a+b)+3ab^2x^2}{\left(1+x^2\right)^2}\right)\,dx,x,\sinh(c+d$$

$$=\frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{a^3\sinh^3(c+dx)}{3d}+\frac{\operatorname{Subst}\left(\int\frac{b^2(3a+b)+}{(1+x^2}\right.}{}$$

$$=\frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{a^3\sinh^3(c+dx)}{3d}+\frac{b^3\operatorname{sech}(c+dx)\tan}{2d}$$

$$=\frac{b^2(6a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a^2(a+3b)\sinh(c+dx)}{d}+\frac{a^3}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.67, size = 483, normalized size = 5.96

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Coth[c + d*x]^3*Csch[c + d*x]^2*(a*Cosh[c + d*x] + b*Sech[c + d*x])^3*(-25
6*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*S
inh[c + d*x]^8*(a + b + a*Sinh[c + d*x]^2)^3 - (315*ArcTanh[Sqrt[-Sinh[c +
d*x]^2]]*(b^3*(2401 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 - 47*Sinh[
c + d*x]^6) + 3*a^2*b*Cosh[c + d*x]^4*(2401 + 1875*Sinh[c + d*x]^2 + 243*Si
nh[c + d*x]^4 + Sinh[c + d*x]^6) + a^3*Cosh[c + d*x]^6*(2401 + 1875*Sinh[c
+ d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + 3*a*b^2*(2401 + 4276*Si
nh[c + d*x]^2 + 2118*Sinh[c + d*x]^4 + 148*Sinh[c + d*x]^6 + Sinh[c + d*x]^
8)))/Sqrt[-Sinh[c + d*x]^2] + 21*(b^3*(36015 + 16120*Sinh[c + d*x]^2 + 1473
*Sinh[c + d*x]^4) + 3*a*b^2*(36015 + 52135*Sinh[c + d*x]^2 + 17593*Sinh[c +
 d*x]^4 + 753*Sinh[c + d*x]^6) + 3*a^2*b*(36015 + 88150*Sinh[c + d*x]^2 + 6
9728*Sinh[c + d*x]^4 + 19786*Sinh[c + d*x]^6 + 753*Sinh[c + d*x]^8) + a^3*(
```

```
36015 + 124165*Sinh[c + d*x]^2 + 157878*Sinh[c + d*x]^4 + 89514*Sinh[c + d*
x]^6 + 19579*Sinh[c + d*x]^8 + 753*Sinh[c + d*x]^10))))/(3780*d*(a + 2*b +
a*Cosh[2*c + 2*d*x])^3)
```

Maple [C] Result contains complex when optimal does not.
time = 1.91, size = 215, normalized size = 2.65

method	result
risch	$\frac{e^{3dx+3c}a^3}{24d} + \frac{3\,e^{dx+c}a^3}{8d} + \frac{3\,e^{dx+c}a^2b}{2d} - \frac{3\,e^{-dx-c}a^3}{8d} - \frac{3\,e^{-dx-c}a^2b}{2d} - \frac{e^{-3dx-3c}a^3}{24d} + \frac{b^3e^{dx+c}\left(e^{2dx+2c}-1\right)}{d\left(1+e^{2dx+2c}\right)^2} + \frac{3ib^2\ln(e^{dx}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*exp(3*d*x+3*c)*a^3+3/8/d*exp(d*x+c)*a^3+3/2/d*exp(d*x+c)*a^2*b-3/8/d
*exp(-d*x-c)*a^3-3/2/d*exp(-d*x-c)*a^2*b-1/24/d*exp(-3*d*x-3*c)*a^3+b^3*exp
(d*x+c)*(exp(2*d*x+2*c)-1)/d/(1+exp(2*d*x+2*c))^2+3*I*b^2/d*ln(exp(d*x+c)+I
)*a+1/2*I*b^3/d*ln(exp(d*x+c)+I)-3*I*b^2/d*ln(exp(d*x+c)-I)*a-1/2*I*b^3/d*l
n(exp(d*x+c)-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(75) = 150.
time = 0.52, size = 179, normalized size = 2.21

$$-b^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) + \frac{1}{24}\,a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} + \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} - \frac{e^{(-3\,dx-3\,c)}}{d}\right) + \frac{3}{2}\,a^2b\left(\frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right) - \frac{6\,ab^2\arctan\left(e^{(-dx-c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -b^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-
2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/24*a^3*(e^(3*d*x + 3*c)/d + 9*e^
(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 3/2*a^2*b*(e^(d*x +
c)/d - e^(-d*x - c)/d) - 6*a*b^2*arctan(e^(-d*x - c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1409 vs. 2(75) = 150.
time = 0.42, size = 1409, normalized size = 17.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(a^3*cosh(d*x + c)^10 + 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + a^3*sin
h(d*x + c)^10 + (11*a^3 + 36*a^2*b)*cosh(d*x + c)^8 + (45*a^3*cosh(d*x + c)
^2 + 11*a^3 + 36*a^2*b)*sinh(d*x + c)^8 + 8*(15*a^3*cosh(d*x + c)^3 + (11*a
```

```
^3 + 36*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(5*a^3 + 18*a^2*b + 12*b^
3)*cosh(d*x + c)^6 + 2*(105*a^3*cosh(d*x + c)^4 + 5*a^3 + 18*a^2*b + 12*b^3
 + 14*(11*a^3 + 36*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*a^3*cosh
(d*x + c)^5 + 14*(11*a^3 + 36*a^2*b)*cosh(d*x + c)^3 + 3*(5*a^3 + 18*a^2*b
+ 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(5*a^3 + 18*a^2*b + 12*b^3)*co
sh(d*x + c)^4 + 2*(105*a^3*cosh(d*x + c)^6 + 35*(11*a^3 + 36*a^2*b)*cosh(d*
x + c)^4 - 5*a^3 - 18*a^2*b - 12*b^3 + 15*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*a^3*cosh(d*x + c)^7 + 7*(11*a^3 + 36*a^
2*b)*cosh(d*x + c)^5 + 5*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c)^3 - (5*a
^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - (11*a^3 + 36
*a^2*b)*cosh(d*x + c)^2 + (45*a^3*cosh(d*x + c)^8 + 28*(11*a^3 + 36*a^2*b)*
cosh(d*x + c)^6 + 30*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c)^4 - 11*a^3 -
 36*a^2*b - 12*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + 24*((6*a*b^2 + b^3)*cosh(d*x + c)^7 + 7*(6*a*b^2 + b^3)*cosh(d*x + c)*si
nh(d*x + c)^6 + (6*a*b^2 + b^3)*sinh(d*x + c)^7 + 2*(6*a*b^2 + b^3)*cosh(d*
x + c)^5 + (12*a*b^2 + 2*b^3 + 21*(6*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^5 + 5*(7*(6*a*b^2 + b^3)*cosh(d*x + c)^3 + 2*(6*a*b^2 + b^3)*cosh(d*x
 + c))*sinh(d*x + c)^4 + (6*a*b^2 + b^3)*cosh(d*x + c)^3 + (35*(6*a*b^2 + b
^3)*cosh(d*x + c)^4 + 6*a*b^2 + b^3 + 20*(6*a*b^2 + b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^3 + (21*(6*a*b^2 + b^3)*cosh(d*x + c)^5 + 20*(6*a*b^2 + b^3)*c
osh(d*x + c)^3 + 3*(6*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(6*a
*b^2 + b^3)*cosh(d*x + c)^6 + 10*(6*a*b^2 + b^3)*cosh(d*x + c)^4 + 3*(6*a*b
^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x +
 c)) + 2*(5*a^3*cosh(d*x + c)^9 + 4*(11*a^3 + 36*a^2*b)*cosh(d*x + c)^7 + 6
*(5*a^3 + 18*a^2*b + 12*b^3)*cosh(d*x + c)^5 - 4*(5*a^3 + 18*a^2*b + 12*b^3
)*cosh(d*x + c)^3 - (11*a^3 + 36*a^2*b)*cosh(d*x + c))*sinh(d*x + c))/(d*co
sh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 2*d
*cosh(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^5 + 5*(7*d*co
sh(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (3
5*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (21*d*cos
h(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 +
(7*d*cosh(d*x + c)^6 + 10*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x
 + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(75) = 150.

time = 0.42, size = 163, normalized size = 2.01

$$\frac{a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+12\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)+36\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+\frac{24\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4}+6\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(6\,ab^2+b^3)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*(e^(d*x + c) - e^(-d*x - c))^3 + 12*a^3*(e^(d*x + c) - e^(-d*x -
c)) + 36*a^2*b*(e^(d*x + c) - e^(-d*x - c)) + 24*b^3*(e^(d*x + c) - e^(-d*x
 - c))/((e^(d*x + c) - e^(-d*x - c))^2 + 4) + 6*(pi + 2*arctan(1/2*(e^(2*d*
x + 2*c) - 1)*e^(-d*x - c)))*(6*a*b^2 + b^3))/d
```

Mupad [B]
time = 0.22, size = 218, normalized size = 2.69

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(b^3\,\sqrt{d^2}+6\,a\,b^2\,\sqrt{d^2}\right)}{d\,\sqrt{36\,a^2\,b^4+12\,a\,b^5+b^6}}\right)\,\sqrt{36\,a^2\,b^4+12\,a\,b^5+b^6}}{\sqrt{d^2}}-\frac{a^3\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}+\frac{a^3\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}-\frac{3\,a^2\,\mathrm{e}^{-c-d\,x}\,(a+4\,b)}{8\,d}+\frac{3\,a^2\,\mathrm{e}^{c+d\,x}\,(a+4\,b)}{8\,d}+\frac{b^3\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,b^3\,\mathrm{e}^{c+d\,x}}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(b^3*(d^2)^(1/2) + 6*a*b^2*(d^2)^(1/2)))/(d*(12*a*b^
5 + b^6 + 36*a^2*b^4)^(1/2)))*(12*a*b^5 + b^6 + 36*a^2*b^4)^(1/2))/(d^2)^(1
/2) - (a^3*exp(- 3*c - 3*d*x))/(24*d) + (a^3*exp(3*c + 3*d*x))/(24*d) - (3*
a^2*exp(- c - d*x)*(a + 4*b))/(8*d) + (3*a^2*exp(c + d*x)*(a + 4*b))/(8*d)
+ (b^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (2*b^3*exp(c + d*x))/(d*(
2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.67 $\int \cosh^2(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=72

$$\frac{1}{2}a^2(a+6b)x+\frac{a^3\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\tanh^3(c+dx)}{3d}$$

```
[Out] 1/2*a^2*(a+6*b)*x+1/2*a^3*cosh(d*x+c)*sinh(d*x+c)/d+b^2*(3*a+b)*tanh(d*x+c)
/d-1/3*b^3*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4231, 398, 393, 212}

$$\frac{a^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{1}{2}a^2x(a+6b)+\frac{b^2(3a+b)\tanh(c+dx)}{d}-\frac{b^3\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (a^2*(a + 6*b)*x)/2 + (a^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (b^2*(3*a +
 b)*Tanh[c + d*x])/d - (b^3*Tanh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^3}{(1-x^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(b^2(3a+b)-b^3x^2+\frac{a^2(a+3b)-3a^2bx^2}{(1-x^2)^2}\right)\,dx, x, \tanh(c+\right.}{d}$$

$$= \frac{b^2(3a+b)\tanh(c+dx)}{d} - \frac{b^3\tanh^3(c+dx)}{3d} + \frac{\text{Subst}\left(\int \frac{a^2(a+3}{(1-}\right.}{}$$

$$= \frac{a^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2(3a+b)\tanh(c+dx)}{d} - \frac{b^3\,\text{t}}{}$$

$$= \frac{1}{2}a^2(a+6b)x + \frac{a^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2(3a+b)\tan}{d}$$

Mathematica [A]
time = 0.36, size = 64, normalized size = 0.89

$$\frac{6a^2(a+6b)(c+dx)+3a^3\sinh(2(c+dx))+4b^2\left(9a+2b+b\text{sech}^2(c+dx)\right)\tanh(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (6*a^2*(a + 6*b)*(c + d*x) + 3*a^3*Sinh[2*(c + d*x)] + 4*b^2*(9*a + 2*b + b
*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)
```

Maple [A]
time = 2.14, size = 113, normalized size = 1.57

method	result	size
risch	$\frac{a^3x}{2}+3a^2bx+\frac{a^3\mathrm{e}^{2dx+2c}}{8d}-\frac{a^3\mathrm{e}^{-2dx-2c}}{8d}-\frac{2b^2\left(9a\,\mathrm{e}^{4dx+4c}+18a\,\mathrm{e}^{2dx+2c}+6b\,\mathrm{e}^{2dx+2c}+9a+2b\right)}{3d\left(1+\mathrm{e}^{2dx+2c}\right)^3}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^3*x+3*a^2*b*x+1/8*a^3/d*exp(2*d*x+2*c)-1/8*a^3/d*exp(-2*d*x-2*c)-2/3*
b^2*(9*a*exp(4*d*x+4*c)+18*a*exp(2*d*x+2*c)+6*b*exp(2*d*x+2*c)+9*a+2*b)/d/(
1+exp(2*d*x+2*c))^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(66) = 132$.
time = 0.29, size = 160, normalized size = 2.22

$$\frac{1}{8}a^3\left(4x + \frac{e^{(2dx+2c)}}{d} - \frac{e^{(-2dx-2c)}}{d}\right) + 3a^2bx + \frac{4}{3}b^3\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} + \frac{1}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}\right) + \frac{6ab^2}{d(e^{(-2dx-2c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^3*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + 3*a^2*b*x + 4/3*b^
3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d
*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x
 - 6*c) + 1))) + 6*a*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(66) = 132$.
time = 0.36, size = 270, normalized size = 3.75

$$\frac{3a^3\sinh(dx+c)^5 - 4(18ab^2 + 4b^3 - 3(a^3 + 6a^2b)dx)\cosh(dx+c)^3 - 12(18ab^2 + 4b^3 - 3(a^3 + 6a^2b)dx)\cosh(dx+c)\sinh(dx+c)^2 + (30a^3\cosh(dx+c)^2 + 9a^3 + 72ab^2 + 16b^3)\sinh(dx+c)^3 - 12(18ab^2 + 4b^3 - 3(a^3 + 6a^2b)dx)\cosh(dx+c) + 3(5a^3\cosh(dx+c)^4 + 2a^3 + 24ab^2 + 16b^3 + (9a^3 + 72ab^2 + 16b^3)\cosh(dx+c)^2)\sinh(dx+c)}{24(d\cosh(dx+c)^3 + 3d\cosh(dx+c)\sinh(dx+c)^2 + 3d\cosh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*a^3*sinh(d*x + c)^5 - 4*(18*a*b^2 + 4*b^3 - 3*(a^3 + 6*a^2*b)*d*x)*
cosh(d*x + c)^3 - 12*(18*a*b^2 + 4*b^3 - 3*(a^3 + 6*a^2*b)*d*x)*cosh(d*x +
c)*sinh(d*x + c)^2 + (30*a^3*cosh(d*x + c)^2 + 9*a^3 + 72*a*b^2 + 16*b^3)*s
inh(d*x + c)^3 - 12*(18*a*b^2 + 4*b^3 - 3*(a^3 + 6*a^2*b)*d*x)*cosh(d*x + c
) + 3*(5*a^3*cosh(d*x + c)^4 + 2*a^3 + 24*a*b^2 + 16*b^3 + (9*a^3 + 72*a*b^
2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d
*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(66) = 132.
time = 0.42, size = 152, normalized size = 2.11

$$\frac{3\,a^3e^{(2\,dx+2\,c)}+12\,(a^3+6\,a^2b)(dx+c)-3\,\left(2\,a^3e^{(2\,dx+2\,c)}+12\,a^2be^{(2\,dx+2\,c)}+a^3\right)e^{(-2\,dx-2\,c)}-\frac{16\,\left(9\,ab^2e^{(4\,dx+4\,c)}+18\,ab^2e^{(2\,dx+2\,c)}+6\,b^3e^{(2\,dx+2\,c)}+9\,ab^2+2\,b^3\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^3*e^(2*d*x + 2*c) + 12*(a^3 + 6*a^2*b)*(d*x + c) - 3*(2*a^3*e^(2*
d*x + 2*c) + 12*a^2*b*e^(2*d*x + 2*c) + a^3)*e^(-2*d*x - 2*c) - 16*(9*a*b^2
*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*d*x + 2*c) + 6*b^3*e^(2*d*x + 2*c) + 9*a*b
^2 + 2*b^3)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B]
time = 0.17, size = 221, normalized size = 3.07

$$\frac{a^2\,x\,(a+6\,b)}{2}-\frac{\frac{2\,a\,b^2}{d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d}+\frac{2\,a\,b^2\,\mathrm{e}^{4\,c+4\,d\,x}}{d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}-\frac{\frac{2\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d}+\frac{2\,a\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}-\frac{a^3\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d}+\frac{a^3\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d}-\frac{2\,a\,b^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (a^2*x*(a + 6*b))/2 - ((2*a*b^2)/d + (4*exp(2*c + 2*d*x)*(3*a*b^2 + 2*b^3))
/(3*d) + (2*a*b^2*exp(4*c + 4*d*x))/d)/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*
d*x) + exp(6*c + 6*d*x) + 1) - ((2*(3*a*b^2 + 2*b^3))/(3*d) + (2*a*b^2*exp(
2*c + 2*d*x))/d)/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (a^3*exp(- 2
*c - 2*d*x))/(8*d) + (a^3*exp(2*c + 2*d*x))/(8*d) - (2*a*b^2)/(d*(exp(2*c +
 2*d*x) + 1))
```

3.68 $\int \cosh(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=93

$$\frac{3b(8a^2+4ab+b^2)\,\text{ArcTan}(\sinh(c+dx))}{8d}+\frac{a^3\sinh(c+dx)}{d}+\frac{3b^2(4a+b)\text{sech}(c+dx)\tanh(c+dx)}{8d}+\frac{b^3\text{sech}^3(}{}$$

[Out] 3/8*b*(8*a^2+4*a*b+b^2)*arctan(sinh(d*x+c))/d+a^3*sinh(d*x+c)/d+3/8*b^2*(4*a+b)*sech(d*x+c)*tanh(d*x+c)/d+1/4*b^3*sech(d*x+c)^3*tanh(d*x+c)/d

Rubi [A]
time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4232, 398, 1171, 393, 209}

$$\frac{a^3\sinh(c+dx)}{d}+\frac{3b(8a^2+4ab+b^2)\,\text{ArcTan}(\sinh(c+dx))}{8d}+\frac{3b^2(4a+b)\tanh(c+dx)\text{sech}(c+dx)}{8d}+\frac{b^3\tanh(c+dx)\text{sech}^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]

[Out] (3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (a^3*Sinh[c + d*x])/d + (3*b^2*(4*a + b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) + (b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a+b+ax^2)^3}{(1+x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(a^3+\frac{b(3a^2+3ab+b^2)+3ab(2a+b)x^2+3a^2bx^4}{(1+x^2)^3}\right)\,dx, x, \sinh(c+ \cdots\right.}{d}$$

$$= \frac{a^3\sinh(c+dx)}{d}+\frac{\operatorname{Subst}\left(\int \frac{b(3a^2+3ab+b^2)+3ab(2a+b)x^2+3a^2bx^4}{(1+x^2)^3}\,dx, x, \cdots\right.}{d}$$

$$= \frac{a^3\sinh(c+dx)}{d}+\frac{b^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}-\frac{\operatorname{Subst}\left(\int \frac{-3\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{a^3\sinh(c+dx)}{d}+\frac{3b^2(4a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d}+\frac{b^3\operatorname{se}\cdots}{\cdots}$$

$$= \frac{3b(8a^2+4ab+b^2)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{a^3\sinh(c+dx)}{d}+\frac{3b\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 7.46, size = 575, normalized size = 6.18

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/7560*(Cosh[c + d*x]*Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^3*(256*Hyper
geometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Si
nh[c + d*x]^8*(a + b + a*Sinh[c + d*x]^2)^3 + 384*HypergeometricPFQ[{3/2, 2
, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b + a*S
inh[c + d*x]^2)^2*(7*b + a*(7 + 5*Sinh[c + d*x]^2)) + (315*ArcTanh[Sqrt[-Si
nh[c + d*x]^2]]*(b^3*(16807 + 15000*Sinh[c + d*x]^2 + 2187*Sinh[c + d*x]^4
- 62*Sinh[c + d*x]^6) + a^3*Cosh[c + d*x]^4*(16807 + 24604*Sinh[c + d*x]^2
+ 11562*Sinh[c + d*x]^4 + 1468*Sinh[c + d*x]^6 + 7*Sinh[c + d*x]^8) + 3*a*b
^2*(16807 + 29406*Sinh[c + d*x]^2 + 15312*Sinh[c + d*x]^4 + 1858*Sinh[c + d
*x]^6 + 9*Sinh[c + d*x]^8) + 3*a^2*b*(16807 + 43812*Sinh[c + d*x]^2 + 40442
*Sinh[c + d*x]^4 + 14956*Sinh[c + d*x]^6 + 1719*Sinh[c + d*x]^8 + 8*Sinh[c
+ d*x]^10)))/Sqrt[-Sinh[c + d*x]^2] - 21*(b^3*(252105 + 140965*Sinh[c + d*x
]^2 + 8226*Sinh[c + d*x]^4) + 3*a*b^2*(252105 + 357055*Sinh[c + d*x]^2 + 13
3071*Sinh[c + d*x]^4 + 6393*Sinh[c + d*x]^6) + 3*a^2*b*(252105 + 573145*Sin
h[c + d*x]^2 + 437991*Sinh[c + d*x]^4 + 120431*Sinh[c + d*x]^6 + 5640*Sinh[
c + d*x]^8) + a^3*(252105 + 789235*Sinh[c + d*x]^2 + 922986*Sinh[c + d*x]^4
 + 491574*Sinh[c + d*x]^6 + 107725*Sinh[c + d*x]^8 + 4887*Sinh[c + d*x]^10)
)))/(d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)
```

Maple [C] Result contains complex when optimal does not.
time = 2.30, size = 257, normalized size = 2.76

method	result
risch	$\frac{e^{dx+c}a^3}{2d} - \frac{e^{-dx-c}a^3}{2d} + \frac{b^2 e^{dx+c}\left(12a\, e^{6dx+6c}+3b\, e^{6dx+6c}+12a\, e^{4dx+4c}+11b\, e^{4dx+4c}-12a\, e^{2dx+2c}-11b\, e^{2dx+2c}-12a-3b\right)}{4d\left(1+e^{2dx+2c}\right)^4} + \frac{3ib}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*exp(d*x+c)*a^3-1/2/d*exp(-d*x-c)*a^3+1/4*b^2*exp(d*x+c)*(12*a*exp(6*d
*x+6*c)+3*b*exp(6*d*x+6*c)+12*a*exp(4*d*x+4*c)+11*b*exp(4*d*x+4*c)-12*a*exp
(2*d*x+2*c)-11*b*exp(2*d*x+2*c)-12*a-3*b)/d/(1+exp(2*d*x+2*c))^4+3*I*b/d*ln
(exp(d*x+c)+I)*a^2+3/2*I*b^2/d*ln(exp(d*x+c)+I)*a+3/8*I*b^3/d*ln(exp(d*x+c)
+I)-3*I*b/d*ln(exp(d*x+c)-I)*a^2-3/2*I*b^2/d*ln(exp(d*x+c)-I)*a-3/8*I*b^3/d
*ln(exp(d*x+c)-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(87) = 174$.
time = 0.53, size = 221, normalized size = 2.38

$$-\frac{1}{4}\,b^3\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{3\,e^{(-dx-c)} + 11\,e^{(-3\,dx-3\,c)} - 11\,e^{(-5\,dx-5\,c)} - 3\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)} + 6\,e^{(-4\,dx-4\,c)} + 4\,e^{(-6\,dx-6\,c)} + e^{(-8\,dx-8\,c)} + 1)}\right) - 3\,ab^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) - \frac{6\,a^2b\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{a^3\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^3*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c)
- 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-
4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 3*a*b^2*(arct
an(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*
c) + e^(-4*d*x - 4*c) + 1))) - 6*a^2*b*arctan(e^(-d*x - c))/d + a^3*sinh(d*
x + c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1992 vs. $2(87) = 174$.
time = 0.39, size = 1992, normalized size = 21.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^3*cosh(d*x + c)^10 + 20*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + 2*a^3*
sinh(d*x + c)^10 + 3*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^8 + 3*(30*a^3*co
sh(d*x + c)^2 + 2*a^3 + 4*a*b^2 + b^3)*sinh(d*x + c)^8 + 24*(10*a^3*cosh(d*
x + c)^3 + (2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (4*a^3
+ 12*a*b^2 + 11*b^3)*cosh(d*x + c)^6 + (420*a^3*cosh(d*x + c)^4 + 4*a^3 + 1
2*a*b^2 + 11*b^3 + 84*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^6 + 6*(84*a^3*cosh(d*x + c)^5 + 28*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^
3 + (4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - (4*a^3 + 1
2*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + (420*a^3*cosh(d*x + c)^6 + 210*(2*a^3 +
 4*a*b^2 + b^3)*cosh(d*x + c)^4 - 4*a^3 - 12*a*b^2 - 11*b^3 + 15*(4*a^3 + 1
2*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(60*a^3*cosh(d*x + c
)^7 + 42*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(4*a^3 + 12*a*b^2 + 11
*b^3)*cosh(d*x + c)^3 - (4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^3 - 2*a^3 - 3*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^2 + 3*(30*a^3*cos
h(d*x + c)^8 + 28*(2*a^3 + 4*a*b^2 + b^3)*cosh(d*x + c)^6 + 5*(4*a^3 + 12*a
*b^2 + 11*b^3)*cosh(d*x + c)^4 - 2*a^3 - 4*a*b^2 - b^3 - 2*(4*a^3 + 12*a*b^
2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((8*a^2*b + 4*a*b^2 + b^3)
*cosh(d*x + c)^9 + 9*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^
8 + (8*a^2*b + 4*a*b^2 + b^3)*sinh(d*x + c)^9 + 4*(8*a^2*b + 4*a*b^2 + b^3)
*cosh(d*x + c)^7 + 4*(8*a^2*b + 4*a*b^2 + b^3 + 9*(8*a^2*b + 4*a*b^2 + b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 28*(3*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*
x + c)^3 + (8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 6*(8*
a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^5 + 6*(21*(8*a^2*b + 4*a*b^2 + b^3)*co
sh(d*x + c)^4 + 8*a^2*b + 4*a*b^2 + b^3 + 14*(8*a^2*b + 4*a*b^2 + b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c
)^5 + 70*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^3 + 15*(8*a^2*b + 4*a*b^2
+ b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*
x + c)^3 + 4*(21*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(8*a^2*b +
4*a*b^2 + b^3)*cosh(d*x + c)^4 + 8*a^2*b + 4*a*b^2 + b^3 + 15*(8*a^2*b + 4*
```

```
a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(3*(8*a^2*b + 4*a*b^2 +
b^3)*cosh(d*x + c)^7 + 7*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(8*a
^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^3 + (8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x
+ c))*sinh(d*x + c)^2 + (8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c) + (9*(8*a^2
*b + 4*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x
 + c)^6 + 30*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^4 + 8*a^2*b + 4*a*b^2
+ b^3 + 12*(8*a^2*b + 4*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*arctan
(cosh(d*x + c) + sinh(d*x + c)) + 2*(10*a^3*cosh(d*x + c)^9 + 12*(2*a^3 + 4
*a*b^2 + b^3)*cosh(d*x + c)^7 + 3*(4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c)
^5 - 2*(4*a^3 + 12*a*b^2 + 11*b^3)*cosh(d*x + c)^3 - 3*(2*a^3 + 4*a*b^2 + b
^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*si
nh(d*x + c)^8 + d*sinh(d*x + c)^9 + 4*d*cosh(d*x + c)^7 + 4*(9*d*cosh(d*x +
 c)^2 + d)*sinh(d*x + c)^7 + 28*(3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sin
h(d*x + c)^6 + 6*d*cosh(d*x + c)^5 + 6*(21*d*cosh(d*x + c)^4 + 14*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^5 + 2*(63*d*cosh(d*x + c)^5 + 70*d*cosh(d*x + c
)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^3 + 4*(21*d*c
osh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c)^3 + 12*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^
3 + d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (9*d*cosh(d*x + c)
^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + d
)*sinh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^3 \cosh(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*cosh(c + d*x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(87) = 174.
time = 0.43, size = 199, normalized size = 2.14

$$\frac{8\,a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 3\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(8\,a^2b + 4\,ab^2 + b^3\right) + \frac{4\left(12\,ab^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 3\,b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 48\,ab^2\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 20\,b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(8*a^3*(e^(d*x + c) - e^(-d*x - c)) + 3*(pi + 2*arctan(1/2*(e^(2*d*x +
 2*c) - 1)*e^(-d*x - c)))*(8*a^2*b + 4*a*b^2 + b^3) + 4*(12*a*b^2*(e^(d*x +
 c) - e^(-d*x - c))^3 + 3*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 48*a*b^2*(e^
```

```
(d*x + c) - e^(-d*x - c)) + 20*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x +
 c) - e^(-d*x - c))^2 + 4)^2)/d
```

Mupad [B]
time = 0.19, size = 344, normalized size = 3.70

$$\frac{a^3\,\mathrm{e}^{c+d\,x}}{2\,d}-\frac{a^3\,\mathrm{e}^{-c-d\,x}}{2\,d}+\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(b^3\,\sqrt{d^2}+4\,a\,b^2\,\sqrt{d^2}+8\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{64\,a^4\,b^2+64\,a^3\,b^3+32\,a^2\,b^4+8\,a\,b^5+b^6}}\right)\,\sqrt{64\,a^4\,b^2+64\,a^3\,b^3+32\,a^2\,b^4+8\,a\,b^5+b^6}}{4\,\sqrt{d^2}}-\frac{6\,b^3\,\mathrm{e}^{c+d\,x}}{d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}-\frac{\mathrm{e}^{c+d\,x}\,\left(12\,a\,b^2-b^3\right)}{2\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}+\frac{4\,b^3\,\mathrm{e}^{c+d\,x}}{d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}+\frac{3\,\mathrm{e}^{c+d\,x}\,\left(b^3+4\,a\,b^2\right)}{4\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (a^3*exp(c + d*x))/(2*d) - (a^3*exp(- c - d*x))/(2*d) + (3*atan((exp(d*x)*e
xp(c)*(b^3*(d^2)^(1/2) + 4*a*b^2*(d^2)^(1/2) + 8*a^2*b*(d^2)^(1/2)))/(d*(8*
a*b^5 + b^6 + 32*a^2*b^4 + 64*a^3*b^3 + 64*a^4*b^2)^(1/2)))*(8*a*b^5 + b^6
+ 32*a^2*b^4 + 64*a^3*b^3 + 64*a^4*b^2)^(1/2))/(4*(d^2)^(1/2)) - (6*b^3*exp
(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) +
 1)) - (exp(c + d*x)*(12*a*b^2 - b^3))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c +
 4*d*x) + 1)) + (4*b^3*exp(c + d*x))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4
*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (3*exp(c + d*x)*(4*a*
b^2 + b^3))/(4*d*(exp(2*c + 2*d*x) + 1))
```

3.69 $\int \textbf{sech}(c+dx)\left(a+b\textbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=147

$$\frac{(2a+b)\left(8a^2+8ab+5b^2\right)\text{ArcTan}(\sinh(c+dx))}{16d}+\frac{b\left(44a^2+44ab+15b^2\right)\text{sech}(c+dx)\tanh(c+dx)}{48d}+\frac{5b(2a-$$

```
[Out] 1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*arctan(sinh(d*x+c))/d+1/48*b*(44*a^2+44*a*
b+15*b^2)*sech(d*x+c)*tanh(d*x+c)/d+5/24*b*(2*a+b)*sech(d*x+c)^3*(a+b+a*sin
h(d*x+c)^2)*tanh(d*x+c)/d+1/6*b*sech(d*x+c)^5*(a+b+a*sinh(d*x+c)^2)^2*tanh(
d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4232, 424, 540, 393, 209}

$$\frac{(2a+b)\left(8a^2+8ab+5b^2\right)\text{ArcTan}(\sinh(c+dx))}{16d}+\frac{b\left(44a^2+44ab+15b^2\right)\tanh(c+dx)\text{sech}(c+dx)}{48d}+\frac{b\tanh(c+dx)\text{sech}^5(c+dx)\left(a\sinh^2(c+dx)+a+b\right)^2}{6d}+\frac{5b(2a+b)\tanh(c+dx)\text{sech}^3(c+dx)\left(a\sinh^2(c+dx)+a+b\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + (b*(44*a
^2 + 44*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) + (5*b*(2*a + b)*
Sech[c + d*x]^3*(a + b + a*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) + (b*Sech
[c + d*x]^5*(a + b + a*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
```

```
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx = \frac{\operatorname{Subst}\left(\int \frac{(a+b+ax^2)^3}{(1+x^2)^4}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\operatorname{sech}^5(c+dx)\left(a+b+a\sinh^2(c+dx)\right)^2\tanh(c+dx)}{6d} + \frac{\text{Subs}}{}$$

$$= \frac{5b(2a+b)\operatorname{sech}^3(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx)}{24d}$$

$$= \frac{b(44a^2+44ab+15b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{48d} + \frac{5b(2a+b)\text{s}}{}$$

$$= \frac{(2a+b)(8a^2+8ab+5b^2)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{b(44a^2+44a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 9.05, size = 1430, normalized size = 9.73

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
[Out] (Coth[c + d*x]^6*Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3*(117228825*(a + b)
^3*Sinh[c + d*x]^2 + 274542345*a*(a + b)^2*Sinh[c + d*x]^4 + 70189350*(a +
b)^3*Sinh[c + d*x]^4 + 215549775*a^2*(a + b)*Sinh[c + d*x]^6 + 168951510*a*
(a + b)^2*Sinh[c + d*x]^6 + 4093425*(a + b)^3*Sinh[c + d*x]^6 + 58009455*a^
3*Sinh[c + d*x]^8 + 135323370*a^2*(a + b)*Sinh[c + d*x]^8 + 9514449*a*(a +
b)^2*Sinh[c + d*x]^8 + 36772890*a^3*Sinh[c + d*x]^10 + 7808535*a^2*(a + b)*
Sinh[c + d*x]^10 - 75520*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1,
 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^10 - 13824*(a + b)^3*Hypergeo
metricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[
c + d*x]^10 - 1024*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1,
 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^10 + 2160711*a^3*Sinh[c
 + d*x]^12 - 189696*a*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1,
 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12 - 38400*a*(a + b)^2*Hypergeom
etricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c
 + d*x]^12 - 3072*a*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1
, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12 - 158976*a^2*(a + b
)*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*S
inh[c + d*x]^14 - 35328*a^2*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2},
 {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^14 - 3072*a^2*(a + b)*
HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c +
 d*x]^2]*Sinh[c + d*x]^14 - 44800*a^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2},
{1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^16 - 10752*a^3*Hypergeomet
ricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c +
 d*x]^16 - 1024*a^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1,
 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^16 + 117228825*(a + b)^3*ArcTanh
[Sqrt[-Sinh[c + d*x]^2]]*Sqrt[-Sinh[c + d*x]^2] + 215549775*a^2*(a + b)*Arc
Tanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 26046
5625*a*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4*Sqrt[-Sinh
[c + d*x]^2] + 17069535*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c +
d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 58009455*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]
]*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 207173295*a^2*(a + b)*ArcTanh[Sq
rt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 41427855*a*(
a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x
]^2] + 56109375*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8*Sqrt[-S
inh[c + d*x]^2] + 33756345*a^2*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh
[c + d*x]^8*Sqrt[-Sinh[c + d*x]^2] + 210735*a*(a + b)^2*ArcTanh[Sqrt[-Sinh[
c + d*x]^2]]*Sinh[c + d*x]^8*Sqrt[-Sinh[c + d*x]^2] + 9261945*a^3*ArcTanh[S
qrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10*Sqrt[-Sinh[c + d*x]^2] + 174825*a^2
*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10*Sqrt[-Sinh[c + d*
x]^2] + 48825*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^12*Sqrt[-Si
nh[c + d*x]^2] - 274542345*a*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(-Si
nh[c + d*x]^2)^(3/2) - 109265625*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*
(-Sinh[c + d*x]^2)^(3/2) + 142065*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]
```

```
*Sinh[c + d*x]^4*(-Sinh[c + d*x]^2)^(3/2)))/(90720*d*(a + 2*b + a*Cosh[2*c
+ 2*d*x])^3)
```

Maple [C] Result contains complex when optimal does not.
time = 1.82, size = 403, normalized size = 2.74

method	result
risch	$\frac{b\,\mathrm{e}^{dx+c}\left(72a^{2}\mathrm{e}^{10dx+10c}+54ab\,\mathrm{e}^{10dx+10c}+15b^{2}\mathrm{e}^{10dx+10c}+216a^{2}\mathrm{e}^{8dx+8c}+306ab\,\mathrm{e}^{8dx+8c}+85b^{2}\mathrm{e}^{8dx+8c}+144a^{2}\mathrm{e}^{6dx+6c}+252ab\,\mathrm{e}^{6dx+}\right.}{24d\left(1+\mathrm{e}^{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*b*exp(d*x+c)*(72*a^2*exp(10*d*x+10*c)+54*a*b*exp(10*d*x+10*c)+15*b^2*e
xp(10*d*x+10*c)+216*a^2*exp(8*d*x+8*c)+306*a*b*exp(8*d*x+8*c)+85*b^2*exp(8*
d*x+8*c)+144*a^2*exp(6*d*x+6*c)+252*a*b*exp(6*d*x+6*c)+198*b^2*exp(6*d*x+6*
c)-144*a^2*exp(4*d*x+4*c)-252*a*b*exp(4*d*x+4*c)-198*b^2*exp(4*d*x+4*c)-216
*a^2*exp(2*d*x+2*c)-306*a*b*exp(2*d*x+2*c)-85*b^2*exp(2*d*x+2*c)-72*a^2-54*
a*b-15*b^2)/d/(1+exp(2*d*x+2*c))^6+I/d*ln(exp(d*x+c)+I)*a^3+3/2*I/d*ln(exp(
d*x+c)+I)*a^2*b+9/8*I/d*ln(exp(d*x+c)+I)*a*b^2+5/16*I/d*ln(exp(d*x+c)+I)*b^
3-I/d*ln(exp(d*x+c)-I)*a^3-3/2*I/d*ln(exp(d*x+c)-I)*a^2*b-9/8*I/d*ln(exp(d*
x+c)-I)*a*b^2-5/16*I/d*ln(exp(d*x+c)-I)*b^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(139) = 278.
time = 0.50, size = 365, normalized size = 2.48

$$-\frac{1}{24}\,b^{3}\left(\frac{15\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{15\,e^{(-dx-c)}+85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}-198\,e^{(-7\,dx-7\,c)}-85\,e^{(-9\,dx-9\,c)}-15\,e^{(-11\,dx-11\,c)}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}\right)-\frac{3}{4}\,ab^{2}\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}+11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}-3\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)-3\,a^{2}b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+\frac{a^{3}\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*b^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3*
c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) - 15
*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-
6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) - 3/4*a*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(
-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x -
 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) -
 3*a^2*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*
e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^3*arctan(sinh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3465 vs. 2(139) = 278.
time = 0.38, size = 3465, normalized size = 23.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/24*(3*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^11 + 33*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(24*a^2*b + 18*a*b^2 + 5*b^3)*sinh(d*x + c)^11 + (216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^9 + (216*a^2*b + 306*a*b^2 + 85*b^3 + 165*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 9*(55*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 + 18*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^7 + 18*(55*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 14*a*b^2 + 11*b^3 + 2*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 42*(33*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 2*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^3 + 3*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 18*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^5 + 18*(77*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 7*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^4 - 8*a^2*b - 14*a*b^2 - 11*b^3 + 21*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 18*(55*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 7*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^5 + 35*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^3 - 5*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - (216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^3 + (495*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 84*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^6 + 630*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^4 - 216*a^2*b - 306*a*b^2 - 85*b^3 - 180*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 3*(55*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 12*(216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c)^7 + 126*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^5 - 60*(8*a^2*b + 14*a*b^2 + 11*b^3)*cosh(d*x + c)^3 - (216*a^2*b + 306*a*b^2 + 85*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^12 + 12*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*sinh(d*x + c)^12 + 6*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^10 + 6*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 15*(33*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 18*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 24*(33*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 30*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 4*(231*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 315*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 80*a^3 + 120*a^2*b + 90*a*b^2 + 25*b^3 + 10

```
5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 +
 24*(33*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 63*(16*a^3
 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 35*(16*a^3 + 24*a^2*b + 1
8*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)
*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)
*cosh(d*x + c)^4 + 15*(33*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
 c)^8 + 84*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 70*(16*
a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b + 18
*a*b^2 + 5*b^3 + 20*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^4 + 20*(11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x +
 c)^9 + 36*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 42*(16*
a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 20*(16*a^3 + 24*a^2*b
+ 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b
^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 +
 6*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^2 + 6*(11*(16*a^3 +
 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^10 + 45*(16*a^3 + 24*a^2*b + 18
*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 70*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)
*cosh(d*x + c)^6 + 50*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^
4 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2
 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((16*a^3 + 24*a^2*b + 18*a*
b^2 + 5*b^3)*cosh(d*x + c)^11 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*co
sh(d*x + c)^9 + 10*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 +
 10*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 5*(16*a^3 + 24
*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (16*a^3 + 24*a^2*b + 18*a*b^2
+ 5*b^3)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)
) - 3*(24*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)^3\mathrm{sech}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(139) = 278.
time = 0.42, size = 310, normalized size = 2.11

$$\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)\left(16a^3+24a^2b+18ab^2+5b^3\right)+\frac{4\left(72a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+54ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+15b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+576a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+576ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+160b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+1152a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+1440ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+528b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^3}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(16*a^3 + 2
4*a^2*b + 18*a*b^2 + 5*b^3) + 4*(72*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5 +
54*a*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 15*b^3*(e^(d*x + c) - e^(-d*x - c
))^5 + 576*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 + 576*a*b^2*(e^(d*x + c) -
e^(-d*x - c))^3 + 160*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 1152*a^2*b*(e^(d
*x + c) - e^(-d*x - c)) + 1440*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 528*b^3
*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3)/d
```

Mupad [B]
time = 1.47, size = 535, normalized size = 3.64

$$\frac{\mathrm{atan}\left(\frac{e^{dx}\,e^{c}\left(16a^3\sqrt{d^2}+5b^3\sqrt{d^2}+18ab^2\sqrt{d^2}+24a^2b\sqrt{d^2}\right)}{d\sqrt{256a^6+768a^5b+1152a^4b^2+1024a^3b^3+564a^2b^4+180ab^5+25b^6}}\right)\sqrt{256a^6+768a^5b+1152a^4b^2+1024a^3b^3+564a^2b^4+180ab^5+25b^6}}{8\sqrt{d^2}}-\frac{e^{c+dx}\,(54ab^2-b^3)}{3d\,(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1)}+\frac{80b^3e^{c+dx}}{3d\,(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1)}+\frac{6e^{c+dx}\,(2ab^2-3b^3)}{d\,(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1)}-\frac{32b^3e^{c+dx}}{3d\,(6e^{2c+2dx}+15e^{4c+4dx}+20e^{6c+6dx}+15e^{8c+8dx}+6e^{10c+10dx}+e^{12c+12dx}+1)}+\frac{e^{c+dx}\,(24a^2b+18ab^2+5b^3)}{8d\,(e^{2c+2dx}+1)}+\frac{e^{c+dx}\,(-72a^2b+18ab^2+5b^3)}{12d\,(2e^{2c+2dx}+e^{4c+4dx}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/cosh(c + d*x),x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(16*a^3*(d^2)^(1/2) + 5*b^3*(d^2)^(1/2) + 18*a*b^2*(
d^2)^(1/2) + 24*a^2*b*(d^2)^(1/2)))/(d*(180*a*b^5 + 768*a^5*b + 256*a^6 + 2
5*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^(1/2)))*(180*a*b^5 + 768
*a^5*b + 256*a^6 + 25*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^(1/2
))/(8*(d^2)^(1/2)) - (exp(c + d*x)*(54*a*b^2 - b^3))/(3*d*(3*exp(2*c + 2*d*
x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (80*b^3*exp(c + d*x))/(3
*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(
8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (6*exp(c + d*x)*(2*a*b^2 - 3*b^3)
)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*
c + 8*d*x) + 1)) - (32*b^3*exp(c + d*x))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(
4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*
d*x) + exp(12*c + 12*d*x) + 1)) + (exp(c + d*x)*(18*a*b^2 + 24*a^2*b + 5*b^
3))/(8*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(18*a*b^2 - 72*a^2*b + 5*b
^3))/(12*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.70 $\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=74

$$\frac{(a+b)^3\tanh(c+dx)}{d}-\frac{b(a+b)^2\tanh^3(c+dx)}{d}+\frac{3b^2(a+b)\tanh^5(c+dx)}{5d}-\frac{b^3\tanh^7(c+dx)}{7d}$$

```
[Out] (a+b)^3*tanh(d*x+c)/d-b*(a+b)^2*tanh(d*x+c)^3/d+3/5*b^2*(a+b)*tanh(d*x+c)^5
/d-1/7*b^3*tanh(d*x+c)^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4231, 200}

$$\frac{3b^2(a+b)\tanh^5(c+dx)}{5d}-\frac{b(a+b)^2\tanh^3(c+dx)}{d}+\frac{(a+b)^3\tanh(c+dx)}{d}-\frac{b^3\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Tanh[c + d*x])/d - (b*(a + b)^2*Tanh[c + d*x]^3)/d + (3*b^2*(a +
 b)*Tanh[c + d*x]^5)/(5*d) - (b^3*Tanh[c + d*x]^7)/(7*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \left(a+b-bx^2\right)^3\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a^3\left(1+\frac{b\left(3a^2+3ab+b^2\right)}{a^3}\right)-3b(a+b)^2x^2+3b^2(a+b)x^4\right.\right.}{d}\\ &= \frac{(a+b)^3\tanh(c+dx)}{d}-\frac{b(a+b)^2\tanh^3(c+dx)}{d}+\frac{3b^2(a+b)}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 319 vs. 2(74) = 148.
time = 0.99, size = 319, normalized size = 4.31

$$\frac{\operatorname{sech}(c)\operatorname{sech}(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\left(140(5a^3+11a^2b+10ab^2+4b^3)\sinh(dx)-35a(15a^2+26ab+16b^2)\sinh(2c+dx)+525a^3\sinh(2c+3dx)+1260a^2b\sinh(2c+3dx)+1176ab^2\sinh(2c+3dx)+336b^3\sinh(2c+3dx)-210a^3\sinh(4c+3dx)-210a^2b\sinh(4c+3dx)+210a^3\sinh(4c+5dx)+490a^2b\sinh(4c+5dx)+392ab^2\sinh(4c+5dx)+112b^3\sinh(4c+5dx)-35a^3\sinh(6c+5dx)+35a^3\sinh(6c+7dx)+70a^2b\sinh(6c+7dx)+56ab^2\sinh(6c+7dx)+16b^3\sinh(6c+7dx)\right)}{280d(a+2b+a\cosh(2(c+dx)))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]*(a + b*Sech[c + d*x]^2)^3*(140*(5*a^3 + 11*a^2*b + 1
0*a*b^2 + 4*b^3)*Sinh[d*x] - 35*a*(15*a^2 + 26*a*b + 16*b^2)*Sinh[2*c + d*x
] + 525*a^3*Sinh[2*c + 3*d*x] + 1260*a^2*b*Sinh[2*c + 3*d*x] + 1176*a*b^2*S
inh[2*c + 3*d*x] + 336*b^3*Sinh[2*c + 3*d*x] - 210*a^3*Sinh[4*c + 3*d*x] -
210*a^2*b*Sinh[4*c + 3*d*x] + 210*a^3*Sinh[4*c + 5*d*x] + 490*a^2*b*Sinh[4*
c + 5*d*x] + 392*a*b^2*Sinh[4*c + 5*d*x] + 112*b^3*Sinh[4*c + 5*d*x] - 35*a
^3*Sinh[6*c + 5*d*x] + 35*a^3*Sinh[6*c + 7*d*x] + 70*a^2*b*Sinh[6*c + 7*d*x
] + 56*a*b^2*Sinh[6*c + 7*d*x] + 16*b^3*Sinh[6*c + 7*d*x]))/(280*d*(a + 2*b
 + a*Cosh[2*(c + d*x)])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(70) = 140.
time = 1.87, size = 303, normalized size = 4.09

method	result
risch	$-\frac{2\left(35a^3 e^{12dx+12c}+210a^3 e^{10dx+10c}+210a^2b\, e^{10dx+10c}+525a^3 e^{8dx+8c}+910a^2b\, e^{8dx+8c}+560a\, b^2 e^{8dx+8c}+700a^3 e^{6dx+6c}+1540a^2b\, e^{6\cdots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*(35*a^3*exp(12*d*x+12*c)+210*a^3*exp(10*d*x+10*c)+210*a^2*b*exp(10*d*
x+10*c)+525*a^3*exp(8*d*x+8*c)+910*a^2*b*exp(8*d*x+8*c)+560*a*b^2*exp(8*d*x
+8*c)+700*a^3*exp(6*d*x+6*c)+1540*a^2*b*exp(6*d*x+6*c)+1400*a*b^2*exp(6*d*x
+6*c)+560*b^3*exp(6*d*x+6*c)+525*a^3*exp(4*d*x+4*c)+1260*a^2*b*exp(4*d*x+4*
c)+1176*a*b^2*exp(4*d*x+4*c)+336*b^3*exp(4*d*x+4*c)+210*a^3*exp(2*d*x+2*c)+
490*a^2*b*exp(2*d*x+2*c)+392*a*b^2*exp(2*d*x+2*c)+112*b^3*exp(2*d*x+2*c)+35
*a^3+70*a^2*b+56*a*b^2+16*b^3)/d/(1+exp(2*d*x+2*c))^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 695 vs. 2(70) = 140.
time = 0.28, size = 695, normalized size = 9.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 32/35*b^3*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(
-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-
2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8
*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1
)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*
e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d
*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4
*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 1
0*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 16/5*a*b^2*(5*e^(
-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d*x - 4*c)
/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1))) + 4*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d
*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x
 - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2*a^3/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(70) = 140$.
time = 0.34, size = 816, normalized size = 11.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/35*((35*a^3 + 35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^6 - 6*(35*a^2*b
 + 28*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (35*a^3 + 35*a^2*b + 2
8*a*b^2 + 8*b^3)*sinh(d*x + c)^6 + 14*(15*a^3 + 25*a^2*b + 14*a*b^2 + 4*b^3
)*cosh(d*x + c)^4 + (210*a^3 + 350*a^2*b + 196*a*b^2 + 56*b^3 + 15*(35*a^3
+ 35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 4*(5*(35*
a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + 28*(5*a^2*b + 7*a*b^2 + 2*b^3)*
cosh(d*x + c))*sinh(d*x + c)^3 + 350*a^3 + 770*a^2*b + 700*a*b^2 + 280*b^3
+ 7*(75*a^3 + 155*a^2*b + 124*a*b^2 + 24*b^3)*cosh(d*x + c)^2 + (15*(35*a^3
 + 35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 525*a^3 + 1085*a^2*b + 86
8*a*b^2 + 168*b^3 + 84*(15*a^3 + 25*a^2*b + 14*a*b^2 + 4*b^3)*cosh(d*x + c)
^2)*sinh(d*x + c)^2 - 2*(3*(35*a^2*b + 28*a*b^2 + 8*b^3)*cosh(d*x + c)^5 +
56*(5*a^2*b + 7*a*b^2 + 2*b^3)*cosh(d*x + c)^3 + 7*(25*a^2*b + 44*a*b^2 + 2
4*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)
*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 8*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*
x + c)^2 + 2*d)*sinh(d*x + c)^6 + 4*(14*d*cosh(d*x + c)^3 + 9*d*cosh(d*x +
c))*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 60*d
```

```
*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 15*d*co
sh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^3 + 56*d*cosh(d*x + c)^2 +
 4*(7*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 42*d*cosh(d*x + c)^2 + 14*
d)*sinh(d*x + c)^2 + 4*(2*d*cosh(d*x + c)^7 + 9*d*cosh(d*x + c)^5 + 14*d*co
sh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c) + 35*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^3\operatorname{sech}^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(70) = 140.
time = 0.43, size = 302, normalized size = 4.08

$$-\frac{2\left(35a^3e^{(12dx+12c)}+210a^3e^{(10dx+10c)}+210a^2be^{(10dx+10c)}+525a^3e^{(8dx+8c)}+910a^2be^{(8dx+8c)}+560ab^2e^{(8dx+8c)}+700a^3e^{(6dx+6c)}+1540a^2be^{(6dx+6c)}+1400ab^2e^{(6dx+6c)}+560b^3e^{(6dx+6c)}+525a^3e^{(4dx+4c)}+1260a^2be^{(4dx+4c)}+1176ab^2e^{(4dx+4c)}+336b^3e^{(4dx+4c)}+210a^3e^{(2dx+2c)}+490a^2be^{(2dx+2c)}+392ab^2e^{(2dx+2c)}+112b^3e^{(2dx+2c)}+35a^3+70a^2b+56ab^2+16b^3\right)}{35d\left(e^{(2dx+2c)}+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*a^3*e^(12*d*x + 12*c) + 210*a^3*e^(10*d*x + 10*c) + 210*a^2*b*e^(
10*d*x + 10*c) + 525*a^3*e^(8*d*x + 8*c) + 910*a^2*b*e^(8*d*x + 8*c) + 560*
a*b^2*e^(8*d*x + 8*c) + 700*a^3*e^(6*d*x + 6*c) + 1540*a^2*b*e^(6*d*x + 6*c
) + 1400*a*b^2*e^(6*d*x + 6*c) + 560*b^3*e^(6*d*x + 6*c) + 525*a^3*e^(4*d*x
 + 4*c) + 1260*a^2*b*e^(4*d*x + 4*c) + 1176*a*b^2*e^(4*d*x + 4*c) + 336*b^3
*e^(4*d*x + 4*c) + 210*a^3*e^(2*d*x + 2*c) + 490*a^2*b*e^(2*d*x + 2*c) + 39
2*a*b^2*e^(2*d*x + 2*c) + 112*b^3*e^(2*d*x + 2*c) + 35*a^3 + 70*a^2*b + 56*
a*b^2 + 16*b^3)/(d*(e^(2*d*x + 2*c) + 1)^7)
```

Mupad [B]
time = 1.50, size = 978, normalized size = 13.22

$$-\frac{\frac{2\left(5a^3+18a^2b+24ab^2+16b^3\right)}{35d}+\frac{2a^3e^{6c+6dx}}{7d}+\frac{6ae^{2c+2dx}\left(5a^2+16ab+16b^2\right)}{35d}+\frac{6a^2e^{4c+4dx}\left(a+2b\right)}{7d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}-\frac{\frac{2a^2\left(a+2b\right)}{7d}+\frac{2a^3e^{2c+2dx}}{7d}}{2e^{2c+2dx}+e^{4c+4dx}+1}-\frac{\frac{2a\left(5a^2+16ab+16b^2\right)}{35d}+\frac{8e^{2c+2dx}\left(5a^3+18a^2b+24ab^2+16b^3\right)}{35d}+\frac{2a^3e^{8c+8dx}}{7d}+\frac{12ae^{4c+4dx}\left(5a^2+16ab+16b^2\right)}{35d}+\frac{8a^2e^{6c+6dx}\left(a+2b\right)}{7d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}-\frac{\frac{2a^3}{7d}+\frac{8e^{6c+6dx}\left(5a^3+18a^2b+24ab^2+16b^3\right)}{7d}+\frac{2a^3e^{12c+12dx}}{7d}+\frac{6ae^{4c+4dx}\left(5a^2+16ab+16b^2\right)}{7d}+\frac{6ae^{8c+8dx}\left(5a^2+16ab+16b^2\right)}{7d}+\frac{12a^2e^{2c+2dx}\left(a+2b\right)}{7d}+\frac{12a^2e^{10c+10dx}\left(a+2b\right)}{7d}}{7e^{2c+2dx}+21e^{4c+4dx}+35e^{6c+6dx}+35e^{8c+8dx}+21e^{10c+10dx}+7e^{12c+12dx}+e^{14c+14dx}+1}-\frac{\frac{2a\left(5a^2+16ab+16b^2\right)}{35d}+\frac{2a^3e^{4c+4dx}}{7d}+\frac{4a^2e^{2c+2dx}\left(a+2b\right)}{7d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}-\frac{\frac{2a^2\left(a+2b\right)}{7d}+\frac{4e^{4c+4dx}\left(5a^3+18a^2b+24ab^2+16b^3\right)}{7d}+\frac{2a^3e^{10c+10dx}}{7d}+\frac{2ae^{2c+2dx}\left(5a^2+16ab+16b^2\right)}{7d}+\frac{4ae^{6c+6dx}\left(5a^2+16ab+16b^2\right)}{7d}+\frac{10a^2e^{8c+8dx}\left(a+2b\right)}{7d}}{6e^{2c+2dx}+15e^{4c+4dx}+20e^{6c+6dx}+15e^{8c+8dx}+6e^{10c+10dx}+e^{12c+12dx}+1}-\frac{2a^3}{7d\left(e^{2c+2dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/cosh(c + d*x)^2,x)
```

```
[Out] - ((2*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(35*d) + (2*a^3*exp(6*c + 6*d
*x))/(7*d) + (6*a*exp(2*c + 2*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(35*d) + (6*a
^2*exp(4*c + 4*d*x)*(a + 2*b))/(7*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d
```

```
*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*a^2*(a + 2*b))/(7*d)
 + (2*a^3*exp(2*c + 2*d*x))/(7*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) +
 1) - ((2*a*(16*a*b + 5*a^2 + 16*b^2))/(35*d) + (8*exp(2*c + 2*d*x)*(24*a*b
^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(35*d) + (2*a^3*exp(8*c + 8*d*x))/(7*d) +
(12*a*exp(4*c + 4*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(35*d) + (8*a^2*exp(6*c +
 6*d*x)*(a + 2*b))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*ex
p(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*a^3)/(7
*d) + (8*exp(6*c + 6*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(7*d) + (
2*a^3*exp(12*c + 12*d*x))/(7*d) + (6*a*exp(4*c + 4*d*x)*(16*a*b + 5*a^2 + 1
6*b^2))/(7*d) + (6*a*exp(8*c + 8*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(7*d) + (1
2*a^2*exp(2*c + 2*d*x)*(a + 2*b))/(7*d) + (12*a^2*exp(10*c + 10*d*x)*(a + 2
*b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x)
 + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp
(14*c + 14*d*x) + 1) - ((2*a*(16*a*b + 5*a^2 + 16*b^2))/(35*d) + (2*a^3*exp
(4*c + 4*d*x))/(7*d) + (4*a^2*exp(2*c + 2*d*x)*(a + 2*b))/(7*d))/(3*exp(2*c
 + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((2*a^2*(a + 2*b))
/(7*d) + (4*exp(4*c + 4*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(7*d)
+ (2*a^3*exp(10*c + 10*d*x))/(7*d) + (2*a*exp(2*c + 2*d*x)*(16*a*b + 5*a^2
+ 16*b^2))/(7*d) + (4*a*exp(6*c + 6*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(7*d) +
 (10*a^2*exp(8*c + 8*d*x)*(a + 2*b))/(7*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*
c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*
x) + exp(12*c + 12*d*x) + 1) - (2*a^3)/(7*d*(exp(2*c + 2*d*x) + 1))
```

3.71 $\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=196

$$\frac{(64a^3+144a^2b+120ab^2+35b^3)\operatorname{ArcTan}(\sinh(c+dx))}{128d}+\frac{(64a^3+144a^2b+120ab^2+35b^3)\operatorname{sech}(c+dx)\tanh$$

```
[Out] 1/128*(64*a^3+144*a^2*b+120*a*b^2+35*b^3)*arctan(sinh(d*x+c))/d+1/128*(64*a
^3+144*a^2*b+120*a*b^2+35*b^3)*sech(d*x+c)*tanh(d*x+c)/d+1/192*b*(72*a^2+92
*a*b+35*b^2)*sech(d*x+c)^3*tanh(d*x+c)/d+1/48*b*(12*a+7*b)*sech(d*x+c)^5*(a
+b+a*sinh(d*x+c)^2)*tanh(d*x+c)/d+1/8*b*sech(d*x+c)^7*(a+b+a*sinh(d*x+c)^2)
^2*tanh(d*x+c)/d
```

Rubi [A]
time = 0.15, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$,
Rules used = {4232, 424, 540, 393, 205, 209}

$$\frac{b(72a^2+92ab+35b^2)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{192d}+\frac{(64a^3+144a^2b+120ab^2+35b^3)\operatorname{ArcTan}(\sinh(c+dx))}{128d}+\frac{(64a^3+144a^2b+120ab^2+35b^3)\tanh(c+dx)\operatorname{sech}(c+dx)}{128d}+\frac{b\tanh(c+dx)\operatorname{sech}^7(c+dx)\left(a\sinh^2(c+dx)+a+b\right)^2}{8d}+\frac{b(12a+7b)\tanh(c+dx)\operatorname{sech}^5(c+dx)\left(a\sinh^2(c+dx)+a+b\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*ArcTan[Sinh[c + d*x]])/(128*d) +
 ((64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*Sech[c + d*x]*Tanh[c + d*x])/(1
28*d) + (b*(72*a^2 + 92*a*b + 35*b^2)*Sech[c + d*x]^3*Tanh[c + d*x])/(192*d
) + (b*(12*a + 7*b)*Sech[c + d*x]^5*(a + b + a*Sinh[c + d*x]^2)*Tanh[c + d*
x])/(48*d) + (b*Sech[c + d*x]^7*(a + b + a*Sinh[c + d*x]^2)^2*Tanh[c + d*x]
)/(8*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{(a+b+ax^2)^3}{(1+x^2)^5} dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b\operatorname{sech}^7(c+dx)\left(a+b+a\sinh^2(c+dx)\right)^2\tanh(c+dx)}{8d} + \frac{\text{Subst}}{}$$

$$= \frac{b(12a+7b)\operatorname{sech}^5(c+dx)\left(a+b+a\sinh^2(c+dx)\right)\tanh(c+dx}{48d}$$

$$= \frac{b(72a^2+92ab+35b^2)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d} + \frac{b(12a+7b}{}$$

$$= \frac{(64a^3+144a^2b+120ab^2+35b^3)\operatorname{sech}(c+dx)\tanh(c+dx)}{128d} + \frac{b}{}$$

$$= \frac{(64a^3+144a^2b+120ab^2+35b^3)\tan^{-1}(\sinh(c+dx))}{128d} + \frac{(64a^3}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.22, size = 1618, normalized size = 8.26

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Coth[c + d*x]^6*Csch[c + d*x]*(a + b*Sech[c + d*x]^2)^3*(344123325*(a + b)
^3*Sinh[c + d*x]^2 + 760096575*a*(a + b)^2*Sinh[c + d*x]^4 + 213089100*(a +
 b)^3*Sinh[c + d*x]^4 + 578580975*a^2*(a + b)*Sinh[c + d*x]^6 + 481962600*a
*(a + b)^2*Sinh[c + d*x]^6 + 12757815*(a + b)^3*Sinh[c + d*x]^6 + 153475245
*a^3*Sinh[c + d*x]^8 + 372265740*a^2*(a + b)*Sinh[c + d*x]^8 + 28676025*a*(
a + b)^2*Sinh[c + d*x]^8 + 99450960*a^3*Sinh[c + d*x]^10 + 22639365*a^2*(a
+ b)*Sinh[c + d*x]^10 - 257600*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2
}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^10 - 65408*(a + b)^3*Hy
pergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]
*Sinh[c + d*x]^10 - 8960*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2
}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^10 - 512*(a + b)^
3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 1, 11/2}, -
Sinh[c + d*x]^2]*Sinh[c + d*x]^10 + 6134499*a^3*Sinh[c + d*x]^12 - 613440*a
*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c +
d*x]^2]*Sinh[c + d*x]^12 - 171648*a*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2,
 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12 - 25344*a
*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}
```

```
, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12 - 1536*a*(a + b)^2*HypergeometricPFQ[{
3/2, 2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh
[c + d*x]^12 - 495552*a^2*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1,
1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^14 - 150144*a^2*(a + b)*Hyperg
eometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sin
h[c + d*x]^14 - 23808*a^2*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}
, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^14 - 1536*a^2*(a +
 b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 1, 11/2},
 -Sinh[c + d*x]^2]*Sinh[c + d*x]^14 - 136640*a^3*HypergeometricPFQ[{3/2, 2,
 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^16 - 43904*a^3*
HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^
2]*Sinh[c + d*x]^16 - 7424*a^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {
1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^16 - 512*a^3*Hypergeo
metricPFQ[{3/2, 2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 1, 11/2}, -Sinh[c + d
*x]^2]*Sinh[c + d*x]^16 + 344123325*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2
]]*Sqrt[-Sinh[c + d*x]^2] + 578580975*a^2*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*
x]^2]]*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 735328125*a*(a + b)^2*ArcTa
nh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 5319877
5*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4*Sqrt[-Sinh[c +
d*x]^2] + 153475245*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6*Sqr
t[-Sinh[c + d*x]^2] + 565126065*a^2*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]
*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 121766085*a*(a + b)^2*ArcTanh[Sqr
t[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 150609375*a^3
*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8*Sqrt[-Sinh[c + d*x]^2] + 9
5298525*a^2*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8*Sqrt[-S
inh[c + d*x]^2] + 656775*a*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c
 + d*x]^8*Sqrt[-Sinh[c + d*x]^2] + 25642575*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]
^2]]*Sinh[c + d*x]^10*Sqrt[-Sinh[c + d*x]^2] + 524475*a^2*(a + b)*ArcTanh[S
qrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10*Sqrt[-Sinh[c + d*x]^2] + 143325*a^3
*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^12*Sqrt[-Sinh[c + d*x]^2] -
760096575*a*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(-Sinh[c + d*x]^2)^(3
/2) - 327796875*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(-Sinh[c + d*x]^2
)^(3/2) + 117495*(a + b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4*
(-Sinh[c + d*x]^2)^(3/2)))/(181440*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)
```

Maple [C] Result contains complex when optimal does not.
time = 1.82, size = 611, normalized size = 3.12

method	result
risch	$\frac{e^{dx+c}\left(-360a\,b^2+360a\,b^2e^{14dx+14c}+3312a^2b\,e^{12dx+12c}+2760a\,b^2e^{12dx+12c}+7344a^2b\,e^{10dx+10c}+9192a\,b^2e^{10dx+10c}+432a^2b\,e^{14dx+\cdots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*exp(d*x+c)*(-360*a*b^2+360*a*b^2*exp(14*d*x+14*c)+3312*a^2*b*exp(12*d
*x+12*c)+2760*a*b^2*exp(12*d*x+12*c)+7344*a^2*b*exp(10*d*x+10*c)+9192*a*b^2
*exp(10*d*x+10*c)+432*a^2*b*exp(14*d*x+14*c)-7344*a^2*b*exp(4*d*x+4*c)-3312
*a^2*b*exp(2*d*x+2*c)-432*a^2*b-192*a^3-105*b^3+6792*a*b^2*exp(8*d*x+8*c)-6
792*a*b^2*exp(6*d*x+6*c)-9192*a*b^2*exp(4*d*x+4*c)+4464*a^2*b*exp(8*d*x+8*c
)-2760*a*b^2*exp(2*d*x+2*c)-4464*a^2*b*exp(6*d*x+6*c)+192*a^3*exp(14*d*x+14
*c)+105*b^3*exp(14*d*x+14*c)-960*a^3*exp(2*d*x+2*c)+960*a^3*exp(12*d*x+12*c
)+805*b^3*exp(12*d*x+12*c)+1728*a^3*exp(10*d*x+10*c)-805*b^3*exp(2*d*x+2*c)
+5053*b^3*exp(8*d*x+8*c)-1728*a^3*exp(4*d*x+4*c)-2681*b^3*exp(4*d*x+4*c)+26
81*b^3*exp(10*d*x+10*c)+960*a^3*exp(8*d*x+8*c)-960*a^3*exp(6*d*x+6*c)-5053*
b^3*exp(6*d*x+6*c))/d/(1+exp(2*d*x+2*c))^8+1/2*I/d*ln(exp(d*x+c)+I)*a^3+9/8
*I/d*ln(exp(d*x+c)+I)*a^2*b+15/16*I/d*ln(exp(d*x+c)+I)*a*b^2+35/128*I/d*ln(
exp(d*x+c)+I)*b^3-1/2*I/d*ln(exp(d*x+c)-I)*a^3-9/8*I/d*ln(exp(d*x+c)-I)*a^2
*b-15/16*I/d*ln(exp(d*x+c)-I)*a*b^2-35/128*I/d*ln(exp(d*x+c)-I)*b^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(186) = 372$.
time = 0.54, size = 556, normalized size = 2.84

$$-\frac{1}{192}\,b^3\left(\frac{105\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{105\,e^{(-dx-c)}+805\,e^{(-3\,dx-3\,c)}+2681\,e^{(-5\,dx-5\,c)}+5053\,e^{(-7\,dx-7\,c)}-5053\,e^{(-9\,dx-9\,c)}-2681\,e^{(-11\,dx-11\,c)}-805\,e^{(-13\,dx-13\,c)}-105\,e^{(-15\,dx-15\,c)}}{d(8\,e^{(-2\,dx-2\,c)}+28\,e^{(-4\,dx-4\,c)}+56\,e^{(-6\,dx-6\,c)}+70\,e^{(-8\,dx-8\,c)}+56\,e^{(-10\,dx-10\,c)}+28\,e^{(-12\,dx-12\,c)}+8\,e^{(-14\,dx-14\,c)}+e^{(-16\,dx-16\,c)}+1)}\right)-\frac{1}{8}\,ab^2\left(\frac{15\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{15\,e^{(-dx-c)}+85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}-198\,e^{(-7\,dx-7\,c)}-85\,e^{(-9\,dx-9\,c)}-15\,e^{(-11\,dx-11\,c)}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}\right)-\frac{3}{4}\,a^2b\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}+11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}-3\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)-a^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/192*b^3*(105*arctan(e^(-d*x - c))/d - (105*e^(-d*x - c) + 805*e^(-3*d*x
 - 3*c) + 2681*e^(-5*d*x - 5*c) + 5053*e^(-7*d*x - 7*c) - 5053*e^(-9*d*x - 9
*c) - 2681*e^(-11*d*x - 11*c) - 805*e^(-13*d*x - 13*c) - 105*e^(-15*d*x - 1
5*c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) +
70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(
-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 1/8*a*b^2*(15*arctan(e^(-d*x
 - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) - 1
98*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) - 15*e^(-11*d*x - 11*c))/(d*(6*e^
(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x -
 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1))) - 3/4*a^2*b*(3*arc
tan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x
 - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) +
4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a^3*(arctan(e^(-d*x - c))/d
 - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*
c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6114 vs. $2(186) = 372$.
time = 0.40, size = 6114, normalized size = 31.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/192*(3*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^15 + 45*(6
4*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)*sinh(d*x + c)^14 + 3*
(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*sinh(d*x + c)^15 + (960*a^3 + 331
2*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^13 + (960*a^3 + 3312*a^2*b +
2760*a*b^2 + 805*b^3 + 315*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^13 + 13*(105*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*
b^3)*cosh(d*x + c)^3 + (960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d
*x + c))*sinh(d*x + c)^12 + (1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)
*cosh(d*x + c)^11 + (4095*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*
x + c)^4 + 1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3 + 78*(960*a^3 + 33
12*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 11*(81
9*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^5 + 26*(960*a^3 +
 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^3 + (1728*a^3 + 7344*a^2*
b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + (960*a^3 + 446
4*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^9 + (15015*(64*a^3 + 144*a^2
*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^6 + 715*(960*a^3 + 3312*a^2*b + 2760
*a*b^2 + 805*b^3)*cosh(d*x + c)^4 + 960*a^3 + 4464*a^2*b + 6792*a*b^2 + 505
3*b^3 + 55*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^9 + 3*(6435*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d
x + c)^7 + 429(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)
^5 + 55*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^3 + 3
*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c))*sinh(d*x + c
)^8 - (960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^7 + (193
05*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^8 + 1716*(960*a^
3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^6 + 330*(1728*a^3 + 73
44*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^4 - 960*a^3 - 4464*a^2*b -
6792*a*b^2 - 5053*b^3 + 36*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^7 + (15015*(64*a^3 + 144*a^2*b + 120*a*b^2 +
35*b^3)*cosh(d*x + c)^9 + 1716*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3
)*cosh(d*x + c)^7 + 462*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cos
h(d*x + c)^5 + 84*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x +
 c)^3 - 7*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c))*sin
h(d*x + c)^6 - (1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c
)^5 + (9009*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^10 + 12
87*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^8 + 462*(172
8*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^6 + 126*(960*a^3
+ 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^4 - 1728*a^3 - 7344*a^2
*b - 9192*a*b^2 - 2681*b^3 - 21*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b
^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + (4095*(64*a^3 + 144*a^2*b + 120*a*b^
2 + 35*b^3)*cosh(d*x + c)^11 + 715*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805
*b^3)*cosh(d*x + c)^9 + 330*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)
*cosh(d*x + c)^7 + 126*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(
d*x + c)^5 - 35*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c

```
)^3 - 5*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c))*sinh
(d*x + c)^4 - (960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^3
 + (1365*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^12 + 286*(
960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c)^10 + 165*(1728*a
^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^8 + 84*(960*a^3 + 44
64*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^6 - 35*(960*a^3 + 4464*a^2*
b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^4 - 960*a^3 - 3312*a^2*b - 2760*a*
b^2 - 805*b^3 - 10*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^3 + (315*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*c
osh(d*x + c)^13 + 78*(960*a^3 + 3312*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x
 + c)^11 + 55*(1728*a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)
^9 + 36*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^7 - 21
*(960*a^3 + 4464*a^2*b + 6792*a*b^2 + 5053*b^3)*cosh(d*x + c)^5 - 10*(1728*
a^3 + 7344*a^2*b + 9192*a*b^2 + 2681*b^3)*cosh(d*x + c)^3 - 3*(960*a^3 + 33
12*a^2*b + 2760*a*b^2 + 805*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((64*a^
3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^16 + 16*(64*a^3 + 144*a^2
*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)*sinh(d*x + c)^15 + (64*a^3 + 144*a^2
*b + 120*a*b^2 + 35*b^3)*sinh(d*x + c)^16 + 8*(64*a^3 + 144*a^2*b + 120*a*b
^2 + 35*b^3)*cosh(d*x + c)^14 + 8*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3
+ 15*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^14 + 112*(5*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^3 +
(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 +
 28*(64*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x + c)^12 + 28*(65*(64
*a^3 + 144*a^2*b + 120*a*b^2 + 35*b^3)*cosh(d*x...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{sech}^2\left(c+dx\right)\right)^3\operatorname{sech}^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 485 vs. 2(186) = 372.
time = 0.42, size = 485, normalized size = 2.47

$$\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)\left(64a^3+144a^2b+120ab^2+35b^3\right)+\frac{4\left(192a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^7+432a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^7+360ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^7+105b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^7+2304a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+6336a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+5280ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+1540b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+9216a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+29952a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+28032ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+8176b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+12288a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)+46080a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+50688ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+17856b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^4}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/768*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(64*a^3 +
144*a^2*b + 120*a*b^2 + 35*b^3) + 4*(192*a^3*(e^(d*x + c) - e^(-d*x - c))^7
```

```
 + 432*a^2*b*(e^(d*x + c) - e^(-d*x - c))^7 + 360*a*b^2*(e^(d*x + c) - e^(-
d*x - c))^7 + 105*b^3*(e^(d*x + c) - e^(-d*x - c))^7 + 2304*a^3*(e^(d*x + c
) - e^(-d*x - c))^5 + 6336*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5 + 5280*a*b^
2*(e^(d*x + c) - e^(-d*x - c))^5 + 1540*b^3*(e^(d*x + c) - e^(-d*x - c))^5
+ 9216*a^3*(e^(d*x + c) - e^(-d*x - c))^3 + 29952*a^2*b*(e^(d*x + c) - e^(-
d*x - c))^3 + 28032*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 8176*b^3*(e^(d*x
 + c) - e^(-d*x - c))^3 + 12288*a^3*(e^(d*x + c) - e^(-d*x - c)) + 46080*a^
2*b*(e^(d*x + c) - e^(-d*x - c)) + 50688*a*b^2*(e^(d*x + c) - e^(-d*x - c))
 + 17856*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2
+ 4)^4)/d
```

Mupad [B]
time = 1.60, size = 931, normalized size = 4.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/cosh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(64*a^3*(d^2)^(1/2) + 35*b^3*(d^2)^(1/2) + 120*a*b^2
*(d^2)^(1/2) + 144*a^2*b*(d^2)^(1/2)))/(d*(8400*a*b^5 + 18432*a^5*b + 4096*
a^6 + 1225*b^6 + 24480*a^2*b^4 + 39040*a^3*b^3 + 36096*a^4*b^2)^(1/2)))*(84
00*a*b^5 + 18432*a^5*b + 4096*a^6 + 1225*b^6 + 24480*a^2*b^4 + 39040*a^3*b^
3 + 36096*a^4*b^2)^(1/2))/(64*(d^2)^(1/2)) - ((a^3*exp(c + d*x))/(2*d) + (2
*exp(7*c + 7*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/d + (a^3*exp(13*c
 + 13*d*x))/(2*d) + (3*a*exp(5*c + 5*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(2*d)
+ (3*a*exp(9*c + 9*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(2*d) + (3*a^2*exp(3*c +
 3*d*x)*(a + 2*b))/d + (3*a^2*exp(11*c + 11*d*x)*(a + 2*b))/d)/(8*exp(2*c +
 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) +
 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp
(16*c + 16*d*x) + 1) + (2*exp(c + d*x)*(48*a*b^2 - 37*b^3))/(3*d*(5*exp(2*c
 + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x)
+ exp(10*c + 10*d*x) + 1)) + (exp(c + d*x)*(24*a^2*b - 120*a*b^2 + b^3))/(4
*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c
+ 8*d*x) + 1)) - (16*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c
+ 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x
) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) + (exp(c + d*x)*(120*a*
b^2 + 144*a^2*b + 64*a^3 + 35*b^3))/(64*d*(exp(2*c + 2*d*x) + 1)) - (4*exp(
c + d*x)*(6*a*b^2 - 29*b^3))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x)
 + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(1
2*c + 12*d*x) + 1)) + (exp(c + d*x)*(120*a*b^2 + 144*a^2*b - 144*a^3 + 35*b
^3))/(96*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (exp(c + d*x)*(24
*a*b^2 - 288*a^2*b + 7*b^3))/(24*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x)
 + exp(6*c + 6*d*x) + 1))
```

3.72 $\int \mathbf{sech}^4(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^3 dx$

Optimal. Leaf size=108

$$\frac{(a+b)^3\tanh(c+dx)}{d}-\frac{(a+b)^2(a+4b)\tanh^3(c+dx)}{3d}+\frac{3b(a+b)(a+2b)\tanh^5(c+dx)}{5d}-\frac{b^2(3a+4b)\tanh^7(}{7d}$$

```
[Out] (a+b)^3*tanh(d*x+c)/d-1/3*(a+b)^2*(a+4*b)*tanh(d*x+c)^3/d+3/5*b*(a+b)*(a+2*
b)*tanh(d*x+c)^5/d-1/7*b^2*(3*a+4*b)*tanh(d*x+c)^7/d+1/9*b^3*tanh(d*x+c)^9/
d
```

Rubi [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4231, 380}

$$-\frac{b^2(3a+4b)\tanh^7(c+dx)}{7d}+\frac{3b(a+b)(a+2b)\tanh^5(c+dx)}{5d}-\frac{(a+b)^2(a+4b)\tanh^3(c+dx)}{3d}+\frac{(a+b)^3\tanh(c+dx)}{d}+\frac{b^3\tanh^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)^3*Tanh[c + d*x])/d - ((a + b)^2*(a + 4*b)*Tanh[c + d*x]^3)/(3*d) +
 (3*b*(a + b)*(a + 2*b)*Tanh[c + d*x]^5)/(5*d) - (b^2*(3*a + 4*b)*Tanh[c +
d*x]^7)/(7*d) + (b^3*Tanh[c + d*x]^9)/(9*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \operatorname{sech}^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int\left(1-x^2\right)\left(a+b-bx^2\right)^3\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int\left((a+b)^3-(a+b)^2(a+4b)x^2+3b(a+b)(a+2b)x^4\right.\right.}{d}$$

$$= \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{(a+b)^2(a+4b)\tanh^3(c+dx)}{3d} + \frac{3b(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 348 vs. 2(108) = 216.
time = 1.22, size = 348, normalized size = 3.22

$$\frac{\operatorname{sech}(c)\operatorname{sech}^9(c+dx)\left(63\left(125a^3+324a^2b+312ab^2+128b^3\right)\sinh(dx)-315a\left(17a^2+36ab+24b^2\right)\sinh(2c+dx)+6825a^3\sinh(2c+3dx)+18648a^2b\sinh(2c+3dx)+18144ab^2\sinh(2c+3dx)+5376b^3\sinh(2c+3dx)-1995a^3\sinh(4c+3dx)-2520a^2b\sinh(4c+3dx)+3465a^3\sinh(4c+5dx)+9072a^2b\sinh(4c+5dx)+7776ab^2\sinh(4c+5dx)+2304b^3\sinh(4c+5dx)-315a^3\sinh(6c+5dx)+945a^3\sinh(6c+7dx)+2268a^2b\sinh(6c+7dx)+1944ab^2\sinh(6c+7dx)+576b^3\sinh(6c+7dx)+105a^3\sinh(8c+9dx)+252a^2b\sinh(8c+9dx)+216ab^2\sinh(8c+9dx)+64b^3\sinh(8c+9dx)\right)}{40320d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^9*(63*(125*a^3 + 324*a^2*b + 312*a*b^2 + 128*b^3)*Si
nh[d*x] - 315*a*(17*a^2 + 36*a*b + 24*b^2)*Sinh[2*c + d*x] + 6825*a^3*Sinh[
2*c + 3*d*x] + 18648*a^2*b*Sinh[2*c + 3*d*x] + 18144*a*b^2*Sinh[2*c + 3*d*x
] + 5376*b^3*Sinh[2*c + 3*d*x] - 1995*a^3*Sinh[4*c + 3*d*x] - 2520*a^2*b*Si
nh[4*c + 3*d*x] + 3465*a^3*Sinh[4*c + 5*d*x] + 9072*a^2*b*Sinh[4*c + 5*d*x]
 + 7776*a*b^2*Sinh[4*c + 5*d*x] + 2304*b^3*Sinh[4*c + 5*d*x] - 315*a^3*Sinh
[6*c + 5*d*x] + 945*a^3*Sinh[6*c + 7*d*x] + 2268*a^2*b*Sinh[6*c + 7*d*x] +
1944*a*b^2*Sinh[6*c + 7*d*x] + 576*b^3*Sinh[6*c + 7*d*x] + 105*a^3*Sinh[8*c
 + 9*d*x] + 252*a^2*b*Sinh[8*c + 9*d*x] + 216*a*b^2*Sinh[8*c + 9*d*x] + 64*
b^3*Sinh[8*c + 9*d*x]))/(40320*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(100) = 200.
time = 1.76, size = 361, normalized size = 3.34

method	result
risch	$-\frac{4\left(216a\,b^2+2520a^2b\,e^{12dx+12c}+11340a^2b\,e^{10dx+10c}+7560a\,b^2e^{10dx+10c}+9072a^2b\,e^{4dx+4c}+2268a^2b\,e^{2dx+2c}+252a^2b+105a^3+64\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4/315*(216*a*b^2+2520*a^2*b*exp(12*d*x+12*c)+11340*a^2*b*exp(10*d*x+10*c)+
7560*a*b^2*exp(10*d*x+10*c)+9072*a^2*b*exp(4*d*x+4*c)+2268*a^2*b*exp(2*d*x+
2*c)+252*a^2*b+105*a^3+64*b^3+19656*a*b^2*exp(8*d*x+8*c)+18144*a*b^2*exp(6*
```

```
d*x+6*c)+7776*a*b^2*exp(4*d*x+4*c)+20412*a^2*b*exp(8*d*x+8*c)+1944*a*b^2*ex
p(2*d*x+2*c)+18648*a^2*b*exp(6*d*x+6*c)+315*a^3*exp(14*d*x+14*c)+945*a^3*ex
p(2*d*x+2*c)+1995*a^3*exp(12*d*x+12*c)+5355*a^3*exp(10*d*x+10*c)+576*b^3*ex
p(2*d*x+2*c)+8064*b^3*exp(8*d*x+8*c)+3465*a^3*exp(4*d*x+4*c)+2304*b^3*exp(4
*d*x+4*c)+7875*a^3*exp(8*d*x+8*c)+6825*a^3*exp(6*d*x+6*c)+5376*b^3*exp(6*d*
x+6*c))/d/(1+exp(2*d*x+2*c))^9
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1245 vs. $2(100) = 200$.
time = 0.29, size = 1245, normalized size = 11.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 256/315*b^3*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c
) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 8
4*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-1
8*d*x - 18*c) + 1)) + 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4
*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x -
 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*
c) + e^(-18*d*x - 18*c) + 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x - 2*c)
+ 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^
(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16
*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 126*e^(-8*d*x - 8*c)/(d*(9*e^(-2*
d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*
c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c)
 + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 1/(d*(9*e^(-2*d*x - 2*
c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126
*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(
-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1))) + 96/35*a*b^2*(7*e^(-2*d*x - 2*
c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*
e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*
x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x
 - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
 + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 35*e^(-6*d*x - 6*c)/(d
*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8
*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 1
4*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x
- 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c)
 + e^(-14*d*x - 14*c) + 1))) + 16/5*a^2*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d
*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
+ e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
```

```
- 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 4/3*a^3*(3*e^(
-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c
) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)
+ 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(100) = 200$.
time = 0.41, size = 1190, normalized size = 11.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -8/315*(2*(105*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cosh(d*x + c)^7 + 14*(10
5*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + (105*
a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*sinh(d*x + c)^7 + 6*(245*a^3 + 399*a^
2*b + 162*a*b^2 + 48*b^3)*cosh(d*x + c)^5 + 3*(175*a^3 + 42*a^2*b - 324*a*b
^2 - 96*b^3 + 7*(105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^5 + 10*(7*(105*a^3 + 63*a^2*b + 54*a*b^2 + 16*b^3)*cosh(d*x
+ c)^3 + 3*(245*a^3 + 399*a^2*b + 162*a*b^2 + 48*b^3)*cosh(d*x + c))*sinh(d
*x + c)^4 + 18*(245*a^3 + 567*a^2*b + 426*a*b^2 + 64*b^3)*cosh(d*x + c)^3 +
 (35*(105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*cosh(d*x + c)^4 + 945*a^3 +
 1134*a^2*b - 108*a*b^2 - 1152*b^3 + 30*(175*a^3 + 42*a^2*b - 324*a*b^2 - 9
6*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 6*(7*(105*a^3 + 63*a^2*b + 54*a*b
^2 + 16*b^3)*cosh(d*x + c)^5 + 10*(245*a^3 + 399*a^2*b + 162*a*b^2 + 48*b^3
)*cosh(d*x + c)^3 + 9*(245*a^3 + 567*a^2*b + 426*a*b^2 + 64*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^2 + 210*(35*a^3 + 93*a^2*b + 90*a*b^2 + 32*b^3)*cosh(d*x
 + c) + (7*(105*a^3 - 126*a^2*b - 108*a*b^2 - 32*b^3)*cosh(d*x + c)^6 + 15*
(175*a^3 + 42*a^2*b - 324*a*b^2 - 96*b^3)*cosh(d*x + c)^4 + 525*a^3 + 882*a
^2*b + 756*a*b^2 + 1344*b^3 + 27*(105*a^3 + 126*a^2*b - 12*a*b^2 - 128*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)*s
inh(d*x + c)^10 + d*sinh(d*x + c)^11 + 9*d*cosh(d*x + c)^9 + (55*d*cosh(d*x
 + c)^2 + 9*d)*sinh(d*x + c)^9 + 3*(55*d*cosh(d*x + c)^3 + 27*d*cosh(d*x +
c))*sinh(d*x + c)^8 + 37*d*cosh(d*x + c)^7 + (330*d*cosh(d*x + c)^4 + 324*d
*cosh(d*x + c)^2 + 35*d)*sinh(d*x + c)^7 + 7*(66*d*cosh(d*x + c)^5 + 108*d*
cosh(d*x + c)^3 + 37*d*cosh(d*x + c))*sinh(d*x + c)^6 + 93*d*cosh(d*x + c)^
5 + 3*(154*d*cosh(d*x + c)^6 + 378*d*cosh(d*x + c)^4 + 245*d*cosh(d*x + c)^
2 + 25*d)*sinh(d*x + c)^5 + (330*d*cosh(d*x + c)^7 + 1134*d*cosh(d*x + c)^5
 + 1295*d*cosh(d*x + c)^3 + 465*d*cosh(d*x + c))*sinh(d*x + c)^4 + 162*d*co
sh(d*x + c)^3 + (165*d*cosh(d*x + c)^8 + 756*d*cosh(d*x + c)^6 + 1225*d*cos
h(d*x + c)^4 + 750*d*cosh(d*x + c)^2 + 90*d)*sinh(d*x + c)^3 + (55*d*cosh(d
*x + c)^9 + 324*d*cosh(d*x + c)^7 + 777*d*cosh(d*x + c)^5 + 930*d*cosh(d*x
+ c)^3 + 486*d*cosh(d*x + c))*sinh(d*x + c)^2 + 210*d*cosh(d*x + c) + (11*d
```

```
*cosh(d*x + c)^10 + 81*d*cosh(d*x + c)^8 + 245*d*cosh(d*x + c)^6 + 375*d*co
sh(d*x + c)^4 + 270*d*cosh(d*x + c)^2 + 42*d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^{2}\left(c + dx\right)\right)^{3}\operatorname{sech}^{4}\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*sech(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(100) = 200.
time = 0.43, size = 360, normalized size = 3.33

$$-\frac{4\left(315\,a^{3}e^{(14\,dx+14\,c)}+1995\,a^{3}e^{(12\,dx+12\,c)}+2520\,a^{2}be^{(12\,dx+12\,c)}+5355\,a^{3}e^{(10\,dx+10\,c)}+11340\,a^{2}be^{(10\,dx+10\,c)}+7560\,ab^{2}e^{(10\,dx+10\,c)}+7875\,a^{3}e^{(8\,dx+8\,c)}+20412\,a^{2}be^{(8\,dx+8\,c)}+19656\,ab^{2}e^{(8\,dx+8\,c)}+8064\,b^{3}e^{(8\,dx+8\,c)}+6825\,a^{3}e^{(6\,dx+6\,c)}+18648\,a^{2}be^{(6\,dx+6\,c)}+18144\,ab^{2}e^{(6\,dx+6\,c)}+5376\,b^{3}e^{(6\,dx+6\,c)}+3465\,a^{3}e^{(4\,dx+4\,c)}+9072\,a^{2}be^{(4\,dx+4\,c)}+7776\,ab^{2}e^{(4\,dx+4\,c)}+2304\,b^{3}e^{(4\,dx+4\,c)}+945\,a^{3}e^{(2\,dx+2\,c)}+2268\,a^{2}be^{(2\,dx+2\,c)}+1944\,ab^{2}e^{(2\,dx+2\,c)}+576\,b^{3}e^{(2\,dx+2\,c)}+105\,a^{3}+252\,a^{2}b+216\,ab^{2}+64\,b^{3}\right)}{315\,d\left(e^{(2\,dx+2\,c)}+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -4/315*(315*a^3*e^(14*d*x + 14*c) + 1995*a^3*e^(12*d*x + 12*c) + 2520*a^2*b
*e^(12*d*x + 12*c) + 5355*a^3*e^(10*d*x + 10*c) + 11340*a^2*b*e^(10*d*x + 1
0*c) + 7560*a*b^2*e^(10*d*x + 10*c) + 7875*a^3*e^(8*d*x + 8*c) + 20412*a^2*
b*e^(8*d*x + 8*c) + 19656*a*b^2*e^(8*d*x + 8*c) + 8064*b^3*e^(8*d*x + 8*c)
+ 6825*a^3*e^(6*d*x + 6*c) + 18648*a^2*b*e^(6*d*x + 6*c) + 18144*a*b^2*e^(6
*d*x + 6*c) + 5376*b^3*e^(6*d*x + 6*c) + 3465*a^3*e^(4*d*x + 4*c) + 9072*a^
2*b*e^(4*d*x + 4*c) + 7776*a*b^2*e^(4*d*x + 4*c) + 2304*b^3*e^(4*d*x + 4*c)
 + 945*a^3*e^(2*d*x + 2*c) + 2268*a^2*b*e^(2*d*x + 2*c) + 1944*a*b^2*e^(2*d
*x + 2*c) + 576*b^3*e^(2*d*x + 2*c) + 105*a^3 + 252*a^2*b + 216*a*b^2 + 64*
b^3)/(d*(e^(2*d*x + 2*c) + 1)^9)
```

Mupad [B]
time = 1.47, size = 1333, normalized size = 12.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3/cosh(c + d*x)^4,x)
```

```
[Out] - ((16*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(315*d) + (4*a^3*exp(6*c + 6
*d*x))/(9*d) + (4*a*exp(2*c + 2*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(21*d) + (8
*a^2*exp(4*c + 4*d*x)*(a + 2*b))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c +
4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)
```

```
 - ((32*exp(8*c + 8*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(9*d) + (8
*a^3*exp(2*c + 2*d*x))/(9*d) + (8*a^3*exp(14*c + 14*d*x))/(9*d) + (8*a*exp(
6*c + 6*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(3*d) + (8*a*exp(10*c + 10*d*x)*(16
*a*b + 5*a^2 + 16*b^2))/(3*d) + (16*a^2*exp(4*c + 4*d*x)*(a + 2*b))/(3*d) +
 (16*a^2*exp(12*c + 12*d*x)*(a + 2*b))/(3*d))/(9*exp(2*c + 2*d*x) + 36*exp(
4*c + 4*d*x) + 84*exp(6*c + 6*d*x) + 126*exp(8*c + 8*d*x) + 126*exp(10*c +
10*d*x) + 84*exp(12*c + 12*d*x) + 36*exp(14*c + 14*d*x) + 9*exp(16*c + 16*d
*x) + exp(18*c + 18*d*x) + 1) - ((4*a^2*(a + 2*b))/(21*d) + (2*a^3*exp(2*c
+ 2*d*x))/(9*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x
) + 1) - ((a*(16*a*b + 5*a^2 + 16*b^2))/(21*d) + (16*exp(2*c + 2*d*x)*(24*a
*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(63*d) + (5*a^3*exp(8*c + 8*d*x))/(9*d)
+ (10*a*exp(4*c + 4*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(21*d) + (40*a^2*exp(6*
c + 6*d*x)*(a + 2*b))/(21*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 2
0*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c
+ 12*d*x) + 1) - (a^3/(9*d) + (16*exp(6*c + 6*d*x)*(24*a*b^2 + 18*a^2*b + 5
*a^3 + 16*b^3))/(9*d) + (7*a^3*exp(12*c + 12*d*x))/(9*d) + (a*exp(4*c + 4*d
*x)*(16*a*b + 5*a^2 + 16*b^2))/d + (5*a*exp(8*c + 8*d*x)*(16*a*b + 5*a^2 +
16*b^2))/(3*d) + (4*a^2*exp(2*c + 2*d*x)*(a + 2*b))/(3*d) + (4*a^2*exp(10*c
 + 10*d*x)*(a + 2*b))/d)/(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp
(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c +
 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1) - ((a*(16*a*b + 5
*a^2 + 16*b^2))/(21*d) + (a^3*exp(4*c + 4*d*x))/(3*d) + (4*a^2*exp(2*c + 2*
d*x)*(a + 2*b))/(7*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c
 + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((4*a^2*(a + 2*b))/(21*d) + (16*exp(4*c
 + 4*d*x)*(24*a*b^2 + 18*a^2*b + 5*a^3 + 16*b^3))/(21*d) + (2*a^3*exp(10*c
+ 10*d*x))/(3*d) + (2*a*exp(2*c + 2*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(7*d) +
 (20*a*exp(6*c + 6*d*x)*(16*a*b + 5*a^2 + 16*b^2))/(21*d) + (20*a^2*exp(8*c
 + 8*d*x)*(a + 2*b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*
exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c
 + 12*d*x) + exp(14*c + 14*d*x) + 1) - a^3/(9*d*(2*exp(2*c + 2*d*x) + exp(4
*c + 4*d*x) + 1))
```

3.73 $\int \frac{\cosh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{(3a^2-4ab+8b^2)\,x}{8a^3}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}\,d}+\frac{(3a-4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d}+\frac{\cosh^3(c+dx)\sinh}{4ad}$$

```
[Out] 1/8*(3*a^2-4*a*b+8*b^2)*x/a^3+1/8*(3*a-4*b)*cosh(d*x+c)*sinh(d*x+c)/a^2/d+1
/4*cosh(d*x+c)^3*sinh(d*x+c)/a/d-b^(5/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^
(1/2))/a^3/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$,
Rules used = {4231, 425, 541, 536, 212, 214}

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a+b}}+\frac{(3a-4b)\sinh(c+dx)\cosh(c+dx)}{8a^2d}+\frac{x(3a^2-4ab+8b^2)}{8a^3}+\frac{\sinh(c+dx)\cosh^3(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((3*a^2 - 4*a*b + 8*b^2)*x)/(8*a^3) - (b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*
x])/Sqrt[a + b]])/(a^3*Sqrt[a + b]*d) + ((3*a - 4*b)*Cosh[c + d*x]*Sinh[c +
 d*x])/(8*a^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cosh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} + \frac{\text{Subst}\left(\int \frac{3a-b-3bx^2}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= \frac{(3a-4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} + \frac{\text{Subst}\left(\int 3\right.}{}$$

$$= \frac{(3a-4b)\cosh(c+dx)\sinh(c+dx)}{8a^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4ad} - \frac{b^3\text{Subst}\left(\int\right.}{}$$

$$= \frac{(3a^2-4ab+8b^2)\,x}{8a^3} - \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}\,d} + \frac{(3a-4b)\cosh(c+dx)\,\text{si}}{8a^2d}$$

Mathematica [A]
time = 0.36, size = 95, normalized size = 0.81

$$\frac{4(3a^2-4ab+8b^2)(c+dx)-\frac{32b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+8a(a-b)\sinh(2(c+dx))+a^2\sinh(4(c+dx))}{32a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (4*(3*a^2 - 4*a*b + 8*b^2)*(c + d*x) - (32*b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c
+ d*x])/Sqrt[a + b]])/Sqrt[a + b] + 8*a*(a - b)*Sinh[2*(c + d*x)] + a^2*Sin
h[4*(c + d*x)])/(32*a^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(103) = 206$.
time = 3.35, size = 347, normalized size = 2.97

method	result
risch	$\frac{3x}{8a}-\frac{bx}{2a^2}+\frac{x\,b^2}{a^3}+\frac{e^{4dx+4c}}{64ad}+\frac{e^{2dx+2c}}{8ad}-\frac{e^{2dx+2c}b}{8a^2d}-\frac{e^{-2dx-2c}}{8ad}+\frac{e^{-2dx-2c}b}{8a^2d}-\frac{e^{-4dx-4c}}{64ad}+\frac{\sqrt{b\,(a+b)}}{}$
derivativedivides	$\frac{\frac{2b^3\left(-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4\sqrt{b}\,\sqrt{a+b}}\right)}{a^3}}{}$
default	$\frac{\frac{2b^3\left(-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4\sqrt{b}\,\sqrt{a+b}}\right)}{a^3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^3/a^3*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)
^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a
+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)))
+1/4/a/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/a/(tanh(1/2*d*x+1/2*c)-1)^3-1/8*(-7*a+
4*b)/a^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/8*(-5*a+4*b)/a^2/(tanh(1/2*d*x+1/2*c)-
1)+1/8/a^3*(-3*a^2+4*a*b-8*b^2)*ln(tanh(1/2*d*x+1/2*c)-1)-1/4/a/(tanh(1/2*d
*x+1/2*c)+1)^4+1/2/a/(tanh(1/2*d*x+1/2*c)+1)^3-1/8*(-5*a+4*b)/a^2/(tanh(1/2
*d*x+1/2*c)+1)-1/8*(7*a-4*b)/a^2/(tanh(1/2*d*x+1/2*c)+1)^2+1/8*(3*a^2-4*a*b
+8*b^2)/a^3*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(103) = 206$.
time = 0.54, size = 526, normalized size = 4.50

$$\frac{3\,b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,\sqrt{(a+b)b}\,ad}+\frac{3\,(dx+c)}{8\,ad}-\frac{\left(8\,be^{(-2\,dx-2\,c)}-a\right)e^{(4\,dx+4\,c)}}{64\,a^2d}+\frac{e^{(2\,dx+2\,c)}}{8\,ad}-\frac{e^{(-2\,dx-2\,c)}}{8\,ad}-\frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{4\,a^2d}+\frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{4\,a^2d}+\frac{(ab+2\,b^2)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}\,a^2d}-\frac{(ab+2\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}\,a^2d}+\frac{(ab+2\,b^2)(dx+c)}{2\,a^3d}+\frac{8\,be^{(-2\,dx-2\,c)}-ae^{(-4\,dx-4\,c)}}{64\,a^2d}+\frac{(a^2b+8\,ab^2+8\,b^3)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,\sqrt{(a+b)b}\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 3/16*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
- 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + 3/8*(d*x + c
)/(a*d) - 1/64*(8*b*e^(-2*d*x - 2*c) - a)*e^(4*d*x + 4*c)/(a^2*d) + 1/8*e^(
2*d*x + 2*c)/(a*d) - 1/8*e^(-2*d*x - 2*c)/(a*d) - 1/4*b*log(a*e^(4*d*x + 4*
c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^2*d) + 1/4*b*log(2*(a + 2*b)*e^(-2
*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d) + 1/8*(a*b + 2*b^2)*log((a*e^
(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b +
 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) - 1/8*(a*b + 2*b^2)*log((a*e^(
-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b
+ 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) + 1/2*(a*b + 2*b^2)*(d*x + c)
/(a^3*d) + 1/64*(8*b*e^(-2*d*x - 2*c) - a*e^(-4*d*x - 4*c))/(a^2*d) + 1/16*
(a^2*b + 8*a*b^2 + 8*b^3)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b
)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*
a^3*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 718 vs. $2(103) = 206$.
time = 0.41, size = 1713, normalized size = 14.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh
(d*x + c)^8 + 8*(3*a^2 - 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 - a*b)
*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + 2*a^2 - 2*a*b)*sinh(d*x + c)^
6 + 8*(7*a^2*cosh(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(35*a^2*cosh(d*x + c)^4 + 4*(3*a^2 - 4*a*b + 8*b^2)*d*x + 60*(a^2 - a*
b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2 -
 4*a*b + 8*b^2)*d*x*cosh(d*x + c) + 20*(a^2 - a*b)*cosh(d*x + c)^3)*sinh(d*
x + c)^3 - 8*(a^2 - a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12*(3
*a^2 - 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^2 + 30*(a^2 - a*b)*cosh(d*x + c)^4
- 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 32*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x
 + c)^3*sinh(d*x + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*cosh(
d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(b/(a + b))*log((a^2*co
```

```
sh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 +
 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*
sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a
*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2
+ a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*
a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d
*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d
*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cos
h(d*x + c))*sinh(d*x + c) + a)) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 -
 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c)^5 - 2*(a^
2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cos
h(d*x + c)^3*sinh(d*x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^
3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4), 1/64*(a^2*cosh(
d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 8*
(3*a^2 - 4*a*b + 8*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 - a*b)*cosh(d*x + c)^6
 + 4*(7*a^2*cosh(d*x + c)^2 + 2*a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cos
h(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cos
h(d*x + c)^4 + 4*(3*a^2 - 4*a*b + 8*b^2)*d*x + 60*(a^2 - a*b)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 4*(3*a^2 - 4*a*b + 8*b^2)*
d*x*cosh(d*x + c) + 20*(a^2 - a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 - 8*(a^
2 - a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 12*(3*a^2 - 4*a*b + 8
*b^2)*d*x*cosh(d*x + c)^2 + 30*(a^2 - a*b)*cosh(d*x + c)^4 - 2*a^2 + 2*a*b)
*sinh(d*x + c)^2 - 64*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)^3*sinh(d*x
 + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*cosh(d*x + c)*sinh(d*
x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c
)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-
b/(a + b))/b) - a^2 + 8*(a^2*cosh(d*x + c)^7 + 4*(3*a^2 - 4*a*b + 8*b^2)*d*
x*cosh(d*x + c)^3 + 6*(a^2 - a*b)*cosh(d*x + c)^5 - 2*(a^2 - a*b)*cosh(d*x
+ c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)^3*sinh(
d*x + c) + 6*a^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*d*cosh(d*x + c)*
sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^4(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(103) = 206.

time = 1.41, size = 208, normalized size = 1.78

$$-\frac{\frac{64\,b^{3}\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^{2}}}\right)}{\sqrt{-ab-b^{2}}\,a^{3}}-\frac{8\,(3\,a^{2}-4\,ab+8\,b^{2})(dx+c)}{a^{3}}-\frac{ae^{(4\,dx+4\,c)}+8\,ae^{(2\,dx+2\,c)}-8\,be^{(2\,dx+2\,c)}}{a^{2}}+\frac{(18\,a^{2}e^{(4\,dx+4\,c)}-24\,abe^{(4\,dx+4\,c)}+48\,b^{2}e^{(4\,dx+4\,c)}+8\,a^{2}e^{(2\,dx+2\,c)}-8\,abe^{(2\,dx+2\,c)}+a^{2})e^{(-4\,dx-4\,c)}}{a^{3}}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/64*(64*b^3*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(s
qrt(-a*b - b^2)*a^3) - 8*(3*a^2 - 4*a*b + 8*b^2)*(d*x + c)/a^3 - (a*e^(4*d*
x + 4*c) + 8*a*e^(2*d*x + 2*c) - 8*b*e^(2*d*x + 2*c))/a^2 + (18*a^2*e^(4*d*
x + 4*c) - 24*a*b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x
 + 2*c) - 8*a*b*e^(2*d*x + 2*c) + a^2)*e^(-4*d*x - 4*c)/a^3)/d
```

Mupad [B]
time = 2.00, size = 260, normalized size = 2.22

$$\frac{x\,(3\,a^{2}-4\,a\,b+8\,b^{2})}{8\,a^{3}}-\frac{\mathrm{e}^{-4\,c-4\,d\,x}}{64\,a\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}}{64\,a\,d}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,(a-b)}{8\,a^{2}\,d}+\frac{\mathrm{e}^{2\,c+2\,d\,x}\,(a-b)}{8\,a^{2}\,d}+\frac{b^{5/2}\,\ln\left(\frac{4\,b^{3}\,\mathrm{e}^{2\,c+2\,d\,x}}{a^{4}}-\frac{2\,b^{5/2}\,(a\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}+2\,b\,d\,\mathrm{e}^{2\,c+2\,d\,x})}{a^{4}\,d\,\sqrt{a+b}}\right)}{2\,a^{3}\,d\,\sqrt{a+b}}-\frac{b^{5/2}\,\ln\left(\frac{4\,b^{3}\,\mathrm{e}^{2\,c+2\,d\,x}}{a^{4}}+\frac{2\,b^{5/2}\,(a\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}+2\,b\,d\,\mathrm{e}^{2\,c+2\,d\,x})}{a^{4}\,d\,\sqrt{a+b}}\right)}{2\,a^{3}\,d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (x*(3*a^2 - 4*a*b + 8*b^2))/(8*a^3) - exp(- 4*c - 4*d*x)/(64*a*d) + exp(4*c
 + 4*d*x)/(64*a*d) - (exp(- 2*c - 2*d*x)*(a - b))/(8*a^2*d) + (exp(2*c + 2*
d*x)*(a - b))/(8*a^2*d) + (b^(5/2)*log((4*b^3*exp(2*c + 2*d*x))/a^4 - (2*b^
(5/2)*(a*d + a*d*exp(2*c + 2*d*x) + 2*b*d*exp(2*c + 2*d*x)))/(a^4*d*(a + b)
^(1/2))))/(2*a^3*d*(a + b)^(1/2)) - (b^(5/2)*log((4*b^3*exp(2*c + 2*d*x))/a
^4 + (2*b^(5/2)*(a*d + a*d*exp(2*c + 2*d*x) + 2*b*d*exp(2*c + 2*d*x)))/(a^4
*d*(a + b)^(1/2))))/(2*a^3*d*(a + b)^(1/2))
```

3.74 $\int \frac{\cosh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{b^2\operatorname{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}\,d}+\frac{(a-b)\sinh(c+dx)}{a^2d}+\frac{\sinh^3(c+dx)}{3ad}$$

```
[Out] (a-b)*sinh(d*x+c)/a^2/d+1/3*sinh(d*x+c)^3/a/d+b^2*arctan(sinh(d*x+c)*a^(1/2
)/(a+b)^(1/2))/a^(5/2)/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4232, 398, 211}

$$\frac{b^2\operatorname{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}d\sqrt{a+b}}+\frac{(a-b)\sinh(c+dx)}{a^2d}+\frac{\sinh^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]*d) +
 ((a - b)*Sinh[c + d*x])/(a^2*d) + Sinh[c + d*x]^3/(3*a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a-b}{a^2}+\frac{x^2}{a}+\frac{b^2}{a^2(a+b+ax^2)}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{(a-b)\sinh(c+dx)}{a^2d}+\frac{\sinh^3(c+dx)}{3ad}+\frac{b^2\text{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{a^2d}$$
$$= \frac{b^2\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}\,d}+\frac{(a-b)\sinh(c+dx)}{a^2d}+\frac{\sinh^3(c+dx)}{3ad}$$

Mathematica [A]
time = 0.20, size = 79, normalized size = 1.04

$$\frac{-\frac{12b^2\text{ArcTan}\left(\frac{\sqrt{a+b}\,\text{csch}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}+3\sqrt{a}\,(3a-4b)\sinh(c+dx)+a^{3/2}\sinh(3(c+dx))}{12a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((-12*b^2*ArcTan[(Sqrt[a + b]*Csch[c + d*x])/Sqrt[a]])/Sqrt[a + b] + 3*Sqrt
[a]*(3*a - 4*b)*Sinh[c + d*x] + a^(3/2)*Sinh[3*(c + d*x)])/(12*a^(5/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(66) = 132.
time = 3.03, size = 209, normalized size = 2.75

method	result
derivativedivides	$\frac{\frac{2b^2\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}\right)}{a^2}-\frac{1}{3a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}}{d}$

default	$\frac{\frac{2b^2\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}\right)}{a^2}-\frac{1}{3a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-}{d}$
risch	$\frac{e^{3dx+3c}}{24ad}+\frac{3\,e^{dx+c}}{8ad}-\frac{e^{dx+c}b}{2a^2d}-\frac{3\,e^{-dx-c}}{8ad}+\frac{e^{-dx-c}b}{2a^2d}-\frac{e^{-3dx-3c}}{24ad}-\frac{b^2\ln\left(e^{2dx+2c}-\frac{2(a+b)e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,d\,a^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^2/a^2*(1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*
d*x+1/2*c)+2*b^(1/2))/a^(1/2))+1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^
(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2)))-1/3/a/(tanh(1/2*d*x+1/2*c)-1
)^3-1/2/a/(tanh(1/2*d*x+1/2*c)-1)^2-(a-b)/a^2/(tanh(1/2*d*x+1/2*c)-1)-1/3/a
/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/a/(tanh(1/2*d*x+1/2*c)+1)^2-(a-b)/a^2/(tanh(
1/2*d*x+1/2*c)+1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/24*(3*(3*a*e^(4*c) - 4*b*e^(4*c))*e^(4*d*x) - 3*(3*a*e^(2*c) - 4*b*e^(2*c
))*e^(2*d*x) + a*e^(6*d*x + 6*c) - a)*e^(-3*d*x - 3*c)/(a^2*d) + 1/8*integr
ate(16*(b^2*e^(3*d*x + 3*c) + b^2*e^(d*x + c))/(a^3*e^(4*d*x + 4*c) + a^3 +
 2*(a^3*e^(2*c) + 2*a^2*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 749 vs. 2(66) = 132.
time = 0.39, size = 1616, normalized size = 21.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*((a^3 + a^2*b)*cosh(d*x + c)^6 + 6*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d
*x + c)^5 + (a^3 + a^2*b)*sinh(d*x + c)^6 + 3*(3*a^3 - a^2*b - 4*a*b^2)*cos
h(d*x + c)^4 + 3*(3*a^3 - a^2*b - 4*a*b^2 + 5*(a^3 + a^2*b)*cosh(d*x + c)^2
```

```
)*sinh(d*x + c)^4 + 4*(5*(a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(3*a^3 - a^2*b -
 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - a^2*b - 3*(3*a^3 - a^2*b -
 4*a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + a^2*b)*cosh(d*x + c)^4 - 3*a^3 + a^
2*b + 4*a*b^2 + 6*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 - 12*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^2*c
osh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(-a^2 - a*b)*log((a
*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 -
2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*
x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c)
- 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 +
(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a
)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^
4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*
x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) +
a)) + 6*((a^3 + a^2*b)*cosh(d*x + c)^5 + 2*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d
*x + c)^3 - (3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 +
 a^3*b)*d*cosh(d*x + c)^3 + 3*(a^4 + a^3*b)*d*cosh(d*x + c)^2*sinh(d*x + c)
 + 3*(a^4 + a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^4 + a^3*b)*d*sinh(d
*x + c)^3), 1/24*((a^3 + a^2*b)*cosh(d*x + c)^6 + 6*(a^3 + a^2*b)*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a^3 + a^2*b)*sinh(d*x + c)^6 + 3*(3*a^3 - a^2*b - 4
*a*b^2)*cosh(d*x + c)^4 + 3*(3*a^3 - a^2*b - 4*a*b^2 + 5*(a^3 + a^2*b)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(3*a^
3 - a^2*b - 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - a^2*b - 3*(3*a^
3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + a^2*b)*cosh(d*x + c)^4 -
 3*a^3 + a^2*b + 4*a*b^2 + 6*(3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c)^2)*sin
h(d*x + c)^2 + 24*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x + c
) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(a^2 + a
*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*s
inh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4
*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + 24*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(
d*x + c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d
*x + c)^3)*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh
(d*x + c))/(a + b)) + 6*((a^3 + a^2*b)*cosh(d*x + c)^5 + 2*(3*a^3 - a^2*b -
 4*a*b^2)*cosh(d*x + c)^3 - (3*a^3 - a^2*b - 4*a*b^2)*cosh(d*x + c))*sinh(d
*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^3 + 3*(a^4 + a^3*b)*d*cosh(d*x + c)
^2*sinh(d*x + c) + 3*(a^4 + a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^4 +
 a^3*b)*d*sinh(d*x + c)^3)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 2.21, size = 332, normalized size = 4.37

$$\frac{e^{3c+3dx}}{24ad}-\frac{e^{-3c-3dx}}{24ad}-\frac{\sqrt{b^4}\left(2\,\mathrm{atan}\left(\left(\left(e^{dx}\,e^{c}\left(\frac{2b^2}{a^8d\,(a+b)^2\sqrt{b^4}}-\frac{4\left(2a^2b^4d\sqrt{b^4}+2a^3b^3d\sqrt{b^4}\right)}{a^6b^5\,(a+b)\sqrt{a^6d^2+b\,a^5d^2}\,\sqrt{a^5d^2\,(a+b)}}\right)-\frac{2b^2e^{3c}e^{3dx}}{a^8d\,(a+b)^2\sqrt{b^4}}\right)\left(\frac{a^7\sqrt{a^6d^2+b\,a^5d^2}}{4}+\frac{a^6b\sqrt{a^6d^2+b\,a^5d^2}}{4}\right)\right)-2\,\mathrm{atan}\left(\frac{b^2e^{dx}e^{c}\sqrt{a^5d^2\,(a+b)}}{2a^2d\,(a+b)\sqrt{b^4}}\right)\right)}{2\sqrt{a^6d^2+b\,a^5d^2}}+\frac{e^{c+dx}\,(3a-4b)}{8a^2d}-\frac{e^{-c-dx}\,(3a-4b)}{8a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] exp(3*c + 3*d*x)/(24*a*d) - exp(- 3*c - 3*d*x)/(24*a*d) - ((b^4)^(1/2)*(2*a
tan((exp(d*x)*exp(c)*((2*b^2)/(a^8*d*(a + b)^2*(b^4)^(1/2)) - (4*(2*a^2*b^4
*d*(b^4)^(1/2) + 2*a^3*b^3*d*(b^4)^(1/2)))/(a^6*b^5*(a + b)*(a^6*d^2 + a^5*
b*d^2)^(1/2)*(a^5*d^2*(a + b))^(1/2))) - (2*b^2*exp(3*c)*exp(3*d*x))/(a^8*d
*(a + b)^2*(b^4)^(1/2)))*((a^7*(a^6*d^2 + a^5*b*d^2)^(1/2))/4 + (a^6*b*(a^6
*d^2 + a^5*b*d^2)^(1/2))/4)) - 2*atan((b^2*exp(d*x)*exp(c)*(a^5*d^2*(a + b)
)^(1/2))/(2*a^2*d*(a + b)*(b^4)^(1/2)))))/(2*(a^6*d^2 + a^5*b*d^2)^(1/2)) +
 (exp(c + d*x)*(3*a - 4*b))/(8*a^2*d) - (exp(- c - d*x)*(3*a - 4*b))/(8*a^2
*d)
```

3.75 $\int \frac{\cosh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{(a-2b)x}{2a^2}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}\,d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2ad}$$

```
[Out] 1/2*(a-2*b)*x/a^2+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d+b^(3/2)*arctanh(b^(1/2)*t
anh(d*x+c)/(a+b)^(1/2))/a^2/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4231, 425, 536, 212, 214}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a+b}}+\frac{x(a-2b)}{2a^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a - 2*b)*x)/(2*a^2) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b
]])/(a^2*Sqrt[a + b]*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{\operatorname{Subst}\left(\int \frac{a-b-bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{2ad}$$
$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{(a-2b)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2a^2d} + \frac{b^2\text{Sub}}{}$$
$$= \frac{(a-2b)x}{2a^2} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}\,d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2ad}$$

Mathematica [A]
time = 0.16, size = 67, normalized size = 0.89

$$\frac{2(a-2b)(c+dx) + \frac{4b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + a\sinh(2(c+dx))}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (2*(a - 2*b)*(c + d*x) + (4*b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a
+ b]])/Sqrt[a + b] + a*Sinh[2*(c + d*x)])/(4*a^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(63) = 126$.
time = 3.52, size = 229, normalized size = 3.05

method	result
risch	$-\frac{bx}{a^2}+\frac{x}{2a}+\frac{e^{2dx+2c}}{8ad}-\frac{e^{-2dx-2c}}{8ad}+\frac{\sqrt{b(a+b)}\,b\ln\left(e^{2dx+2c}-\frac{2\sqrt{b(a+b)}-a-2b}{a}\right)}{2(a+b)d\,a^2}-\frac{\sqrt{b(a+}$
derivativedivides	$\frac{-\frac{2b^2\left(-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\tanh\left(\frac{dx}{2}+\right.\right.}{4\sqrt{b}\,\sqrt{a+b}}\right.}{a^2}}{}$
default	$\frac{-\frac{2b^2\left(-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\tanh\left(\frac{dx}{2}+\right.\right.}{4\sqrt{b}\,\sqrt{a+b}}\right.}{a^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^2/a^2*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c
)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((
a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))
)+1/2/a/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/a/(tanh(1/2*d*x+1/2*c)-1)+1/2/a^2*(2*
b-a)*ln(tanh(1/2*d*x+1/2*c)-1)-1/2/a/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/a/(tanh(
1/2*d*x+1/2*c)+1)+1/2*(-2*b+a)/a^2*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(63) = 126$.
time = 0.49, size = 352, normalized size = 4.69

$$\frac{b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,\sqrt{(a+b)b}\,ad}+\frac{dx+c}{2\,ad}+\frac{e^{(2\,dx+2\,c)}}{8\,ad}-\frac{e^{(-2\,dx-2\,c)}}{8\,ad}-\frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{4\,a^2d}+\frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{4\,a^2d}+\frac{(ab+2\,b^2)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}\,a^2d}-\frac{(ab+2\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,\sqrt{(a+b)b}\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x -
 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + 1/2*(d*x + c)
/(a*d) + 1/8*e^(2*d*x + 2*c)/(a*d) - 1/8*e^(-2*d*x - 2*c)/(a*d) - 1/4*b*log
(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^2*d) + 1/4*b*log(2
*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d) + 1/8*(a*b +
```

```
2*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x +
2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d) - 1/8*(a*b + 2
*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x -
 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(63) = 126$.
time = 0.41, size = 829, normalized size = 11.05

$$\left[\frac{4(a-2b)dx\cosh(dx+c)^2 + a\cosh(dx+c)^4 + 4a\cosh(dx+c)\sinh(dx+c)^3 + a\sinh(dx+c)^4 + 2\left(2(a-2b)dx + 3a\cosh(dx+c)^2\right)\sinh(dx+c)^2 + 4\left(b\cosh(dx+c)^2 + 2b\cosh(dx+c)\sinh(dx+c) + b\sinh(dx+c)^2\right)\sqrt{\frac{b}{a+b}}\log\left(\frac{a^2\cosh(dx+c)^4 + 4a^2\cosh(dx+c)\sinh(dx+c)^3 + a^2\sinh(dx+c)^4 + 2(a^2+2ab)\cosh(dx+c)^2 + 2\left(3a^2\cosh(dx+c)^2 + a^2 + 2ab\right)\sinh(dx+c)^2 + a^2 + 8ab + 8b^2 + 4\left(a^2\cosh(dx+c)^3 + (a^2+2ab)\cosh(dx+c)\right)\sinh(dx+c) - 4\left((a^2+ab)\cosh(dx+c)^2 + 2(a^2+ab)\cosh(dx+c)\sinh(dx+c) + (a^2+ab)\sinh(dx+c)^2 + a^2 + 3ab + 2b^2\right)\sqrt{\frac{b}{a+b}}}{a\cosh(dx+c)^4 + 4a\cosh(dx+c)\sinh(dx+c)^3 + a\sinh(dx+c)^4 + 2(a+2b)\cosh(dx+c)^2 + 2\left(3a\cosh(dx+c)^2 + a + 2b\right)\sinh(dx+c)^2 + 4\left(a\cosh(dx+c)^3 + (a+2b)\cosh(dx+c)\right)\sinh(dx+c) + a}\right) + 4\left(2(a-2b)dx\cosh(dx+c) + a\cosh(dx+c)^3\right)\sinh(dx+c) - a}{8\left(a^2d\cosh(dx+c)^2 + 2a^2d\cosh(dx+c)\sinh(dx+c) + a^2d\sinh(dx+c)^2\right)}, \frac{4(a-2b)dx\cosh(dx+c)^2 + a\cosh(dx+c)^4 + 4a\cosh(dx+c)\sinh(dx+c)^3 + a\sinh(dx+c)^4 + 2\left(2(a-2b)dx + 3a\cosh(dx+c)^2\right)\sinh(dx+c)^2 + 8\left(b\cosh(dx+c)^2 + 2b\cosh(dx+c)\sinh(dx+c) + b\sinh(dx+c)^2\right)\sqrt{-\frac{b}{a+b}}\arctan\left(\frac{\left(a\cosh(dx+c)^2 + 2a\cosh(dx+c)\sinh(dx+c) + a\sinh(dx+c)^2 + a + 2b\right)\sqrt{-\frac{b}{a+b}}}{2b}\right) + 4\left(2(a-2b)dx\cosh(dx+c) + a\cosh(dx+c)^3\right)\sinh(dx+c) - a}{8\left(a^2d\cosh(dx+c)^2 + 2a^2d\cosh(dx+c)\sinh(dx+c) + a^2d\sinh(dx+c)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a - 2*b)*d*x*cosh(d*x + c)^2 + a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(2*(a - 2*b)*d*x + 3*a*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
 + c) + b*sinh(d*x + c)^2)*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2
*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh
(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2
 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*si
nh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(
a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*
x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*
sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x
+ c) + a)) + 4*(2*(a - 2*b)*d*x*cosh(d*x + c) + a*cosh(d*x + c)^3)*sinh(d*x
 + c) - a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x + c) + a
^2*d*sinh(d*x + c)^2), 1/8*(4*(a - 2*b)*d*x*cosh(d*x + c)^2 + a*cosh(d*x +
c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(2*(a - 2*
b)*d*x + 3*a*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(b*cosh(d*x + c)^2 + 2*b*
cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)*sqrt(-b/(a + b))*arctan(1/
2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2
+ a + 2*b)*sqrt(-b/(a + b))/b) + 4*(2*(a - 2*b)*d*x*cosh(d*x + c) + a*cosh(
d*x + c)^3)*sinh(d*x + c) - a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x +
c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

Giac [A]
time = 0.88, size = 125, normalized size = 1.67

$$\frac{\frac{8\,b^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^2}+\frac{4\,(dx+c)(a-2\,b)}{a^2}+\frac{e^{(2\,dx+2\,c)}}{a}-\frac{\left(2\,ae^{(2\,dx+2\,c)}-4\,be^{(2\,dx+2\,c)}+a\right)e^{(-2\,dx-2\,c)}}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^2*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt
(-a*b - b^2)*a^2) + 4*(d*x + c)*(a - 2*b)/a^2 + e^(2*d*x + 2*c)/a - (2*a*e^
(2*d*x + 2*c) - 4*b*e^(2*d*x + 2*c) + a)*e^(-2*d*x - 2*c)/a^2)/d
```

Mupad [B]
time = 1.97, size = 206, normalized size = 2.75

$$\frac{x\,\left(a-2\,b\right)}{2\,a^2}-\frac{{\mathrm{e}}^{-2\,c-2\,d\,x}}{8\,a\,d}+\frac{{\mathrm{e}}^{2\,c+2\,d\,x}}{8\,a\,d}+\frac{b^{3/2}\,\ln\left(-\frac{4\,b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{a^3}-\frac{2\,b^{3/2}\,\left(a\,d+a\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}+2\,b\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{a^3\,d\,\sqrt{a+b}}\right)}{2\,a^2\,d\,\sqrt{a+b}}-\frac{b^{3/2}\,\ln\left(\frac{2\,b^{3/2}\,\left(a\,d+a\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}+2\,b\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{a^3\,d\,\sqrt{a+b}}-\frac{4\,b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{a^3}\right)}{2\,a^2\,d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (x*(a - 2*b))/(2*a^2) - exp(- 2*c - 2*d*x)/(8*a*d) + exp(2*c + 2*d*x)/(8*a*
d) + (b^(3/2)*log(- (4*b^2*exp(2*c + 2*d*x))/a^3 - (2*b^(3/2)*(a*d + a*d*ex
p(2*c + 2*d*x) + 2*b*d*exp(2*c + 2*d*x)))/(a^3*d*(a + b)^(1/2))))/(2*a^2*d*
(a + b)^(1/2)) - (b^(3/2)*log((2*b^(3/2)*(a*d + a*d*exp(2*c + 2*d*x) + 2*b*
d*exp(2*c + 2*d*x)))/(a^3*d*(a + b)^(1/2)) - (4*b^2*exp(2*c + 2*d*x))/a^3))
/(2*a^2*d*(a + b)^(1/2))
```

3.76 $\int \frac{\cosh(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$-\frac{b\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}\,d}+\frac{\sinh(c+dx)}{ad}$$

```
[Out] sinh(d*x+c)/a/d-b*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/d/(a+b)^(
1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4232, 396, 211}

$$\frac{\sinh(c+dx)}{ad}-\frac{b\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((b*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b]*d))
+ Sinh[c + d*x]/(a*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\sinh(c+dx)}{ad} - \frac{b\text{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{ad}$$
$$= -\frac{b\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}\,d} + \frac{\sinh(c+dx)}{ad}$$

Mathematica [A]
time = 0.08, size = 52, normalized size = 1.00

$$\frac{-\frac{b\text{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + \sqrt{a}\sinh(c+dx)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
[Out] (-((b*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/Sqrt[a + b]) + Sqrt[a]*S
inh[c + d*x])/(a^(3/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(44) = 88.
time = 2.71, size = 125, normalized size = 2.40

method	result
derivativedivides	$\frac{-\frac{2b\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\sqrt{a}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\sqrt{a}}\right)}{a}-\frac{1}{a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-}{d}$
default	$\frac{-\frac{2b\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\sqrt{a}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\sqrt{a}}\right)}{a}-\frac{1}{a\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-}{d}$

risch	$\frac{e^{dx+c}}{2ad} - \frac{e^{-dx-c}}{2ad} - \frac{b \ln\left(e^{2dx+2c} + \frac{2(a+b)e^{dx+c}}{\sqrt{-a^2-ab}} - 1\right)}{2\sqrt{-a^2-ab}\, da} + \frac{b \ln\left(e^{2dx+2c} - \frac{2(a+b)e^{dx+c}}{\sqrt{-a^2-ab}} - 1\right)}{2\sqrt{-a^2-ab}\, da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a*b*(1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x
+1/2*c)+2*b^(1/2))/a^(1/2))+1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/
2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2)))-1/a/(tanh(1/2*d*x+1/2*c)+1)-1/a
/(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)/(a*d) - 1/2*integrate(4*(b*e^(3*d*x
+ 3*c) + b*e^(d*x + c))/(a^2*e^(4*d*x + 4*c) + a^2 + 2*(a^2*e^(2*c) + 2*a*b
*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(44) = 88$.
time = 0.39, size = 718, normalized size = 13.81

$$\left[\frac{(a^2+ab)\cosh(dx+c)^2 + 2(a^2+ab)\cosh(dx+c)\sinh(dx+c) + (a^2+ab)\sinh(dx+c)^2 - \sqrt{-a^2-ab}\,(b\cosh(dx+c) + b\sinh(dx+c))\log(\ldots) - a^2 - ab}{2((a^3+a^2b)d\cosh(dx+c) + (a^3+a^2b)d\sinh(dx+c))}, \text{[illegible]}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x +
c) + (a^2 + a*b)*sinh(d*x + c)^2 - sqrt(-a^2 - a*b)*(b*cosh(d*x + c) + b*si
nh(d*x + c))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a
*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 -
 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 +
 sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*s
qrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2
 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c
))*sinh(d*x + c) + a)) - a^2 - a*b)/((a^3 + a^2*b)*d*cosh(d*x + c) + (a^3 +
 a^2*b)*d*sinh(d*x + c)), 1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*
```

```
cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 - 2*sqrt(a^2 + a*
b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*
cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x +
c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) - 2*
sqrt(a^2 + a*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(1/2*sqrt(a^2 + a
*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b)) - a^2 - a*b)/((a^3 + a^2*b)*d*
cosh(d*x + c) + (a^3 + a^2*b)*d*sinh(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 1.74, size = 277, normalized size = 5.33

$$\frac{e^{c+dx}}{2\,a\,d}-\frac{e^{-c-dx}}{2\,a\,d}-\frac{\left(2\,\mathrm{atan}\left(\frac{b^3\,e^{d\,x}\,e^{c}\,\sqrt{a^3\,d^2\,\left(a+b\right)}}{2\,a\,d\,(a+b)\,(b^2)^{3/2}}\right)-2\,\mathrm{atan}\left(\left(e^{d\,x}\,e^{c}\,\left(\frac{2\,b^3}{a^5\,d\,(a+b)^2\,(b^2)^{3/2}}-\frac{4\,\left(2\,a^2\,d\,(b^2)^{3/2}+2\,a\,b\,d\,(b^2)^{3/2}\right)}{a^4\,b^3\,(a+b)\,\sqrt{a^4\,d^2+b\,a^3\,d^2}\,\sqrt{a^3\,d^2\,\left(a+b\right)}}\right)-\frac{2\,b^3\,e^{3\,c}\,e^{3\,d\,x}}{a^5\,d\,(a+b)^2\,(b^2)^{3/2}}\right)\,\left(\frac{a^5\,\sqrt{a^4\,d^2+b\,a^3\,d^2}}{4}+\frac{a^4\,b\,\sqrt{a^4\,d^2+b\,a^3\,d^2}}{4}\right)\right)\right)\,\sqrt{b^2}}{2\,\sqrt{a^4\,d^2+b\,a^3\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] exp(c + d*x)/(2*a*d) - exp(- c - d*x)/(2*a*d) - ((2*atan((b^3*exp(d*x)*exp(
c)*(a^3*d^2*(a + b))^(1/2))/(2*a*d*(a + b)*(b^2)^(3/2))) - 2*atan((exp(d*x)
*exp(c)*((2*b^3)/(a^5*d*(a + b)^2*(b^2)^(3/2)) - (4*(2*a^2*d*(b^2)^(3/2) +
2*a*b*d*(b^2)^(3/2)))/(a^4*b^3*(a + b)*(a^4*d^2 + a^3*b*d^2)^(1/2)*(a^3*d^2
*(a + b))^(1/2))) - (2*b^3*exp(3*c)*exp(3*d*x))/(a^5*d*(a + b)^2*(b^2)^(3/2
)))*((a^5*(a^4*d^2 + a^3*b*d^2)^(1/2))/4 + (a^4*b*(a^4*d^2 + a^3*b*d^2)^(1/
2))/4)))*(b^2)^(1/2))/(2*(a^4*d^2 + a^3*b*d^2)^(1/2))
```

3.77 $\int \frac{\text{sech}(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,\sqrt{a+b}\,d}$$

```
[Out] arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/d/a^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4232, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 36, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,\sqrt{a+b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(28) = 56$.
time = 1.78, size = 80, normalized size = 2.22

method	result	size
derivativedivides	$\frac{\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{\sqrt{a+b}\,\sqrt{a}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{\sqrt{a+b}\,\sqrt{a}}}{d}$	80
default	$\frac{\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{\sqrt{a+b}\,\sqrt{a}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{\sqrt{a+b}\,\sqrt{a}}}{d}$	80
risch	$-\frac{\ln\left(e^{2dx+2c}-\frac{2(a+b)e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,d}+\frac{\ln\left(e^{2dx+2c}+\frac{2(a+b)e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,d}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*
b^(1/2))/a^(1/2))+1/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*
d*x+1/2*c)-2*b^(1/2))/a^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(sech(d*x + c)/(b*sech(d*x + c)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(28) = 56$.
time = 0.36, size = 487, normalized size = 13.53

$$\left[-\frac{\sqrt{-a^2-ab}\log\left(\frac{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4-2(3a+2b)\cosh(dx+c)^2+2\left(3a\cosh(dx+c)^2-3a-2b\right)\sinh(dx+c)^2+4\left(a\cosh(dx+c)^3-(3a+2b)\cosh(dx+c)\right)\sinh(dx+c)-4\left(\cosh(dx+c)^3+3\cosh(dx+c)\sinh(dx+c)^2+\sinh(dx+c)^3+\left(3\cosh(dx+c)^2-1\right)\sinh(dx+c)-\cosh(dx+c)\right)\sqrt{-a^2-ab}+a}{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2(a+2b)\cosh(dx+c)^2+2\left(3a\cosh(dx+c)^2+a+2b\right)\sinh(dx+c)^2+4\left(a\cosh(dx+c)^3+(a+2b)\cosh(dx+c)\right)\sinh(dx+c)+a}\right)}{2(a^2+ab)d}, \frac{\sqrt{a^2+ab}\arctan\left(\frac{a\cosh(dx+c)^3+3a\cosh(dx+c)\sinh(dx+c)^2+a\sinh(dx+c)^3+(3a+4b)\cosh(dx+c)+\left(3a\cosh(dx+c)^2+3a+4b\right)\sinh(dx+c)}{2\sqrt{a^2+ab}}\right)+\sqrt{a^2+ab}\arctan\left(\frac{\sqrt{a^2+ab}(\cosh(dx+c)+\sinh(dx+c))}{2(a+b)}\right)}{(a^2+ab)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
+ c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*
x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*
cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*
x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d
*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(
d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(
d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*co
sh(d*x + c))*sinh(d*x + c) + a))/((a^2 + a*b)*d), (sqrt(a^2 + a*b)*arctan(1
/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)
^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x
 + c))/sqrt(a^2 + a*b)) + sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(
d*x + c) + sinh(d*x + c))/(a + b)))/((a^2 + a*b)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.90, size = 108, normalized size = 3.00

$$-\frac{\ln\left(-\frac{4\,\left(b-b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^2\,\left(a+b\right)}-\frac{8\,b\,\mathrm{e}^{c+d\,x}}{{\left(-a\right)}^{5/2}\,\sqrt{a+b}}\right)-\ln\left(\frac{8\,b\,\mathrm{e}^{c+d\,x}}{{\left(-a\right)}^{5/2}\,\sqrt{a+b}}-\frac{4\,\left(b-b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^2\,\left(a+b\right)}\right)}{2\,\sqrt{-a}\,d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] -(log(- (4*(b - b*exp(2*c + 2*d*x)))/(a^2*(a + b)) - (8*b*exp(c + d*x))/((-
a)^(5/2)*(a + b)^(1/2))) - log((8*b*exp(c + d*x))/((-a)^(5/2)*(a + b)^(1/2)
) - (4*(b - b*exp(2*c + 2*d*x)))/(a^2*(a + b))))/(2*(-a)^(1/2)*d*(a + b)^(1
/2))
```

3.78 $\int \frac{\text{sech}^2(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a+b}\,d}$$

```
[Out] arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/d/b^(1/2)/(a+b)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4231, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}^2(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 36, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a+b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(28) = 56$.
time = 1.72, size = 102, normalized size = 2.83

method	result
derivativedivides	$\frac{-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{2\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\right.}}{2\sqrt{b}\,\sqrt{a+b}}}{d}$
default	$\frac{-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{2\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\right.}}{2\sqrt{b}\,\sqrt{a+b}}}{d}$
risch	$\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^2}+2b\sqrt{ab+b^2}-2ab-2b^2}{a\sqrt{ab+b^2}}\right)}{2\sqrt{ab+b^2}\,d}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^2}+2b\sqrt{ab+b^2}+2ab+2b^2}{a\sqrt{ab+b^2}}\right.}{2\sqrt{ab+b^2}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1
/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/2/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*t
anh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(28) = 56$.
time = 0.51, size = 66, normalized size = 1.83

$$-\frac{\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,\sqrt{(a+b)b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

[Out] -1/2*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*d)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(28) = 56$.
time = 0.36, size = 411, normalized size = 11.42

$$\left[\frac{\log\left(\frac{a^2\cosh(dx+c)^4+4a^2\cosh(dx+c)\sinh(dx+c)^3+a^2\sinh(dx+c)^4+2\left(a^2+2ab\right)\cosh(dx+c)^2+2\left(3a^2\cosh(dx+c)^2+a^2+2ab\right)\sinh(dx+c)^2+a^2+8ab+8b^2+4\left(a^2\cosh(dx+c)^3+\left(a^2+2ab\right)\cosh(dx+c)\right)\sinh(dx+c)-4\left(a\cosh(dx+c)^2+2a\cosh(dx+c)\sinh(dx+c)+a\sinh(dx+c)^2+a+2b\right)\sqrt{ab+b^2}}{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2\left(a+2b\right)\cosh(dx+c)^2+2\left(3a\cosh(dx+c)^2+a+2b\right)\sinh(dx+c)^2+4\left(a\cosh(dx+c)^3+\left(a+2b\right)\cosh(dx+c)\right)\sinh(dx+c)+a}\right)}{2\sqrt{ab+b^2}\,d}, \frac{\sqrt{-ab-b^2}\arctan\left(\frac{\left(a\cosh(dx+c)^2+2a\cosh(dx+c)\sinh(dx+c)+a\sinh(dx+c)^2+a+2b\right)\sqrt{-ab-b^2}}{2\left(ab+b^2\right)}\right)}{\left(ab+b^2\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")

[Out] [1/2*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a))/(sqrt(a*b + b^2)*d), sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a*b + b^2))/((a*b + b^2)*d)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**2/(a+b*sech(d*x+c)**2),x)

[Out] Integral(sech(c + d*x)**2/(a + b*sech(c + d*x)**2), x)

Giac [A]
time = 0.53, size = 47, normalized size = 1.31

$$\frac{\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")

```
[Out] arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2
)*d)
```

Mupad [B]
time = 0.57, size = 125, normalized size = 3.47

$$\frac{\operatorname{atan}\left(\frac{d\,(a+2\,b)}{2\,\sqrt{-b\,d^2\,(a+b)}}+\frac{a\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\left(\frac{4}{a^2\,d}+\frac{(a+2\,b)\,(a\,d+2\,b\,d)}{a^2\,\sqrt{-b^2\,d^2-a\,b\,d^2}\,\sqrt{-b\,d^2\,(a+b)}}\right)\,\sqrt{-b^2\,d^2-a\,b\,d^2}}{2}\right)}{\sqrt{-b^2\,d^2-a\,b\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] atan((d*(a + 2*b))/(2*(-b*d^2*(a + b))^(1/2)) + (a*exp(2*c)*exp(2*d*x)*(4/(
a^2*d) + ((a + 2*b)*(a*d + 2*b*d))/(a^2*(- b^2*d^2 - a*b*d^2)^(1/2)*(-b*d^2
*(a + b))^(1/2)))*(- b^2*d^2 - a*b*d^2)^(1/2))/2)/(- b^2*d^2 - a*b*d^2)^(1/
2)
```

3.79 $\int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\operatorname{ArcTan}(\sinh(c+dx))}{bd}-\frac{\sqrt{a}\operatorname{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}\,d}$$

```
[Out] arctan(sinh(d*x+c))/b/d-arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b/d
/(a+b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4232, 400, 209, 211}

$$\frac{\operatorname{ArcTan}(\sinh(c+dx))}{bd}-\frac{\sqrt{a}\operatorname{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/(b*d) - (Sqrt[a]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[
a + b]])/(b*Sqrt[a + b]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+b+ax^2)}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{bd}\\ &= \frac{\tan^{-1}(\sinh(c+dx))}{bd} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 194 vs. $2(55) = 110$.
time = 0.48, size = 194, normalized size = 3.53

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^2(c+dx)\left(\sqrt{a}\,\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\,\operatorname{csch}(c+dx)\sqrt{(\cosh(c)-\sinh(c))^2}\,(\cosh(c)+\sinh(c))}{\sqrt{a}}\right)\cosh(c)+2\sqrt{a+b}\,\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{(\cosh(c)-\sinh(c))^2}-\sqrt{a}\,\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\,\operatorname{csch}(c+dx)\sqrt{(\cosh(c)-\sinh(c))^2}\,(\cosh(c)+\sinh(c))}{\sqrt{a}}\right)\sinh(c)\right)}{2b\sqrt{a+b}\,d\left(a+b\operatorname{sech}^2(c+dx)\right)\sqrt{(\cosh(c)-\sinh(c))^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[a]*ArcTan[(Sqrt[a +
b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*
Cosh[c] + 2*Sqrt[a + b]*ArcTan[Tanh[(c + d*x)/2]]*Sqrt[(Cosh[c] - Sinh[c])^
2] - Sqrt[a]*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*
(Cosh[c] + Sinh[c]))/Sqrt[a]]*Sinh[c]))/(2*b*Sqrt[a + b]*d*(a + b*Sech[c +
d*x]^2)*Sqrt[(Cosh[c] - Sinh[c])^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(47) = 94$.
time = 1.71, size = 104, normalized size = 1.89

method	result

derivativedivides	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{2a\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}\right)}{b}}{d}$
default	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{2a\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\,\sqrt{a}}\right)}{b}}{d}$
risch	$\frac{i\ln\left(e^{dx+c}+i\right)}{db}-\frac{i\ln\left(e^{dx+c}-i\right)}{db}+\frac{\sqrt{-a\,(a+b)}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-a\,(a+b)}\,e^{dx+c}}{a}-1\right)}{2(a+b)db}-\frac{\sqrt{-a\,(a+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b*arctan(tanh(1/2*d*x+1/2*c))-2*a/b*(1/2/(a+b)^(1/2)/a^(1/2)*arctan(
1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+1/2/(a+b)^(1/2)/
a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(d*x + c))/(b*d) - 8*integrate(1/4*(a*e^(3*d*x + 3*c) + a*e^(d*x
 + c))/(a*b*e^(4*d*x + 4*c) + a*b + 2*(a*b*e^(2*c) + 2*b^2*e^(2*c))*e^(2*d*
x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(47) = 94.
time = 0.39, size = 526, normalized size = 9.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/(a + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
+ c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*
```

```
x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*
cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(
d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh(d*x + c)
+ (3*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a + b)) + a)/
(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4
+ 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x
+ c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)
) + 4*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d), -(sqrt(a/(a + b))*arcta
n(1/2*sqrt(a/(a + b))*(cosh(d*x + c) + sinh(d*x + c))) + sqrt(a/(a + b))*ar
ctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*
x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*si
nh(d*x + c))*sqrt(a/(a + b))/a) - 2*arctan(cosh(d*x + c) + sinh(d*x + c)))/
(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.81, size = 307, normalized size = 5.58

$$\frac{2\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\left(9\,a^2\,\sqrt{b^2\,d^2}+16\,b^2\,\sqrt{b^2\,d^2}+24\,a\,b\,\sqrt{b^2\,d^2}\right)}{9\,d\,a^2\,b+24\,d\,a\,b^2+16\,d\,b^3}\right)}{\sqrt{b^2\,d^2}}-\frac{\sqrt{a}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{a}\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\sqrt{b^2\,d^2\,\left(a+b\right)}}{2\,b\,d\,\left(a+b\right)}\right)+2\,\mathrm{atan}\left(\frac{4\,b^4\,d^2\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c+4\,a^2\,b^2\,d^2\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c-a\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\sqrt{b^3\,d^2+a\,b^2\,d^2}\,\sqrt{b^2\,d^2\,\left(a+b\right)}+8\,a\,b^3\,d^2\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c+a\,{\mathrm{e}}^{3\,c}\,{\mathrm{e}}^{3\,d\,x}\,\sqrt{b^3\,d^2+a\,b^2\,d^2}\,\sqrt{b^2\,d^2\,\left(a+b\right)}}{\sqrt{a}\,d\,\left(2\,b^2+2\,a\,b\right)\,\sqrt{b^2\,d^2\,\left(a+b\right)}}\right)\right)}{2\,\sqrt{b^3\,d^2+a\,b^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] (2*atan((exp(d*x)*exp(c)*(9*a^2*(b^2*d^2)^(1/2) + 16*b^2*(b^2*d^2)^(1/2) +
24*a*b*(b^2*d^2)^(1/2)))/(16*b^3*d + 24*a*b^2*d + 9*a^2*b*d)))/(b^2*d^2)^(1
/2) - (a^(1/2)*(2*atan((a^(1/2)*exp(d*x)*exp(c)*(b^2*d^2*(a + b))^(1/2))/(2
*b*d*(a + b))) + 2*atan((4*b^4*d^2*exp(d*x)*exp(c) + 4*a^2*b^2*d^2*exp(d*x)
*exp(c) - a*exp(d*x)*exp(c)*(b^3*d^2 + a*b^2*d^2)^(1/2)*(b^2*d^2*(a + b))^(
1/2) + 8*a*b^3*d^2*exp(d*x)*exp(c) + a*exp(3*c)*exp(3*d*x)*(b^3*d^2 + a*b^2
*d^2)^(1/2)*(b^2*d^2*(a + b))^(1/2))/(a^(1/2)*d*(2*a*b + 2*b^2)*(b^2*d^2*(a
 + b))^(1/2)))))/(2*(b^3*d^2 + a*b^2*d^2)^(1/2))
```

3.80 $\int \frac{\text{sech}^4(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}\,d}+\frac{\tanh(c+dx)}{bd}$$

```
[Out] -a*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(3/2)/d/(a+b)^(1/2)+tanh(d*x+
c)/b/d
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4231, 396, 214}

$$\frac{\tanh(c+dx)}{bd}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((a*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b]*d))
 + Tanh[c + d*x]/(b*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\tanh(c+dx)}{bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{bd}\\ &= -\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}\,d} + \frac{\tanh(c+dx)}{bd}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(52) = 104.
time = 0.48, size = 182, normalized size = 3.50

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^2(c+dx)\left(a\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\,\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(-\cosh(2c)+\sinh(2c))+\sqrt{a+b}\,\operatorname{sech}(c)\operatorname{sech}(c+dx)\sqrt{b(\cosh(c)-\sinh(c))^4}\,\sinh(dx)\right)}{2b\sqrt{a+b}\,d\,(a+b\operatorname{sech}^2(c+dx))\,\sqrt{b(\cosh(c)-\sinh(c))^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(a*ArcTanh[(Sech[d*x]*(Cos
h[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a +
b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + Sqrt[a + b]*S
ech[c]*Sech[c + d*x]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*Sinh[d*x]))/(2*b*Sqrt[a
+ b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(44) = 88.
time = 1.54, size = 138, normalized size = 2.65

method	result
derivativedivides	$\frac{\frac{2a\left(-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\right.}{4\sqrt{b}\,\sqrt{a+b}}}{b}}{d}$
default	$\frac{\frac{2a\left(-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\right.}{4\sqrt{b}\,\sqrt{a+b}}}{b}}{d}$

risch	$-\frac{2}{bd(1+e^{2dx+2c})}+\frac{a\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^2}+2b\sqrt{ab+b^2}+2ab+2b^2}{a\sqrt{ab+b^2}}\right)}{2\sqrt{ab+b^2}\,db}-\frac{a\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^2}+2}{a\sqrt{}}\right)}{2\sqrt{ab+b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a/b*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2
*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^
(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)))+2/b
*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(44) = 88$.
time = 0.50, size = 91, normalized size = 1.75

$$\frac{a\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,\sqrt{(a+b)b}\,bd}+\frac{2}{(be^{(-2\,dx-2\,c)}+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x -
 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*b*d) + 2/((b*e^(-2*d
*x - 2*c) + b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(44) = 88$.
time = 0.38, size = 645, normalized size = 12.40

$$\left[\frac{\left(a\cosh\left(dx+c\right)^2+2\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a\sinh\left(dx+c\right)^2+a\right)\sqrt{ab+b^2}\log\left(\frac{a^2\cosh(dx+c)^4+4\,a^2\cosh(dx+c)\sinh(dx+c)^3+a^2\sinh(dx+c)^4+2\,(a^2+2\,ab)\cosh(dx+c)^2+2\,\left(3\,a^2\cosh(dx+c)^2+a^2+2\,ab\right)\sinh(dx+c)^2+a^2+8\,ab+8\,b^2+4\,\left(a^2\cosh(dx+c)^3+(a^2+2\,ab)\cosh(dx+c)\right)\sinh(dx+c)+4\,\left(a\cosh(dx+c)^2+2\,a\cosh(dx+c)\sinh(dx+c)+a\sinh(dx+c)^2+a+2\,b\right)\sqrt{ab+b^2}}{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2\,(a+2\,b)\cosh(dx+c)^2+2\,\left(3\,a\cosh(dx+c)^2+a+2\,b\right)\sinh(dx+c)^2+4\,\left(a\cosh(dx+c)^3+(a+2\,b)\cosh(dx+c)\right)\sinh(dx+c)+a}\right)-4\,ab-4\,b^2}{2\,\left((ab^2+b^3)d\cosh\left(dx+c\right)^2+2\,(ab^2+b^3)d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(ab^2+b^3)d\sinh\left(dx+c\right)^2+(ab^2+b^3)d\right)},\ -\frac{\left(a\cosh\left(dx+c\right)^2+2\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a\sinh\left(dx+c\right)^2+a\right)\sqrt{-ab-b^2}\arctan\left(\frac{\left(a\cosh(dx+c)^2+2\,a\cosh(dx+c)\sinh(dx+c)+a\sinh(dx+c)^2+a+2\,b\right)\sqrt{-ab-b^2}}{2\,(ab+b^2)}\right)+2\,ab+2\,b^2}{(ab^2+b^3)d\cosh\left(dx+c\right)^2+2\,(ab^2+b^3)d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(ab^2+b^3)d\sinh\left(dx+c\right)^2+(ab^2+b^3)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c
)^2 + a)*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sin
h(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3
*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 +
 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a
*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a
+ 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c
)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c
)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x
```

```
+ c))*sinh(d*x + c) + a)) - 4*a*b - 4*b^2)/((a*b^2 + b^3)*d*cosh(d*x + c)^2
 + 2*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b^2 + b^3)*d*sinh(d*x
 + c)^2 + (a*b^2 + b^3)*d), -((a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d
*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x +
c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(
-a*b - b^2)/(a*b + b^2)) + 2*a*b + 2*b^2)/((a*b^2 + b^3)*d*cosh(d*x + c)^2
+ 2*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b^2 + b^3)*d*sinh(d*x
+ c)^2 + (a*b^2 + b^3)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

Giac [A]
time = 0.64, size = 72, normalized size = 1.38

$$-\frac{\frac{a\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,b}+\frac{2}{b\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(a*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b -
 b^2)*b) + 2/(b*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B]
time = 0.48, size = 166, normalized size = 3.19

$$\frac{a\,\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}}{b}-\frac{2\,\left(a\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}+2\,b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{b^{3/2}\,d\,\sqrt{a+b}}\right)}{2\,b^{3/2}\,d\,\sqrt{a+b}}-\frac{2}{b\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{a\,\ln\left(\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}}{b}+\frac{2\,\left(a\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}+2\,b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{b^{3/2}\,d\,\sqrt{a+b}}\right)}{2\,b^{3/2}\,d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] (a*log((4*exp(2*c + 2*d*x))/b - (2*(a*d + a*d*exp(2*c + 2*d*x) + 2*b*d*exp(
2*c + 2*d*x)))/(b^(3/2)*d*(a + b)^(1/2))))/(2*b^(3/2)*d*(a + b)^(1/2)) - 2/
(b*d*(exp(2*c + 2*d*x) + 1)) - (a*log((4*exp(2*c + 2*d*x))/b + (2*(a*d + a*
d*exp(2*c + 2*d*x) + 2*b*d*exp(2*c + 2*d*x)))/(b^(3/2)*d*(a + b)^(1/2))))/(
2*b^(3/2)*d*(a + b)^(1/2))
```

3.81 $\int \frac{\text{sech}^5(c+dx)}{a+b\text{sech}^2(c+dx)} dx$

Optimal. Leaf size=86

$$-\frac{(2a-b)\text{ArcTan}(\sinh(c+dx))}{2b^2d}+\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}\,d}+\frac{\text{sech}(c+dx)\tanh(c+dx)}{2bd}$$

```
[Out] -1/2*(2*a-b)*arctan(sinh(d*x+c))/b^2/d+a^(3/2)*arctan(sinh(d*x+c)*a^(1/2)/(
a+b)^(1/2))/b^2/d/(a+b)^(1/2)+1/2*sech(d*x+c)*tanh(d*x+c)/b/d
```

Rubi [A]
time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4232, 425, 536, 209, 211}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a+b}}-\frac{(2a-b)\text{ArcTan}(\sinh(c+dx))}{2b^2d}+\frac{\tanh(c+dx)\text{sech}(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/2*((2*a - b)*ArcTan[Sinh[c + d*x]])/(b^2*d) + (a^(3/2)*ArcTan[(Sqrt[a]*S
inh[c + d*x])/Sqrt[a + b]])/(b^2*Sqrt[a + b]*d) + (Sech[c + d*x]*Tanh[c + d
*x])/(2*b*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+ax^2)}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd} - \frac{\operatorname{Subst}\left(\int \frac{a-b-ax^2}{(1+x^2)(a+b+ax^2)}\,dx, x, \sinh(c+dx)\right)}{2bd}$$

$$= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{b^2d} - \frac{(2a-b)\mathrm{S}}{}$$

$$= -\frac{(2a-b)\tan^{-1}(\sinh(c+dx))}{2b^2d} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}\,d} + \frac{\operatorname{sech}(c+dx)\tan}{2bd}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 213 vs. 2(86) = 172.
time = 1.27, size = 213, normalized size = 2.48

$$\frac{\cosh(c)(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^2(c+dx)\left(b\sqrt{a+b}\operatorname{sech}^2(c)\operatorname{sech}^2(c+dx)\sqrt{(\cosh(c)-\sinh(c))^2}\sinh(dx)+2a^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\operatorname{csch}(c+dx)\sqrt{(\cosh(c)-\sinh(c))^2}(\cosh(c)+\sinh(c))}{\sqrt{a}}\right)(-1+\tanh(c))-\sqrt{a+b}\operatorname{sech}(c)\sqrt{(\cosh(c)-\sinh(c))^2}\left(2(2a-b)\operatorname{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-b\operatorname{sech}(c+dx)\tanh(c)\right)\right)}{4b^2\sqrt{a+b}\,d\,(a+b\operatorname{sech}^2(c+dx))\sqrt{(\cosh(c)-\sinh(c))^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (Cosh[c]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(b*Sqrt[a + b]*Sec
h[c]^2*Sech[c + d*x]^2*Sqrt[(Cosh[c] - Sinh[c])^2]*Sinh[d*x] + 2*a^(3/2)*Ar
```

```
cTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh
[c]))/Sqrt[a]]*(-1 + Tanh[c]) - Sqrt[a + b]*Sech[c]*Sqrt[(Cosh[c] - Sinh[c]
)^2]*(2*(2*a - b)*ArcTan[Tanh[(c + d*x)/2]] - b*Sech[c + d*x]*Tanh[c])))/(4
*b^2*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[(Cosh[c] - Sinh[c])^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(74) = 148$.
time = 1.95, size = 159, normalized size = 1.85

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{(-b+2a)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^2}+\frac{2a^2\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\sqrt{a}}\right.}{}}{d}$
default	$\frac{-\frac{2\left(\frac{\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{(-b+2a)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^2}+\frac{2a^2\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{b}}{2\sqrt{a}}\right)}{2\sqrt{a+b}\sqrt{a}}\right.}{}}{d}$
risch	$\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{db\left(1+e^{2dx+2c}\right)^2}-\frac{i\ln\left(e^{dx+c}-i\right)}{2db}+\frac{i\ln\left(e^{dx+c}-i\right)a}{d\,b^2}+\frac{i\ln\left(e^{dx+c}+i\right)}{2db}-\frac{i\ln\left(e^{dx+c}+i\right)a}{d\,b^2}+\frac{\sqrt{-a\left(a+b\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^2*((1/2*b*tanh(1/2*d*x+1/2*c)^3-1/2*b*tanh(1/2*d*x+1/2*c))/(tanh(
1/2*d*x+1/2*c)^2+1)^2+1/2*(-b+2*a)*arctan(tanh(1/2*d*x+1/2*c)))+2*a^2/b^2*(
1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(
1/2))/a^(1/2))+1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d
*x+1/2*c)-2*b^(1/2))/a^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (e^(3*d*x + 3*c) - e^(d*x + c))/(b*d*e^(4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2*c
) + b*d) - (2*a*e^c - b*e^c)*arctan(e^(d*x + c))*e^(-c)/(b^2*d) + 32*integr
ate(1/16*(a^2*e^(3*d*x + 3*c) + a^2*e^(d*x + c))/(a*b^2*e^(4*d*x + 4*c) + a
*b^2 + 2*(a*b^2*e^(2*c) + 2*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(74) = 148$.
time = 0.40, size = 1518, normalized size = 17.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*cosh(d*x + c)^3 + 6*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*b*sinh(d*
x + c)^3 + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(
d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c
)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(-a/(a
 + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(
d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a -
 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*s
inh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*
x + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh(d*x + c) + (3*(a + b)*cos
h(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a + b)) + a)/(a*cosh(d*x + c)
^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*co
sh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*co
sh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 2*((2*a - b)
*cosh(d*x + c)^4 + 4*(2*a - b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a - b)*si
nh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^
2 + 2*a - b)*sinh(d*x + c)^2 + 4*((2*a - b)*cosh(d*x + c)^3 + (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + 2*a - b)*arctan(cosh(d*x + c) + sinh(d*x + c))
 - 2*b*cosh(d*x + c) + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(b^2*d*cos
h(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*d*sinh(d*x + c)^
4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d)*s
inh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x +
 c)), (b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x +
 c)^3 + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x
 + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2
 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a/(a + b
))*arctan(1/2*sqrt(a/(a + b))*(cosh(d*x + c) + sinh(d*x + c))) + (a*cosh(d*
x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh
(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x +
 c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a/(a + b))*arctan(1/2*(a*c
osh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3
*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))*
sqrt(a/(a + b))/a) - ((2*a - b)*cosh(d*x + c)^4 + 4*(2*a - b)*cosh(d*x + c)
*sinh(d*x + c)^3 + (2*a - b)*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2
+ 2*(3*(2*a - b)*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*((2*a - b)*
cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + 2*a - b)*arctan(
cosh(d*x + c) + sinh(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 - b
```

```
)*sinh(d*x + c))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x +
c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d
*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 + b^2*
d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**5/(a + b*sech(c + d*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 3.50, size = 946, normalized size = 11.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] ((a^3)^(1/2)*(2*atan((exp(d*x)*exp(c)*((64*(6*b^3*d*(a^3)^(3/2) + 2*b^6*d*(
a^3)^(1/2) + 6*a*b^2*d*(a^3)^(3/2) - 4*a*b^5*d*(a^3)^(1/2) - 6*a^2*b^4*d*(a
^3)^(1/2)))/(a^4*b^4*(a + b)*(a*b + b^2)*(b^5*d^2 + a*b^4*d^2)^(1/2)*(b^4*d
^2*(a + b))^(1/2)*(3*a^3 - 3*a*b^2 + b^3)) - (32*(3*a^5*(b^5*d^2 + a*b^4*d^
2)^(1/2) + a^2*b^3*(b^5*d^2 + a*b^4*d^2)^(1/2) - 3*a^3*b^2*(b^5*d^2 + a*b^4
*d^2)^(1/2)))/(a^2*b^6*d*(a + b)^2*(a*b + b^2)*(b^5*d^2 + a*b^4*d^2)^(1/2)*
(a^3)^(1/2)*(3*a^3 - 3*a*b^2 + b^3))) + (32*exp(3*c)*exp(3*d*x)*(3*a^5*(b^5
*d^2 + a*b^4*d^2)^(1/2) + a^2*b^3*(b^5*d^2 + a*b^4*d^2)^(1/2) - 3*a^3*b^2*(
b^5*d^2 + a*b^4*d^2)^(1/2)))/(a^2*b^6*d*(a + b)^2*(a*b + b^2)*(b^5*d^2 + a*
```

```
b^4*d^2)^(1/2)*(a^3)^(1/2)*(3*a^3 - 3*a*b^2 + b^3)))*((a^2*b^7*(b^5*d^2 + a
*b^4*d^2)^(1/2))/64 + (a^3*b^6*(b^5*d^2 + a*b^4*d^2)^(1/2))/32 + (a^4*b^5*(
b^5*d^2 + a*b^4*d^2)^(1/2))/64)) + 2*atan((a^2*exp(d*x)*exp(c)*(b^4*d^2*(a
+ b))^(1/2))/(2*b^2*d*(a + b)*(a^3)^(1/2)))))/(2*(b^5*d^2 + a*b^4*d^2)^(1/2
)) - (atan((exp(d*x)*exp(c)*(18*a^7*(b^4*d^2)^(1/2) - b^7*(b^4*d^2)^(1/2) -
 21*a^2*b^5*(b^4*d^2)^(1/2) + 12*a^3*b^4*(b^4*d^2)^(1/2) + 30*a^4*b^3*(b^4*
d^2)^(1/2) - 36*a^5*b^2*(b^4*d^2)^(1/2) + 8*a*b^6*(b^4*d^2)^(1/2) - 9*a^6*b
*(b^4*d^2)^(1/2)))/(b^8*d*(4*a^2 - 4*a*b + b^2)^(1/2) + 9*a^2*b^6*d*(4*a^2
- 4*a*b + b^2)^(1/2) + 6*a^3*b^5*d*(4*a^2 - 4*a*b + b^2)^(1/2) - 18*a^4*b^4
*d*(4*a^2 - 4*a*b + b^2)^(1/2) + 9*a^6*b^2*d*(4*a^2 - 4*a*b + b^2)^(1/2) -
6*a*b^7*d*(4*a^2 - 4*a*b + b^2)^(1/2)))*(4*a^2 - 4*a*b + b^2)^(1/2))/(b^4*d
^2)^(1/2) + exp(c + d*x)/(b*d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x))/(b
*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.82 $\int \frac{\text{sech}^6(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}\,d}-\frac{(a-b)\tanh(c+dx)}{b^2d}-\frac{\tanh^3(c+dx)}{3bd}$$

[Out] a^2*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(5/2)/d/(a+b)^(1/2)-(a-b)*tanh(d*x+c)/b^2/d-1/3*tanh(d*x+c)^3/b/d

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4231, 398, 214}

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}d\sqrt{a+b}}-\frac{(a-b)\tanh(c+dx)}{b^2d}-\frac{\tanh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2),x]

[Out] (a^2*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]*d) - ((a - b)*Tanh[c + d*x])/(b^2*d) - Tanh[c + d*x]^3/(3*b*d)

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^6(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(-\frac{a-b}{b^2}-\frac{x^2}{b}+\frac{a^2}{b^2(a+b-bx^2)}\right)\,dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{(a-b)\tanh(c+dx)}{b^2d}-\frac{\tanh^3(c+dx)}{3bd}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx\right)}{b^2d} \\
&= \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}\,d}-\frac{(a-b)\tanh(c+dx)}{b^2d}-\frac{\tanh^3(c+dx)}{3bd}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 214 vs. 2(77) = 154.
time = 1.50, size = 214, normalized size = 2.78

$$
\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^2(c+dx)\left(3a^2\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(\cosh(2c)-\sinh(2c))+\sqrt{a+b}\operatorname{sech}(c+dx)\sqrt{b(\cosh(c)-\sinh(c))^4}\left(\operatorname{sech}(c)\left(-3a+2b+b\operatorname{sech}^2(c+dx)\right)\sinh(dx)+b\operatorname{sech}(c+dx)\tanh(c)\right)\right)}{6b^2\sqrt{a+b}\,d\left(a+b\operatorname{sech}^2(c+dx)\right)\sqrt{b(\cosh(c)-\sinh(c))^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(3*a^2*ArcTanh[(Sech[d*x]*
(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]) + Sqrt[a + b
]*Sech[c + d*x]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(Sech[c]*(-3*a + 2*b + b*Sech
[c + d*x]^2)*Sinh[d*x] + b*Sech[c + d*x]*Tanh[c])))/(6*b^2*Sqrt[a + b]*d*(a
 + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(67) = 134.
time = 1.84, size = 183, normalized size = 2.38

method	result
derivativedivides	$\frac{\frac{2(-a+b)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(-2a+\frac{2b}{3}\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2(-a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{2a^2\left(-\frac{\ln\left(\sqrt{a+b}\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tan\right.}{4\sqrt{b}\sqrt{a-}}\right.}{}}{d}$

default	$\frac{\frac{2(-a+b)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(-2a+\frac{2b}{3}\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2(-a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{2a^2\left(-\frac{\ln\left(\sqrt{a+b}\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\,\mathrm{t}\cdots\right.}{4\sqrt{b}\sqrt{a\cdots}}\cdots\right.}{\cdots}}{d}$
risch	$\frac{2a\,e^{4dx+4c}+4a\,e^{2dx+2c}-4b\,e^{2dx+2c}+2a-\frac{4b}{3}}{b^2d\left(1+e^{2dx+2c}\right)^3}+\frac{a^2\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^2}+2b\sqrt{ab+b^2}-2ab-2b^2}{a\sqrt{ab+b^2}}\right)}{2\sqrt{ab+b^2}\,db^2}-\frac{a^2\ln\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^2*((-a+b)*tanh(1/2*d*x+1/2*c)^5+(-2*a+2/3*b)*tanh(1/2*d*x+1/2*c)^3
+(-a+b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+1)^3-2*a^2/b^2*(-1/4/b^
(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c
)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+
1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(67) = 134$.
time = 0.51, size = 160, normalized size = 2.08

$$-\frac{a^2\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,\sqrt{(a+b)b}\,b^2d}-\frac{2\left(6\,(a-b)e^{(-2\,dx-2\,c)}+3\,ae^{(-4\,dx-4\,c)}+3\,a-2\,b\right)}{3\,(3\,b^2e^{(-2\,dx-2\,c)}+3\,b^2e^{(-4\,dx-4\,c)}+b^2e^{(-6\,dx-6\,c)}+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*
x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*b^2*d) - 2/3*(6*(
a - b)*e^(-2*d*x - 2*c) + 3*a*e^(-4*d*x - 4*c) + 3*a - 2*b)/((3*b^2*e^(-2*d
*x - 2*c) + 3*b^2*e^(-4*d*x - 4*c) + b^2*e^(-6*d*x - 6*c) + b^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(67) = 134$.
time = 0.39, size = 1905, normalized size = 24.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b + a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + 12*(a^2*b + a*b^2)*sinh(d*x + c)^4 + 12*a^2*b + 4*a*b^2
```

- 8*b^3 + 24*(a^2*b - b^3)*cosh(d*x + c)^2 + 24*(a^2*b - b^3 + 3*(a^2*b + a
*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(a^2*cosh(d*x + c)^6 + 6*a^2*cos
h(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x + c)^4 +
3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2 + 4
*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a^2*c
osh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 6*(a^
2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c
))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*c
osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^
2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(
d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)
*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 +
a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 +
a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*
sinh(d*x + c) + a)) + 48*((a^2*b + a*b^2)*cosh(d*x + c)^3 + (a^2*b - b^3)*c
osh(d*x + c))*sinh(d*x + c))/((a*b^3 + b^4)*d*cosh(d*x + c)^6 + 6*(a*b^3 +
b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a*b^3 + b^4)*d*sinh(d*x + c)^6 + 3*
(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 3*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^2 + (a
*b^3 + b^4)*d)*sinh(d*x + c)^4 + 3*(a*b^3 + b^4)*d*cosh(d*x + c)^2 + 4*(5*(
a*b^3 + b^4)*d*cosh(d*x + c)^3 + 3*(a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 6*(a*b^3 + b^4)*d*cosh(d*x
+ c)^2 + (a*b^3 + b^4)*d)*sinh(d*x + c)^2 + (a*b^3 + b^4)*d + 6*((a*b^3 + b
^4)*d*cosh(d*x + c)^5 + 2*(a*b^3 + b^4)*d*cosh(d*x + c)^3 + (a*b^3 + b^4)*d
*cosh(d*x + c))*sinh(d*x + c)), 1/3*(6*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 24
*(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 6*(a^2*b + a*b^2)*sinh(d*x
+ c)^4 + 6*a^2*b + 2*a*b^2 - 4*b^3 + 12*(a^2*b - b^3)*cosh(d*x + c)^2 + 12
*(a^2*b - b^3 + 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(a^2
*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^
6 + 3*a^2*cosh(d*x + c)^4 + 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4
+ 3*a^2*cosh(d*x + c)^2 + 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*(5*a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*s
inh(d*x + c)^2 + a^2 + 6*(a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2
*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)
^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a
*b - b^2)/(a*b + b^2)) + 24*((a^2*b + a*b^2)*cosh(d*x + c)^3 + (a^2*b - b^3
)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3 + b^4)*d*cosh(d*x + c)^6 + 6*(a*b^3
+ b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a*b^3 + b^4)*d*sinh(d*x + c)^6 +
3*(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 3*(5*(a*b^3 + b^4)*d*cosh(d*x + c)^2 +
(a*b^3 + b^4)*d)*sinh(d*x + c)^4 + 3*(a*b^3 + b^4)*d*cosh(d*x + c)^2 + 4*(
5*(a*b^3 + b^4)*d*cosh(d*x + c)^3 + 3*(a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d
x + c)^3 + 3(5*(a*b^3 + b^4)*d*cosh(d*x + c)^4 + 6*(a*b^3 + b^4)*d*cosh(d
*x + c)^2 + (a*b^3 + b^4)*d)*sinh(d*x + c)^2 + (a*b^3 + b^4)*d + 6*((a*b^3
+ b^4)*d*cosh(d*x + c)^5 + 2*(a*b^3 + b^4)*d*cosh(d*x + c)^3 + (a*b^3 + b^4
)*d*cosh(d*x + c))*sinh(d*x + c))]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**6/(a + b*sech(c + d*x)**2), x)
```

Giac [A]
time = 0.61, size = 118, normalized size = 1.53

$$\frac{\frac{3\,a^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,b^2}+\frac{2\left(3\,ae^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}+3\,a-2\,b\right)}{b^2\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt
(-a*b - b^2)*b^2) + 2*(3*a*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) - 6*b*e^(2
*d*x + 2*c) + 3*a - 2*b)/(b^2*(e^(2*d*x + 2*c) + 1)^3))/d
```

Mupad [B]
time = 1.99, size = 334, normalized size = 4.34

$$\frac{8}{3\,b\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}-\frac{4}{b\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}+\frac{2\,a}{b^2\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{a^2\,\ln\left(\frac{4\,a^2\,\left(2\,a\,b+a^2+a^2\,\mathrm{e}^{2\,c+2\,d\,x}+8\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}+8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{b^5\,\left(a+b\right)}-\frac{8\,a^2\,\left(a+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}+4\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{b^{9/2}\,\sqrt{a+b}}\right)}{2\,b^{5/2}\,d\,\sqrt{a+b}}+\frac{a^2\,\ln\left(\frac{8\,a^2\,\left(a+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}+4\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{b^{9/2}\,\sqrt{a+b}}+\frac{4\,a^2\,\left(2\,a\,b+a^2+a^2\,\mathrm{e}^{2\,c+2\,d\,x}+8\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}+8\,a\,b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{b^5\,\left(a+b\right)}\right)}{2\,b^{5/2}\,d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)),x)
```

```
[Out] 8/(3*b*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
- 4/(b*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (2*a)/(b^2*d*(exp(2
*c + 2*d*x) + 1)) - (a^2*log((4*a^2*(2*a*b + a^2 + a^2*exp(2*c + 2*d*x) + 8
*b^2*exp(2*c + 2*d*x) + 8*a*b*exp(2*c + 2*d*x)))/(b^5*(a + b)) - (8*a^2*(a
+ 2*a*exp(2*c + 2*d*x) + 4*b*exp(2*c + 2*d*x)))/(b^(9/2)*(a + b)^(1/2))))/(
2*b^(5/2)*d*(a + b)^(1/2)) + (a^2*log((8*a^2*(a + 2*a*exp(2*c + 2*d*x) + 4*
b*exp(2*c + 2*d*x)))/(b^(9/2)*(a + b)^(1/2)) + (4*a^2*(2*a*b + a^2 + a^2*ex
p(2*c + 2*d*x) + 8*b^2*exp(2*c + 2*d*x) + 8*a*b*exp(2*c + 2*d*x)))/(b^5*(a
+ b))))/(2*b^(5/2)*d*(a + b)^(1/2))
```

3.83 $\int \frac{\cosh^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=125

$$\frac{b^2(6a+5b)\text{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}d}+\frac{(a-2b)\sinh(c+dx)}{a^3d}+\frac{\sinh^3(c+dx)}{3a^2d}-\frac{b^3\sinh(c+dx)}{2a^3(a+b)d\left(a+b+a\sinh^2(c-\right.}$$

```
[Out] 1/2*b^2*(6*a+5*b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(7/2)/(a+b)^(3/
2)/d+(a-2*b)*sinh(d*x+c)/a^3/d+1/3*sinh(d*x+c)^3/a^2/d-1/2*b^3*sinh(d*x+c)/
a^3/(a+b)/d/(a+b+a*sinh(d*x+c)^2)
```

Rubi [A]
time = 0.12, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4232, 398, 393, 211}

$$\frac{b^2(6a+5b)\text{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{7/2}d(a+b)^{3/2}}-\frac{b^3\sinh(c+dx)}{2a^3d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{(a-2b)\sinh(c+dx)}{a^3d}+\frac{\sinh^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (b^2*(6*a + 5*b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*a^(7/2)*(a
 + b)^(3/2)*d) + ((a - 2*b)*Sinh[c + d*x])/(a^3*d) + Sinh[c + d*x]^3/(3*a^2
*d) - (b^3*Sinh[c + d*x])/(2*a^3*(a + b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a-2b}{a^3}+\frac{x^2}{a^2}+\frac{b^2(3a+2b)+3ab^2x^2}{a^3\left(a+b+ax^2\right)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a-2b)\sinh(c+dx)}{a^3d}+\frac{\sinh^3(c+dx)}{3a^2d}+\frac{\text{Subst}\left(\int \frac{b^2(3a+2b)+3ab^2x^2}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh\right.}{a^3d}$$

$$= \frac{(a-2b)\sinh(c+dx)}{a^3d}+\frac{\sinh^3(c+dx)}{3a^2d}-\frac{b^3\sinh(c+dx)}{2a^3(a+b)d\left(a+b+a\sinh^2(c+d\right.}$$

$$= \frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}d}+\frac{(a-2b)\sinh(c+dx)}{a^3d}+\frac{\sinh^3(c+dx)}{3a^2d}$$

Mathematica [A]
time = 0.64, size = 113, normalized size = 0.90

$$\frac{-\frac{6b^2(6a+5b)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\text{csch}(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}}+3\sqrt{a}\left(3a-8b-\frac{4b^3}{(a+b)(a+2b+a\cosh(2(c+dx)))}\right)\sinh(c+dx)+a^{3/2}\sinh(3(c+dx))}{12a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((-6*b^2*(6*a + 5*b)*ArcTan[(Sqrt[a + b]*Csch[c + d*x])/Sqrt[a]])/(a + b)^(
3/2) + 3*Sqrt[a]*(3*a - 8*b - (4*b^3)/((a + b)*(a + 2*b + a*Cosh[2*(c + d*x
)])))*Sinh[c + d*x] + a^(3/2)*Sinh[3*(c + d*x)])/(12*a^(7/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(111) = 222$.

time = 3.11, size = 321, normalized size = 2.57

method	result
derivativedivides	$\frac{2b^2\left(\frac{\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b+2a}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(6a+5b)\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh}{2\sqrt{}}\right)}{2\sqrt{a+b}}\right.}{}\right.}{a^3}$
default	$\frac{2b^2\left(\frac{\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b+2a}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(6a+5b)\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh}{2\sqrt{}}\right)}{2\sqrt{a+b}}\right.}{}\right.}{a^3}$
risch	$\frac{e^{3dx+3c}}{24a^2d}+\frac{3\,e^{dx+c}}{8a^2d}-\frac{e^{dx+c}b}{a^3d}-\frac{3\,e^{-dx-c}}{8a^2d}+\frac{e^{-dx-c}b}{a^3d}-\frac{e^{-3dx-3c}}{24a^2d}-\frac{b^3e^{dx+c}\left(e^{2dx+2c}-1\right)}{a^3(a+b)d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^2/a^3*((1/2*b/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/2*b/(a+b)*tanh(1/2*d*x
+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+
1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)+1/2*(6*a+5*b)/(a+b)*(1/2/(a+b)^(1/2
)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))
+1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^
(1/2))/a^(1/2))))-1/3/a^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/a^2/(tanh(1/2*d*x+1
/2*c)-1)^2-(-2*b+a)/a^3/(tanh(1/2*d*x+1/2*c)-1)-1/3/a^2/(tanh(1/2*d*x+1/2*c
)+1)^3+1/2/a^2/(tanh(1/2*d*x+1/2*c)+1)^2-(-2*b+a)/a^3/(tanh(1/2*d*x+1/2*c)+
1))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(a^3 + a^2*b - (a^3*e^(10*c) + a^2*b*e^(10*c))*e^(10*d*x) - (11*a^3*e
^(8*c) - 9*a^2*b*e^(8*c) - 20*a*b^2*e^(8*c))*e^(8*d*x) - 2*(5*a^3*e^(6*c) +
 11*a^2*b*e^(6*c) - 42*a*b^2*e^(6*c) - 60*b^3*e^(6*c))*e^(6*d*x) + 2*(5*a^3
*e^(4*c) + 11*a^2*b*e^(4*c) - 42*a*b^2*e^(4*c) - 60*b^3*e^(4*c))*e^(4*d*x)
+ (11*a^3*e^(2*c) - 9*a^2*b*e^(2*c) - 20*a*b^2*e^(2*c))*e^(2*d*x))/((a^5*d*
e^(7*c) + a^4*b*d*e^(7*c))*e^(7*d*x) + 2*(a^5*d*e^(5*c) + 3*a^4*b*d*e^(5*c)
 + 2*a^3*b^2*d*e^(5*c))*e^(5*d*x) + (a^5*d*e^(3*c) + a^4*b*d*e^(3*c))*e^(3*
d*x)) + 1/8*integrate(8*((6*a*b^2*e^(3*c) + 5*b^3*e^(3*c))*e^(3*d*x) + (6*a
*b^2*e^c + 5*b^3*e^c)*e^(d*x))/(a^5 + a^4*b + (a^5*e^(4*c) + a^4*b*e^(4*c))
*e^(4*d*x) + 2*(a^5*e^(2*c) + 3*a^4*b*e^(2*c) + 2*a^3*b^2*e^(2*c))*e^(2*d*x
)), x)
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3084 vs. $2(111) = 222$.
time = 0.45, size = 5842, normalized size = 46.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^10 + 10*(a^5 + 2*a^4*b + a^3
*b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(d*x +
c)^10 + (11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^8 + (11*
a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3 + 45*(a^5 + 2*a^4*b + a^3*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c
)^3 + (11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^7 + 2*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*
x + c)^6 + 2*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4 + 105*
(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^4 + 14*(11*a^5 + 2*a^4*b - 29*a^3*b
^2 - 20*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^5 + 2*a^4*b +
a^3*b^2)*cosh(d*x + c)^5 + 14*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*
cosh(d*x + c)^3 + 3*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4
)*cosh(d*x + c))*sinh(d*x + c)^5 - a^5 - 2*a^4*b - a^3*b^2 - 2*(5*a^5 + 16*
a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^4 + 2*(105*(a^5
+ 2*a^4*b + a^3*b^2)*cosh(d*x + c)^6 - 5*a^5 - 16*a^4*b + 31*a^3*b^2 + 102*
a^2*b^3 + 60*a*b^4 + 35*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d
*x + c)^4 + 15*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cos
h(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x +
c)^7 + 7*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^5 + 5*(
5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c)^3 - (
5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x + c))*sinh
```

(d*x + c)^3 - (11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^2
+ (45*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^8 + 28*(11*a^5 + 2*a^4*b - 29
*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c)^6 - 11*a^5 - 2*a^4*b + 29*a^3*b^2 + 20
*a^2*b^3 + 30*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh
(d*x + c)^4 - 12*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 - 6*((6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^7
+ 7*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + (6*a^2*b^2 + 5*a*
b^3)*sinh(d*x + c)^7 + 2*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)^5 +
(12*a^2*b^2 + 34*a*b^3 + 20*b^4 + 21*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^5 + 5*(7*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^3 + 2*(6*a^2*b^
2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + (6*a^2*b^2 + 5*a*b^
3)*cosh(d*x + c)^3 + (35*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^4 + 6*a^2*b^2
+ 5*a*b^3 + 20*(6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cosh(d*x + c)^2)*sinh(d*x +
c)^3 + (21*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^5 + 20*(6*a^2*b^2 + 17*a*b^3
 + 10*b^4)*cosh(d*x + c)^3 + 3*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c))*sinh(d*
x + c)^2 + (7*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^6 + 10*(6*a^2*b^2 + 17*a*
b^3 + 10*b^4)*cosh(d*x + c)^4 + 3*(6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*si
nh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*
cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a
+ 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*
sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) -
 cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c
)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*
a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a +
2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 2*(5*(a^5 + 2*a^4*b + a^3*b^2)*co
sh(d*x + c)^9 + 4*(11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c
)^7 + 6*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x +
 c)^5 - 4*(5*a^5 + 16*a^4*b - 31*a^3*b^2 - 102*a^2*b^3 - 60*a*b^4)*cosh(d*x
 + c)^3 - (11*a^5 + 2*a^4*b - 29*a^3*b^2 - 20*a^2*b^3)*cosh(d*x + c))*sinh(
d*x + c))/((a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^7 + 7*(a^7 + 2*a^6*b +
 a^5*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^6 + (a^7 + 2*a^6*b + a^5*b^2)*d*sin
h(d*x + c)^7 + 2*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^5
+ (21*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^2 + 2*(a^7 + 4*a^6*b + 5*a^
5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^5 + (a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*
x + c)^3 + 5*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^3 + 2*(a^7 + 4*a^
6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35*(a^7 +
2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^4 + 20*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^
4*b^3)*d*cosh(d*x + c)^2 + (a^7 + 2*a^6*b + a^5*b^2)*d)*sinh(d*x + c)^3 + (
21*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^5 + 20*(a^7 + 4*a^6*b + 5*a^5*
b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^3 + 3*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x
 + c))*sinh(d*x + c)^2 + (7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^6 + 1
0*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^4 + 3*(a^7 + 2*a^
6*b + a^5*b^2)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/24*((a^5 + 2*a^4*b + a^
3*b^2)*cosh(d*x + c)^10 + 10*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)*sinh(d

```
*x + c)^9 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(d*x ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3\left(c+dx\right)}{\left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(cosh(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh\left(c+d\,x\right)}^3}{{\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2, x)
```

3.84 $\int \frac{\cosh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=144

$$\frac{(a-4b)x}{2a^3}+\frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)}+\frac{b(a+2b)\tanh(c+dx)}{2a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

```
[Out] 1/2*(a-4*b)*x/a^3+1/2*b^(3/2)*(5*a+4*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(
1/2))/a^3/(a+b)^(3/2)/d+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)^
2)+1/2*b*(a+2*b)*tanh(d*x+c)/a^2/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.16, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4231, 425, 541, 536, 212, 214}

$$\frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3d(a+b)^{3/2}}+\frac{x(a-4b)}{2a^3}+\frac{b(a+2b)\tanh(c+dx)}{2a^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a - 4*b)*x)/(2*a^3) + (b^(3/2)*(5*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a + b]])/(2*a^3*(a + b)^(3/2)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*
a*d*(a + b - b*Tanh[c + d*x]^2)) + (b*(a + 2*b)*Tanh[c + d*x])/(2*a^2*(a +
b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a-b-3bx^2}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b(a+2b)\tanh(c+dx)}{2a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Sul}}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{b(a+2b)\tanh(c+dx)}{2a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{(a}{}$$

$$= \frac{(a-4b)x}{2a^3} + \frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}d} + \frac{\cosh(c+dx)\sinh(c+}{2ad\left(a+b-b\tanh^2(c-\right.}$$

Mathematica [A]
time = 0.93, size = 103, normalized size = 0.72

$$\frac{2(a-4b)(c+dx) + \frac{2b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \left(a + \frac{2ab^2}{(a+b)(a+2b+a\cosh(2(c+dx)))}\right)\sinh(2(c+dx))}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (2*(a - 4*b)*(c + d*x) + (2*b^(3/2)*(5*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a + b]])/(a + b)^(3/2) + (a + (2*a*b^2)/((a + b)*(a + 2*b + a*Cos
h[2*(c + d*x)])))*Sinh[2*(c + d*x)])/(4*a^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(128) = 256$.
time = 3.12, size = 341, normalized size = 2.37

method	result
risch	$\frac{x}{2a^2} - \frac{2xb}{a^3} + \frac{e^{2dx+2c}}{8a^2d} - \frac{e^{-2dx-2c}}{8a^2d} - \frac{b^2\left(a\,e^{2dx+2c}+2b\,e^{2dx+2c}+a\right)}{a^3(a+b)d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)} + \frac{5\sqrt{b\,(a+b)}\,b\ln\left(\right.}{}$

derivativedivides	$\frac{-\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^3}-\frac{2b^2\left(\frac{-\frac{a\left(\tanh^3\left(\frac{dx}{2}\cdots\right.\right.}{2(a+b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{1}{2a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^3}-\frac{2b^2\left(\frac{-\frac{a\left(\tanh^3\left(\frac{dx}{2}\cdots\right.\right.}{2(a+b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\cdots}\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^2/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/a^2/(tanh(1/2*d*x+1/2*c)+1)+1/2
*(a-4*b)/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-2*b^2/a^3*((-1/2/(a+b)*a*tanh(1/2*d*
x+1/2*c)^3-1/2/(a+b)*a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh
(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)+
1/2*(5*a+4*b)/(a+b)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1
/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*
ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1
/2))))+1/2/a^2/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/a^2/(tanh(1/2*d*x+1/2*c)-1)+1/
2/a^3*(-a+4*b)*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(134) = 268$.
time = 0.52, size = 696, normalized size = 4.83

$$\frac{(3a^2b+12ab^2+8b^3)\log\left(\frac{ae^{(2dx+2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(2dx+2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{16(a^4+a^3b)\sqrt{(a+b)b}\,d}-\frac{(3a^2b+12ab^2+8b^3)\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{16(a^4+a^3b)\sqrt{(a+b)b}\,d}+\frac{(3ab+2b^2)\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{8(a^3+a^2b)\sqrt{(a+b)b}\,d}-\frac{a^2b+2ab^2+(a^2b+8ab^2+8b^3)e^{(2dx+2c)}}{4(a^5+a^4b+(a^5+a^4b)e^{(4dx+4c)}+2(a^5+3a^4b+2a^3b^2)e^{(2dx+2c)})d}+\frac{a^2b+2ab^2+(a^2b+8ab^2+8b^3)e^{(-2dx-2c)}}{4(a^5+a^4b+2(a^5+3a^4b+2a^3b^2)e^{(-2dx-2c)}+(a^5+a^4b)e^{(-4dx-4c)})d}-\frac{ab+(ab+2b^2)e^{(-2dx-2c)}}{2(a^4+a^3b+2(a^4+3a^3b+2a^2b^2)e^{(-2dx-2c)}+(a^4+a^3b)e^{(-4dx-4c)})d}+\frac{dx+c}{2a^2d}+\frac{e^{(2dx+2c)}}{8a^2d}-\frac{e^{(-2dx-2c)}}{8a^2d}-\frac{b\log\left(ae^{(4dx+4c)}+2(a+2b)e^{(2dx+2c)}+a\right)}{2a^3d}+\frac{b\log\left(2(a+2b)e^{(-2dx-2c)}+ae^{(-4dx-4c)}+a\right)}{2a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt
((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + a^3
*b)*sqrt((a + b)*b)*d) - 1/16*(3*a^2*b + 12*a*b^2 + 8*b^3)*log((a*e^(-2*d*x
 - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sq
rt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)*d) + 1/8*(3*a*b + 2*b^2)*log
((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a
 + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((a + b)*b)*d) - 1/4*(a^2*b
 + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(2*d*x + 2*c))/((a^5 + a^4*b + (a^
```

```
5 + a^4*b)*e^(4*d*x + 4*c) + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(2*d*x + 2*c))
*d) + 1/4*(a^2*b + 2*a*b^2 + (a^2*b + 8*a*b^2 + 8*b^3)*e^(-2*d*x - 2*c))/((
a^5 + a^4*b + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*e^(-2*d*x - 2*c) + (a^5 + a^4*b
)*e^(-4*d*x - 4*c))*d) - 1/2*(a*b + (a*b + 2*b^2)*e^(-2*d*x - 2*c))/((a^4 +
 a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2*c) + (a^4 + a^3*b)*e^(
-4*d*x - 4*c))*d) + 1/2*(d*x + c)/(a^2*d) + 1/8*e^(2*d*x + 2*c)/(a^2*d) - 1
/8*e^(-2*d*x - 2*c)/(a^2*d) - 1/2*b*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(
2*d*x + 2*c) + a)/(a^3*d) + 1/2*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-
4*d*x - 4*c) + a)/(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1731 vs. $2(134) = 268$.
time = 0.45, size = 3739, normalized size = 25.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((a^3 + a^2*b)*cosh(d*x + c)^8 + 8*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d*
x + c)^7 + (a^3 + a^2*b)*sinh(d*x + c)^8 + 2*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(
a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^6 + 2*(a^3 + 3*a^2*b + 2*a*b^2
+ 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x + 14*(a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(
d*x + c)^6 + 4*(14*(a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 2*a*b
^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(a
*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^4 + 2*(3
5*(a^3 + a^2*b)*cosh(d*x + c)^4 - 4*a*b^2 - 8*b^3 + 4*(a^3 - a^2*b - 10*a*b
^2 - 8*b^3)*d*x + 15*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)
*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + a^2*b)*cosh(d*x + c)^5
 + 5*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x +
 c)^3 - 4*(a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x +
 c))*sinh(d*x + c)^3 - a^3 - a^2*b - 2*(a^3 + 3*a^2*b + 6*a*b^2 - 2*(a^3 -
3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + a^2*b)*cosh(d*x + c)
^6 + 15*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*
x + c)^4 - a^3 - 3*a^2*b - 6*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x - 24*(
a*b^2 + 2*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 2*((5*a^2*b + 4*a*b^2)*cosh(d*x + c)^6 + 6*(5*a^2*b + 4*a*b^2
)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^2*b + 4*a*b^2)*sinh(d*x + c)^6 + 2*(
5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + (10*a^2*b + 28*a*b^2 + 16*b^3
 + 15*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b
+ 4*a*b^2)*cosh(d*x + c)^3 + 2*(5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c))*
sinh(d*x + c)^3 + (5*a^2*b + 4*a*b^2)*cosh(d*x + c)^2 + (15*(5*a^2*b + 4*a*
b^2)*cosh(d*x + c)^4 + 5*a^2*b + 4*a*b^2 + 12*(5*a^2*b + 14*a*b^2 + 8*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)^5
 + 4*(5*a^2*b + 14*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (5*a^2*b + 4*a*b^2)*cos
```

```
h(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2
*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh
(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2
 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*si
nh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(
a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*
x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*
sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x
+ c) + a)) + 4*(2*(a^3 + a^2*b)*cosh(d*x + c)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^
2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^5 - 8*(a*b^2 + 2*b^3 - (
a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^3 - (a^3 + 3*a^2*b + 6*a
*b^2 - 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5
 + a^4*b)*d*cosh(d*x + c)^6 + 6*(a^5 + a^4*b)*d*cosh(d*x + c)*sinh(d*x + c)
^5 + (a^5 + a^4*b)*d*sinh(d*x + c)^6 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh
(d*x + c)^4 + (15*(a^5 + a^4*b)*d*cosh(d*x + c)^2 + 2*(a^5 + 3*a^4*b + 2*a^
3*b^2)*d)*sinh(d*x + c)^4 + (a^5 + a^4*b)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + a
^4*b)*d*cosh(d*x + c)^3 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + (15*(a^5 + a^4*b)*d*cosh(d*x + c)^4 + 12*(a^5 + 3*a^4*b + 2
*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + a^4*b)*d)*sinh(d*x + c)^2 + 2*(3*(a^5
+ a^4*b)*d*cosh(d*x + c)^5 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cosh(d*x + c)^
3 + (a^5 + a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*((a^3 + a^2*b)*cosh(
d*x + c)^8 + 8*(a^3 + a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + a^2*b)*
sinh(d*x + c)^8 + 2*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*
d*x)*cosh(d*x + c)^6 + 2*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*
b^2)*d*x + 14*(a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(14*(a^3 +
 a^2*b)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4
*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(a*b^2 + 2*b^3 - (a^3 - a^2
*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + a^2*b)*cosh(d*x
+ c)^4 - 4*a*b^2 - 8*b^3 + 4*(a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x + 15*(a^3
 + 3*a^2*b + 2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^2)*si
nh(d*x + c)^4 + 8*(7*(a^3 + a^2*b)*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 2*a
*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^3 - 4*(a*b^2 + 2*b^3
- (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - a^
3 - a^2*b - 2*(a^3 + 3*a^2*b + 6*a*b^2 - 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*c
osh(d*x + c)^2 + 2*(14*(a^3 + a^2*b)*cosh(d*x + c)^6 + 15*(a^3 + 3*a^2*b +
2*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x)*cosh(d*x + c)^4 - a^3 - 3*a^2*b
- 6*a*b^2 + 2*(a^3 - 3*a^2*b - 4*a*b^2)*d*x - 24*(a*b^2 + 2*b^3 - (a^3 - a^
2*b - 10*a*b^2 - 8*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((5*a^2*b
 + 4*a*b^2)*cosh(d*x + c)^6 + 6*(5*a^2*b + 4*a*b^2)*cosh(d*x + c)*sinh(d*x
+ c)^5 + (5*a^2*b + 4*a*b^2)*sinh(d*x + c)^6 + ...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(cosh(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(134) = 268.
time = 1.06, size = 323, normalized size = 2.24

$$\frac{\frac{12\,(5\,ab^2+4\,b^3)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{(a^4+a^3b)\sqrt{-ab-b^2}}-\frac{2\,a^3e^{(6\,dx+6\,c)}-6\,a^2be^{(6\,dx+6\,c)}-8\,ab^2e^{(6\,dx+6\,c)}+7\,a^3e^{(4\,dx+4\,c)}-a^2be^{(4\,dx+4\,c)}-16\,ab^2e^{(4\,dx+4\,c)}+16\,b^3e^{(4\,dx+4\,c)}+8\,a^3e^{(2\,dx+2\,c)}+12\,a^2be^{(2\,dx+2\,c)}+28\,ab^2e^{(2\,dx+2\,c)}+3\,a^3+3\,a^2b}{(a^4+a^3b)\left(ae^{(6\,dx+6\,c)}+2\,ae^{(4\,dx+4\,c)}+4\,be^{(4\,dx+4\,c)}+ae^{(2\,dx+2\,c)}\right)}+\frac{12\,(dx+c)(a-4\,b)}{a^3}+\frac{3\,e^{(2\,dx+2\,c)}}{a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(12*(5*a*b^2 + 4*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a
*b - b^2))/((a^4 + a^3*b)*sqrt(-a*b - b^2)) - (2*a^3*e^(6*d*x + 6*c) - 6*a^
2*b*e^(6*d*x + 6*c) - 8*a*b^2*e^(6*d*x + 6*c) + 7*a^3*e^(4*d*x + 4*c) - a^2
*b*e^(4*d*x + 4*c) - 16*a*b^2*e^(4*d*x + 4*c) + 16*b^3*e^(4*d*x + 4*c) + 8*
a^3*e^(2*d*x + 2*c) + 12*a^2*b*e^(2*d*x + 2*c) + 28*a*b^2*e^(2*d*x + 2*c) +
 3*a^3 + 3*a^2*b)/((a^4 + a^3*b)*(a*e^(6*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) +
 4*b*e^(4*d*x + 4*c) + a*e^(2*d*x + 2*c))) + 12*(d*x + c)*(a - 4*b)/a^3 + 3
*e^(2*d*x + 2*c)/a^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^2}{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2, x)
```

3.85 $\int \frac{\cosh(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=100

$$-\frac{b(4a+3b)\text{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}d}+\frac{\sinh(c+dx)}{a^2d}+\frac{b^2\sinh(c+dx)}{2a^2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

```
[Out] -1/2*b*(4*a+3*b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/(a+b)^(3/2
)/d+sinh(d*x+c)/a^2/d+1/2*b^2*sinh(d*x+c)/a^2/(a+b)/d/(a+b+a*sinh(d*x+c)^2)
```

Rubi [A]
time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4232, 398, 393, 211}

$$-\frac{b(4a+3b)\text{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{5/2}d(a+b)^{3/2}}+\frac{b^2\sinh(c+dx)}{2a^2d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\sinh(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/2*(b*(4*a + 3*b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(a^(5/2)*(
a + b)^(3/2)*d) + Sinh[c + d*x]/(a^2*d) + (b^2*Sinh[c + d*x])/(2*a^2*(a + b
)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+b+ax^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2}-\frac{b(2a+b)+2abx^2}{a^2(a+b+ax^2)^2}\right)\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\sinh(c+dx)}{a^2d}-\frac{\text{Subst}\left(\int \frac{b(2a+b)+2abx^2}{(a+b+ax^2)^2}\,dx, x, \sinh(c+dx)\right)}{a^2d}\\ &= \frac{\sinh(c+dx)}{a^2d}+\frac{b^2\sinh(c+dx)}{2a^2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}-\frac{(b(4a+3b))\text{Subst}\left(\int \right.}{2a\ldots}\\ &= -\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}d}+\frac{\sinh(c+dx)}{a^2d}+\frac{b^2\sinh(c+\ldots}{2a^2(a+b)d\left(a+b+a\ldots\right.}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 234 vs. 2(100) = 200.

time = 1.25, size = 234, normalized size = 2.34

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^3(c+dx)\left(\frac{b(4a+3b)\text{ArcTan}\left(\frac{\sqrt{a+b}\operatorname{csch}(c+dx)\sqrt{(\cosh(c)-\sinh(c))^2}(\cosh(c)+\sinh(c))}{\sqrt{a}}\right)(a+2b+a\cosh(2(c+dx)))\operatorname{sech}(c+dx)(\cosh(c)-\sinh(c))}{(a+b)^{3/2}\sqrt{(\cosh(c)-\sinh(c))^2}}+2\sqrt{a}\cosh(dx)(a+2b+a\cosh(2(c+dx)))\operatorname{sech}(c+dx)\sinh(c)+2\sqrt{a}\cosh(c)(a+2b+a\cosh(2(c+dx)))\operatorname{sech}(c+dx)\sinh(dx)+\frac{2\sqrt{a}\,b^2\tanh(c+dx)}{a+b}\right)}{8a^{5/2}d\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*((b*(4*a + 3*b)*ArcTan[(Sq
rt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sq
rt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]*(Cosh[c] - Sinh[c]))/(
(a + b)^(3/2)*Sqrt[(Cosh[c] - Sinh[c])^2]) + 2*Sqrt[a]*Cosh[d*x]*(a + 2*b +
 a*Cosh[2*(c + d*x)])*Sech[c + d*x]*Sinh[c] + 2*Sqrt[a]*Cosh[c]*(a + 2*b +
```

```
a*Cosh[2*(c + d*x)])*Sech[c + d*x]*Sinh[d*x] + (2*Sqrt[a]*b^2*Tanh[c + d*x]
)/(a + b)))/(8*a^(5/2)*d*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(88) = 176$.

time = 2.85, size = 237, normalized size = 2.37

method	result
derivativedivides	$-\frac{\frac{2b\left(\frac{\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b+2a}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(4a+3b)\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\mathrm{ta}}{2}\right.}{2\sqrt{a+}}\right.}{}\right.}{a^2}}{d}$
default	$-\frac{\frac{2b\left(\frac{\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b+2a}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(4a+3b)\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\mathrm{ta}}{2}\right.}{2\sqrt{a+}}\right.}{}\right.}{a^2}}{d}$
risch	$\frac{e^{dx+c}}{2a^2d}-\frac{e^{-dx-c}}{2a^2d}+\frac{b^2e^{dx+c}\left(e^{2dx+2c}-1\right)}{a^2d(a+b)\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)}-\frac{b\ln\left(e^{2dx+2c}+\frac{2(a+b)e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{\sqrt{-a^2-ab}\,(a+b)da}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b/a^2*((1/2*b/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/2*b/(a+b)*tanh(1/2*d*x+
1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1
/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)+1/2*(4*a+3*b)/(a+b)*(1/2/(a+b)^(1/2)
/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+
1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(
1/2))/a^(1/2))))-1/a^2/(tanh(1/2*d*x+1/2*c)-1)-1/a^2/(tanh(1/2*d*x+1/2*c)+1
))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2 + a*b - (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) - (a^2*e^(4*c) + 5*
a*b*e^(4*c) + 6*b^2*e^(4*c))*e^(4*d*x) + (a^2*e^(2*c) + 5*a*b*e^(2*c) + 6*b
^2*e^(2*c))*e^(2*d*x))/((a^4*d*e^(5*c) + a^3*b*d*e^(5*c))*e^(5*d*x) + 2*(a^
4*d*e^(3*c) + 3*a^3*b*d*e^(3*c) + 2*a^2*b^2*d*e^(3*c))*e^(3*d*x) + (a^4*d*e
^c + a^3*b*d*e^c)*e^(d*x)) - 1/2*integrate(2*((4*a*b*e^(3*c) + 3*b^2*e^(3*c
))*e^(3*d*x) + (4*a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^4 + a^3*b + (a^4*e^(4*c)
 + a^3*b*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 3*a^3*b*e^(2*c) + 2*a^2*b^2*
e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1636 vs. $2(88) = 176$.
time = 0.43, size = 3154, normalized size = 31.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 12*(a^4 + 2*a^3*b + a^2
*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a^4 + 2*a^3*b + a^2*b^2)*sinh(d*x
+ c)^6 + 2*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^4 + 2*(a^4
+ 6*a^3*b + 11*a^2*b^2 + 6*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 - 2*a^4 - 4*a^3*b - 2*a^2*b^2 + 8*(5*(a^4 + 2*a^3*b +
 a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x
 + c))*sinh(d*x + c)^3 - 2*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x
+ c)^2 + 2*(15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - a^4 - 6*a^3*b -
11*a^2*b^2 - 6*a*b^3 + 6*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 - ((4*a^2*b + 3*a*b^2)*cosh(d*x + c)^5 + 5*(4*a^2*b +
 3*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (4*a^2*b + 3*a*b^2)*sinh(d*x + c)
^5 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b + 11*a*b^2
 + 6*b^3 + 5*(4*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(4
*a^2*b + 3*a*b^2)*cosh(d*x + c)^3 + 3*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x
 + c))*sinh(d*x + c)^2 + (4*a^2*b + 3*a*b^2)*cosh(d*x + c) + (5*(4*a^2*b +
3*a*b^2)*cosh(d*x + c)^4 + 4*a^2*b + 3*a*b^2 + 6*(4*a^2*b + 11*a*b^2 + 6*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4
+ 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cos
h(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*c
osh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(cosh(d*x + c
)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^
2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*c
```

```
osh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*(3*(a^4 +
 2*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 2*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b
^3)*cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))
*sinh(d*x + c))/((a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)^5 + 5*(a^6 + 2*a
^5*b + a^4*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^4 + (a^6 + 2*a^5*b + a^4*b^2)
*d*sinh(d*x + c)^5 + 2*(a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*d*cosh(d*x +
 c)^3 + 2*(5*(a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)^2 + (a^6 + 4*a^5*b +
 5*a^4*b^2 + 2*a^3*b^3)*d)*sinh(d*x + c)^3 + (a^6 + 2*a^5*b + a^4*b^2)*d*co
sh(d*x + c) + 2*(5*(a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)^3 + 3*(a^6 + 4
*a^5*b + 5*a^4*b^2 + 2*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*(a^6
+ 2*a^5*b + a^4*b^2)*d*cosh(d*x + c)^4 + 6*(a^6 + 4*a^5*b + 5*a^4*b^2 + 2*a
^3*b^3)*d*cosh(d*x + c)^2 + (a^6 + 2*a^5*b + a^4*b^2)*d)*sinh(d*x + c)), 1/
2*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 6*(a^4 + 2*a^3*b + a^2*b^2)*
cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(d*x + c)^6 +
 (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^4 + (a^4 + 6*a^3*b +
11*a^2*b^2 + 6*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^2)*sinh(d
*x + c)^4 - a^4 - 2*a^3*b - a^2*b^2 + 4*(5*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d
*x + c)^3 + (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^3 - (a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2 + (15*(a^4
+ 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - a^4 - 6*a^3*b - 11*a^2*b^2 - 6*a*b^3
 + 6*(a^4 + 6*a^3*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 - ((4*a^2*b + 3*a*b^2)*cosh(d*x + c)^5 + 5*(4*a^2*b + 3*a*b^2)*cosh(d*x +
 c)*sinh(d*x + c)^4 + (4*a^2*b + 3*a*b^2)*sinh(d*x + c)^5 + 2*(4*a^2*b + 11
*a*b^2 + 6*b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3 + 5*(4*a^2*
b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(4*a^2*b + 3*a*b^2)*co
sh(d*x + c)^3 + 3*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c))*sinh(d*x + c)
^2 + (4*a^2*b + 3*a*b^2)*cosh(d*x + c) + (5*(4*a^2*b + 3*a*b^2)*cosh(d*x +
c)^4 + 4*a^2*b + 3*a*b^2 + 6*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x
 + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*
a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) - ((4*a^2*b
+ 3*a*b^2)*cosh(d*x + c)^5 + 5*(4*a^2*b + 3*a*b^2)*cosh(d*x + c)*sinh(d*x +
 c)^4 + (4*a^2*b + 3*a*b^2)*sinh(d*x + c)^5 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3
)*cosh(d*x + c)^3 + 2*(4*a^2*b + 11*a*b^2 + 6*b^3 + 5*(4*a^2*b + 3*a*b^2)*c
osh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(4*a^2*b + 3*a*b^2)*cosh(d*x + c)^3
+ 3*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (4*a^2*b
+ 3*a*b^2)*cosh(d*x + c) + (5*(4*a^2*b + 3*a*b^2)*cosh(d*x + c)^4 + 4*a^2*b
 + 3*a*b^2 + 6*(4*a^2*b + 11*a*b^2 + 6*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))
*sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))
/(a + b)) + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 2*(a^4 + 6*a^3
*b + 11*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^3 - (a^4 + 6*a^3*b + 11*a^2*b^2 +
6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d*
x + c)^5 + 5*(a^6 + 2*a^5*b + a^4*b^2)*d*cosh(d...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(cosh(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)}{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^2, x)
```

3.86 $\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=82

$$\frac{(2a+b)\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}d}-\frac{b\sinh(c+dx)}{2a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*(2*a+b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(3/2)/d-1
/2*b*sinh(d*x+c)/a/(a+b)/d/(a+b+a*sinh(d*x+c)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4232, 393, 211}

$$\frac{(2a+b)\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{3/2}d(a+b)^{3/2}}-\frac{b\sinh(c+dx)}{2ad(a+b)\left(a\sinh^2(c+dx)+a+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((2*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b)^
(3/2)*d) - (b*Sinh[c + d*x])/(2*a*(a + b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
```

```
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{(a+b+ax^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\sinh(c+dx)}{2a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{(2a+b)\operatorname{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(\right.}{2a(a+b)d}$$

$$= \frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}d} - \frac{b\sinh(c+dx)}{2a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

Mathematica [A]
time = 0.21, size = 124, normalized size = 1.51

$$\frac{(2a^2+3ab+b^2)\operatorname{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right) - \sqrt{a}\,b\sqrt{a+b}\sinh(c+dx) + a(2a+b)\operatorname{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)\sinh^2(c+dx)}{a^{3/2}(a+b)^{3/2}d(a+2b+a\cosh(2(c+dx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((2*a^2 + 3*a*b + b^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]] - Sqrt[a
]*b*Sqrt[a + b]*Sinh[c + d*x] + a*(2*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/
Sqrt[a + b]]*Sinh[c + d*x]^2)/(a^(3/2)*(a + b)^(3/2)*d*(a + 2*b + a*Cosh[2*
(c + d*x)]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(70) = 140$.
time = 2.15, size = 201, normalized size = 2.45

method	result
derivativedivides	$\frac{\frac{\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a(a+b)} - \frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(a+b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b} + (2a+b)\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\tanh\left(\frac{dx}{2}+\right.}{2\sqrt{a}}\right.}{2\sqrt{a+b}\sqrt{a}}\right.}{d}$

default	$\frac{\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a(a+b)}-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(a+b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(2a+b)\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}\right.}{2\sqrt{a}}\right.}{2\sqrt{a+b}\,\sqrt{}}\right.}{d}$
risch	$-\frac{e^{dx+c}b\left(e^{2dx+2c}-1\right)}{ad(a+b)\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2(a+b)e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,(a+b)d}-\frac{b\ln\left(e^{2dx+2c}-\frac{2(a+b}{\sqrt{-a^2}}\right.}{4\sqrt{-a^2-ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(1/2/a*b/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/2/a*b/(a+b)*tanh(1/2*d*x+1/2*
c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c
)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)+(2*a+b)/a/(a+b)*(1/2/(a+b)^(1/2)/a^(1/2)
*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+1/2/(a+b
)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^
(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3*d + a^2*b*d + (a^3*d*e^(4*c) + a^
2*b*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c) + 3*a^2*b*d*e^(2*c) + 2*a*b^2*d
*e^(2*c))*e^(2*d*x)) + 2*integrate(1/2*((2*a*e^(3*c) + b*e^(3*c))*e^(3*d*x)
 + (2*a*e^c + b*e^c)*e^(d*x))/(a^3 + a^2*b + (a^3*e^(4*c) + a^2*b*e^(4*c))*
e^(4*d*x) + 2*(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 2*a*b^2*e^(2*c))*e^(2*d*x)),
 x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(70) = 140$.
time = 0.40, size = 1856, normalized size = 22.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b + a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^2 + 4*(a^2*b + a*b^2)*sinh(d*x + c)^3 + ((2*a^2 + a*b)*cosh(
```

```
d*x + c)^4 + 4*(2*a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + a*b)*
sinh(d*x + c)^4 + 2*(2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 +
 a*b)*cosh(d*x + c)^2 + 2*a^2 + 5*a*b + 2*b^2)*sinh(d*x + c)^2 + 2*a^2 + a*
b + 4*((2*a^2 + a*b)*cosh(d*x + c)^3 + (2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c
))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*
(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 -
(3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x
+ c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x +
 c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*
x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 +
2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 +
(a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*(a^2*b + a*b^2)*cosh(d*x +
 c) - 4*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/
((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*
d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + 2*a^4*b + a^3*b^2)*d*sinh(d*x + c)
^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^
5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a
^2*b^3)*d)*sinh(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b^2)*d + 4*((a^5 + 2*a^4*
b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*
cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 6*
(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2*b + a*b^2)*sinh(d*x
+ c)^3 - ((2*a^2 + a*b)*cosh(d*x + c)^4 + 4*(2*a^2 + a*b)*cosh(d*x + c)*sin
h(d*x + c)^3 + (2*a^2 + a*b)*sinh(d*x + c)^4 + 2*(2*a^2 + 5*a*b + 2*b^2)*co
sh(d*x + c)^2 + 2*(3*(2*a^2 + a*b)*cosh(d*x + c)^2 + 2*a^2 + 5*a*b + 2*b^2)
*sinh(d*x + c)^2 + 2*a^2 + a*b + 4*((2*a^2 + a*b)*cosh(d*x + c)^3 + (2*a^2
+ 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b)*arctan(1/2*(
a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 +
 (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*b)*sinh(d*x + c
))/sqrt(a^2 + a*b)) - ((2*a^2 + a*b)*cosh(d*x + c)^4 + 4*(2*a^2 + a*b)*cosh
(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + a*b)*sinh(d*x + c)^4 + 2*(2*a^2 + 5*a*
b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + a*b)*cosh(d*x + c)^2 + 2*a^2 + 5
*a*b + 2*b^2)*sinh(d*x + c)^2 + 2*a^2 + a*b + 4*((2*a^2 + a*b)*cosh(d*x + c
)^3 + (2*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b)
*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b)) - 2*(a
^2*b + a*b^2)*cosh(d*x + c) - 2*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*cosh(d*x
 + c)^2)*sinh(d*x + c))/((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a
^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + 2*a^4*b +
a^3*b^2)*d*sinh(d*x + c)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*co
sh(d*x + c)^2 + 2*(3*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 4
*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d)*sinh(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b
^2)*d + 4*((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 4*a^4*b + 5
*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+d\,x)\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^2), x)
```

3.87 $\int \frac{\text{sech}^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2} dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,(a+b)^{3/2}d}+\frac{\tanh(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

```
[Out] 1/2*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/(a+b)^(3/2)/d/b^(1/2)+1/2*tanh
(d*x+c)/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4231, 205, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,d(a+b)^{3/2}}+\frac{\tanh(c+dx)}{2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)*d) +
Tanh[c + d*x]/(2*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
```

```
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\tanh(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}+\frac{\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{2(a+b)d}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,(a+b)^{3/2}d}+\frac{\tanh(c+dx)}{2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 187 vs. $2(74) = 148$.
time = 0.75, size = 187, normalized size = 2.53

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^4(c+dx)\left(\frac{\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\,\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(a+2b+a\cosh(2(c+dx)))(\cosh(2c)-\sinh(2c))}{\sqrt{a+b}\,\sqrt{b(\cosh(c)-\sinh(c))^4}}+\operatorname{sech}(2c)\sinh(2dx)-\frac{(a+2b)\tanh(2c)}{a}\right)}{8(a+b)d\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((ArcTanh[(Sech[d*x]*(Cosh
[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b
]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c
] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + Sech[2*c]*Sin
h[2*d*x] - ((a + 2*b)*Tanh[2*c])/a))/(8*(a + b)*d*(a + b*Sech[c + d*x]^2)^2
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(62) = 124$.
time = 1.61, size = 206, normalized size = 2.78

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}-\frac{-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\right.}{4\sqrt{b}\,\sqrt{}}}{}}{d}$

default	$-\frac{\frac{2\left(-\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}-\frac{-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\cdots\right.}{4\sqrt{b}\,\sqrt{a\cdots}}}{\cdots}}{d}$
risch	$-\frac{a\,\mathrm{e}^{2dx+2c}+2b\,\mathrm{e}^{2dx+2c}+a}{ad(a+b)\left(a\,\mathrm{e}^{4dx+4c}+2a\,\mathrm{e}^{2dx+2c}+4b\,\mathrm{e}^{2dx+2c}+a\right)}+\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{a\sqrt{ab+b^2}+2b\sqrt{ab+b^2}-2ab-2b^2}{a\sqrt{ab+b^2}}\right)}{4\sqrt{ab+b^2}\,(a+b)d}-\frac{\ln\left(\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2/(a+b)*tanh(1/2*d*x+1/2*c)^3-1/2/(a+b)*tanh(1/2*d*x+1/2*c))/(a
*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*
b*tanh(1/2*d*x+1/2*c)^2+a+b)-1/(a+b)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/
2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(
1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)
*b^(1/2)+(a+b)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(65) = 130$.
time = 0.56, size = 150, normalized size = 2.03

$$\frac{(a+2\,b)e^{(-2\,dx-2\,c)}+a}{\left(a^3+a^2b+2\,(a^3+3\,a^2b+2\,ab^2)e^{(-2\,dx-2\,c)}+(a^3+a^2b)e^{(-4\,dx-4\,c)}\right)d}-\frac{\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,\sqrt{(a+b)b}\,(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((a + 2*b)*e^(-2*d*x - 2*c) + a)/((a^3 + a^2*b + 2*(a^3 + 3*a^2*b + 2*a*b^2
)*e^(-2*d*x - 2*c) + (a^3 + a^2*b)*e^(-4*d*x - 4*c))*d) - 1/4*log((a*e^(-2*
d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2
*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a + b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(65) = 130$.
time = 0.37, size = 1489, normalized size = 20.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 8*
(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b + 3*a*b^2
```

```
+ 2*b^3)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(
d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a
^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x +
 c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^
2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)
^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a
*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 +
 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x
+ c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(
d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a +
 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 +
 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))/((a^4
*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b + 2*a^3*b^2 + a^2*b^
3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d*sinh(d
*x + c)^4 + 2*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*cosh(d*x + c)^2 +
 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^4*b + 4*a^3*b^2
+ 5*a^2*b^3 + 2*a*b^4)*d)*sinh(d*x + c)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d
 + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^4*b + 4*a^3*b^2
+ 5*a^2*b^3 + 2*a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a^2*b + 2*a
*b^2 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 4*(a^2*b + 3*a*b^2 + 2
*b^3)*cosh(d*x + c)*sinh(d*x + c) + 2*(a^2*b + 3*a*b^2 + 2*b^3)*sinh(d*x +
c)^2 - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sin
h(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 +
 a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c
)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-
a*b - b^2)/(a*b + b^2)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 +
 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b +
 2*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3
+ 2*a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*
x + c)^2 + (a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d)*sinh(d*x + c)^2 + (
a^4*b + 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*
x + c)^3 + (a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*cosh(d*x + c))*sinh(
d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
[Out] Integral(sech(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [A]
time = 0.70, size = 130, normalized size = 1.76

$$\frac{\frac{\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,(a+b)}-\frac{2\,\left(ae^{(2\,dx+2\,c)}+2\,be^{(2\,dx+2\,c)}+a\right)}{(a^2+ab)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b
- b^2)*(a + b)) - 2*(a*e^(2*d*x + 2*c) + 2*b*e^(2*d*x + 2*c) + a)/((a^2 + a
*b)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^2\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2), x)
```

3.88 $\int \frac{\mathrm{sech}^3(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=73

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}\,(a+b)^{3/2}d}+\frac{\sinh(c+dx)}{2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*sinh(d*x+c)/(a+b)/d/(a+b+a*sinh(d*x+c)^2)+1/2*arctan(sinh(d*x+c)*a^(1/2
)/(a+b)^(1/2))/(a+b)^(3/2)/d/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4232, 205, 211}

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}\,d(a+b)^{3/2}}+\frac{\sinh(c+dx)}{2d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)*d) + S
inh[c + d*x]/(2*(a + b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
```

```
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+b+ax^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+b+ax^2}\,dx, x, \sinh(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{a}\,(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

Mathematica [A]
time = 0.16, size = 108, normalized size = 1.48

$$\frac{(a+2b+a\cosh(2c+2dx))^2\operatorname{sech}^4(c+dx)\left(\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{3/2}} + \frac{\sinh(c+dx)}{(a+b)\left(a+b+a\sinh^2(c+dx)\right)}\right)}{8d\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*Sech[c + d*x]^4*(ArcTan[(Sqrt[a]*Sinh[c
+ d*x])/Sqrt[a + b]]/(Sqrt[a]*(a + b)^(3/2)) + Sinh[c + d*x]/((a + b)*(a +
b + a*Sinh[c + d*x]^2))))/(8*d*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(61) = 122$.
time = 1.71, size = 185, normalized size = 2.53

method	result
risch	$\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{d(a+b)\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)} - \frac{\ln\left(e^{2dx+2c}-\frac{2(a+b)e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{4\sqrt{-a^2-ab}\,(a+b)d} + \frac{\ln\left(e^{2dx+2c}+\frac{2(a+b)e^{dx+c}}{\sqrt{-a^2-ab}}\right.}{4\sqrt{-a^2-ab}\,(a+b}$

derivativedivides	$\frac{\frac{-\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{a+b}+\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2b+2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2}{2\sqrt{a}}\right.}{2\sqrt{a+b}\,\sqrt{a}}}{d}$
default	$\frac{\frac{-\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{a+b}+\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2b+2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2}{2\sqrt{a}}\right.}{2\sqrt{a+b}\,\sqrt{a}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/2/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/2/(a+b)*tanh(1/2*d*x+1/2*c))/(a*
tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b
*tanh(1/2*d*x+1/2*c)^2+a+b)+1/(a+b)*(1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*
(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+1/2/(a+b)^(1/2)/a^(1/2)
*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (e^(3*d*x + 3*c) - e^(d*x + c))/(a^2*d + a*b*d + (a^2*d*e^(4*c) + a*b*d*e^(
4*c))*e^(4*d*x) + 2*(a^2*d*e^(2*c) + 3*a*b*d*e^(2*c) + 2*b^2*d*e^(2*c))*e^(
2*d*x)) + 8*integrate(1/8*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^2 + a*b + (a^2
*e^(4*c) + a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) + 3*a*b*e^(2*c) + 2*b^2*
e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(61) = 122$.
time = 0.41, size = 1570, normalized size = 21.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2 + a*b)*cosh(d*x + c)^3 + 12*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x
 + c)^2 + 4*(a^2 + a*b)*sinh(d*x + c)^3 - (a*cosh(d*x + c)^4 + 4*a*cosh(d*x
```

```
 + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2
*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (
a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(-a^2 - a*b)*log((a*cosh(d*x
 + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a +
2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2
+ 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh
(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d
*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh
(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a
+ 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2
+ 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*(
a^2 + a*b)*cosh(d*x + c) + 4*(3*(a^2 + a*b)*cosh(d*x + c)^2 - a^2 - a*b)*si
nh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*
b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*
sinh(d*x + c)^4 + 2*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c)^2
 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 5*a^
2*b^2 + 2*a*b^3)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4
 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*
b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a^2 + a*b)*cosh(d*x + c)^3 +
6*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2 + a*b)*sinh(d*x + c)^3
 + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)
^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d
*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) +
 a)*sqrt(a^2 + a*b)*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(
d*x + c)^2 + a*sinh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x
+ c)^2 + 3*a + 4*b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + (a*cosh(d*x + c)^4 +
4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*
x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*
x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a^2 + a*b)*arct
an(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/(a + b)) - 2*(a^2 +
a*b)*cosh(d*x + c) + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x
 + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*b + a^
2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d
*x + c)^4 + 2*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*cosh(d*x + c)^2 + 2*(
3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 5*a^2*b^2
+ 2*a*b^3)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a
^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d
*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^3\,\left(a+\frac{b}{\cosh\left(c+d\,x\right)^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2), x)
```

3.89 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2} dx$

Optimal. Leaf size=83

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}d}-\frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

```
[Out] 1/2*(a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(3/2)/(a+b)^(3/2)/d-
1/2*a*tanh(d*x+c)/b/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4231, 393, 214}

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}d(a+b)^{3/2}}-\frac{a\tanh(c+dx)}{2bd(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*b^(3/2)*(a + b)
^(3/2)*d) - (a*Tanh[c + d*x])/(2*b*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
```

```
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{(a+2b)\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tan\right.}{2b(a+b)d}$$

$$= \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}d} - \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

Mathematica [A]
time = 0.13, size = 88, normalized size = 1.06

$$4\left(\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{3/2}d} - \frac{a\sinh(2(c+dx))}{8b(a+b)d(a+2b+a\cosh(2(c+dx)))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] 4*(((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b^(3/2)*(a +
 b)^(3/2)*d) - (a*Sinh[2*(c + d*x)])/(8*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c
+ d*x)])))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(71) = 142.
time = 1.92, size = 222, normalized size = 2.67

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b(a+b)}+\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2b(a+b)}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}-\frac{(2b+a)\left(-\frac{\ln\left(\sqrt{a+b}\left(\tanh^2\left(\frac{dx}{2}\right.\right.\right.}{4}\right.}{}}{d}$

default	$-\frac{\frac{2\left(\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b(a+b)}+\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2b(a+b)}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}-\frac{(2b+a)\left(-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\right.\right.\right.}{4\sqrt{}}\right.}{}}{d}$
risch	$\frac{a\,e^{2dx+2c}+2b\,e^{2dx+2c}+a}{db(a+b)\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)}+\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^2}+2b\sqrt{ab+b^2}-2ab-2b^2}{a\sqrt{ab+b^2}}\right)}{2\sqrt{ab+b^2}\,(a+b)d}+\frac{\ln\left(e^{2c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(1/2*a/b/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/2*a/b/(a+b)*tanh(1/2*d*x+1/2
*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*
c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)-(2*b+a)/b/(a+b)*(-1/4/b^(1/2)/(a+b)^(1/
2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)
^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh
(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(74) = 148$.
time = 0.56, size = 165, normalized size = 1.99

$$-\frac{(a+2\,b)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,\sqrt{(a+b)b}\,(ab+b^2)d}-\frac{(a+2\,b)e^{(-2\,dx-2\,c)}+a}{(a^2b+ab^2+2\,(a^2b+3\,ab^2+2\,b^3)e^{(-2\,dx-2\,c)}+(a^2b+ab^2)e^{(-4\,dx-4\,c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a + 2*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^
(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a*b + b^2)
*d) - ((a + 2*b)*e^(-2*d*x - 2*c) + a)/((a^2*b + a*b^2 + 2*(a^2*b + 3*a*b^2
 + 2*b^3)*e^(-2*d*x - 2*c) + (a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 664 vs. $2(74) = 148$.
time = 0.40, size = 1569, normalized size = 18.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 8*(
a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b + 3*a*b^2 +
```

```
2*b^3)*sinh(d*x + c)^2 + ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*
cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*
a*b + 4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4
*a*b + 4*b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c
)^3 + (a^2 + 4*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*l
og((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*
x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2
 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 +
(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cos
h(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a
*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 +
2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x +
c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))
/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 + 2*a^2*b^3
+ a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*
sinh(d*x + c)^4 + 2*(a^3*b^2 + 4*a^2*b^3 + 5*a*b^4 + 2*b^5)*d*cosh(d*x + c)
^2 + 2*(3*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^3*b^2 + 4*a^
2*b^3 + 5*a*b^4 + 2*b^5)*d)*sinh(d*x + c)^2 + (a^3*b^2 + 2*a^2*b^3 + a*b^4)
*d + 4*((a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^3*b^2 + 4*a^2*
b^3 + 5*a*b^4 + 2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a^2*b + 2*a*
b^2 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 4*(a^2*b + 3*a*b^2 + 2*
b^3)*cosh(d*x + c)*sinh(d*x + c) + 2*(a^2*b + 3*a*b^2 + 2*b^3)*sinh(d*x + c
)^2 + ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d
*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*a*b + 4*b^2)*cosh(d*
x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*sinh(d
*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2 + 4*a*b +
 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d
*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*
sqrt(-a*b - b^2)/(a*b + b^2)))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x +
c)^4 + 4*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a
^3*b^2 + 2*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^4 + 2*(a^3*b^2 + 4*a^2*b^3 + 5*
a*b^4 + 2*b^5)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cos
h(d*x + c)^2 + (a^3*b^2 + 4*a^2*b^3 + 5*a*b^4 + 2*b^5)*d)*sinh(d*x + c)^2 +
 (a^3*b^2 + 2*a^2*b^3 + a*b^4)*d + 4*((a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*cosh(
d*x + c)^3 + (a^3*b^2 + 4*a^2*b^3 + 5*a*b^4 + 2*b^5)*d*cosh(d*x + c))*sinh(
d*x + c))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [A]
time = 0.60, size = 139, normalized size = 1.67

$$\frac{\frac{(a+2\,b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{(ab+b^2)\sqrt{-ab-b^2}}+\frac{2\,\left(ae^{(2\,dx+2\,c)}+2\,be^{(2\,dx+2\,c)}+a\right)}{(ab+b^2)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a + 2*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(
(a*b + b^2)*sqrt(-a*b - b^2)) + 2*(a*e^(2*d*x + 2*c) + 2*b*e^(2*d*x + 2*c)
+ a)/((a*b + b^2)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x +
 2*c) + a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^4\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2), x)
```

3.90 $\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=101

$$\frac{\operatorname{ArcTan}(\sinh(c+dx))}{b^2d}-\frac{\sqrt{a}\,(2a+3b)\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^2(a+b)^{3/2}d}-\frac{a\sinh(c+dx)}{2b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)}$$

```
[Out] arctan(sinh(d*x+c))/b^2/d-1/2*a*sinh(d*x+c)/b/(a+b)/d/(a+b+a*sinh(d*x+c)^2)
-1/2*(2*a+3*b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b^2/(a+b)^(3
/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4232, 425, 536, 209, 211}

$$-\frac{\sqrt{a}\,(2a+3b)\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^2d(a+b)^{3/2}}-\frac{a\sinh(c+dx)}{2bd(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\operatorname{ArcTan}(\sinh(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/(b^2*d) - (Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a]*Sinh[c
 + d*x])/Sqrt[a + b]])/(2*b^2*(a + b)^(3/2)*d) - (a*Sinh[c + d*x])/(2*b*(a
+ b)*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\text{sech}^5(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+ax^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{a\sinh(c+dx)}{2b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a+2b-ax^2}{(1+x^2)(a+b+ax^2)}\,dx, x, \sinh(c\right.}{2b(a+b)d}$$

$$= -\frac{a\sinh(c+dx)}{2b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{b^2 d} -$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{b^2 d} - \frac{\sqrt{a}\,(2a+3b)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^2(a+b)^{3/2}d} - \frac{a\,\text{s}}{2b(a+b)d\,(a}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 282 vs. 2(101) = 202.
time = 1.74, size = 282, normalized size = 2.79

$$\frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^3(c+dx)\left(\sqrt{a}\,(2a+3b)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\text{csch}(c+dx)\sqrt{(\cosh(c)-\sinh(c))^2}\,(\cosh(c)+\sinh(c))}{\sqrt{a}}\right)\cosh(c)(a+2b+a\cosh(2(c+dx)))\text{sech}(c+dx)-(a+2b+a\cosh(2(c+dx)))\text{sech}(c+dx)\left(-4(a+b)^{3/2}\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{(\cosh(c)-\sinh(c))^2}+\sqrt{a}\,(2a+3b)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\text{csch}(c+dx)\sqrt{(\cosh(c)-\sinh(c))^2}\,(\cosh(c)+\sinh(c))}{\sqrt{a}}\right)\sinh(c)\right)-2ab\sqrt{a+b}\,\sqrt{(\cosh(c)-\sinh(c))^2}\,\tanh(c+dx)\right)}{8b^2(a+b)^{3/2}d\left(a+b\,\text{sech}^2(c+dx)\right)^2\sqrt{(\cosh(c)-\sinh(c))^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^3*(Sqrt[a]*(2*a + 3*b)*ArcTa
n[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]
))/Sqrt[a]]*Cosh[c]*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x] - (a + 2*
b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]*(-4*(a + b)^(3/2)*ArcTan[Tanh[(c + d
*x)/2]]*Sqrt[(Cosh[c] - Sinh[c])^2] + Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a +
b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*
Sinh[c]) - 2*a*b*Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[c + d*x]))/(8
*b^2*(a + b)^(3/2)*d*(a + b*Sech[c + d*x]^2)^2*Sqrt[(Cosh[c] - Sinh[c])^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(89) = 178$.

time = 1.86, size = 216, normalized size = 2.14

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{2a\left(\frac{-\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2b+2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(2a+3b)\left(\right.}{}\right.}{b^2}}{d}$
default	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{2a\left(\frac{-\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2b+2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(2a+3b)\left(\right.}{}\right.}{b^2}}{d}$
risch	$-\frac{a\,e^{dx+c}\left(e^{2dx+2c}-1\right)}{bd(a+b)\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)}+\frac{i\ln\left(e^{dx+c}+i\right)}{d\,b^2}-\frac{i\ln\left(e^{dx+c}-i\right)}{d\,b^2}+\frac{\sqrt{-a\,(a+b)}\,\ln\left(e^{2d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^2*arctan(tanh(1/2*d*x+1/2*c))-2*a/b^2*((-1/2*b/(a+b)*tanh(1/2*d*x+
1/2*c)^3+1/2*b/(a+b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1
/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)+1/
2*(2*a+3*b)/(a+b)*(1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1
```

```
/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+
b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(a*e^(3*d*x + 3*c) - a*e^(d*x + c))/(a^2*b*d + a*b^2*d + (a^2*b*d*e^(4*c)
+ a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^2*b*d*e^(2*c) + 3*a*b^2*d*e^(2*c) + 2*b
^3*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(b^2*d) - 32*integrate(1/3
2*((2*a^2*e^(3*c) + 3*a*b*e^(3*c))*e^(3*d*x) + (2*a^2*e^c + 3*a*b*e^c)*e^(d
*x))/(a^2*b^2 + a*b^3 + (a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*e^(4*d*x) + 2*(a^
2*b^2*e^(2*c) + 3*a*b^3*e^(2*c) + 2*b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(89) = 178$.
time = 0.43, size = 2069, normalized size = 20.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b
*sinh(d*x + c)^3 - 4*a*b*cosh(d*x + c) - ((2*a^2 + 3*a*b)*cosh(d*x + c)^4 +
 4*(2*a^2 + 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b)*sinh(d*x
 + c)^4 + 2*(2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b)*
cosh(d*x + c)^2 + 2*a^2 + 7*a*b + 6*b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b +
4*((2*a^2 + 3*a*b)*cosh(d*x + c)^3 + (2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(-a/(a + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)
*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3
*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3
*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a
+ b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh
(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a +
 b)) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d
*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)
*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x
 + c) + a)) - 8*((a^2 + a*b)*cosh(d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^2 + a*b)*sinh(d*x + c)^4 + 2*(a^2 + 3*a*b + 2*b^2)*cos
h(d*x + c)^2 + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sinh
(d*x + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(d*x + c)^3 + (a^2 + 3*a*b + 2
```

```
*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) +
 4*(3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c))/((a^2*b^2 + a*b^3)*d*cosh(d
*x + c)^4 + 4*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2
+ a*b^3)*d*sinh(d*x + c)^4 + 2*(a^2*b^2 + 3*a*b^3 + 2*b^4)*d*cosh(d*x + c)^
2 + 2*(3*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^2*b^2 + 3*a*b^3 + 2*b^4)*
d)*sinh(d*x + c)^2 + (a^2*b^2 + a*b^3)*d + 4*((a^2*b^2 + a*b^3)*d*cosh(d*x
+ c)^3 + (a^2*b^2 + 3*a*b^3 + 2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*
(2*a*b*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*b*sinh(d
*x + c)^3 - 2*a*b*cosh(d*x + c) + ((2*a^2 + 3*a*b)*cosh(d*x + c)^4 + 4*(2*a
^2 + 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b)*sinh(d*x + c)^4
 + 2*(2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(d*
x + c)^2 + 2*a^2 + 7*a*b + 6*b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b + 4*((2*a
^2 + 3*a*b)*cosh(d*x + c)^3 + (2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c))*sinh(d
*x + c))*sqrt(a/(a + b))*arctan(1/2*sqrt(a/(a + b))*(cosh(d*x + c) + sinh(d
*x + c))) + ((2*a^2 + 3*a*b)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (2*a^2 + 3*a*b)*sinh(d*x + c)^4 + 2*(2*a^2 + 7*a*b +
6*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(d*x + c)^2 + 2*a^2 + 7*a
*b + 6*b^2)*sinh(d*x + c)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*cosh(d*x +
 c)^3 + (2*a^2 + 7*a*b + 6*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/(a + b
))*arctan(1/2*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*si
nh(d*x + c)^3 + (3*a + 4*b)*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 + 3*a + 4*
b)*sinh(d*x + c))*sqrt(a/(a + b))/a) - 4*((a^2 + a*b)*cosh(d*x + c)^4 + 4*(
a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + a*b)*sinh(d*x + c)^4 + 2*
(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 +
a^2 + 3*a*b + 2*b^2)*sinh(d*x + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(d*x
+ c)^3 + (a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*
x + c) + sinh(d*x + c)) + 2*(3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c))/((
a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4 + 4*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2*(a^2*b^2 + 3*a*b^3
+ 2*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2 + (a^
2*b^2 + 3*a*b^3 + 2*b^4)*d)*sinh(d*x + c)^2 + (a^2*b^2 + a*b^3)*d + 4*((a^2
*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (a^2*b^2 + 3*a*b^3 + 2*b^4)*d*cosh(d*x +
c))*sinh(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**5/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^5\,{\left(a+\frac{b}{\cosh\left(c+d\,x\right)^2}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^2), x)
```

3.91 $\int \frac{\text{sech}^6(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2} \, dx$

Optimal. Leaf size=101

$$-\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}d}+\frac{\tanh(c+dx)}{b^2d}+\frac{a^2\tanh(c+dx)}{2b^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

```
[Out] -1/2*a*(3*a+4*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(5/2)/(a+b)^(3/
2)/d+tanh(d*x+c)/b^2/d+1/2*a^2*tanh(d*x+c)/b^2/(a+b)/d/(a+b-b*tanh(d*x+c)^2
)
```

Rubi [A]
time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4231, 398, 393, 214}

$$\frac{a^2\tanh(c+dx)}{2b^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}d(a+b)^{3/2}}+\frac{\tanh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/2*(a*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(5/2)*
(a + b)^(3/2)*d) + Tanh[c + d*x]/(b^2*d) + (a^2*Tanh[c + d*x])/(2*b^2*(a +
b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a(a+2b)-2abx^2}{b^2\left(a+b-bx^2\right)^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{b^2d} - \frac{\operatorname{Subst}\left(\int \frac{a(a+2b)-2abx^2}{\left(a+b-bx^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{b^2d}$$

$$= \frac{\tanh(c+dx)}{b^2d} + \frac{a^2\tanh(c+dx)}{2b^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{(a(3a+4b))\operatorname{Subst}\left(\int\right.}{2}$$

$$= -\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}d} + \frac{\tanh(c+dx)}{b^2d} + \frac{a^2\tanh(c}{2b^2(a+b)d\left(a+b-\right.}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 229 vs. 2(101) = 202.
time = 2.52, size = 229, normalized size = 2.27

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^4(c+dx)\left(-\frac{a(3a+4b)\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\,\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(a+2b+a\cosh(2(c+dx)))(\cosh(2c)-\sinh(2c))}{(a+b)^{3/2}\sqrt{b(\cosh(c)-\sinh(c))^4}}+2(a+2b+a\cosh(2(c+dx)))\operatorname{sech}(c)\operatorname{sech}(c+dx)\sinh(dx)+\frac{a(a\operatorname{sech}(2c)\sinh(2dx)-(a+2b)\tanh(2c))}{a+b}\right)}{8b^2d\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(-((a*(3*a + 4*b)*ArcTanh[
(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]
))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c +
d*x)])*(Cosh[2*c] - Sinh[2*c]))/((a + b)^(3/2)*Sqrt[b*(Cosh[c] - Sinh[c])^4
```

```
])) + 2*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c]*Sech[c + d*x]*Sinh[d*x] + (
a*(a*Sech[2*c]*Sinh[2*d*x] - (a + 2*b)*Tanh[2*c]))/(a + b)))/(8*b^2*d*(a +
b*Sech[c + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(89) = 178$.
time = 1.75, size = 250, normalized size = 2.48

method	result
derivativedivides	$\frac{2a\left(\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b+2a}+\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2b+2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(3a+4b)\left(-\frac{\ln\left(\sqrt{a+b}\left(\tanh^2(\right.\right.}{}\right.}{}\right.}{b^2}$
default	$\frac{2a\left(\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b+2a}+\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2b+2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(3a+4b)\left(-\frac{\ln\left(\sqrt{a+b}\left(\tanh^2(\right.\right.}{}\right.}{}\right.}{b^2}$
risch	$-\frac{3a^2e^{4dx+4c}+4ab\,e^{4dx+4c}+6a^2e^{2dx+2c}+14ab\,e^{2dx+2c}+8b^2e^{2dx+2c}+3a^2+2ab}{b^2d\left(1+e^{2dx+2c}\right)(a+b)\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)}+\frac{3a^2\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^2}+\cdots}{a\sqrt{\cdots}}\right.}{4\sqrt{ab+b^2}\,(\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a/b^2*((1/2/(a+b)*a*tanh(1/2*d*x+1/2*c)^3+1/2/(a+b)*a*tanh(1/2*d*x+1
/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/
2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)+1/2*(3*a+4*b)/(a+b)*(-1/4/b^(1/2)/(a+
b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)
+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-
2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))+2/b^2*tanh(1/2*d*x+1/2*c)/(tan
h(1/2*d*x+1/2*c)^2+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(92) = 184$.
time = 0.58, size = 244, normalized size = 2.42

$$\frac{(3\,a+4\,b)a\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,(ab^2+b^3)\sqrt{(a+b)b}\,d}+\frac{3\,a^2+2\,ab+2\,(3\,a^2+7\,ab+4\,b^2)e^{(-2\,dx-2\,c)}+(3\,a^2+4\,ab)e^{(-4\,dx-4\,c)}}{(a^2b^2+ab^3+(3\,a^2b^2+7\,ab^3+4\,b^4)e^{(-2\,dx-2\,c)}+(3\,a^2b^2+7\,ab^3+4\,b^4)e^{(-4\,dx-4\,c)}+(a^2b^2+ab^3)e^{(-6\,dx-6\,c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(3*a + 4*b)*a*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a
*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a*b^2 + b^3)*sqrt((a +
b)*b)*d) + (3*a^2 + 2*a*b + 2*(3*a^2 + 7*a*b + 4*b^2)*e^(-2*d*x - 2*c) + (3
*a^2 + 4*a*b)*e^(-4*d*x - 4*c))/((a^2*b^2 + a*b^3 + (3*a^2*b^2 + 7*a*b^3 +
4*b^4)*e^(-2*d*x - 2*c) + (3*a^2*b^2 + 7*a*b^3 + 4*b^4)*e^(-4*d*x - 4*c) +
(a^2*b^2 + a*b^3)*e^(-6*d*x - 6*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1358 vs. $2(92) = 184$.
time = 0.43, size = 2958, normalized size = 29.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^4 + 16*(3*a^3*b + 7*
a^2*b^2 + 4*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(3*a^3*b + 7*a^2*b^2 +
 4*a*b^3)*sinh(d*x + c)^4 + 12*a^3*b + 20*a^2*b^2 + 8*a*b^3 + 8*(3*a^3*b +
10*a^2*b^2 + 11*a*b^3 + 4*b^4)*cosh(d*x + c)^2 + 8*(3*a^3*b + 10*a^2*b^2 +
11*a*b^3 + 4*b^4 + 3*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 - ((3*a^3 + 4*a^2*b)*cosh(d*x + c)^6 + 6*(3*a^3 + 4*a^2*b)*cosh(
d*x + c)*sinh(d*x + c)^5 + (3*a^3 + 4*a^2*b)*sinh(d*x + c)^6 + (9*a^3 + 24*
a^2*b + 16*a*b^2)*cosh(d*x + c)^4 + (9*a^3 + 24*a^2*b + 16*a*b^2 + 15*(3*a^
3 + 4*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(3*a^3 + 4*a^2*b)*cosh
(d*x + c)^3 + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3
+ 3*a^3 + 4*a^2*b + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^2 + (15*(3*
a^3 + 4*a^2*b)*cosh(d*x + c)^4 + 9*a^3 + 24*a^2*b + 16*a*b^2 + 6*(9*a^3 + 2
4*a^2*b + 16*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^3 + 4*a^2*
b)*cosh(d*x + c)^5 + 2*(9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^3 + (9*a
^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log
((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x
+ c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 +
 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a
^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(
d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*c
osh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*
(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)
^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) +
16*((3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^3 + (3*a^3*b + 10*a^2*b^2
 + 11*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b^3 + 2*a^2*b^4 +
a*b^5)*d*cosh(d*x + c)^6 + 6*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)*
sinh(d*x + c)^5 + (a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*sinh(d*x + c)^6 + (3*a^3*
b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^4 + (15*(a^3*b^3 + 2*a
```

```
^2*b^4 + a*b^5)*d*cosh(d*x + c)^2 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*
b^6)*d)*sinh(d*x + c)^4 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cos
h(d*x + c)^2 + 4*(5*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^3 + (3*a^
3*b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (
15*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^4 + 6*(3*a^3*b^3 + 10*a^2*
b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^2 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*
b^5 + 4*b^6)*d)*sinh(d*x + c)^2 + (a^3*b^3 + 2*a^2*b^4 + a*b^5)*d + 2*(3*(a
^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^5 + 2*(3*a^3*b^3 + 10*a^2*b^4 +
 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^3 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 +
 4*b^6)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(3*a^3*b + 7*a^2*b^2 + 4*a
*b^3)*cosh(d*x + c)^4 + 8*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)*sin
h(d*x + c)^3 + 2*(3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*sinh(d*x + c)^4 + 6*a^3*b
+ 10*a^2*b^2 + 4*a*b^3 + 4*(3*a^3*b + 10*a^2*b^2 + 11*a*b^3 + 4*b^4)*cosh(d
*x + c)^2 + 4*(3*a^3*b + 10*a^2*b^2 + 11*a*b^3 + 4*b^4 + 3*(3*a^3*b + 7*a^2
*b^2 + 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^3 + 4*a^2*b)*cosh(
d*x + c)^6 + 6*(3*a^3 + 4*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^3 + 4
*a^2*b)*sinh(d*x + c)^6 + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^4 + (
9*a^3 + 24*a^2*b + 16*a*b^2 + 15*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 4*(5*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^3 + (9*a^3 + 24*a^2*b + 16*
a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^3 + 4*a^2*b + (9*a^3 + 24*a^2*b
 + 16*a*b^2)*cosh(d*x + c)^2 + (15*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^4 + 9*a^
3 + 24*a^2*b + 16*a*b^2 + 6*(9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 2*(3*(3*a^3 + 4*a^2*b)*cosh(d*x + c)^5 + 2*(9*a^3 + 24*a^
2*b + 16*a*b^2)*cosh(d*x + c)^3 + (9*a^3 + 24*a^2*b + 16*a*b^2)*cosh(d*x +
c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cos
h(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-a*b - b^2)/(a
*b + b^2)) + 8*((3*a^3*b + 7*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^3 + (3*a^3*b
+ 10*a^2*b^2 + 11*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b^3 +
2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^6 + 6*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*co
sh(d*x + c)*sinh(d*x + c)^5 + (a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*sinh(d*x + c)
^6 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^4 + (15*(a
^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^2 + (3*a^3*b^3 + 10*a^2*b^4 + 1
1*a*b^5 + 4*b^6)*d)*sinh(d*x + c)^4 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 +
4*b^6)*d*cosh(d*x + c)^2 + 4*(5*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x +
c)^3 + (3*a^3*b^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c))*sinh(d*
x + c)^3 + (15*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(d*x + c)^4 + 6*(3*a^3*b
^3 + 10*a^2*b^4 + 11*a*b^5 + 4*b^6)*d*cosh(d*x + c)^2 + (3*a^3*b^3 + 10*a^2
*b^4 + 11*a*b^5 + 4*b^6)*d)*sinh(d*x + c)^2 + (a^3*b^3 + 2*a^2*b^4 + a*b^5)
*d + 2*(3*(a^3*b^3 + 2*a^2*b^4 + a*b^5)*d*cosh(...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**6/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(92) = 184.
time = 0.60, size = 225, normalized size = 2.23

$$-\frac{\frac{(3a^2+4ab)\arctan\left(\frac{ae^{(2dx+2c)}+a+2b}{2\sqrt{-ab-b^2}}\right)}{(ab^2+b^3)\sqrt{-ab-b^2}}+\frac{2\left(3a^2e^{(4dx+4c)}+4abe^{(4dx+4c)}+6a^2e^{(2dx+2c)}+14abe^{(2dx+2c)}+8b^2e^{(2dx+2c)}+3a^2+2ab\right)}{(ab^2+b^3)\left(ae^{(6dx+6c)}+3ae^{(4dx+4c)}+4be^{(4dx+4c)}+3ae^{(2dx+2c)}+4be^{(2dx+2c)}+a\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a^2 + 4*a*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b -
b^2))/((a*b^2 + b^3)*sqrt(-a*b - b^2)) + 2*(3*a^2*e^(4*d*x + 4*c) + 4*a*b*e
^(4*d*x + 4*c) + 6*a^2*e^(2*d*x + 2*c) + 14*a*b*e^(2*d*x + 2*c) + 8*b^2*e^(
2*d*x + 2*c) + 3*a^2 + 2*a*b)/((a*b^2 + b^3)*(a*e^(6*d*x + 6*c) + 3*a*e^(4*
d*x + 4*c) + 4*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c
) + a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+dx)^6\left(a+\frac{b}{\cosh(c+dx)^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^2), x)
```

3.92 $\int \frac{\text{sech}^7(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=153

$$-\frac{(4a-b)\text{ArcTan}(\sinh(c+dx))}{2b^3d}+\frac{a^{3/2}(4a+5b)\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^3(a+b)^{3/2}d}+\frac{a(2a+b)\sinh(c+dx)}{2b^2(a+b)d\left(a+b+a\sinh^2(c+\right.}$$

```
[Out] -1/2*(4*a-b)*arctan(sinh(d*x+c))/b^3/d+1/2*a^(3/2)*(4*a+5*b)*arctan(sinh(d*
x+c)*a^(1/2)/(a+b)^(1/2))/b^3/(a+b)^(3/2)/d+1/2*a*(2*a+b)*sinh(d*x+c)/b^2/(
a+b)/d/(a+b+a*sinh(d*x+c)^2)+1/2*sech(d*x+c)*tanh(d*x+c)/b/d/(a+b+a*sinh(d*
x+c)^2)
```

Rubi [A]
time = 0.16, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4232, 425, 541, 536, 209, 211}

$$\frac{a^{3/2}(4a+5b)\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^3d(a+b)^{3/2}}-\frac{(4a-b)\text{ArcTan}(\sinh(c+dx))}{2b^3d}+\frac{a(2a+b)\sinh(c+dx)}{2b^2d(a+b)\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\tanh(c+dx)\text{sech}(c+dx)}{2bd\left(a\sinh^2(c+dx)+a+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((4*a - b)*ArcTan[Sinh[c + d*x]])/(b^3*d) + (a^(3/2)*(4*a + 5*b)*ArcTa
n[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(2*b^3*(a + b)^(3/2)*d) + (a*(2*a +
 b)*Sinh[c + d*x])/(2*b^2*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)) + (Sech[c
+ d*x]*Tanh[c + d*x])/(2*b*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+ax^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+b+a\sinh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{a-b-3ax^2}{(1+x^2)(a+b+ax^2)^2}\,dx, x, \sinh(c+dx)\right.}{2bd}$$

$$= \frac{a(2a+b)\sinh(c+dx)}{2b^2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+b+a\sinh^2(c+dx)\right)} - \frac{\operatorname{Su}}{}$$

$$= \frac{a(2a+b)\sinh(c+dx)}{2b^2(a+b)d\left(a+b+a\sinh^2(c+dx)\right)} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+b+a\sinh^2(c+dx)\right)} - \frac{(4a}{}$$

$$= -\frac{(4a-b)\tan^{-1}(\sinh(c+dx))}{2b^3d} + \frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{2b^3(a+b)^{3/2}d} + \frac{}{2b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 489 vs. 2(153) = 306.
time = 3.42, size = 489, normalized size = 3.20

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c]*Sech[c + d*x]^3*(-(a^(3/2)*(4*a +
5*b)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c]
 + Sinh[c]))/Sqrt[a]]*Cosh[c]^2*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*
x]) + b*(a + b)^(3/2)*(a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*Sqrt[
(Cosh[c] - Sinh[c])^2]*Sinh[c] - Cosh[c]*Sech[c + d*x]*(2*Sqrt[a + b]*(4*a^
2 + 3*a*b - b^2)*ArcTan[Tanh[(c + d*x)/2]]*(a + 2*b + a*Cosh[2*(c + d*x)])*
Sqrt[(Cosh[c] - Sinh[c])^2] - a^(5/2)*b*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*S
qrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(13 + 5*Cosh[2*(c
+ d*x)])*Sinh[c]) + a^(3/2)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c]
 - Sinh[c])^2]*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(2*a^2 + 5*b^2 + 2*a^2*Cosh[2*
(c + d*x)])*Sech[c + d*x]*Sinh[2*c] + b*(a + b)^(3/2)*(a + 2*b + a*Cosh[2*(
c + d*x)])*Sech[c + d*x]^3*Sqrt[(Cosh[c] - Sinh[c])^2]*Sinh[d*x] + 2*a^2*b*
Sqrt[a + b]*Cosh[c]*Sqrt[(Cosh[c] - Sinh[c])^2]*Tanh[c + d*x]))/(8*b^3*(a +
 b)^(3/2)*d*(a + b*Sech[c + d*x]^2)^2*Sqrt[(Cosh[c] - Sinh[c])^2])
```

Maple [A]
time = 1.93, size = 271, normalized size = 1.77

method	result
derivativedivides	$\frac{2a^2\left(\frac{-\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2b+2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(5b+4a)\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh}{2\sqrt{}}\right)}{2\sqrt{a+b}}\right.}{}\right.}{b^3}$
default	$\frac{2a^2\left(\frac{-\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2b+2a}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(5b+4a)\left(\frac{\arctan\left(\frac{2\sqrt{a+b}\,\tanh}{2\sqrt{}}\right)}{2\sqrt{a+b}}\right.}{}\right.}{b^3}$
risch	$\frac{e^{dx+c}\left(2a^2 e^{6dx+6c}+ab\,e^{6dx+6c}+2a^2 e^{4dx+4c}+5ab\,e^{4dx+4c}+4b^2 e^{4dx+4c}-2a^2 e^{2dx+2c}-5ab\,e^{2dx+2c}-4b^2 e^{2dx+2c}-2a^2-ab\right)}{b^2 d(a+b)\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)\left(1+e^{2dx+2c}\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^2/b^3*((-1/2*b/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/2*b/(a+b)*tanh(1/2*d*
x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x
+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)+1/2*(5*b+4*a)/(a+b)*(1/2/(a+b)^(1/
2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2)
)+1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b
^(1/2))/a^(1/2))))-2/b^3*((1/2*b*tanh(1/2*d*x+1/2*c)^3-1/2*b*tanh(1/2*d*x+1
/2*c))/(tanh(1/2*d*x+1/2*c)^2+1)^2+1/2*(4*a-b)*arctan(tanh(1/2*d*x+1/2*c)))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] ((2*a^2*e^(7*c) + a*b*e^(7*c))*e^(7*d*x) + (2*a^2*e^(5*c) + 5*a*b*e^(5*c) +
 4*b^2*e^(5*c))*e^(5*d*x) - (2*a^2*e^(3*c) + 5*a*b*e^(3*c) + 4*b^2*e^(3*c))
*e^(3*d*x) - (2*a^2*e^c + a*b*e^c)*e^(d*x))/(a^2*b^2*d + a*b^3*d + (a^2*b^2
*d*e^(8*c) + a*b^3*d*e^(8*c))*e^(8*d*x) + 4*(a^2*b^2*d*e^(6*c) + 2*a*b^3*d*
e^(6*c) + b^4*d*e^(6*c))*e^(6*d*x) + 2*(3*a^2*b^2*d*e^(4*c) + 7*a*b^3*d*e^(
4*c) + 4*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^2*b^2*d*e^(2*c) + 2*a*b^3*d*e^(2*c
) + b^4*d*e^(2*c))*e^(2*d*x)) - (4*a*e^c - b*e^c)*arctan(e^(d*x + c))*e^(-c
)/(b^3*d) + 128*integrate(1/128*((4*a^3*e^(3*c) + 5*a^2*b*e^(3*c))*e^(3*d*x
) + (4*a^3*e^c + 5*a^2*b*e^c)*e^(d*x))/(a^2*b^3 + a*b^4 + (a^2*b^3*e^(4*c)
+ a*b^4*e^(4*c))*e^(4*d*x) + 2*(a^2*b^3*e^(2*c) + 3*a*b^4*e^(2*c) + 2*b^5*e
^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3476 vs. $2(137) = 274$.
time = 0.47, size = 6499, normalized size = 42.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(2*a^2*b + a*b^2)*cosh(d*x + c)^7 + 28*(2*a^2*b + a*b^2)*cosh(d*x +
 c)*sinh(d*x + c)^6 + 4*(2*a^2*b + a*b^2)*sinh(d*x + c)^7 + 4*(2*a^2*b + 5*
a*b^2 + 4*b^3)*cosh(d*x + c)^5 + 4*(2*a^2*b + 5*a*b^2 + 4*b^3 + 21*(2*a^2*b
 + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(2*a^2*b + a*b^2)*cosh(d
*x + c)^3 + (2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*
(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + 4*(35*(2*a^2*b + a*b^2)*cosh(
d*x + c)^4 - 2*a^2*b - 5*a*b^2 - 4*b^3 + 10*(2*a^2*b + 5*a*b^2 + 4*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 1
0*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^3 - 3*(2*a^2*b + 5*a*b^2 + 4*b^
3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((4*a^3 + 5*a^2*b)*cosh(d*x + c)^8 + 8*
(4*a^3 + 5*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + 5*a^2*b)*sinh(d*
x + c)^8 + 4*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^6 + 4*(4*a^3 + 9*a^2
*b + 5*a*b^2 + 7*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*
(4*a^3 + 5*a^2*b)*cosh(d*x + c)^3 + 3*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^5 + 2*(12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^4 +
2*(35*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^4 + 12*a^3 + 31*a^2*b + 20*a*b^2 + 30
*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3
 + 5*a^2*b)*cosh(d*x + c)^5 + 10*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^
3 + (12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 +
 5*a^2*b + 4*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 5*
a^2*b)*cosh(d*x + c)^6 + 15*(4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^4 + 4
*a^3 + 9*a^2*b + 5*a*b^2 + 3*(12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 8*((4*a^3 + 5*a^2*b)*cosh(d*x + c)^7 + 3*(4*a^3 + 9*a^2
*b + 5*a*b^2)*cosh(d*x + c)^5 + (12*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c
```

)^3 + (4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a/(a + b))*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 - (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c))*sqrt(-a/(a + b)) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*((4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^8 + 8*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + 3*a^2*b - a*b^2)*sinh(d*x + c)^8 + 4*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^6 + 4*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 + 7*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^3 + 3*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^4 + 12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3 + 30*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^5 + 10*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^3 + (12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + 3*a^2*b - a*b^2 + 4*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^6 + 15*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^4 + 4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 + 3*(12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + 3*a^2*b - a*b^2)*cosh(d*x + c)^7 + 3*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c)^5 + (12*a^3 + 25*a^2*b + 9*a*b^2 - 4*b^3)*cosh(d*x + c)^3 + (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 4*(2*a^2*b + a*b^2)*cosh(d*x + c) + 4*(7*(2*a^2*b + a*b^2)*cosh(d*x + c)^6 + 5*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^4 - 2*a^2*b - a*b^2 - 3*(2*a^2*b + 5*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^2*b^3 + a*b^4)*d*cosh(d*x + c)^8 + 8*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b^3 + a*b^4)*d*sinh(d*x + c)^8 + 4*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^2*b^3 + 2*a*b^4 + b^5)*d)*sinh(d*x + c)^6 + 2*(3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + 3*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 30*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d)*sinh(d*x + c)^4 + 4*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5 + 10*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c)^3 + (3*a^2*b^3 + 7*a*b^4 + 4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 15*(a^2*b^3 + 2*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 3*(3*a^2*b^3 + 7*...

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(sech(c + d*x)**7/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+d\,x)^7\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^7*(a + b/cosh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^7*(a + b/cosh(c + d*x)^2)^2), x)
```

3.93 $\int \frac{\cosh^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=204

$$\frac{(a-6b)x}{2a^4}+\frac{b^{3/2}(35a^2+56ab+24b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{5/2}d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2}+\frac{b(2a}{4a^2(a+b)d}$$

```
[Out] 1/2*(a-6*b)*x/a^4+1/8*b^(3/2)*(35*a^2+56*a*b+24*b^2)*arctanh(b^(1/2)*tanh(d
*x+c)/(a+b)^(1/2))/a^4/(a+b)^(5/2)/d+1/2*cosh(d*x+c)*sinh(d*x+c)/a/d/(a+b-b
*tanh(d*x+c)^2)^2+1/4*b*(2*a+3*b)*tanh(d*x+c)/a^2/(a+b)/d/(a+b-b*tanh(d*x+c
)^2)^2+1/8*b*(4*a+3*b)*(a+4*b)*tanh(d*x+c)/a^3/(a+b)^2/d/(a+b-b*tanh(d*x+c)
^2)
```

Rubi [A]

time = 0.28, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4231, 425, 541, 536, 212, 214}

$$\frac{x(a-6b)}{2a^4}+\frac{b(4a+3b)(a+4b)\tanh(c+dx)}{8a^3d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{b(2a+3b)\tanh(c+dx)}{4a^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}+\frac{b^{3/2}(35a^2+56ab+24b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4d(a+b)^{5/2}}+\frac{\sinh(c+dx)\cosh(c+dx)}{2ad\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a - 6*b)*x)/(2*a^4) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTanh[(Sqrt[b
]*Tanh[c + d*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(5/2)*d) + (Cosh[c + d*x]*Sin
h[c + d*x])/(2*a*d*(a + b - b*Tanh[c + d*x]^2)^2) + (b*(2*a + 3*b)*Tanh[c +
 d*x])/(4*a^2*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2) + (b*(4*a + 3*b)*(a
+ 4*b)*Tanh[c + d*x])/(8*a^3*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\mathrm{Subst}\left(\int \frac{a-b-5bx^2}{(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx\right.}{2ad}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(2a+3b)\tanh(c+dx)}{4a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \cdots$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(2a+3b)\tanh(c+dx)}{4a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \cdots$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{b(2a+3b)\tanh(c+dx)}{4a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \cdots$$

$$= \frac{(a-6b)x}{2a^4} + \frac{b^{3/2}(35a^2+56ab+24b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{5/2}d} + \frac{\cosh(c+\cdots}{2ad\left(a+b-\cdots\right.}$$

Mathematica [A]
time = 2.80, size = 156, normalized size = 0.76

$$\frac{4(a-6b)(c+dx) + \frac{b^{3/2}(35a^2+56ab+24b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + a\left(2 + \frac{13ab^2}{(a+b)^2(a+2b+a\cosh(2(c+dx)))} + \frac{2b^3(3a+8b+5a\cosh(2(c+dx)))}{(a+b)^2(a+2b+a\cosh(2(c+dx)))^2}\right)\sinh(2(c+dx))}{8a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (4*(a - 6*b)*(c + d*x) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTanh[(Sqrt[
b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) + a*(2 + (13*a*b^2)/((a + b)^
2*(a + 2*b + a*Cosh[2*(c + d*x)])) + (2*b^3*(3*a + 8*b + 5*a*Cosh[2*(c + d*
x)]))/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2))*Sinh[2*(c + d*x)])/(8*
a^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(186) = 372.
time = 3.56, size = 439, normalized size = 2.15 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^2/a^4*((-1/8*a*(13*a+8*b)/(a+b)*tanh(1/2*d*x+1/2*c)^7-1/8*a*(39*a
^2+19*a*b-8*b^2)/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-1/8*a*(39*a^2+19*a*b-8*b^2)/
(a+b)^2*tanh(1/2*d*x+1/2*c)^3-1/8*a*(13*a+8*b)/(a+b)*tanh(1/2*d*x+1/2*c))/(
a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2
*b*tanh(1/2*d*x+1/2*c)^2+a+b)^2+1/8*(35*a^2+56*a*b+24*b^2)/(a^2+2*a*b+b^2)*
(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d
*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(
1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))-1/2/a^3/(tanh
(1/2*d*x+1/2*c)+1)^2+1/2/a^3/(tanh(1/2*d*x+1/2*c)+1)+1/2*(a-6*b)/a^4*ln(tan
h(1/2*d*x+1/2*c)+1)+1/2/a^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/a^3/(tanh(1/2*d*x
+1/2*c)-1)+1/2/a^4*(-a+6*b)*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. $2(195) = 390$.
time = 0.58, size = 1373, normalized size = 6.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3/64*(5*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a
+ 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)
))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)*d) - 3/64*(5*a^3*b + 30*a^2*b
^2 + 40*a*b^3 + 16*b^4)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*
b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 2*a^5*b + a
^4*b^2)*sqrt((a + b)*b)*d) + 1/32*(15*a^2*b + 20*a*b^2 + 8*b^3)*log((a*e^(-
2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b +
 2*sqrt((a + b)*b)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d) - 1/16*(
9*a^4*b + 32*a^3*b^2 + 20*a^2*b^3 + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b^3 +
32*a*b^4)*e^(6*d*x + 6*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 + 832*a*b
^4 + 320*b^5)*e^(4*d*x + 4*c) + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b^3 + 160
*a*b^4)*e^(2*d*x + 2*c))/((a^8 + 2*a^7*b + a^6*b^2 + (a^8 + 2*a^7*b + a^6*b
^2)*e^(8*d*x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(6*d*x +
6*c) + 2*(3*a^8 + 14*a^7*b + 27*a^6*b^2 + 24*a^5*b^3 + 8*a^4*b^4)*e^(4*d*x
+ 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 + 2*a^5*b^3)*e^(2*d*x + 2*c))*d) + 1/
16*(9*a^4*b + 32*a^3*b^2 + 20*a^2*b^3 + (27*a^4*b + 194*a^3*b^2 + 336*a^2*b
^3 + 160*a*b^4)*e^(-2*d*x - 2*c) + (27*a^4*b + 264*a^3*b^2 + 740*a^2*b^3 +
832*a*b^4 + 320*b^5)*e^(-4*d*x - 4*c) + 3*(3*a^4*b + 34*a^3*b^2 + 64*a^2*b^
3 + 32*a*b^4)*e^(-6*d*x - 6*c))/((a^8 + 2*a^7*b + a^6*b^2 + 4*(a^8 + 4*a^7*
b + 5*a^6*b^2 + 2*a^5*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^8 + 14*a^7*b + 27*a^6*
b^2 + 24*a^5*b^3 + 8*a^4*b^4)*e^(-4*d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b
^2 + 2*a^5*b^3)*e^(-6*d*x - 6*c) + (a^8 + 2*a^7*b + a^6*b^2)*e^(-8*d*x - 8*
c))*d) - 1/8*(9*a^3*b + 6*a^2*b^2 + (27*a^3*b + 68*a^2*b^2 + 32*a*b^3)*e^(-
2*d*x - 2*c) + 3*(9*a^3*b + 30*a^2*b^2 + 40*a*b^3 + 16*b^4)*e^(-4*d*x - 4*c
```

```
) + (9*a^3*b + 28*a^2*b^2 + 16*a*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a
^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*
a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*
(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b +
 a^5*b^2)*e^(-8*d*x - 8*c))*d) + 1/2*(d*x + c)/(a^3*d) + 1/8*e^(2*d*x + 2*c
)/(a^3*d) - 1/8*e^(-2*d*x - 2*c)/(a^3*d) - 3/4*b*log(a*e^(4*d*x + 4*c) + 2*
(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^4*d) + 3/4*b*log(2*(a + 2*b)*e^(-2*d*x -
2*c) + a*e^(-4*d*x - 4*c) + a)/(a^4*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5729 vs. $2(195) = 390$.
time = 0.51, size = 11740, normalized size = 57.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^12 + 24*(a^5 + 2*a^4*b + a
^3*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + 2*(a^5 + 2*a^4*b + a^3*b^2)*sinh(d
*x + c)^12 + 8*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11
*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^10 + 4*(2*a^5 + 8*a^4*b + 10*a^3*b
^2 + 4*a^2*b^3 + 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x + 33*(a^5 +
 2*a^4*b + a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 40*(11*(a^5 + 2*a^4
*b + a^3*b^2)*cosh(d*x + c)^3 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 +
(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^
9 + 2*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*
a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^8 + 2*(5*a^5
 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 495*(a^5 + 2*a^4*b + a^3
*b^2)*cosh(d*x + c)^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*
b^4)*d*x + 180*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11
*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(99*(a^5 +
 2*a^4*b + a^3*b^2)*cosh(d*x + c)^5 + 60*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2
*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^3 + (5*a
^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19
*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(
39*a^3*b^2 + 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3
*b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^6 + 4*(462*(a^5
 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^6 - 39*a^3*b^2 - 134*a^2*b^3 - 184*a*b^
4 - 80*b^5 + 420*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b -
11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 4*(3*a^5 - 4*a^4*b - 57*a^3*
b^2 - 138*a^2*b^3 - 136*a*b^4 - 48*b^5)*d*x + 14*(5*a^5 + 26*a^4*b + 27*a^3
*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3
- 12*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(198*(a^5 + 2*a^4*b +
 a^3*b^2)*cosh(d*x + c)^7 + 252*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a
```

```
^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^5 + 14*(5*a^5 + 2
6*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b
^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^3 - 3*(39*a^3*b^2 + 134*a^2*
b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 -
136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*a^5 - 4*a^4*b -
 2*a^3*b^2 - 2*(5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 + 128*a*b^4 -
16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^
4 + 2*(495*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^8 + 840*(a^5 + 4*a^4*b +
 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh
(d*x + c)^6 - 5*a^5 - 26*a^4*b - 131*a^3*b^2 - 256*a^2*b^3 - 128*a*b^4 + 70
*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b
 - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^4 + 16*(a^5 - 2*a
^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x - 30*(39*a^3*b^2 + 134*a^2*b
^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2*b^3 - 1
36*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(55*(a^5 + 2*a
^4*b + a^3*b^2)*cosh(d*x + c)^9 + 120*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^
3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*cosh(d*x + c)^7 + 14*(5*a
^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 - 32*a*b^4 + 16*(a^5 - 2*a^4*b - 19
*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cosh(d*x + c)^5 - 10*(39*a^3*b^2 + 1
34*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a^2
*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^3 - (5*a^5 + 26*a^4*b + 131*a
^3*b^2 + 256*a^2*b^3 + 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*
b^3 - 12*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(2*a^5 + 8*a^4*b +
23*a^3*b^2 + 14*a^2*b^3 - 2*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*d*x)*c
osh(d*x + c)^2 + 4*(33*(a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^10 + 90*(a^5
 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + (a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^
3)*d*x)*cosh(d*x + c)^8 + 14*(5*a^5 + 26*a^4*b + 27*a^3*b^2 - 32*a^2*b^3 -
32*a*b^4 + 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*cos
h(d*x + c)^6 - 2*a^5 - 8*a^4*b - 23*a^3*b^2 - 14*a^2*b^3 - 15*(39*a^3*b^2 +
 134*a^2*b^3 + 184*a*b^4 + 80*b^5 - 4*(3*a^5 - 4*a^4*b - 57*a^3*b^2 - 138*a
^2*b^3 - 136*a*b^4 - 48*b^5)*d*x)*cosh(d*x + c)^4 + 2*(a^5 - 4*a^4*b - 11*a
^3*b^2 - 6*a^2*b^3)*d*x - 3*(5*a^5 + 26*a^4*b + 131*a^3*b^2 + 256*a^2*b^3 +
 128*a*b^4 - 16*(a^5 - 2*a^4*b - 19*a^3*b^2 - 28*a^2*b^3 - 12*a*b^4)*d*x)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 + ((35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cos
h(d*x + c)^10 + 10*(35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cosh(d*x + c)*sinh(
d*x + c)^9 + (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*sinh(d*x + c)^10 + 4*(35*
a^4*b + 126*a^3*b^2 + 136*a^2*b^3 + 48*a*b^4)*cosh(d*x + c)^8 + (140*a^4*b
+ 504*a^3*b^2 + 544*a^2*b^3 + 192*a*b^4 + 45*(35*a^4*b + 56*a^3*b^2 + 24*a^
2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 56*a^3*b^2 + 24
*a^2*b^3)*cosh(d*x + c)^3 + 4*(35*a^4*b + 126*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(195) = 390.
time = 1.78, size = 395, normalized size = 1.94

$$\frac{\frac{(35\,a^2b^2+56\,ab^3+24\,b^4)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{(a^6+2\,a^5b+a^4b^2)\sqrt{-ab-b^2}}-\frac{2\,(13\,a^3b^2e^{(6\,dx+6\,c)}+40\,a^2b^3e^{(6\,dx+6\,c)}+24\,ab^4e^{(6\,dx+6\,c)}+39\,a^3b^2e^{(4\,dx+4\,c)}+134\,a^2b^3e^{(4\,dx+4\,c)}+184\,ab^4e^{(4\,dx+4\,c)}+80\,b^5e^{(4\,dx+4\,c)}+39\,a^3b^2e^{(2\,dx+2\,c)}+104\,a^2b^3e^{(2\,dx+2\,c)}+56\,ab^4e^{(2\,dx+2\,c)}+13\,a^3b^2+10\,a^2b^3)}{(a^6+2\,a^5b+a^4b^2)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)^2}+\frac{4\,(dx+c)(a-6\,b)}{a^4}+\frac{e^{(2\,dx+2\,c)}}{a^3}-\frac{(2\,ae^{(2\,dx+2\,c)}-12\,be^{(2\,dx+2\,c)}+a)e^{(-2\,dx-2\,c)}}{a^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((35*a^2*b^2 + 56*a*b^3 + 24*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2
*b)/sqrt(-a*b - b^2))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-a*b - b^2)) - 2*(13*
a^3*b^2*e^(6*d*x + 6*c) + 40*a^2*b^3*e^(6*d*x + 6*c) + 24*a*b^4*e^(6*d*x +
6*c) + 39*a^3*b^2*e^(4*d*x + 4*c) + 134*a^2*b^3*e^(4*d*x + 4*c) + 184*a*b^4
*e^(4*d*x + 4*c) + 80*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d*x + 2*c) + 10
4*a^2*b^3*e^(2*d*x + 2*c) + 56*a*b^4*e^(2*d*x + 2*c) + 13*a^3*b^2 + 10*a^2*
b^3)/((a^6 + 2*a^5*b + a^4*b^2)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) +
4*b*e^(2*d*x + 2*c) + a)^2) + 4*(d*x + c)*(a - 6*b)/a^4 + e^(2*d*x + 2*c)/a
^3 - (2*a*e^(2*d*x + 2*c) - 12*b*e^(2*d*x + 2*c) + a)*e^(-2*d*x - 2*c)/a^4)
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cosh\left(c+d\,x\right)}^2}{{\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3, x)
```

3.94 $\int \frac{\cosh(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=154

$$-\frac{3b(4(a+b)^2+(2a+b)^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}d}+\frac{\sinh(c+dx)}{a^3d}-\frac{b^3\sinh(c+dx)}{4a^3(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2}$$

```
[Out] -3/8*b*(4*(a+b)^2+(2*a+b)^2)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(7/2
)/(a+b)^(5/2)/d+sinh(d*x+c)/a^3/d-1/4*b^3*sinh(d*x+c)/a^3/(a+b)/d/(a+b+a*si
nh(d*x+c)^2)^2+3/8*b^2*(4*a+3*b)*sinh(d*x+c)/a^3/(a+b)^2/d/(a+b+a*sinh(d*x+
c)^2)
```

Rubi [A]
time = 0.14, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {4232, 398, 1171, 393, 211}

$$-\frac{3b(4(a+b)^2+(2a+b)^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{7/2}d(a+b)^{5/2}}-\frac{b^3\sinh(c+dx)}{4a^3d(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}+\frac{3b^2(4a+3b)\sinh(c+dx)}{8a^3d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\sinh(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b
]])/(8*a^(7/2)*(a + b)^(5/2)*d) + Sinh[c + d*x]/(a^3*d) - (b^3*Sinh[c + d*x
])/(4*a^3*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) + (3*b^2*(4*a + 3*b)*Sin
h[c + d*x])/(8*a^3*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{(a+b+ax^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{a^3}-\frac{b(3a^2+3ab+b^2)+3ab(2a+b)x^2+3a^2bx^4}{a^3(a+b+ax^2)^3}\right)\,dx, x, \sinh(c+dx)\right)}{d}\\
&= \frac{\sinh(c+dx)}{a^3d}-\frac{\operatorname{Subst}\left(\int \frac{b(3a^2+3ab+b^2)+3ab(2a+b)x^2+3a^2bx^4}{(a+b+ax^2)^3}\,dx, x, \sinh(c+dx)\right)}{a^3d}\\
&= \frac{\sinh(c+dx)}{a^3d}-\frac{b^3\sinh(c+dx)}{4a^3(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2}+\frac{\operatorname{Subst}\left(\int \frac{-3b(2a+b)^2-}{(a+b+}\right.}{4}\\
&= \frac{\sinh(c+dx)}{a^3d}-\frac{b^3\sinh(c+dx)}{4a^3(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2}+\frac{3b^2(4a+3b)}{8a^3(a+b)^2d\left(a+b\right.}\\
&= -\frac{3b(4(a+b)^2+(2a+b)^2)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}d}+\frac{\sinh(c+dx)}{a^3d}-\frac{}{4a^3(a+}
\end{aligned}
$$

Mathematica [A]

time = 2.46, size = 292, normalized size = 1.90

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^5(c+dx)\left(\frac{3b(8a^2+12ab+5b^2)\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\operatorname{csch}(c+dx)\sqrt{(\cosh(c)-\sinh(c))^2}(\cosh(c)+\sinh(c))}{\sqrt{a}}\right)(a+2b+a\cosh(2(c+dx)))^2\operatorname{sech}(c+dx)(\cosh(c)-\sinh(c))}{(a+b)^{5/2}\sqrt{(\cosh(c)-\sinh(c))^2}}+8\sqrt{a}\cosh(dx)(a+2b+a\cosh(2(c+dx)))^2\operatorname{sech}(c+dx)\sinh(c)+8\sqrt{a}\cosh(c)(a+2b+a\cosh(2(c+dx)))^2\operatorname{sech}(c+dx)\sinh(dx)-\frac{8\sqrt{a}\,b^3\tanh(c+dx)}{a+b}+\frac{6\sqrt{a}\,b^2(4a+3b)(a+2b+a\cosh(2(c+dx)))\tanh(c+dx)}{(a+b)^2}\right)}{64a^{7/2}d\left(a+b\operatorname{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*((3*b*(8*a^2 + 12*a*b + 5*
b^2)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c]
 + Sinh[c]))/Sqrt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*(Cosh
[c] - Sinh[c]))/((a + b)^(5/2)*Sqrt[(Cosh[c] - Sinh[c])^2]) + 8*Sqrt[a]*Cos
h[d*x]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*Sinh[c] + 8*Sqrt[a]*
Cosh[c]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*Sinh[d*x] - (8*Sqrt
[a]*b^3*Tanh[c + d*x])/(a + b) + (6*Sqrt[a]*b^2*(4*a + 3*b)*(a + 2*b + a*Co
sh[2*(c + d*x)])*Tanh[c + d*x])/(a + b)^2))/(64*a^(7/2)*d*(a + b*Sech[c + d
*x]^2)^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(140) = 280$.
time = 3.30, size = 335, normalized size = 2.18

method	result
derivativedivides	$-\frac{1}{a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2b\left(\frac{\frac{b(12a+7b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a+8b}+\frac{3b\left(4a^2-7ab-7b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2}-\frac{3b\left(4a^2-7ab-7b^2\right)\left(\tanh^3\left(\frac{dx}{2}-\right.\right.}{8(a+b)^2}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\right.\right.\right.}\right.}{}$
default	$-\frac{1}{a^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2b\left(\frac{\frac{b(12a+7b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a+8b}+\frac{3b\left(4a^2-7ab-7b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2}-\frac{3b\left(4a^2-7ab-7b^2\right)\left(\tanh^3\left(\frac{dx}{2}-\right.\right.}{8(a+b)^2}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\right.\right.\right.}\right.}{}$

risch	$\frac{e^{dx+c}}{2a^3d} - \frac{e^{-dx-c}}{2a^3d} + \frac{b^2e^{dx+c}(12a^2e^{6dx+6c}+9ab\,e^{6dx+6c}+12a^2e^{4dx+4c}+49ab\,e^{4dx+4c}+28b^2e^{4dx+4c}-12a^2e^{2dx+2c}-49a}{4a^3d(a+b)^2(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^3/(tanh(1/2*d*x+1/2*c)+1)-2/a^3*b*((1/8*b*(12*a+7*b)/(a+b)*tanh(1
/2*d*x+1/2*c)^7+3/8*b*(4*a^2-7*a*b-7*b^2)/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-3/8
*b*(4*a^2-7*a*b-7*b^2)/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-1/8*b*(12*a+7*b)/(a+b)
*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*
tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)^2+3/8*(8*a^2+12*a*b+5*
b^2)/(a^2+2*a*b+b^2)*(1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tan
h(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*
(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))))-1/a^3/(tanh(1/2*d*x+1
/2*c)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^4 + 4*a^3*b + 2*a^2*b^2 - 2*(a^4*e^(10*c) + 2*a^3*b*e^(10*c) + a^
2*b^2*e^(10*c))*e^(10*d*x) - (6*a^4*e^(8*c) + 28*a^3*b*e^(8*c) + 50*a^2*b^2
*e^(8*c) + 25*a*b^3*e^(8*c))*e^(8*d*x) - (4*a^4*e^(6*c) + 24*a^3*b*e^(6*c)
+ 80*a^2*b^2*e^(6*c) + 129*a*b^3*e^(6*c) + 60*b^4*e^(6*c))*e^(6*d*x) + (4*a
^4*e^(4*c) + 24*a^3*b*e^(4*c) + 80*a^2*b^2*e^(4*c) + 129*a*b^3*e^(4*c) + 60
*b^4*e^(4*c))*e^(4*d*x) + (6*a^4*e^(2*c) + 28*a^3*b*e^(2*c) + 50*a^2*b^2*e^
(2*c) + 25*a*b^3*e^(2*c))*e^(2*d*x))/((a^7*d*e^(9*c) + 2*a^6*b*d*e^(9*c) +
a^5*b^2*d*e^(9*c))*e^(9*d*x) + 4*(a^7*d*e^(7*c) + 4*a^6*b*d*e^(7*c) + 5*a^5
*b^2*d*e^(7*c) + 2*a^4*b^3*d*e^(7*c))*e^(7*d*x) + 2*(3*a^7*d*e^(5*c) + 14*a
^6*b*d*e^(5*c) + 27*a^5*b^2*d*e^(5*c) + 24*a^4*b^3*d*e^(5*c) + 8*a^3*b^4*d*
e^(5*c))*e^(5*d*x) + 4*(a^7*d*e^(3*c) + 4*a^6*b*d*e^(3*c) + 5*a^5*b^2*d*e^(
3*c) + 2*a^4*b^3*d*e^(3*c))*e^(3*d*x) + (a^7*d*e^c + 2*a^6*b*d*e^c + a^5*b^
2*d*e^c)*e^(d*x)) - 1/2*integrate(3/2*((8*a^2*b*e^(3*c) + 12*a*b^2*e^(3*c)
+ 5*b^3*e^(3*c))*e^(3*d*x) + (8*a^2*b*e^c + 12*a*b^2*e^c + 5*b^3*e^c)*e^(d*
x))/(a^6 + 2*a^5*b + a^4*b^2 + (a^6*e^(4*c) + 2*a^5*b*e^(4*c) + a^4*b^2*e^(
4*c))*e^(4*d*x) + 2*(a^6*e^(2*c) + 4*a^5*b*e^(2*c) + 5*a^4*b^2*e^(2*c) + 2*
a^3*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5414 vs. $2(140) = 280$.

time = 0.50, size = 9856, normalized size = 64.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

```
[Out] [1/16*(8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^10 + 80*(a^6 +
 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + 8*(a^6 + 3*
a^5*b + 3*a^4*b^2 + a^3*b^3)*sinh(d*x + c)^10 + 4*(6*a^6 + 34*a^5*b + 78*a^
4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^8 + 4*(6*a^6 + 34*a^5*b + 78
*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4 + 90*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 32*(30*(a^6 + 3*a^5*b + 3*a^4*b^2 + a
^3*b^3)*cosh(d*x + c)^3 + (6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*
a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(4*a^6 + 28*a^5*b + 104*a^4*b^2
 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^6 + 4*(4*a^6 + 28*a^
5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5 + 420*(a^6 + 3*a^5
*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^4 + 28*(6*a^6 + 34*a^5*b + 78*a^4*b
^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 8*a^6 - 24
*a^5*b - 24*a^4*b^2 - 8*a^3*b^3 + 8*(252*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b
^3)*cosh(d*x + c)^5 + 28*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a
^2*b^4)*cosh(d*x + c)^3 + 3*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 +
 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(4*a^6 + 28*a^5
*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^4 +
4*(420*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^6 - 4*a^6 - 28*a
^5*b - 104*a^4*b^2 - 209*a^3*b^3 - 189*a^2*b^4 - 60*a*b^5 + 70*(6*a^6 + 34*
a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^4 + 15*(4*a^6 +
 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 + 16*(60*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d
*x + c)^7 + 14*(6*a^6 + 34*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*co
sh(d*x + c)^5 + 5*(4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b
^4 + 60*a*b^5)*cosh(d*x + c)^3 - (4*a^6 + 28*a^5*b + 104*a^4*b^2 + 209*a^3*
b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(6*a^6 + 3
4*a^5*b + 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^2 + 4*(90*(a^
6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^8 + 28*(6*a^6 + 34*a^5*b +
 78*a^4*b^2 + 75*a^3*b^3 + 25*a^2*b^4)*cosh(d*x + c)^6 - 6*a^6 - 34*a^5*b -
 78*a^4*b^2 - 75*a^3*b^3 - 25*a^2*b^4 + 15*(4*a^6 + 28*a^5*b + 104*a^4*b^2
+ 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^4 - 6*(4*a^6 + 28*a^5
*b + 104*a^4*b^2 + 209*a^3*b^3 + 189*a^2*b^4 + 60*a*b^5)*cosh(d*x + c)^2)*s
inh(d*x + c)^2 - 3*((8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^9 + 9*
(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + (8*a^4*b
 + 12*a^3*b^2 + 5*a^2*b^3)*sinh(d*x + c)^9 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a
^2*b^3 + 10*a*b^4)*cosh(d*x + c)^7 + 4*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 +
 10*a*b^4 + 9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^7 + 28*(3*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^3 + (8*a^4*
```

```
b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^6 + 2*
(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^5
 + 2*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5 + 63*(8*a^4
*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^4 + 42*(8*a^4*b + 28*a^3*b^2 + 2
9*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(8*a^4*b + 1
2*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^5 + 70*(8*a^4*b + 28*a^3*b^2 + 29*a^2*
b^3 + 10*a*b^4)*cosh(d*x + c)^3 + 5*(24*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 +
 136*a*b^4 + 40*b^5)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(8*a^4*b + 28*a^3*b
^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^3 + 4*(21*(8*a^4*b + 12*a^3*b^2 +
 5*a^2*b^3)*cosh(d*x + c)^6 + 8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4
+ 35*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^4 + 5*(24
*a^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^2)*s
inh(d*x + c)^3 + 4*(9*(8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^7 +
21*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^5 + 5*(24*a
^4*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^3 + 3*
(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c))*sinh(d*x + c)
^2 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c) + (9*(8*a^4*b + 12*a^
3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^8 + 28*(8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3
+ 10*a*b^4)*cosh(d*x + c)^6 + 8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3 + 10*(24*a^4
*b + 100*a^3*b^2 + 175*a^2*b^3 + 136*a*b^4 + 40*b^5)*cosh(d*x + c)^4 + 12*(
8*a^4*b + 28*a^3*b^2 + 29*a^2*b^3 + 10*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c
)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x +
 c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cos
h(d*x + c))*sinh(d*x + c) + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x +
 c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x
+ c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
 + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x
 + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)}{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)/(a + b/cosh(c + d*x)^2)^3, x)
```

3.95 $\int \frac{\text{sech}(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3} \, dx$

Optimal. Leaf size=142

$$\frac{(8a^2+8ab+3b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}d}-\frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2}-\frac{3b(2a+b)\sinh(c}{8a^2(a+b)^2d\left(a+b+a\,\mathrm{s}\right.}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/(a+
b)^(5/2)/d-1/4*b*cosh(d*x+c)^2*sinh(d*x+c)/a/(a+b)/d/(a+b+a*sinh(d*x+c)^2)^
2-3/8*b*(2*a+b)*sinh(d*x+c)/a^2/(a+b)^2/d/(a+b+a*sinh(d*x+c)^2)
```

Rubi [A]
time = 0.10, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4232, 424, 393, 211}

$$-\frac{3b(2a+b)\sinh(c+dx)}{8a^2d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}+\frac{(8a^2+8ab+3b^2)\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{5/2}d(a+b)^{5/2}}-\frac{b\sinh(c+dx)\cosh^2(c+dx)}{4ad(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*a^
(5/2)*(a + b)^(5/2)*d) - (b*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*(a + b)*d*(
a + b + a*Sinh[c + d*x]^2)^2) - (3*b*(2*a + b)*Sinh[c + d*x])/(8*a^2*(a + b
)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+b+ax^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{4a+b+(4a+3b)x^2}{\left(a+b+ax^2\right)^2}\,dx, x, \sinh \dots\right)}{4a(a+b)d}$$

$$= -\frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} - \frac{3b(2a+b)\sinh(c+dx)}{8a^2(a+b)^2d\left(a+b+a\sinh^2(c+\dots\right)}$$

$$= \frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}d} - \frac{b\cosh^2(c+dx)\sinh(c+dx\dots}{4a(a+b)d\left(a+b+a\sinh^2(c+\dots\right)}$$

Mathematica [A]

time = 1.99, size = 214, normalized size = 1.51

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^5(c+dx)\left(\frac{(8a^2+8ab+3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{a+b}\,\operatorname{csch}(c+dx)\sqrt{(\cosh(c)-\sinh(c))^2}\,(\cosh(c)+\sinh(c))}{\sqrt{a}}\right)(a+2b+a\cosh(2(c+dx)))^2\operatorname{sech}(c+dx)(-\cosh(c)+\sinh(c))}{\sqrt{a+b}\,\sqrt{(\cosh(c)-\sinh(c))^2}}+8\sqrt{a}\,b^2(a+b)\tanh(c+dx)-2\sqrt{a}\,b(8a+5b)(a+2b+a\cosh(2(c+dx)))\tanh(c+dx)\right)}{64a^{5/2}(a+b)^2d\left(a+b\operatorname{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*(((8*a^2 + 8*a*b + 3*b^2)*
ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]*(Cosh[c] + Si
nh[c]))/Sqrt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]*(-Cosh[c]
+ Sinh[c]))/(Sqrt[a + b]*Sqrt[(Cosh[c] - Sinh[c])^2]) + 8*Sqrt[a]*b^2*(a +
```

```
b)*Tanh[c + d*x] - 2*Sqrt[a]*b*(8*a + 5*b)*(a + 2*b + a*Cosh[2*(c + d*x)])*
Tanh[c + d*x]))/(64*a^(5/2)*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(128) = 256$.

time = 2.40, size = 308, normalized size = 2.17

method	result
derivativedivides	$\frac{\frac{\frac{b(8a+3b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)a^2}+\frac{b\left(8a^2-13ab-9b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2a^2}-\frac{b\left(8a^2-13ab-9b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2a^2}-\frac{b(8a+3b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4(a+b)a^2}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+}{d}$
default	$\frac{\frac{\frac{b(8a+3b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)a^2}+\frac{b\left(8a^2-13ab-9b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2a^2}-\frac{b\left(8a^2-13ab-9b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2a^2}-\frac{b(8a+3b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4(a+b)a^2}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+}{d}$
risch	$-\frac{e^{dx+c}b\left(8a^2e^{6dx+6c}+5ab\,e^{6dx+6c}+8a^2e^{4dx+4c}+29ab\,e^{4dx+4c}+12b^2e^{4dx+4c}-8a^2e^{2dx+2c}-29ab\,e^{2dx+2c}-12b^2e^{2dx+2c}\right.}{4a^2(a+b)^2d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(1/8*b*(8*a+3*b)/(a+b)/a^2*tanh(1/2*d*x+1/2*c)^7+1/8*b*(8*a^2-13*a*b
-9*b^2)/(a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^5-1/8*b*(8*a^2-13*a*b-9*b^2)/(a+b)^
2/a^2*tanh(1/2*d*x+1/2*c)^3-1/8*b*(8*a+3*b)/(a+b)/a^2*tanh(1/2*d*x+1/2*c))/
(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-
2*b*tanh(1/2*d*x+1/2*c)^2+a+b)^2+1/4*(8*a^2+8*a*b+3*b^2)/a^2/(a^2+2*a*b+b^2
)*(1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*
b^(1/2))/a^(1/2))+1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/
2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((8*a^2*b*e^(7*c) + 5*a*b^2*e^(7*c))*e^(7*d*x) + (8*a^2*b*e^(5*c) + 29
*a*b^2*e^(5*c) + 12*b^3*e^(5*c))*e^(5*d*x) - (8*a^2*b*e^(3*c) + 29*a*b^2*e^
```

```
(3*c) + 12*b^3*e^(3*c))*e^(3*d*x) - (8*a^2*b*e^c + 5*a*b^2*e^c)*e^(d*x))/(a
^6*d + 2*a^5*b*d + a^4*b^2*d + (a^6*d*e^(8*c) + 2*a^5*b*d*e^(8*c) + a^4*b^2
*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 4*a^5*b*d*e^(6*c) + 5*a^4*b^2*d*
e^(6*c) + 2*a^3*b^3*d*e^(6*c))*e^(6*d*x) + 2*(3*a^6*d*e^(4*c) + 14*a^5*b*d*
e^(4*c) + 27*a^4*b^2*d*e^(4*c) + 24*a^3*b^3*d*e^(4*c) + 8*a^2*b^4*d*e^(4*c)
)*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 4*a^5*b*d*e^(2*c) + 5*a^4*b^2*d*e^(2*c) +
2*a^3*b^3*d*e^(2*c))*e^(2*d*x)) + 2*integrate(1/8*((8*a^2*e^(3*c) + 8*a*b*e
^(3*c) + 3*b^2*e^(3*c))*e^(3*d*x) + (8*a^2*e^c + 8*a*b*e^c + 3*b^2*e^c)*e^(
d*x))/(a^5 + 2*a^4*b + a^3*b^2 + (a^5*e^(4*c) + 2*a^4*b*e^(4*c) + a^3*b^2*e
^(4*c))*e^(4*d*x) + 2*(a^5*e^(2*c) + 4*a^4*b*e^(2*c) + 5*a^3*b^2*e^(2*c) +
2*a^2*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3753 vs. $2(128) = 256$.
time = 0.49, size = 6806, normalized size = 47.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^7 + 28*(8*a^4*b
+ 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(8*a^4*b + 13*a
^3*b^2 + 5*a^2*b^3)*sinh(d*x + c)^7 + 4*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3
+ 12*a*b^4)*cosh(d*x + c)^5 + 4*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b
^4 + 21*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5
 + 20*(7*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c)^3 + (8*a^4*b + 37
*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(8*a^4
*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^3 - 4*(8*a^4*b + 37*
a^3*b^2 + 41*a^2*b^3 + 12*a*b^4 - 35*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cos
h(d*x + c)^4 - 10*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x +
 c)^2)*sinh(d*x + c)^3 + 4*(21*(8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x
+ c)^5 + 10*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^3
- 3*(8*a^4*b + 37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c))*sinh(d*x
+ c)^2 + ((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^8 + 8*(8*a^4 + 8*a^3*
b + 3*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (8*a^4 + 8*a^3*b + 3*a^2*b^2
)*sinh(d*x + c)^8 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x +
c)^6 + 4*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3 + 7*(8*a^4 + 8*a^3*b + 3*
a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^4 + 8*a^3*b + 3*a^2*b
^2)*cosh(d*x + c)^3 + 3*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^5 + 2*(24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*
b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^4
+ 24*a^4 + 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4 + 30*(8*a^4 + 24*a^3*
b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 8*a^3*
b + 3*a^2*b^2 + 8*(7*(8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^5 + 10*(8*
```

```
a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^3 + (24*a^4 + 88*a^3*b
 + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a
^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^2 + 4*(7*(8*a^4 + 8*a^3
*b + 3*a^2*b^2)*cosh(d*x + c)^6 + 15*(8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b
^3)*cosh(d*x + c)^4 + 8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*a*b^3 + 3*(24*a^4 +
 88*a^3*b + 137*a^2*b^2 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)
^2 + 8*((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^7 + 3*(8*a^4 + 24*a^3*b
 + 19*a^2*b^2 + 6*a*b^3)*cosh(d*x + c)^5 + (24*a^4 + 88*a^3*b + 137*a^2*b^2
 + 88*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (8*a^4 + 24*a^3*b + 19*a^2*b^2 + 6*
a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^
4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a
*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x +
 c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c
)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x +
 c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)
*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a
*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*(8*a^4*
b + 13*a^3*b^2 + 5*a^2*b^3)*cosh(d*x + c) + 4*(7*(8*a^4*b + 13*a^3*b^2 + 5*
a^2*b^3)*cosh(d*x + c)^6 - 8*a^4*b - 13*a^3*b^2 - 5*a^2*b^3 + 5*(8*a^4*b +
37*a^3*b^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^4 - 3*(8*a^4*b + 37*a^3*b
^2 + 41*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^8 + 3*a^7*b
 + 3*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^8 + 3*a^7*b + 3*a^6*b^2 +
a^5*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5
*b^3)*d*sinh(d*x + c)^8 + 4*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*
b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*cosh(
d*x + c)^2 + (a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*
x + c)^6 + 2*(3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4 + 8*a
^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d*co
sh(d*x + c)^3 + 3*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*d
*cosh(d*x + c)^4 + 30*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d
*cosh(d*x + c)^2 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*b^4
 + 8*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3
 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3
)*d*cosh(d*x + c)^5 + 10*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4
)*d*cosh(d*x + c)^3 + (3*a^8 + 17*a^7*b + 41*a^6*b^2 + 51*a^5*b^3 + 32*a^4*
b^4 + 8*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^8 + 3*a^7*b + 3
*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^6 + 15*(a^8 + 5*a^7*b + 9*a^6*b^2 + 7*a
^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^8 + 17*a^7*b + 41*a^6*b^2 +
51*a^5*b^3 + 32*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^2 + (a^8 + 5*a^7*b + 9
*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^2 + (a^8 + 3*a^7*b + 3*a
^6*b^2 + a^5*b^3)*d + 8*((a^8 + 3*a^7*b + 3*a^6...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+d\,x)\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)*(a + b/cosh(c + d*x)^2)^3), x)
```

3.96 $\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=108

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{b}\,(a+b)^{5/2}d}+\frac{\tanh(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}+\frac{3\tanh(c+dx)}{8(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

```
[Out] 3/8*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/(a+b)^(5/2)/d/b^(1/2)+1/4*tanh
(d*x+c)/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2+3/8*tanh(d*x+c)/(a+b)^2/d/(a+b-b*ta
nh(d*x+c)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4231, 205, 214}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{b}\,d(a+b)^{5/2}}+\frac{3\tanh(c+dx)}{8d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}+\frac{\tanh(c+dx)}{4d(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*Sqrt[b]*(a + b)^(5/2)*d
) + Tanh[c + d*x]/(4*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2) + (3*Tanh[c +
 d*x])/(8*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
```

```
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3\operatorname{Subst}\left(\int \frac{1}{(a+b-bx^2)^2}\,dx, x, \tanh(c+$$

$$= \frac{\tanh(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c+dx)}{8(a+b)^2d\left(a+b-b\tanh^2(c+dx)\right)}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{b}\,(a+b)^{5/2}d} + \frac{\tanh(c+dx)}{4(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{}{8(a+b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 258 vs. 2(108) = 216.
time = 1.52, size = 258, normalized size = 2.39

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^6(c+dx)\left(\frac{3\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(a+2b+a\cosh(2(c+dx)))^2(\cosh(2c)-\sinh(2c))}{\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}+\frac{4b(a+b)\operatorname{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{a^2}-\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}(2c)\left((5a^2+16ab+8b^2)\sinh(2c)-a(5a+2b)\sinh(2dx)\right)}{a^2}\right)}{64(a+b)^2d\left(a+b\operatorname{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*((3*ArcTanh[(Sech[d*x]*(Co
sh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a +
 b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*(Cosh
[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (4*b*(a +
 b)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/a^2 - ((a + 2*b + a*Co
sh[2*(c + d*x)])*Sech[2*c]*((5*a^2 + 16*a*b + 8*b^2)*Sinh[2*c] - a*(5*a + 2
*b)*Sinh[2*d*x]))/a^2))/(64*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(94) = 188.
time = 2.13, size = 260, normalized size = 2.41

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{5\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}-\frac{3(5a+b)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2}-\frac{3(5a+b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)}\right)}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}-\frac{3\left(-\frac{\ln\left(\sqrt{a+b}\left(\tanh^2\right.\right.}{}\right.}{}}{d}$
default	$\frac{-\frac{2\left(-\frac{5\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}-\frac{3(5a+b)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2}-\frac{3(5a+b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)}\right)}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}-\frac{3\left(-\frac{\ln\left(\sqrt{a+b}\left(\tanh^2\right.\right.}{}\right.}{}}{d}$
risch	$-\frac{5a^3e^{6dx+6c}+16a^2b\,e^{6dx+6c}+8a\,b^2e^{6dx+6c}+15a^3e^{4dx+4c}+46a^2b\,e^{4dx+4c}+56a\,b^2e^{4dx+4c}+16b^3e^{4dx+4c}+15a^3e^{2dx+2c}+}{4a^2d(a+b)^2\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-5/8/(a+b)*tanh(1/2*d*x+1/2*c)^7-3/8*(5*a+b)/(a+b)^2*tanh(1/2*d*x+
1/2*c)^5-3/8*(5*a+b)/(a+b)^2*tanh(1/2*d*x+1/2*c)^3-5/8/(a+b)*tanh(1/2*d*x+1
/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/
2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)^2-3/4/(a^2+2*a*b+b^2)*(-1/4/b^(1/2)/(
a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/
2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^
2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(100) = 200$.
time = 0.53, size = 353, normalized size = 3.27

$$\frac{5a^3+2a^2b+(15a^3+32a^2b+8ab^2)e^{(-2dx-2c)}+(15a^3+46a^2b+56ab^2+16b^3)e^{(-4dx-4c)}+(5a^3+16a^2b+8ab^2)e^{(-6dx-6c)}}{4\left(a^6+2a^5b+a^4b^2+4\left(a^6+4a^5b+5a^4b^2+2a^3b^3\right)e^{(-2dx-2c)}+2\left(3a^6+14a^5b+27a^4b^2+24a^3b^3+8a^2b^4\right)e^{(-4dx-4c)}+4\left(a^6+4a^5b+5a^4b^2+2a^3b^3\right)e^{(-6dx-6c)}+\left(a^6+2a^5b+a^4b^2\right)e^{(-8dx-8c)}\right)d}-\frac{3\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{16\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(5*a^3 + 2*a^2*b + (15*a^3 + 32*a^2*b + 8*a*b^2)*e^(-2*d*x - 2*c) + (15
*a^3 + 46*a^2*b + 56*a*b^2 + 16*b^3)*e^(-4*d*x - 4*c) + (5*a^3 + 16*a^2*b +
 8*a*b^2)*e^(-6*d*x - 6*c))/((a^6 + 2*a^5*b + a^4*b^2 + 4*(a^6 + 4*a^5*b +
5*a^4*b^2 + 2*a^3*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^6 + 14*a^5*b + 27*a^4*b^2
+ 24*a^3*b^3 + 8*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(a^6 + 4*a^5*b + 5*a^4*b^2 +
 2*a^3*b^3)*e^(-6*d*x - 6*c) + (a^6 + 2*a^5*b + a^4*b^2)*e^(-8*d*x - 8*c))*
d) - 3/16*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d
*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)
*b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2434 vs. $2(100) = 200$.
time = 0.40, size = 5109, normalized size = 47.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^6 + 2
4*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)*sinh(d*x + c)
^5 + 4*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*sinh(d*x + c)^6 + 20*a
^4*b + 28*a^3*b^2 + 8*a^2*b^3 + 4*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72
*a*b^4 + 16*b^5)*cosh(d*x + c)^4 + 4*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 +
 72*a*b^4 + 16*b^5 + 15*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*
a*b^4)*cosh(d*x + c)^3 + (15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 +
16*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4*b + 47*a^3*b^2 + 40*a^2*
b^3 + 8*a*b^4)*cosh(d*x + c)^2 + 4*(15*a^4*b + 47*a^3*b^2 + 40*a^2*b^3 + 8*
a*b^4 + 15*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^4 +
6*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 - 3*(a^4*cosh(d*x + c)^8 + 8*a^4*cosh(d*x + c)*sinh(d*x +
 c)^7 + a^4*sinh(d*x + c)^8 + 4*(a^4 + 2*a^3*b)*cosh(d*x + c)^6 + 4*(7*a^4*
cosh(d*x + c)^2 + a^4 + 2*a^3*b)*sinh(d*x + c)^6 + 8*(7*a^4*cosh(d*x + c)^3
 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 8*a^3*b +
8*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*a^4*cosh(d*x + c)^4 + 3*a^4 + 8*a^3*b +
8*a^2*b^2 + 30*(a^4 + 2*a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 8*(
7*a^4*cosh(d*x + c)^5 + 10*(a^4 + 2*a^3*b)*cosh(d*x + c)^3 + (3*a^4 + 8*a^3
*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b)*cosh(d*x
 + c)^2 + 4*(7*a^4*cosh(d*x + c)^6 + 15*(a^4 + 2*a^3*b)*cosh(d*x + c)^4 + a
^4 + 2*a^3*b + 3*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 8*(a^4*cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b)*cosh(d*x + c)^5 + (3*a^4
+ 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*a^3*b)*cosh(d*x + c))*sin
h(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*
sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2
*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^
2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4
*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 +
 a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x
+ c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x
+ c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d
*x + c))*sinh(d*x + c) + a)) + 8*(3*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*
a*b^4)*cosh(d*x + c)^5 + 2*(15*a^4*b + 61*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4
+ 16*b^5)*cosh(d*x + c)^3 + (15*a^4*b + 47*a^3*b^2 + 40*a^2*b^3 + 8*a*b^4)*
cosh(d*x + c))*sinh(d*x + c))/((a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*
```

```
cosh(d*x + c)^8 + 8*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x +
c)*sinh(d*x + c)^7 + (a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*sinh(d*x +
 c)^8 + 4*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*
x + c)^6 + 4*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^2
 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d)*sinh(d*x + c)
^6 + 2*(3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2
*b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*
cosh(d*x + c)^3 + 3*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)
*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 +
a^4*b^4)*d*cosh(d*x + c)^4 + 30*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4
+ 2*a^3*b^5)*d*cosh(d*x + c)^2 + (3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^
4*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d)*sinh(d*x + c)^4 + 4*(a^7*b + 5*a^6*b^2 +
 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^7*b + 3*a^6
*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^5 + 10*(a^7*b + 5*a^6*b^2 + 9*a
^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^3 + (3*a^7*b + 17*a^6*b^2 +
 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d*cosh(d*x + c))*sinh(d*
x + c)^3 + 4*(7*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^6
 + 15*(a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x +
c)^4 + 3*(3*a^7*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a
^2*b^6)*d*cosh(d*x + c)^2 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*
a^3*b^5)*d)*sinh(d*x + c)^2 + (a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d +
 8*((a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*d*cosh(d*x + c)^7 + 3*(a^7*b
+ 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh(d*x + c)^5 + (3*a^7
*b + 17*a^6*b^2 + 41*a^5*b^3 + 51*a^4*b^4 + 32*a^3*b^5 + 8*a^2*b^6)*d*cosh(
d*x + c)^3 + (a^7*b + 5*a^6*b^2 + 9*a^5*b^3 + 7*a^4*b^4 + 2*a^3*b^5)*d*cosh
(d*x + c))*sinh(d*x + c)), -1/8*(2*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a
*b^4)*cosh(d*x + c)^6 + 12*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^4)*co
sh(d*x + c)*sinh(d*x + c)^5 + 2*(5*a^4*b + 21*a^3*b^2 + 24*a^2*b^3 + 8*a*b^
4)*sinh(d*x + c)^6 + 10*a^4*b + 14*a^3*b^2 + 4*a^2*b^3 + 2*(15*a^4*b + 61*a
^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5)*cosh(d*x + c)^4 + 2*(15*a^4*b + 6
1*a^3*b^2 + 102*a^2*b^3 + 72*a*b^4 + 16*b^5 + 1...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(100) = 200.

time = 0.81, size = 282, normalized size = 2.61

$$\frac{\frac{3 \arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{(a^2+2\,ab+b^2)\sqrt{-ab-b^2}} - \frac{2\,(5\,a^3e^{(6\,dx+6\,c)}+16\,a^2be^{(6\,dx+6\,c)}+8\,ab^2e^{(6\,dx+6\,c)}+15\,a^3e^{(4\,dx+4\,c)}+46\,a^2be^{(4\,dx+4\,c)}+56\,ab^2e^{(4\,dx+4\,c)}+16\,b^3e^{(4\,dx+4\,c)}+15\,a^3e^{(2\,dx+2\,c)}+32\,a^2be^{(2\,dx+2\,c)}+8\,ab^2e^{(2\,dx+2\,c)}+5\,a^3+2\,a^2b)}{(a^4+2\,a^3b+a^2b^2)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^2 + 2
*a*b + b^2)*sqrt(-a*b - b^2)) - 2*(5*a^3*e^(6*d*x + 6*c) + 16*a^2*b*e^(6*d*
x + 6*c) + 8*a*b^2*e^(6*d*x + 6*c) + 15*a^3*e^(4*d*x + 4*c) + 46*a^2*b*e^(4
*d*x + 4*c) + 56*a*b^2*e^(4*d*x + 4*c) + 16*b^3*e^(4*d*x + 4*c) + 15*a^3*e^
(2*d*x + 2*c) + 32*a^2*b*e^(2*d*x + 2*c) + 8*a*b^2*e^(2*d*x + 2*c) + 5*a^3
+ 2*a^2*b)/((a^4 + 2*a^3*b + a^2*b^2)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2
*c) + 4*b*e^(2*d*x + 2*c) + a)^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^2\,\left(a+\frac{b}{\cosh\left(c+d\,x\right)^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3), x)
```

3.97 $\int \frac{\text{sech}^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=123

$$\frac{(4a+b)\text{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{3/2}(a+b)^{5/2}d}-\frac{b\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2}+\frac{(4a+b)\sinh(c+dx)}{8a(a+b)^2d\left(a+b+a\sinh^2(c+d\right.}$$

```
[Out] 1/8*(4*a+b)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(5/2)/d-1
/4*b*sinh(d*x+c)/a/(a+b)/d/(a+b+a*sinh(d*x+c)^2)^2+1/8*(4*a+b)*sinh(d*x+c)/
a/(a+b)^2/d/(a+b+a*sinh(d*x+c)^2)
```

Rubi [A]
time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4232, 393, 205, 211}

$$\frac{(4a+b)\text{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{3/2}d(a+b)^{5/2}}+\frac{(4a+b)\sinh(c+dx)}{8ad(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{b\sinh(c+dx)}{4ad(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((4*a + b)*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*a^(3/2)*(a + b)^
(5/2)*d) - (b*Sinh[c + d*x])/(4*a*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2)
+ ((4*a + b)*Sinh[c + d*x])/(8*a*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{(a+b+ax^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{(4a+b)\operatorname{Subst}\left(\int \frac{1}{(a+b+ax^2)^2}\,dx, x, \right.}{4a(a+b)d}$$

$$= -\frac{b\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{(4a+b)\sinh(c+dx)}{8a(a+b)^2d\left(a+b+a\sinh^2(c+\right.}$$

$$= \frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8a^{3/2}(a+b)^{5/2}d} - \frac{b\sinh(c+dx)}{4a(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} +$$

Mathematica [A]
time = 0.54, size = 159, normalized size = 1.29

$$-\frac{(a+2b+a\cosh(2(c+dx)))^3\operatorname{sech}^6(c+dx)\left(\frac{8\sinh(c+dx)}{\left(a+b+a\sinh^2(c+dx)\right)^2}-(4a+b)\left(\frac{3\operatorname{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{5/2}}+\frac{5(a+b)\sinh(c+dx)+3a\sinh^3(c+dx)}{(a+b)^2\left(a+b+a\sinh^2(c+dx)\right)^2}\right)\right)}{192ad\left(a+b\operatorname{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/192*((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((8*Sinh[c + d*x]
)/(a + b + a*Sinh[c + d*x]^2)^2 - (4*a + b)*((3*ArcTan[(Sqrt[a]*Sinh[c + d*
x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (5*(a + b)*Sinh[c + d*x] + 3*a*
Sinh[c + d*x]^3)/((a + b)^2*(a + b + a*Sinh[c + d*x]^2)^2))))/(a*d*(a + b*S
ech[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(109) = 218$.
time = 2.08, size = 294, normalized size = 2.39

method	result
derivativedivides	$\frac{\frac{-\frac{(4a-b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a(a+b)}-\frac{\left(4a^2-5ab+3b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2a}+\frac{\left(4a^2-5ab+3b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2a}+\frac{(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a+b)}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{(4a+b)}{}}{d}$
default	$\frac{\frac{-\frac{(4a-b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a(a+b)}-\frac{\left(4a^2-5ab+3b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2a}+\frac{\left(4a^2-5ab+3b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2a}+\frac{(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a+b)}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{(4a+b)}{}}{d}$
risch	$\frac{e^{dx+c}\left(4a^2e^{6dx+6c}+ab\,e^{6dx+6c}+4a^2e^{4dx+4c}+9ab\,e^{4dx+4c}-4b^2e^{4dx+4c}-4a^2e^{2dx+2c}-9ab\,e^{2dx+2c}+4b^2e^{2dx+2c}-4a^2-ab\right)}{4a(a+b)^2d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/8*(4*a-b)/a/(a+b)*tanh(1/2*d*x+1/2*c)^7-1/8*(4*a^2-5*a*b+3*b^2)/
(a+b)^2/a*tanh(1/2*d*x+1/2*c)^5+1/8*(4*a^2-5*a*b+3*b^2)/(a+b)^2/a*tanh(1/2*
d*x+1/2*c)^3+1/8*(4*a-b)/a/(a+b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c
)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*
c)^2+a+b)^2+1/4*(4*a+b)/a/(a^2+2*a*b+b^2)*(1/2/(a+b)^(1/2)/a^(1/2)*arctan(1
/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+1/2/(a+b)^(1/2)/a
^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*a^2*e^(7*c) + a*b*e^(7*c))*e^(7*d*x) + (4*a^2*e^(5*c) + 9*a*b*e^(5*
c) - 4*b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) + 9*a*b*e^(3*c) - 4*b^2*e^(3
*c))*e^(3*d*x) - (4*a^2*e^c + a*b*e^c)*e^(d*x))/(a^5*d + 2*a^4*b*d + a^3*b^
2*d + (a^5*d*e^(8*c) + 2*a^4*b*d*e^(8*c) + a^3*b^2*d*e^(8*c))*e^(8*d*x) + 4
*(a^5*d*e^(6*c) + 4*a^4*b*d*e^(6*c) + 5*a^3*b^2*d*e^(6*c) + 2*a^2*b^3*d*e^(
```

```
6*c))*e^(6*d*x) + 2*(3*a^5*d*e^(4*c) + 14*a^4*b*d*e^(4*c) + 27*a^3*b^2*d*e^
(4*c) + 24*a^2*b^3*d*e^(4*c) + 8*a*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^5*d*e^(2
*c) + 4*a^4*b*d*e^(2*c) + 5*a^3*b^2*d*e^(2*c) + 2*a^2*b^3*d*e^(2*c))*e^(2*d
*x)) + 8*integrate(1/32*((4*a*e^(3*c) + b*e^(3*c))*e^(3*d*x) + (4*a*e^c + b
*e^c)*e^(d*x))/(a^4 + 2*a^3*b + a^2*b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) +
a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 4*a^3*b*e^(2*c) + 5*a^2*b^2*e
^(2*c) + 2*a*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3262 vs. $2(109) = 218$.
time = 0.47, size = 6037, normalized size = 49.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^7 + 28*(4*a^4 + 5*a^3*b
+ a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(4*a^4 + 5*a^3*b + a^2*b^2)*si
nh(d*x + c)^7 + 4*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^5
+ 4*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 21*(4*a^4 + 5*a^3*b + a^2*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh
(d*x + c)^3 + (4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(
d*x + c)^4 - 4*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + 4
*(35*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - 4*a^4 - 13*a^3*b - 5*a^2
*b^2 + 4*a*b^3 + 10*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^3 + 4*(21*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 10
*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 - 3*(4*a^4 + 13*a
^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - ((4*a^3 + a^2*
b)*cosh(d*x + c)^8 + 8*(4*a^3 + a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a
^3 + a^2*b)*sinh(d*x + c)^8 + 4*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^6
 + 4*(4*a^3 + 9*a^2*b + 2*a*b^2 + 7*(4*a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d
*x + c)^6 + 8*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^3 + 3*(4*a^3 + 9*a^2*b + 2*a
*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(12*a^3 + 35*a^2*b + 40*a*b^2 + 8*
b^3)*cosh(d*x + c)^4 + 2*(35*(4*a^3 + a^2*b)*cosh(d*x + c)^4 + 12*a^3 + 35*
a^2*b + 40*a*b^2 + 8*b^3 + 30*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 + 8*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^5 + 10*(4*a^3 + 9*a^2*
b + 2*a*b^2)*cosh(d*x + c)^3 + (12*a^3 + 35*a^2*b + 40*a*b^2 + 8*b^3)*cosh(
d*x + c))*sinh(d*x + c)^3 + 4*a^3 + a^2*b + 4*(4*a^3 + 9*a^2*b + 2*a*b^2)*c
osh(d*x + c)^2 + 4*(7*(4*a^3 + a^2*b)*cosh(d*x + c)^6 + 15*(4*a^3 + 9*a^2*b
 + 2*a*b^2)*cosh(d*x + c)^4 + 4*a^3 + 9*a^2*b + 2*a*b^2 + 3*(12*a^3 + 35*a^
2*b + 40*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a^3 + a^2*
b)*cosh(d*x + c)^7 + 3*(4*a^3 + 9*a^2*b + 2*a*b^2)*cosh(d*x + c)^5 + (12*a^
3 + 35*a^2*b + 40*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + (4*a^3 + 9*a^2*b + 2*a*b
^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log((a*cosh(d*x + c)^4 +
```

```
 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*cosh
(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a*co
sh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)
^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2
 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a)/(a*cosh(d*x + c)
^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*co
sh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*co
sh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*(4*a^4 + 5
*a^3*b + a^2*b^2)*cosh(d*x + c) + 4*(7*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x
 + c)^6 + 5*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^4 - 4*a^
4 - 5*a^3*b - a^2*b^2 - 3*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x +
 c)^8 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)*sinh(d*x +
c)^7 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*sinh(d*x + c)^8 + 4*(a^7 + 5
*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^7 +
 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 9*a^5*
b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^7 + 17*a^6*b + 41*
a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^
7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^3 + 3*(a^7 + 5*a^6*b + 9
*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*
(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^4 + 30*(a^7 + 5*a^6*b
 + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^2 + (3*a^7 + 17*a^6*b
 + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d)*sinh(d*x + c)^4 + 4
*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^2 + 8*
(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^5 + 10*(a^7 + 5*a^
6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^3 + (3*a^7 + 17*a^
6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(7*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^
6 + 15*(a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^
4 + 3*(3*a^7 + 17*a^6*b + 41*a^5*b^2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)
*d*cosh(d*x + c)^2 + (a^7 + 5*a^6*b + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d)
*sinh(d*x + c)^2 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d + 8*((a^7 + 3*a^
6*b + 3*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2
 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c)^5 + (3*a^7 + 17*a^6*b + 41*a^5*b^
2 + 51*a^4*b^3 + 32*a^3*b^4 + 8*a^2*b^5)*d*cosh(d*x + c)^3 + (a^7 + 5*a^6*b
 + 9*a^5*b^2 + 7*a^4*b^3 + 2*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*
(2*(4*a^4 + 5*a^3*b + a^2*b^2)*cosh(d*x + c)^7 + 14*(4*a^4 + 5*a^3*b + a^2*
b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(4*a^4 + 5*a^3*b + a^2*b^2)*sinh(d*x
 + c)^7 + 2*(4*a^4 + 13*a^3*b + 5*a^2*b^2 - 4*a...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^3\,{\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3), x)
```

3.98 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=125

$$\frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{5/2}d}-\frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}+\frac{(a+4b)\tanh(c+dx)}{8b(a+b)^2d\left(a+b-b\tanh^2(c+a\right.}$$

```
[Out] 1/8*(a+4*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(3/2)/(a+b)^(5/2)/d-
1/4*a*tanh(d*x+c)/b/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2+1/8*(a+4*b)*tanh(d*x+c)
/b/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4231, 393, 205, 214}

$$\frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}d(a+b)^{5/2}}+\frac{(a+4b)\tanh(c+dx)}{8bd(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a\tanh(c+dx)}{4bd(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 4*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b^(3/2)*(a + b)
^(5/2)*d) - (a*Tanh[c + d*x])/(4*b*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^2)
 + ((a + 4*b)*Tanh[c + d*x])/(8*b*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(a+4b)\operatorname{Subst}\left(\int \frac{1}{(a+b-bx^2)^2}\,dx, x,\right.}{4b(a+b)d}$$

$$= -\frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(a+4b)\tanh(c+dx)}{8b(a+b)^2d\left(a+b-b\tanh^2(c+\right.}$$

$$= \frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{5/2}d} - \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} +$$

Mathematica [A]
time = 2.26, size = 250, normalized size = 2.00

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^6(c+dx)\left(\frac{(a+4b)\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(a+2b+a\cosh(2(c+dx)))^2(\cosh(2c)-\sinh(2c))}{b\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}} - \frac{4(a+b)\operatorname{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{a} + \frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}(2c)((a+4b)\sinh(2c)-(a-2b)\sinh(2dx))}{b}\right)}{64(a+b)^2d\left(a+b\operatorname{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*(((a + 4*b)*ArcTanh[(Sech[
d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*
Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])
^2*(Cosh[2*c] - Sinh[2*c]))/(b*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) -
 (4*(a + b)*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/a + ((a + 2*b
```

```
+ a*Cosh[2*(c + d*x)])*Sech[2*c]*((a + 4*b)*Sinh[2*c] - (a - 2*b)*Sinh[2*d*
x]))/b))/(64*(a + b)^2*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(111) = 222$.
time = 1.79, size = 310, normalized size = 2.48

method	result
derivativedivides	$-\frac{2\left(\frac{(a-4b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8b(a+b)}+\frac{\left(3a^2-5ab+4b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2b}+\frac{\left(3a^2-5ab+4b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2b}+\frac{(a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8b(a+b)}\right)}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2} - \frac{(a}{}$
default	$-\frac{2\left(\frac{(a-4b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8b(a+b)}+\frac{\left(3a^2-5ab+4b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2b}+\frac{\left(3a^2-5ab+4b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2b}+\frac{(a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8b(a+b)}\right)}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2} - \frac{(a}{}$
risch	$\frac{a^3e^{6dx+6c}+4a^2b\,e^{6dx+6c}+3a^3e^{4dx+4c}+2a^2b\,e^{4dx+4c}-8a\,b^2e^{4dx+4c}-16b^3e^{4dx+4c}+3a^3e^{2dx+2c}-4a^2b\,e^{2dx+2c}-16a\,b^2e^{2c}}{4adb(a+b)^2\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(1/8*(a-4*b)/b/(a+b)*tanh(1/2*d*x+1/2*c)^7+1/8*(3*a^2-5*a*b+4*b^2)/
(a+b)^2/b*tanh(1/2*d*x+1/2*c)^5+1/8*(3*a^2-5*a*b+4*b^2)/(a+b)^2/b*tanh(1/2*
d*x+1/2*c)^3+1/8*(a-4*b)/b/(a+b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c
)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*
c)^2+a+b)^2-1/4*(a+4*b)/b/(a^2+2*a*b+b^2)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b
)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/
4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1
/2*c)*b^(1/2)+(a+b)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(117) = 234$.
time = 0.58, size = 369, normalized size = 2.95

$$-\frac{(a+4b)\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{16\left(a^2b+2ab^2+b^3\right)\sqrt{(a+b)b}\,d}-\frac{a^3-2a^2b+\left(3a^3-4a^2b-16ab^2\right)e^{(-2dx-2c)}+\left(3a^3+2a^2b-8ab^2-16b^3\right)e^{(-4dx-4c)}+\left(a^3+4a^2b\right)e^{(-6dx-6c)}}{4\left(a^5b+2a^4b^2+a^3b^3+4\left(a^5b+4a^4b^2+5a^3b^3+2a^2b^4\right)e^{(-2dx-2c)}+2\left(3a^5b+14a^4b^2+27a^3b^3+24a^2b^4+8ab^5\right)e^{(-4dx-4c)}+4\left(a^5b+4a^4b^2+5a^3b^3+2a^2b^4\right)e^{(-6dx-6c)}+\left(a^5b+2a^4b^2+a^3b^3\right)e^{(-8dx-8c)}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(a + 4*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e
^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b + 2*a*b^2 + b^3)*sq
rt((a + b)*b)*d) - 1/4*(a^3 - 2*a^2*b + (3*a^3 - 4*a^2*b - 16*a*b^2)*e^(-2*
```

```
d*x - 2*c) + (3*a^3 + 2*a^2*b - 8*a*b^2 - 16*b^3)*e^(-4*d*x - 4*c) + (a^3 +
 4*a^2*b)*e^(-6*d*x - 6*c))/((a^5*b + 2*a^4*b^2 + a^3*b^3 + 4*(a^5*b + 4*a^
4*b^2 + 5*a^3*b^3 + 2*a^2*b^4)*e^(-2*d*x - 2*c) + 2*(3*a^5*b + 14*a^4*b^2 +
 27*a^3*b^3 + 24*a^2*b^4 + 8*a*b^5)*e^(-4*d*x - 4*c) + 4*(a^5*b + 4*a^4*b^2
 + 5*a^3*b^3 + 2*a^2*b^4)*e^(-6*d*x - 6*c) + (a^5*b + 2*a^4*b^2 + a^3*b^3)*
e^(-8*d*x - 8*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2603 vs. $2(117) = 234$.
time = 0.43, size = 5447, normalized size = 43.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^6 + 24*(a^4*b + 5*a^
3*b^2 + 4*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(a^4*b + 5*a^3*b^2 + 4
*a^2*b^3)*sinh(d*x + c)^6 + 4*a^4*b - 4*a^3*b^2 - 8*a^2*b^3 + 4*(3*a^4*b +
5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^4 + 4*(3*a^4*b + 5
*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5 + 15*(a^4*b + 5*a^3*b^2 + 4*a^2*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)
*cosh(d*x + c)^3 + (3*a^4*b + 5*a^3*b^2 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4*b - a^3*b^2 - 20*a^2*b^3 - 16*a*b^4
)*cosh(d*x + c)^2 + 4*(3*a^4*b - a^3*b^2 - 20*a^2*b^3 - 16*a*b^4 + 15*(a^4*
b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^4 + 6*(3*a^4*b + 5*a^3*b^2 - 6*a^2
*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((a^4 + 4*a^3*
b)*cosh(d*x + c)^8 + 8*(a^4 + 4*a^3*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4
 + 4*a^3*b)*sinh(d*x + c)^8 + 4*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6
 + 4*(a^4 + 6*a^3*b + 8*a^2*b^2 + 7*(a^4 + 4*a^3*b)*cosh(d*x + c)^2)*sinh(d
*x + c)^6 + 8*(7*(a^4 + 4*a^3*b)*cosh(d*x + c)^3 + 3*(a^4 + 6*a^3*b + 8*a^2
*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 20*a^3*b + 40*a^2*b^2 + 3
2*a*b^3)*cosh(d*x + c)^4 + 2*(35*(a^4 + 4*a^3*b)*cosh(d*x + c)^4 + 3*a^4 +
20*a^3*b + 40*a^2*b^2 + 32*a*b^3 + 30*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^4 + a^4 + 4*a^3*b + 8*(7*(a^4 + 4*a^3*b)*cosh(d*x + c
)^5 + 10*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + (3*a^4 + 20*a^3*b +
40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 6*a^3*b +
8*a^2*b^2)*cosh(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b)*cosh(d*x + c)^6 + 15*(a^4
 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + a^4 + 6*a^3*b + 8*a^2*b^2 + 3*(3*
a^4 + 20*a^3*b + 40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
8*((a^4 + 4*a^3*b)*cosh(d*x + c)^7 + 3*(a^4 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x
 + c)^5 + (3*a^4 + 20*a^3*b + 40*a^2*b^2 + 32*a*b^3)*cosh(d*x + c)^3 + (a^4
 + 6*a^3*b + 8*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((
a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x +
c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2
```

```
*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2
 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*
x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cos
h(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a
 + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2
 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*
(3*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^5 + 2*(3*a^4*b + 5*a^3*b^2
 - 6*a^2*b^3 - 24*a*b^4 - 16*b^5)*cosh(d*x + c)^3 + (3*a^4*b - a^3*b^2 - 20
*a^2*b^3 - 16*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 + 3*a^5*b^3 +
3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4
 + a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*
b^4 + a^3*b^5)*d*sinh(d*x + c)^8 + 4*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a
^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b
^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*
b^5 + 2*a^2*b^6)*d)*sinh(d*x + c)^6 + 2*(3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^
4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b^2 +
3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + 3*(a^6*b^2 + 5*a^5*b^3
 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*
(35*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(a^6
*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^2 + (
3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d)
*sinh(d*x + c)^4 + 4*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b
^6)*d*cosh(d*x + c)^2 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*
cosh(d*x + c)^5 + 10*(a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b
^6)*d*cosh(d*x + c)^3 + (3*a^6*b^2 + 17*a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5 +
 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6*b^2 + 3
*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(a^6*b^2 + 5*a^5*b^3
 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^4 + 3*(3*a^6*b^2 + 17
*a^5*b^3 + 41*a^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^
2 + (a^6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d)*sinh(d*x +
 c)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d + 8*((a^6*b^2 + 3*a^5
*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*(a^6*b^2 + 5*a^5*b^3 + 9*
a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c)^5 + (3*a^6*b^2 + 17*a^5*b^
3 + 41*a^4*b^4 + 51*a^3*b^5 + 32*a^2*b^6 + 8*a*b^7)*d*cosh(d*x + c)^3 + (a^
6*b^2 + 5*a^5*b^3 + 9*a^4*b^4 + 7*a^3*b^5 + 2*a^2*b^6)*d*cosh(d*x + c))*sin
h(d*x + c)), 1/8*(2*(a^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)^6 + 12*(a
^4*b + 5*a^3*b^2 + 4*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a^4*b + 5*
a^3*b^2 + 4*a^2*b^3)*sinh(d*x + c)^6 + 2*a^4*b ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(117) = 234$.

time = 1.03, size = 274, normalized size = 2.19

$$\frac{\frac{(a+4b)\arctan\left(\frac{ae^{(2dx+2c)}+a+2b}{2\sqrt{-ab-b^2}}\right)}{(a^2b+2ab^2+b^3)\sqrt{-ab-b^2}}+\frac{2\left(a^3e^{(6dx+6c)}+4a^2be^{(6dx+6c)}+3a^3e^{(4dx+4c)}+2a^2be^{(4dx+4c)}-8ab^2e^{(4dx+4c)}-16b^3e^{(4dx+4c)}+3a^3e^{(2dx+2c)}-4a^2be^{(2dx+2c)}-16ab^2e^{(2dx+2c)}+a^3-2a^2b\right)}{(a^3b+2a^2b^2+ab^3)\left(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((a + 4*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(
(a^2*b + 2*a*b^2 + b^3)*sqrt(-a*b - b^2)) + 2*(a^3*e^(6*d*x + 6*c) + 4*a^2*
b*e^(6*d*x + 6*c) + 3*a^3*e^(4*d*x + 4*c) + 2*a^2*b*e^(4*d*x + 4*c) - 8*a*b
^2*e^(4*d*x + 4*c) - 16*b^3*e^(4*d*x + 4*c) + 3*a^3*e^(2*d*x + 2*c) - 4*a^2
*b*e^(2*d*x + 2*c) - 16*a*b^2*e^(2*d*x + 2*c) + a^3 - 2*a^2*b)/((a^3*b + 2*
a^2*b^2 + a*b^3)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x +
2*c) + a)^2))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^4\,{\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3), x)
```

3.99 $\int \frac{\text{sech}^5(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=106

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{a}\,(a+b)^{5/2}d}+\frac{\sinh(c+dx)}{4(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2}+\frac{3\sinh(c+dx)}{8(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)}$$

```
[Out] 1/4*sinh(d*x+c)/(a+b)/d/(a+b+a*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/(a+b)^2/d/(
a+b+a*sinh(d*x+c)^2)+3/8*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))/(a+b)^(5/2
)/d/a^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4232, 205, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{a}\,d(a+b)^{5/2}}+\frac{3\sinh(c+dx)}{8d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}+\frac{\sinh(c+dx)}{4d(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*Sqrt[a]*(a + b)^(5/2)*d)
 + Sinh[c + d*x]/(4*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) + (3*Sinh[c +
d*x])/(8*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
```

```
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(a+b+ax^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{4(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{3\operatorname{Subst}\left(\int \frac{1}{(a+b+ax^2)^2}\,dx, x, \sinh(c+a\right.}{4(a+b)d}$$

$$= \frac{\sinh(c+dx)}{4(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8(a+b)^2d\left(a+b+a\sinh^2(c+dx)\right)}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{8\sqrt{a}\,(a+b)^{5/2}d} + \frac{\sinh(c+dx)}{4(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{}{8(a+b)^2}$$

Mathematica [A]
time = 0.21, size = 125, normalized size = 1.18

$$\frac{(a+2b+a\cosh(2(c+dx)))^3\operatorname{sech}^6(c+dx)\left(\frac{3\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{5/2}} + \frac{5(a+b)\sinh(c+dx)+3a\sinh^3(c+dx)}{(a+b)^2\left(a+b+a\sinh^2(c+dx)\right)^2}\right)}{64d\left(a+b\operatorname{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((3*ArcTan[(Sqrt[a]*Sinh
[c + d*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (5*(a + b)*Sinh[c + d*x]
 + 3*a*Sinh[c + d*x]^3)/((a + b)^2*(a + b + a*Sinh[c + d*x]^2)^2)))/(64*d*(
a + b*Sech[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(92) = 184.
time = 1.97, size = 240, normalized size = 2.26

method	result

risch	$\frac{e^{dx+c}\left(3a\,e^{6dx+6c}+11a\,e^{4dx+4c}+20b\,e^{4dx+4c}-11a\,e^{2dx+2c}-20b\,e^{2dx+2c}-3a\right)}{4(a+b)^2 d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)^2}-\frac{3\ln\left(e^{2dx+2c}-\frac{2(a+b)e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{16\sqrt{-a^2-ab}\,(a+b)^2 d}$
derivativedivides	$\frac{\frac{-\frac{5\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)}+\frac{3(a+5b)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2}-\frac{3(a+5b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2}+\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4(a+b)}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{3\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\right.}{2\sqrt{a}}\right.}{8\sqrt{a+b}\,\sqrt{a}}}{d}$
default	$\frac{\frac{-\frac{5\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)}+\frac{3(a+5b)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2}-\frac{3(a+5b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^2}+\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4(a+b)}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{3\arctan\left(\frac{2\sqrt{a+b}\,\tanh\left(\frac{dx}{2}+\right.}{2\sqrt{a}}\right.}{8\sqrt{a+b}\,\sqrt{a}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-5/8/(a+b)*tanh(1/2*d*x+1/2*c)^7+3/8*(a+5*b)/(a+b)^2*tanh(1/2*d*x+1
/2*c)^5-3/8*(a+5*b)/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+5/8/(a+b)*tanh(1/2*d*x+1/
2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2
*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)^2+3/4/(a^2+2*a*b+b^2)*(1/2/(a+b)^(1/2)
/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+
1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(
1/2))/a^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((11*a*e^(5*c) + 20*b*e^(5*c))*e^(5*d*x) - (11*a*e^(3*c) + 20*b*e^(3*c)
)*e^(3*d*x) + 3*a*e^(7*d*x + 7*c) - 3*a*e^(d*x + c))/(a^4*d + 2*a^3*b*d + a
^2*b^2*d + (a^4*d*e^(8*c) + 2*a^3*b*d*e^(8*c) + a^2*b^2*d*e^(8*c))*e^(8*d*x
) + 4*(a^4*d*e^(6*c) + 4*a^3*b*d*e^(6*c) + 5*a^2*b^2*d*e^(6*c) + 2*a*b^3*d*
e^(6*c))*e^(6*d*x) + 2*(3*a^4*d*e^(4*c) + 14*a^3*b*d*e^(4*c) + 27*a^2*b^2*d
*e^(4*c) + 24*a*b^3*d*e^(4*c) + 8*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^4*d*e^(2*
c) + 4*a^3*b*d*e^(2*c) + 5*a^2*b^2*d*e^(2*c) + 2*a*b^3*d*e^(2*c))*e^(2*d*x)
) + 32*integrate(3/128*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^3 + 2*a^2*b + a*b
^2 + (a^3*e^(4*c) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(
2*c) + 4*a^2*b*e^(2*c) + 5*a*b^2*e^(2*c) + 2*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2638 vs. $2(92) = 184$.
time = 0.40, size = 5006, normalized size = 47.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(12*(a^3 + a^2*b)*cosh(d*x + c)^7 + 84*(a^3 + a^2*b)*cosh(d*x + c)*si
nh(d*x + c)^6 + 12*(a^3 + a^2*b)*sinh(d*x + c)^7 + 4*(11*a^3 + 31*a^2*b + 2
0*a*b^2)*cosh(d*x + c)^5 + 4*(11*a^3 + 31*a^2*b + 20*a*b^2 + 63*(a^3 + a^2*
b)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(21*(a^3 + a^2*b)*cosh(d*x + c)^3
+ (11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(11*a^3
 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 + 4*(105*(a^3 + a^2*b)*cosh(d*x + c
)^4 - 11*a^3 - 31*a^2*b - 20*a*b^2 + 10*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^3 + 4*(63*(a^3 + a^2*b)*cosh(d*x + c)^5 + 10*(11
*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 - 3*(11*a^3 + 31*a^2*b + 20*a*b
^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*
x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)*cosh(d*x + c
)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*co
sh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 +
 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 + 30*(a^2 + 2*a
*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cos
h(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*c
osh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 4*(7*a^2*
cosh(d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 8*b
^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x
+ c)^7 + 3*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x
 + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b)*log(
(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4
- 2*(3*a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(
d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c
) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3
+ (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) +
 a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c
)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(
d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c)
+ a)) - 12*(a^3 + a^2*b)*cosh(d*x + c) + 4*(21*(a^3 + a^2*b)*cosh(d*x + c)^
6 + 5*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^4 - 3*a^3 - 3*a^2*b - 3*
(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^6 + 3*a^
5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2
 + a^3*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 +
a^3*b^3)*d*sinh(d*x + c)^8 + 4*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a
^2*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*co
```

```
sh(d*x + c)^2 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d)*sinh
(d*x + c)^6 + 2*(3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 +
8*a*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*c
osh(d*x + c)^3 + 3*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*co
sh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*
d*cosh(d*x + c)^4 + 30*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*
d*cosh(d*x + c)^2 + (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^
4 + 8*a*b^5)*d)*sinh(d*x + c)^4 + 4*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3
+ 2*a^2*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)
*d*cosh(d*x + c)^5 + 10*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)
*d*cosh(d*x + c)^3 + (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b
^4 + 8*a*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6 + 3*a^5*b + 3*a^
4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 + 15*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*
b^3 + 2*a^2*b^4)*d*cosh(d*x + c)^4 + 3*(3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*
a^3*b^3 + 32*a^2*b^4 + 8*a*b^5)*d*cosh(d*x + c)^2 + (a^6 + 5*a^5*b + 9*a^4*
b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d)*sinh(d*x + c)^2 + (a^6 + 3*a^5*b + 3*a^4*b^
2 + a^3*b^3)*d + 8*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^7
 + 3*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh(d*x + c)^5
+ (3*a^6 + 17*a^5*b + 41*a^4*b^2 + 51*a^3*b^3 + 32*a^2*b^4 + 8*a*b^5)*d*cos
h(d*x + c)^3 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*cosh(d
*x + c))*sinh(d*x + c)), 1/8*(6*(a^3 + a^2*b)*cosh(d*x + c)^7 + 42*(a^3 + a
^2*b)*cosh(d*x + c)*sinh(d*x + c)^6 + 6*(a^3 + a^2*b)*sinh(d*x + c)^7 + 2*(
11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^5 + 2*(11*a^3 + 31*a^2*b + 20*a
*b^2 + 63*(a^3 + a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(21*(a^3 + a^
2*b)*cosh(d*x + c)^3 + (11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d
*x + c)^4 - 2*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 + 2*(105*(a^3
+ a^2*b)*cosh(d*x + c)^4 - 11*a^3 - 31*a^2*b - 20*a*b^2 + 10*(11*a^3 + 31*a
^2*b + 20*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(63*(a^3 + a^2*b)*cos
h(d*x + c)^5 + 10*(11*a^3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c)^3 - 3*(11*a^
3 + 31*a^2*b + 20*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*(a^2*cosh(d*x +
 c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)**5/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^5\,{\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^5*(a + b/cosh(c + d*x)^2)^3), x)
```

3.100 $\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{(3a^2+8ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{5/2}(a+b)^{5/2}d}-\frac{a\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}-\frac{3a(a+2b)\tanh(c}{8b^2(a+b)^2d\left(a+b-b\tanh\right.}$$

```
[Out] 1/8*(3*a^2+8*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/b^(5/2)/(a
+b)^(5/2)/d-1/4*a*sech(d*x+c)^2*tanh(d*x+c)/b/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
^2-3/8*a*(a+2*b)*tanh(d*x+c)/b^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4231, 424, 393, 214}

$$\frac{(3a^2+8ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{5/2}d(a+b)^{5/2}}-\frac{3a(a+2b)\tanh(c+dx)}{8b^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{a\tanh(c+dx)\operatorname{sech}^2(c+dx)}{4bd(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(8*b
^(5/2)*(a + b)^(5/2)*d) - (a*Sech[c + d*x]^2*Tanh[c + d*x])/(4*b*(a + b)*d*
(a + b - b*Tanh[c + d*x]^2)^2) - (3*a*(a + 2*b)*Tanh[c + d*x])/(8*b^2*(a +
b)^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{-a-4b+(3a+4b)x^2}{(a+b-bx^2)^2}\,dx, x, \tan\right.}{4b(a+b)d}$$

$$= -\frac{a\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4b(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{3a(a+2b)\tanh(c+dx)}{8b^2(a+b)^2d\left(a+b-b\tanh^2(c+\right.}$$

$$= \frac{(3a^2+8ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8b^{5/2}(a+b)^{5/2}d} - \frac{a\operatorname{sech}^2(c+dx)\tanh(c+a}{4b(a+b)d\left(a+b-b\tanh^2(c-\right.}$$

Mathematica [A]
time = 0.68, size = 125, normalized size = 0.87

$$\frac{\frac{(3a^2+8ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}\,(3a^2+16ab+16b^2+3a(a+2b)\cosh(2(c+dx)))\sinh(2(c+dx))}{(a+b)^2(a+2b+a\cosh(2(c+dx)))^2}}{8b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (((3*a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a
+ b)^(5/2) - (a*Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c
+ d*x)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2))/
(8*b^(5/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(130) = 260$.
time = 1.96, size = 328, normalized size = 2.28

method	result
derivativedivides	$-\frac{2\left(\frac{a(3a+8b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)b^2}+\frac{a\left(9a^2+13ab-8b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2b^2}+\frac{a\left(9a^2+13ab-8b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2b^2}+\frac{a(3a+8b)\tanh\left(\frac{dx}{2}+\right.}{8(a+b)b^2}\right.}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}$
default	$-\frac{2\left(\frac{a(3a+8b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)b^2}+\frac{a\left(9a^2+13ab-8b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2b^2}+\frac{a\left(9a^2+13ab-8b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2b^2}+\frac{a(3a+8b)\tanh\left(\frac{dx}{2}+\right.}{8(a+b)b^2}\right.}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}$
risch	$\frac{3a^3e^{6dx+6c}+8a^2b\,e^{6dx+6c}+8a\,b^2e^{6dx+6c}+9a^3e^{4dx+4c}+42a^2b\,e^{4dx+4c}+72a\,b^2e^{4dx+4c}+48b^3e^{4dx+4c}+9a^3e^{2dx+2c}+40a^2b}{4d\,b^2(a+b)^2\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(1/8*a*(3*a+8*b)/(a+b)/b^2*tanh(1/2*d*x+1/2*c)^7+1/8*a*(9*a^2+13*a*
b-8*b^2)/(a+b)^2/b^2*tanh(1/2*d*x+1/2*c)^5+1/8*a*(9*a^2+13*a*b-8*b^2)/(a+b)
^2/b^2*tanh(1/2*d*x+1/2*c)^3+1/8*a*(3*a+8*b)/(a+b)/b^2*tanh(1/2*d*x+1/2*c))
/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2
-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)^2-1/4*(3*a^2+8*a*b+8*b^2)/b^2/(a^2+2*a*b+b^
2)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/
2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*ta
nh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(136) = 272$.
time = 0.61, size = 395, normalized size = 2.74

$$-\frac{(3a^2+8ab+8b^2)\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{16(a^2b^2+2ab^3+b^4)\sqrt{(a+b)b}\,d}-\frac{3a^3+6a^2b+(9a^3+40a^2b+40ab^2)e^{(-2dx-2c)}+3(3a^3+14a^2b+24ab^2+16b^3)e^{(-4dx-4c)}+(3a^3+8a^2b+8ab^2)e^{(-6dx-6c)}}{4(a^4b^2+2a^3b^3+a^2b^4+4(a^4b^2+4a^3b^3+5a^2b^4+2ab^5)e^{(-2dx-2c)}+2(3a^4b^2+14a^3b^3+27a^2b^4+24ab^5+8b^6)e^{(-4dx-4c)}+4(a^4b^2+4a^3b^3+5a^2b^4+2ab^5)e^{(-6dx-6c)}+(a^4b^2+2a^3b^3+a^2b^4)e^{(-8dx-8c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*a^2 + 8*a*b + 8*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a
 + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 + 2
*a*b^3 + b^4)*sqrt((a + b)*b)*d) - 1/4*(3*a^3 + 6*a^2*b + (9*a^3 + 40*a^2*b
 + 40*a*b^2)*e^(-2*d*x - 2*c) + 3*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3)*e^
(-4*d*x - 4*c) + (3*a^3 + 8*a^2*b + 8*a*b^2)*e^(-6*d*x - 6*c))/((a^4*b^2 +
2*a^3*b^3 + a^2*b^4 + 4*(a^4*b^2 + 4*a^3*b^3 + 5*a^2*b^4 + 2*a*b^5)*e^(-2*d
```

```
*x - 2*c) + 2*(3*a^4*b^2 + 14*a^3*b^3 + 27*a^2*b^4 + 24*a*b^5 + 8*b^6)*e^(-
4*d*x - 4*c) + 4*(a^4*b^2 + 4*a^3*b^3 + 5*a^2*b^4 + 2*a*b^5)*e^(-6*d*x - 6*
c) + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2823 vs. $2(136) = 272$.
time = 0.50, size = 5887, normalized size = 40.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^6 + 24
*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^
5 + 4*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*sinh(d*x + c)^6 + 12*a^
4*b + 36*a^3*b^2 + 24*a^2*b^3 + 12*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*
a*b^4 + 16*b^5)*cosh(d*x + c)^4 + 12*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 4
0*a*b^4 + 16*b^5 + 5*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 16*(5*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b
^4)*cosh(d*x + c)^3 + 3*(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*
b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3
+ 40*a*b^4)*cosh(d*x + c)^2 + 4*(9*a^4*b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b
^4 + 15*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^4 + 18*
(3*a^4*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 + ((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^8 + 8*(3*a^4 +
 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 8*a^3*b + 8*
a^2*b^2)*sinh(d*x + c)^8 + 4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cos
h(d*x + c)^6 + 4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3 + 7*(3*a^4 + 8*a
^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 8*a^3*b
+ 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(9*a^4 + 48*a^3*b + 112*a^2*b^2 + 128*a
*b^3 + 64*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d
*x + c)^4 + 9*a^4 + 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4 + 30*(3*a^4
 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a
^4 + 8*a^3*b + 8*a^2*b^2 + 8*(7*(3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)
^5 + 10*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (9*a^4
 + 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(
3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(3*a^4 + 14*a^3*b + 24*a^
2*b^2 + 16*a*b^3)*cosh(d*x + c)^4 + 3*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^
3 + 3*(9*a^4 + 48*a^3*b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 8*((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(3
*a^4 + 14*a^3*b + 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (9*a^4 + 48*a^3*
b + 112*a^2*b^2 + 128*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (3*a^4 + 14*a^3*b +
```

```
 24*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((
a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x +
c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2
*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2
 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*
x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cos
h(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a
 + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2
 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*
(3*(3*a^4*b + 11*a^3*b^2 + 16*a^2*b^3 + 8*a*b^4)*cosh(d*x + c)^5 + 6*(3*a^4
*b + 17*a^3*b^2 + 38*a^2*b^3 + 40*a*b^4 + 16*b^5)*cosh(d*x + c)^3 + (9*a^4*
b + 49*a^3*b^2 + 80*a^2*b^3 + 40*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^5
*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^8 + 8*(a^5*b^3 + 3*
a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^3 +
 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*sinh(d*x + c)^8 + 4*(a^5*b^3 + 5*a^4*b^
4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^3 + 3*
a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^2 + (a^5*b^3 + 5*a^4*b^4 + 9
*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d)*sinh(d*x + c)^6 + 2*(3*a^5*b^3 + 17*a^4*
b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d*cosh(d*x + c)^4 + 8*(7*
(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^3 + 3*(a^5*b^3
+ 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c))*sinh(d*x +
c)^5 + 2*(35*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^4
+ 30*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c
)^2 + (3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)
*d)*sinh(d*x + c)^4 + 4*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*
b^7)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d
*cosh(d*x + c)^5 + 10*(a^5*b^3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^
7)*d*cosh(d*x + c)^3 + (3*a^5*b^3 + 17*a^4*b^4 + 41*a^3*b^5 + 51*a^2*b^6 +
32*a*b^7 + 8*b^8)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^3 + 3*a^4*
b^4 + 3*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^6 + 15*(a^5*b^3 + 5*a^4*b^4 + 9*
a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d*cosh(d*x + c)^4 + 3*(3*a^5*b^3 + 17*a^4*b^
4 + 41*a^3*b^5 + 51*a^2*b^6 + 32*a*b^7 + 8*b^8)*d*cosh(d*x + c)^2 + (a^5*b^
3 + 5*a^4*b^4 + 9*a^3*b^5 + 7*a^2*b^6 + 2*a*b^7)*d)*sinh(d*x + c)^2 + (a^5*
b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*d + 8*((a^5*b^3 + 3*a^4*b^4 + 3*a^3*
b^5 + a^2*b^6)*d*cosh(d*x + c)^7 + 3*(a^5*b^3 +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(sech(c + d*x)**6/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(136) = 272.
time = 0.99, size = 302, normalized size = 2.10

$$\frac{\frac{(3a^2+8ab+8b^2)\arctan\left(\frac{ae^{(2dx+2c)}+a+2b}{2\sqrt{-ab-b^2}}\right)}{(a^2b^2+2ab^3+b^4)\sqrt{-ab-b^2}}+\frac{2\left(3a^3e^{(6dx+6c)}+8a^2be^{(6dx+6c)}+8ab^2e^{(6dx+6c)}+9a^3e^{(4dx+4c)}+42a^2be^{(4dx+4c)}+72ab^2e^{(4dx+4c)}+48b^3e^{(4dx+4c)}+9a^3e^{(2dx+2c)}+40a^2be^{(2dx+2c)}+40ab^2e^{(2dx+2c)}+3a^3+6a^2b\right)}{(a^2b^2+2ab^3+b^4)\left(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(
-a*b - b^2))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt(-a*b - b^2)) + 2*(3*a^3*e^(6*d
*x + 6*c) + 8*a^2*b*e^(6*d*x + 6*c) + 8*a*b^2*e^(6*d*x + 6*c) + 9*a^3*e^(4*
d*x + 4*c) + 42*a^2*b*e^(4*d*x + 4*c) + 72*a*b^2*e^(4*d*x + 4*c) + 48*b^3*e
^(4*d*x + 4*c) + 9*a^3*e^(2*d*x + 2*c) + 40*a^2*b*e^(2*d*x + 2*c) + 40*a*b^
2*e^(2*d*x + 2*c) + 3*a^3 + 6*a^2*b)/((a^2*b^2 + 2*a*b^3 + b^4)*(a*e^(4*d*x
 + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cosh\left(c+d\,x\right)}^6\,{\left(a+\frac{b}{{\cosh\left(c+d\,x\right)}^2}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^6*(a + b/cosh(c + d*x)^2)^3), x)
```

3.101 $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=153

$$\frac{\operatorname{ArcTan}(\sinh(c+dx))}{b^3d}-\frac{\sqrt{a}\,(8a^2+20ab+15b^2)\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{8b^3(a+b)^{5/2}d}-\frac{a\sinh(c+dx)}{4b(a+b)d\left(a+b+a\sinh^2(c+d\right.}$$

```
[Out] arctan(sinh(d*x+c))/b^3/d-1/4*a*sinh(d*x+c)/b/(a+b)/d/(a+b+a*sinh(d*x+c)^2)
^2-1/8*a*(4*a+7*b)*sinh(d*x+c)/b^2/(a+b)^2/d/(a+b+a*sinh(d*x+c)^2)-1/8*(8*a
^2+20*a*b+15*b^2)*arctan(sinh(d*x+c)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b^3/(a+b)
^(5/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4232, 425, 541, 536, 209, 211}

$$-\frac{\sqrt{a}\,(8a^2+20ab+15b^2)\operatorname{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{8b^3d(a+b)^{5/2}}-\frac{a(4a+7b)\sinh(c+dx)}{8b^2d(a+b)^2\left(a\sinh^2(c+dx)+a+b\right)}-\frac{a\sinh(c+dx)}{4bd(a+b)\left(a\sinh^2(c+dx)+a+b\right)^2}+\frac{\operatorname{ArcTan}(\sinh(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/(b^3*d) - (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*ArcTan[(
Sqrt[a]*Sinh[c + d*x])/Sqrt[a + b]])/(8*b^3*(a + b)^(5/2)*d) - (a*Sinh[c +
d*x])/(4*b*(a + b)*d*(a + b + a*Sinh[c + d*x]^2)^2) - (a*(4*a + 7*b)*Sinh[c
 + d*x])/(8*b^2*(a + b)^2*d*(a + b + a*Sinh[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4232

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\text{sech}^7(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+ax^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{a\sinh(c+dx)}{4b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+4b-3ax^2}{(1+x^2)(a+b+ax^2)^2}\,dx, x, \sinh\right.}{4b(a+b)d}$$

$$= -\frac{a\sinh(c+dx)}{4b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} - \frac{a(4a+7b)\sinh(c+dx)}{8b^2(a+b)^2d\left(a+b+a\sinh^2(c+d\right.}$$

$$= -\frac{a\sinh(c+dx)}{4b(a+b)d\left(a+b+a\sinh^2(c+dx)\right)^2} - \frac{a(4a+7b)\sinh(c+dx)}{8b^2(a+b)^2d\left(a+b+a\sinh^2(c+d\right.}$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{b^3d} - \frac{\sqrt{a}\,(8a^2+20ab+15b^2)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{a+b}}\right)}{8b^3(a+b)^{5/2}d} - \frac{}{4b(}$$

Mathematica [A]
time = 3.15, size = 247, normalized size = 1.61

$$\frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^5(c+dx)\left(16\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)(a+2b+a\cosh(2(c+dx)))^2\text{sech}(c+dx)+\frac{\sqrt{a}\,(8a^2+20ab+15b^2)\text{ArcTan}\left(\frac{\sqrt{a+b}\,\text{csch}(c+dx)\sqrt{(\cosh(c)-\sinh(c))^2}\,(\cosh(c)+\sinh(c))}{\sqrt{a}}\right)(a+2b+a\cosh(2(c+dx)))^2\text{sech}(c+dx)(\cosh(c)-\sinh(c))}{(a+b)^{5/2}\sqrt{(\cosh(c)-\sinh(c))^2}}-\frac{8ab^2\tanh(c+dx)}{a+b}-\frac{2ab(4a+7b)(a+2b+a\cosh(2(c+dx)))\tanh(c+dx)}{(a+b)^2}\right)}{64b^3d\left(a+b\text{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^7/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^5*(16*ArcTan[Tanh[(c + d*x)/
2]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x] + (Sqrt[a]*(8*a^2 + 20*
a*b + 15*b^2)*ArcTan[(Sqrt[a + b]*Csch[c + d*x]*Sqrt[(Cosh[c] - Sinh[c])^2]
*(Cosh[c] + Sinh[c]))/Sqrt[a]]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d
*x]*(Cosh[c] - Sinh[c]))/((a + b)^(5/2)*Sqrt[(Cosh[c] - Sinh[c])^2]) - (8*a
*b^2*Tanh[c + d*x])/(a + b) - (2*a*b*(4*a + 7*b)*(a + 2*b + a*Cosh[2*(c + d
*x)])*Tanh[c + d*x])/(a + b)^2))/(64*b^3*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(139) = 278.
time = 2.00, size = 314, normalized size = 2.05

method	result

derivativedivides	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3}-\frac{2a\left(\frac{-\frac{b(9b+4a)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}-\frac{b\left(4a^2-11ab-27b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2}+\frac{b\left(4a^2-11ab-27b^2\right)\left(\tanh^3\left(\right.\right.}{8(a+b)^2}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}\right.\right.\right.}\right.}{}}{}$
default	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3}-\frac{2a\left(\frac{-\frac{b(9b+4a)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}-\frac{b\left(4a^2-11ab-27b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2}+\frac{b\left(4a^2-11ab-27b^2\right)\left(\tanh^3\left(\right.\right.}{8(a+b)^2}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}\right.\right.\right.}\right.}{}}{}$
risch	$-\frac{e^{dx+c}a\left(4a^2e^{6dx+6c}+7ab\,e^{6dx+6c}+4a^2e^{4dx+4c}+31ab\,e^{4dx+4c}+36b^2e^{4dx+4c}-4a^2e^{2dx+2c}-31ab\,e^{2dx+2c}-36b^2e^{2dx+2}\right.}{4b^2(a+b)^2d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^3*arctan(tanh(1/2*d*x+1/2*c))-2/b^3*a*((-1/8*b*(9*b+4*a)/(a+b)*tan
h(1/2*d*x+1/2*c)^7-1/8*b*(4*a^2-11*a*b-27*b^2)/(a+b)^2*tanh(1/2*d*x+1/2*c)^
5+1/8*b*(4*a^2-11*a*b-27*b^2)/(a+b)^2*tanh(1/2*d*x+1/2*c)^3+1/8*b*(9*b+4*a)
/(a+b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^
4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)^2+1/8*(8*a^2+20*
a*b+15*b^2)/(a^2+2*a*b+b^2)*(1/2/(a+b)^(1/2)/a^(1/2)*arctan(1/2*(2*(a+b)^(1
/2)*tanh(1/2*d*x+1/2*c)+2*b^(1/2))/a^(1/2))+1/2/(a+b)^(1/2)/a^(1/2)*arctan(
1/2*(2*(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)-2*b^(1/2))/a^(1/2)))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((4*a^3*e^(7*c) + 7*a^2*b*e^(7*c))*e^(7*d*x) + (4*a^3*e^(5*c) + 31*a^2
*b*e^(5*c) + 36*a*b^2*e^(5*c))*e^(5*d*x) - (4*a^3*e^(3*c) + 31*a^2*b*e^(3*c
) + 36*a*b^2*e^(3*c))*e^(3*d*x) - (4*a^3*e^c + 7*a^2*b*e^c)*e^(d*x))/(a^4*b
^2*d + 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(8*c) + 2*a^3*b^3*d*e^(8*c) +
 a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^4*b^2*d*e^(6*c) + 4*a^3*b^3*d*e^(6*c)
+ 5*a^2*b^4*d*e^(6*c) + 2*a*b^5*d*e^(6*c))*e^(6*d*x) + 2*(3*a^4*b^2*d*e^(4*
c) + 14*a^3*b^3*d*e^(4*c) + 27*a^2*b^4*d*e^(4*c) + 24*a*b^5*d*e^(4*c) + 8*b
^6*d*e^(4*c))*e^(4*d*x) + 4*(a^4*b^2*d*e^(2*c) + 4*a^3*b^3*d*e^(2*c) + 5*a^
2*b^4*d*e^(2*c) + 2*a*b^5*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(b^
3*d) - 128*integrate(1/512*((8*a^3*e^(3*c) + 20*a^2*b*e^(3*c) + 15*a*b^2*e^
(3*c))*e^(3*d*x) + (8*a^3*e^c + 20*a^2*b*e^c + 15*a*b^2*e^c)*e^(d*x))/(a^3*
b^3 + 2*a^2*b^4 + a*b^5 + (a^3*b^3*e^(4*c) + 2*a^2*b^4*e^(4*c) + a*b^5*e^(4
*c))*e^(4*d*x) + 2*(a^3*b^3*e^(2*c) + 4*a^2*b^4*e^(2*c) + 5*a*b^5*e^(2*c) +
 2*b^6*e^(2*c))*e^(2*d*x)), x)
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4335 vs. $2(139) = 278$.
time = 0.50, size = 7993, normalized size = 52.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^7 + 28*(4*a^3*b + 7*a^2*b^2)*
cosh(d*x + c)*sinh(d*x + c)^6 + 4*(4*a^3*b + 7*a^2*b^2)*sinh(d*x + c)^7 + 4
*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^5 + 4*(4*a^3*b + 31*a^2*b^
2 + 36*a*b^3 + 21*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 +
20*(7*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^3 + (4*a^3*b + 31*a^2*b^2 + 36*a*
b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*c
osh(d*x + c)^3 + 4*(35*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^4 - 4*a^3*b - 31
*a^2*b^2 - 36*a*b^3 + 10*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^3 + 4*(21*(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c)^5 + 10*(4*a^3*
b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^3 - 3*(4*a^3*b + 31*a^2*b^2 + 36*a
*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - ((8*a^4 + 20*a^3*b + 15*a^2*b^2)*cos
h(d*x + c)^8 + 8*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c
)^7 + (8*a^4 + 20*a^3*b + 15*a^2*b^2)*sinh(d*x + c)^8 + 4*(8*a^4 + 36*a^3*b
 + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^6 + 4*(8*a^4 + 36*a^3*b + 55*a^2*b^
2 + 30*a*b^3 + 7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 + 8*(7*(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^3 + 3*(8*a^4 +
36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^
4 + 124*a^3*b + 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^4 + 2*(35*
(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^4 + 24*a^4 + 124*a^3*b + 269*
a^2*b^2 + 280*a*b^3 + 120*b^4 + 30*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^4 + 20*a^3*b + 15*a^2*b^2 + 8*(7*
```

(8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^5 + 10*(8*a^4 + 36*a^3*b + 55
*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^3 + (24*a^4 + 124*a^3*b + 269*a^2*b^2 +
280*a*b^3 + 120*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^4 + 36*a^3*b +
55*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^2 + 4*(7*(8*a^4 + 20*a^3*b + 15*a^2*b
^2)*cosh(d*x + c)^6 + 15*(8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3)*cosh(d*
x + c)^4 + 8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*a*b^3 + 3*(24*a^4 + 124*a^3*b
+ 269*a^2*b^2 + 280*a*b^3 + 120*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*
((8*a^4 + 20*a^3*b + 15*a^2*b^2)*cosh(d*x + c)^7 + 3*(8*a^4 + 36*a^3*b + 55
*a^2*b^2 + 30*a*b^3)*cosh(d*x + c)^5 + (24*a^4 + 124*a^3*b + 269*a^2*b^2 +
280*a*b^3 + 120*b^4)*cosh(d*x + c)^3 + (8*a^4 + 36*a^3*b + 55*a^2*b^2 + 30*
a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a/(a + b))*log((a*cosh(d*x + c)^
4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*(3*a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - 3*a - 2*b)*sinh(d*x + c)^2 + 4*(a
*cosh(d*x + c)^3 - (3*a + 2*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*co
sh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x
+ c)^3 - (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d
*x + c))*sqrt(-a/(a + b)) + a)/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(
d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(
d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*co
sh(d*x + c))*sinh(d*x + c) + a)) - 32*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x +
c)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 +
2*a^3*b + a^2*b^2)*sinh(d*x + c)^8 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3
)*cosh(d*x + c)^6 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3
*b + a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*
b^2)*cosh(d*x + c)^3 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c
))*sinh(d*x + c)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*c
osh(d*x + c)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 + 3*a^4 +
14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5*a^2*b^2 +
2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*
(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^5 + 10*(a^4 + 4*a^3*b + 5*a^2*b^2 +
2*a*b^3)*cosh(d*x + c)^3 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b
^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3
)*cosh(d*x + c)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 15*(a^
4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^4 + a^4 + 4*a^3*b + 5*a^2*
b^2 + 2*a*b^3 + 3*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^7 +
3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^5 + (3*a^4 + 14*a^3*
b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 5*a^2
*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d
x + c)) - 4(4*a^3*b + 7*a^2*b^2)*cosh(d*x + c) + 4*(7*(4*a^3*b + 7*a^2*b^
2)*cosh(d*x + c)^6 + 5*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^4 -
4*a^3*b - 7*a^2*b^2 - 3*(4*a^3*b + 31*a^2*b^2 + 36*a*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c))/((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^8 + 8*(a^4*
b^3 + 2*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4*b^3 + 2*a
^3*b^4 + a^2*b^5)*d*sinh(d*x + c)^8 + 4*(a^4*b^3 + 4*a^3*b^4 + 5*a^2*b^5 +

```
2*a*b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^4*b^3 + 2*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7/(a+b*sech(d*x+c)**2)**3,x)
[Out] Integral(sech(c + d*x)**7/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^7\,\left(a+\frac{b}{\cosh\left(c+d\,x\right)^2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^7*(a + b/cosh(c + d*x)^2)^3),x)
[Out] int(1/(cosh(c + d*x)^7*(a + b/cosh(c + d*x)^2)^3), x)
```

3.102 $\int \left(a+b\text{sech}^2(c+dx)\right)\tanh^4(c+dx)\,dx$

Optimal. Leaf size=48

$$ax-\frac{a\tanh(c+dx)}{d}-\frac{a\tanh^3(c+dx)}{3d}+\frac{b\tanh^5(c+dx)}{5d}$$

```
[Out] a*x-a*tanh(d*x+c)/d-1/3*a*tanh(d*x+c)^3/d+1/5*b*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4226, 1816, 212}

$$-\frac{a\tanh^3(c+dx)}{3d}-\frac{a\tanh(c+dx)}{d}+ax+\frac{b\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^4,x]
```

```
[Out] a*x - (a*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^5)
/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int (a+b\text{sech}^2(c+dx))\tanh^4(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+b(1-x^2))}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-a-ax^2+bx^4+\frac{a}{1-x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{a\tanh(c+dx)}{d} - \frac{a\tanh^3(c+dx)}{3d} + \frac{b\tanh^5(c+dx)}{5d} + \frac{a\text{Subst}}{}$$
$$= ax - \frac{a\tanh(c+dx)}{d} - \frac{a\tanh^3(c+dx)}{3d} + \frac{b\tanh^5(c+dx)}{5d}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 1.19

$$\frac{a\tanh^{-1}(\tanh(c+dx))}{d} - \frac{a\tanh(c+dx)}{d} - \frac{a\tanh^3(c+dx)}{3d} + \frac{b\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^4,x]
```

```
[Out] (a*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3
*d) + (b*Tanh[c + d*x]^5)/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(44) = 88.
time = 1.92, size = 102, normalized size = 2.12

method	result	size
risch	$ax+\frac{4a\,e^{8dx+8c}-2b\,e^{8dx+8c}+12a\,e^{6dx+6c}+\frac{44a\,e^{4dx+4c}}{3}-4b\,e^{4dx+4c}+\frac{28a\,e^{2dx+2c}}{3}+\frac{8a}{3}-\frac{2b}{5}}{d\left(1+e^{2dx+2c}\right)^5}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+2/15*(30*a*exp(8*d*x+8*c)-15*b*exp(8*d*x+8*c)+90*a*exp(6*d*x+6*c)+110*a
*exp(4*d*x+4*c)-30*b*exp(4*d*x+4*c)+70*a*exp(2*d*x+2*c)+20*a-3*b)/d/(1+exp(
2*d*x+2*c))^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(44) = 88.
time = 0.29, size = 92, normalized size = 1.92

$$\frac{b\tanh(dx+c)^5}{5\,d} + \frac{1}{3}\,a\left(3\,x + \frac{3\,c}{d} - \frac{4\,(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + 2)}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/5*b*tanh(d*x + c)^5/d + 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^
(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x
 - 6*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(44) = 88$.
time = 0.38, size = 327, normalized size = 6.81

$$\frac{(15\,adx+20\,a-3\,b)\cosh\left(dx+c\right)^{5}+5\,(15\,adx+20\,a-3\,b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}-(20\,a-3\,b)\sinh\left(dx+c\right)^{5}+5\,(15\,adx+20\,a-3\,b)\cosh\left(dx+c\right)^{3}-5\,\left(2\,(20\,a-3\,b)\cosh\left(dx+c\right)^{2}+8\,a+3\,b\right)\sinh\left(dx+c\right)^{3}+5\,\left(2\,(15\,adx+20\,a-3\,b)\cosh\left(dx+c\right)^{3}+3\,(15\,adx+20\,a-3\,b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+10\,(15\,adx+20\,a-3\,b)\cosh\left(dx+c\right)-5\,\left((20\,a-3\,b)\cosh\left(dx+c\right)^{4}+3\,(8\,a+3\,b)\cosh\left(dx+c\right)^{2}+4\,a-6\,b\right)\sinh\left(dx+c\right)}{15\,\left(d\cosh\left(dx+c\right)^{5}+5\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}+5\,d\cosh\left(dx+c\right)^{3}+5\,\left(2\,d\cosh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+10\,d\cosh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/15*((15*a*d*x + 20*a - 3*b)*cosh(d*x + c)^5 + 5*(15*a*d*x + 20*a - 3*b)*c
osh(d*x + c)*sinh(d*x + c)^4 - (20*a - 3*b)*sinh(d*x + c)^5 + 5*(15*a*d*x +
 20*a - 3*b)*cosh(d*x + c)^3 - 5*(2*(20*a - 3*b)*cosh(d*x + c)^2 + 8*a + 3*
b)*sinh(d*x + c)^3 + 5*(2*(15*a*d*x + 20*a - 3*b)*cosh(d*x + c)^3 + 3*(15*a
*d*x + 20*a - 3*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(15*a*d*x + 20*a - 3
*b)*cosh(d*x + c) - 5*((20*a - 3*b)*cosh(d*x + c)^4 + 3*(8*a + 3*b)*cosh(d*
x + c)^2 + 4*a - 6*b)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)
*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(
d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^{2}\left(c + dx\right)\right)\tanh^{4}\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*tanh(c + d*x)**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(44) = 88$.
time = 0.42, size = 108, normalized size = 2.25

$$\frac{15\,(dx+c)a+\frac{2\,\left(30\,ae^{(8\,dx+8\,c)}-15\,be^{(8\,dx+8\,c)}+90\,ae^{(6\,dx+6\,c)}+110\,ae^{(4\,dx+4\,c)}-30\,be^{(4\,dx+4\,c)}+70\,ae^{(2\,dx+2\,c)}+20\,a-3\,b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^{5}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/15*(15*(d*x + c)*a + 2*(30*a*e^(8*d*x + 8*c) - 15*b*e^(8*d*x + 8*c) + 90*
a*e^(6*d*x + 6*c) + 110*a*e^(4*d*x + 4*c) - 30*b*e^(4*d*x + 4*c) + 70*a*e^(
2*d*x + 2*c) + 20*a - 3*b)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 1.54, size = 433, normalized size = 9.02

$$a\,x+\frac{\frac{2\,\left(2\,a-3\,b\right)}{15\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+b\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(2\,a-b\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}+\frac{\frac{2\,\left(2\,a-b\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+b\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a+b\right)}{5\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(2\,a-3\,b\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(2\,a-b\right)}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}+\frac{\frac{2\,\left(a+b\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,a-b\right)}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}+\frac{\frac{2\,\left(a+b\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a+b\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,a-3\,b\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(2\,a-b\right)}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}+\frac{2\,\left(2\,a-b\right)}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] a*x + ((2*(2*a - 3*b))/(15*d) + (4*exp(2*c + 2*d*x)*(a + b))/(5*d) + (2*exp
(4*c + 4*d*x)*(2*a - b))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) +
exp(6*c + 6*d*x) + 1) + ((2*(2*a - b))/(5*d) + (8*exp(2*c + 2*d*x)*(a + b))
/(5*d) + (8*exp(6*c + 6*d*x)*(a + b))/(5*d) + (4*exp(4*c + 4*d*x)*(2*a - 3*
b))/(5*d) + (2*exp(8*c + 8*d*x)*(2*a - b))/(5*d))/(5*exp(2*c + 2*d*x) + 10*
exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10
*d*x) + 1) + ((2*(a + b))/(5*d) + (2*exp(2*c + 2*d*x)*(2*a - b))/(5*d))/(2*
exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((2*(a + b))/(5*d) + (6*exp(4*c
+ 4*d*x)*(a + b))/(5*d) + (2*exp(2*c + 2*d*x)*(2*a - 3*b))/(5*d) + (2*exp(6
*c + 6*d*x)*(2*a - b))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*
exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + (2*(2*a - b))/(5*d*(exp(2*c + 2*
d*x) + 1))
```

3.103 $\int \left(a + b\,\mathrm{sech}^2(c + dx)\right) \tanh^3(c + dx)\, dx$

Optimal. Leaf size=49

$$\frac{a\log(\cosh(c+dx))}{d}+\frac{(a-b)\mathrm{sech}^2(c+dx)}{2d}+\frac{b\,\mathrm{sech}^4(c+dx)}{4d}$$

```
[Out] a*ln(cosh(d*x+c))/d+1/2*(a-b)*sech(d*x+c)^2/d+1/4*b*sech(d*x+c)^4/d
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 457, 77}

$$\frac{(a-b)\mathrm{sech}^2(c+dx)}{2d}+\frac{a\log(\cosh(c+dx))}{d}+\frac{b\,\mathrm{sech}^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^3,x]
```

```
[Out] (a*Log[Cosh[c + d*x]])/d + ((a - b)*Sech[c + d*x]^2)/(2*d) + (b*Sech[c + d*
x]^4)/(4*d)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)\tanh^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)}{x^5}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)(b+ax)}{x^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^3}+\frac{a-b}{x^2}-\frac{a}{x}\right)\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{a\log(\cosh(c+dx))}{d} + \frac{(a-b)\text{sech}^2(c+dx)}{2d} + \frac{b\text{sech}^4(c+dx)}{4d}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.92

$$\frac{a\log(\cosh(c+dx))}{d} - \frac{a\tanh^2(c+dx)}{2d} + \frac{b\tanh^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^3,x]
```

```
[Out] (a*Log[Cosh[c + d*x]])/d - (a*Tanh[c + d*x]^2)/(2*d) + (b*Tanh[c + d*x]^4)/
(4*d)
```

Maple [A]
time = 1.07, size = 57, normalized size = 1.16

method	result	size
derivativedivides	$\frac{a\left(\ln(\cosh(dx+c))-\frac{\left(\tanh^2(dx+c)\right)}{2}\right)+b\left(-\frac{\sinh^2(dx+c)}{2\cosh(dx+c)^4}-\frac{1}{4\cosh(dx+c)^4}\right)}{d}$	57
default	$\frac{a\left(\ln(\cosh(dx+c))-\frac{\left(\tanh^2(dx+c)\right)}{2}\right)+b\left(-\frac{\sinh^2(dx+c)}{2\cosh(dx+c)^4}-\frac{1}{4\cosh(dx+c)^4}\right)}{d}$	57
risch	$-ax-\frac{2ac}{d}+\frac{2\,e^{2dx+2c}\left(a\,e^{4dx+4c}-b\,e^{4dx+4c}+2a\,e^{2dx+2c}+a-b\right)}{d\left(1+e^{2dx+2c}\right)^4}+\frac{a\ln\left(1+e^{2dx+2c}\right)}{d}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(ln(cosh(d*x+c))-1/2*tanh(d*x+c)^2)+b*(-1/2*sinh(d*x+c)^2/cosh(d*x+c
)^4-1/4/cosh(d*x+c)^4))
```

Maxima [A]
time = 0.57, size = 78, normalized size = 1.59

$$\frac{b\tanh{(dx+c)}^4}{4\,d}+a\left(x+\frac{c}{d}+\frac{\log{(e^{(-2\,dx-2\,c)}+1)}}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*tanh(d*x + c)^4/d + a*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-
2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1072 vs. $2(45) = 90$.
time = 0.37, size = 1072, normalized size = 21.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(a*d*x*cosh(d*x + c)^8 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a*d*x*sin
h(d*x + c)^8 + 2*(2*a*d*x - a + b)*cosh(d*x + c)^6 + 2*(14*a*d*x*cosh(d*x +
 c)^2 + 2*a*d*x - a + b)*sinh(d*x + c)^6 + 4*(14*a*d*x*cosh(d*x + c)^3 + 3*
(2*a*d*x - a + b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*cosh(d
*x + c)^4 + 2*(35*a*d*x*cosh(d*x + c)^4 + 3*a*d*x + 15*(2*a*d*x - a + b)*co
sh(d*x + c)^2 - 2*a)*sinh(d*x + c)^4 + 8*(7*a*d*x*cosh(d*x + c)^5 + 5*(2*a*
d*x - a + b)*cosh(d*x + c)^3 + (3*a*d*x - 2*a)*cosh(d*x + c))*sinh(d*x + c)
^3 + a*d*x + 2*(2*a*d*x - a + b)*cosh(d*x + c)^2 + 2*(14*a*d*x*cosh(d*x + c
)^6 + 15*(2*a*d*x - a + b)*cosh(d*x + c)^4 + 2*a*d*x + 6*(3*a*d*x - 2*a)*co
sh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^8 + 8*a*cosh(d*x
+ c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 + 4*a*cosh(d*x + c)^6 + 4*(7*a*cos
h(d*x + c)^2 + a)*sinh(d*x + c)^6 + 8*(7*a*cosh(d*x + c)^3 + 3*a*cosh(d*x +
 c))*sinh(d*x + c)^5 + 6*a*cosh(d*x + c)^4 + 2*(35*a*cosh(d*x + c)^4 + 30*a
*cosh(d*x + c)^2 + 3*a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*x + c)^5 + 10*a*cos
h(d*x + c)^3 + 3*a*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a*cosh(d*x + c)^2 + 4
*(7*a*cosh(d*x + c)^6 + 15*a*cosh(d*x + c)^4 + 9*a*cosh(d*x + c)^2 + a)*sin
h(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 + 3*a*cosh(d*x + c)^5 + 3*a*cosh(d*x +
c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*log(2*cosh(d*x + c)/(cosh(d*x +
c) - sinh(d*x + c))) + 4*(2*a*d*x*cosh(d*x + c)^7 + 3*(2*a*d*x - a + b)*cos
h(d*x + c)^5 + 2*(3*a*d*x - 2*a)*cosh(d*x + c)^3 + (2*a*d*x - a + b)*cosh(d
*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c
)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)
*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c
)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2
```

```
+ 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*
d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x +
c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*
(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x
 + c))*sinh(d*x + c) + d)
```

Sympy [A]
time = 0.35, size = 80, normalized size = 1.63

$$\begin{cases} ax - \frac{a\log(\tanh(c+dx)+1)}{d} - \frac{a\tanh^2(c+dx)}{2d} - \frac{b\tanh^2(c+dx)\operatorname{sech}^2(c+dx)}{4d} - \frac{b\operatorname{sech}^2(c+dx)}{4d} & \text{for } d \neq 0 \\ x\left(a + b\operatorname{sech}^2(c)\right)\tanh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c)**3,x)
```

```
[Out] Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d - a*tanh(c + d*x)**2/(2*d) - b*
tanh(c + d*x)**2*sech(c + d*x)**2/(4*d) - b*sech(c + d*x)**2/(4*d), Ne(d, 0
)), (x*(a + b*sech(c)**2)*tanh(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(45) = 90.
time = 0.43, size = 119, normalized size = 2.43

$$-\frac{12(dx+c)a - 12a\log\left(e^{(2dx+2c)}+1\right) + \frac{25ae^{(8dx+8c)}+76ae^{(6dx+6c)}+24be^{(6dx+6c)}+102ae^{(4dx+4c)}+76ae^{(2dx+2c)}+24be^{(2dx+2c)}+25a}{\left(e^{(2dx+2c)}+1\right)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/12*(12*(d*x + c)*a - 12*a*log(e^(2*d*x + 2*c) + 1) + (25*a*e^(8*d*x + 8*
c) + 76*a*e^(6*d*x + 6*c) + 24*b*e^(6*d*x + 6*c) + 102*a*e^(4*d*x + 4*c) +
76*a*e^(2*d*x + 2*c) + 24*b*e^(2*d*x + 2*c) + 25*a)/(e^(2*d*x + 2*c) + 1)^4
)/d
```

Mupad [B]
time = 0.12, size = 173, normalized size = 3.53

$$\frac{2(a-b)}{d\left(e^{2c+2dx}+1\right)} - ax - \frac{2(a-3b)}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)} - \frac{8b}{d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)} + \frac{4b}{d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)} + \frac{a\ln\left(e^{2c}e^{2dx}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (2*(a - b))/(d*(exp(2*c + 2*d*x) + 1)) - a*x - (2*(a - 3*b))/(d*(2*exp(2*c
+ 2*d*x) + exp(4*c + 4*d*x) + 1)) - (8*b)/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*
c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (4*b)/(d*(4*exp(2*c + 2*d*x) + 6*exp(
4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (a*log(exp(2*c
)*exp(2*d*x) + 1))/d
```

3.104 $\int \left(a+b\,\text{sech}^2(c+dx)\right)\tanh^2(c+dx)\,dx$

Optimal. Leaf size=32

$$ax-\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

```
[Out] a*x-a*tanh(d*x+c)/d+1/3*b*tanh(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4226, 1816, 212}

$$-\frac{a\tanh(c+dx)}{d}+ax+\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^2,x]
```

```
[Out] a*x - (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int (a+b\text{sech}^2(c+dx)) \tanh^2(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+b(1-x^2))}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-a+bx^2+\frac{a}{1-x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}+\frac{a\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh\right.}{d}$$
$$= ax-\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.28

$$\frac{a\tanh^{-1}(\tanh(c+dx))}{d}-\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x]^2,x]
```

```
[Out] (a*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3
*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(30) = 60.
time = 1.60, size = 66, normalized size = 2.06

method	result	size
risch	$ax+\frac{2a\,e^{4dx+4c}-2b\,e^{4dx+4c}+4a\,e^{2dx+2c}+2a-\frac{2b}{3}}{d\left(1+e^{2dx+2c}\right)^3}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+2/3*(3*a*exp(4*d*x+4*c)-3*b*exp(4*d*x+4*c)+6*a*exp(2*d*x+2*c)+3*a-b)/d/
(1+exp(2*d*x+2*c))^3
```

Maxima [A]
time = 0.29, size = 42, normalized size = 1.31

$$\frac{b\tanh{(dx+c)}^3}{3\,d}+a\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b*tanh(d*x + c)^3/d + a*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(30) = 60$.
time = 0.40, size = 155, normalized size = 4.84

$$\frac{(3\,adx + 3\,a - b)\cosh\left(dx + c\right)^{3} + 3\,(3\,adx + 3\,a - b)\cosh\left(dx + c\right)\sinh\left(dx + c\right)^{2} - (3\,a - b)\sinh\left(dx + c\right)^{3} + 3\,(3\,adx + 3\,a - b)\cosh\left(dx + c\right) - 3\,\left((3\,a - b)\cosh\left(dx + c\right)^{2} + a + b\right)\sinh\left(dx + c\right)}{3\,\left(d\cosh\left(dx + c\right)^{3} + 3\,d\cosh\left(dx + c\right)\sinh\left(dx + c\right)^{2} + 3\,d\cosh\left(dx + c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*a*d*x + 3*a - b)*cosh(d*x + c)^3 + 3*(3*a*d*x + 3*a - b)*cosh(d*x +
 c)*sinh(d*x + c)^2 - (3*a - b)*sinh(d*x + c)^3 + 3*(3*a*d*x + 3*a - b)*cos
h(d*x + c) - 3*((3*a - b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))/(d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^{2}\left(c + dx\right)\right)\tanh^{2}\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*tanh(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(30) = 60$.
time = 0.40, size = 72, normalized size = 2.25

$$\frac{3\,(dx + c)a + \frac{2\,\left(3\,ae^{(4\,dx + 4\,c)} - 3\,be^{(4\,dx + 4\,c)} + 6\,ae^{(2\,dx + 2\,c)} + 3\,a - b\right)}{\left(e^{(2\,dx + 2\,c)} + 1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a + 2*(3*a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) + 6*a*e^(
2*d*x + 2*c) + 3*a - b)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B]
time = 1.50, size = 163, normalized size = 5.09

$$\frac{\frac{2\,(a+b)}{3\,d} + \frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(a-b)}{3\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x} + \mathrm{e}^{4\,c+4\,d\,x} + 1} + a\,x + \frac{\frac{2\,(a-b)}{3\,d} + \frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)}{3\,d} + \frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\,(a-b)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x} + 3\,\mathrm{e}^{4\,c+4\,d\,x} + \mathrm{e}^{6\,c+6\,d\,x} + 1} + \frac{2\,(a-b)}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] ((2*(a + b))/(3*d) + (2*exp(2*c + 2*d*x)*(a - b))/(3*d))/(2*exp(2*c + 2*d*x
) + exp(4*c + 4*d*x) + 1) + a*x + ((2*(a - b))/(3*d) + (4*exp(2*c + 2*d*x)*
(a + b))/(3*d) + (2*exp(4*c + 4*d*x)*(a - b))/(3*d))/(3*exp(2*c + 2*d*x) +
3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + (2*(a - b))/(3*d*(exp(2*c + 2*
d*x) + 1))
```

3.105 $\int (a+b\text{sech}^2(c+dx)) \tanh(c+dx) \, dx$

Optimal. Leaf size=29

$$\frac{a\log(\cosh(c+dx))}{d}-\frac{b\text{sech}^2(c+dx)}{2d}$$

```
[Out] a*ln(cosh(d*x+c))/d-1/2*b*sech(d*x+c)^2/d
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4223, 14}

$$\frac{a\log(\cosh(c+dx))}{d}-\frac{b\text{sech}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x],x]
```

```
[Out] (a*Log[Cosh[c + d*x]])/d - (b*Sech[c + d*x]^2)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int (a+b\text{sech}^2(c+dx)) \tanh(c+dx) \, dx &= \frac{\text{Subst}\left(\int \frac{b+ax^2}{x^3} \, dx, x, \cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{b}{x^3}+\frac{a}{x}\right) \, dx, x, \cosh(c+dx)\right)}{d}\\ &= \frac{a\log(\cosh(c+dx))}{d}-\frac{b\text{sech}^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.00

$$\frac{a\log(\cosh(c+dx))}{d}-\frac{b\mathrm{sech}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)*Tanh[c + d*x],x]
```

```
[Out] (a*Log[Cosh[c + d*x]])/d - (b*Sech[c + d*x]^2)/(2*d)
```

Maple [A]
time = 0.63, size = 27, normalized size = 0.93

method	result	size
derivativedivides	$-\frac{\frac{b\,\mathrm{sech}(dx+c)^2}{2}+a\ln(\mathrm{sech}(dx+c))}{d}$	27
default	$-\frac{\frac{b\,\mathrm{sech}(dx+c)^2}{2}+a\ln(\mathrm{sech}(dx+c))}{d}$	27
risch	$-ax-\frac{2ac}{d}-\frac{2b\,\mathrm{e}^{2dx+2c}}{d(1+\mathrm{e}^{2dx+2c})^2}+\frac{a\ln(1+\mathrm{e}^{2dx+2c})}{d}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)*tanh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(1/2*b*sech(d*x+c)^2+a*ln(sech(d*x+c)))
```

Maxima [A]
time = 0.28, size = 27, normalized size = 0.93

$$\frac{b\tanh\left(dx+c\right)^2}{2\,d}+\frac{a\log\left(\cosh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*b*tanh(d*x + c)^2/d + a*log(cosh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(27) = 54.
time = 0.38, size = 359, normalized size = 12.38

$$-\frac{adx\cosh\left(dx+c\right)^4+4\,adx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+adx\sinh\left(dx+c\right)^4+adx+2\left(adx+b\right)\cosh\left(dx+c\right)^2+2\left(3\,adx\cosh\left(dx+c\right)^2+adx+b\right)\sinh\left(dx+c\right)^2-\left(a\cosh\left(dx+c\right)^4+4\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+a\sinh\left(dx+c\right)^4+2\,a\cosh\left(dx+c\right)^2+2\left(3\,a\cosh\left(dx+c\right)^2+a\right)\sinh\left(dx+c\right)^2+4\left(a\cosh\left(dx+c\right)^3+a\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+a\right)\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)+4\left(adx\cosh\left(dx+c\right)^3+\left(adx+b\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{d\cosh\left(dx+c\right)^4+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+d\sinh\left(dx+c\right)^4+2\,d\cosh\left(dx+c\right)^2+2\left(3\,d\cosh\left(dx+c\right)^2+d\right)\sinh\left(dx+c\right)^2+4\left(d\cosh\left(dx+c\right)^3+d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a*d*x*cosh(d*x + c)^4 + 4*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*x*sin
h(d*x + c)^4 + a*d*x + 2*(a*d*x + b)*cosh(d*x + c)^2 + 2*(3*a*d*x*cosh(d*x
+ c)^2 + a*d*x + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c
)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d
*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*si
nh(d*x + c) + a)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(
a*d*x*cosh(d*x + c)^3 + (a*d*x + b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d
*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cos
h(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x
+ c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A]
time = 0.15, size = 42, normalized size = 1.45

$$\begin{cases} ax - \frac{a\log\left(\tanh\left(c+dx\right)+1\right)}{d} - \frac{b\,\mathrm{sech}^2\left(c+dx\right)}{2d} & \text{for } d \neq 0 \\ x\left(a+b\,\mathrm{sech}^2\left(c\right)\right)\tanh\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)*tanh(d*x+c),x)
```

```
[Out] Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d - b*sech(c + d*x)**2/(2*d), Ne(
d, 0)), (x*(a + b*sech(c)**2)*tanh(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(27) = 54.
time = 0.40, size = 83, normalized size = 2.86

$$-\frac{2\,(dx+c)a - 2\,a\log\left(e^{(2\,dx+2\,c)}+1\right) + \frac{3\,ae^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+3\,a}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a - 2*a*log(e^(2*d*x + 2*c) + 1) + (3*a*e^(4*d*x + 4*c) +
 6*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + 3*a)/(e^(2*d*x + 2*c) + 1)^2)/
d
```

Mupad [B]
time = 1.47, size = 72, normalized size = 2.48

$$\frac{2\,b}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)} - \frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - a\,x + \frac{a\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*b)/(d*(exp(2*c +
 2*d*x) + 1)) - a*x + (a*log(exp(2*c)*exp(2*d*x) + 1))/d
```

3.106 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right) \, dx$

Optimal. Leaf size=15

$$ax+\frac{b\tanh(c+dx)}{d}$$

```
[Out] a*x+b*tanh(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3852, 8}

$$ax+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sech[c + d*x]^2,x]
```

```
[Out] a*x + (b*Tanh[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\,\mathrm{sech}^2(c+dx)\right) \, dx &= ax+b\int \mathrm{sech}^2(c+dx)\,dx\\ &= ax+\frac{(ib)\mathrm{Subst}\left(\int 1\,dx,x,-i\tanh(c+dx)\right)}{d}\\ &= ax+\frac{b\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$ax+\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sech[c + d*x]^2,x]
```

```
[Out] a*x + (b*Tanh[c + d*x])/d
```

Maple [A]
time = 1.41, size = 24, normalized size = 1.60

method	result	size
risch	$ax-\frac{2b}{d\left(1+\mathrm{e}^{2dx+2c}\right)}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sech(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*x-2*b/d/(1+exp(2*d*x+2*c))
```

Maxima [A]
time = 0.27, size = 23, normalized size = 1.53

$$ax+\frac{2\,b}{d(e^{(-2\,dx-2\,c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] a*x + 2*b/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.
time = 0.36, size = 36, normalized size = 2.40

$$\frac{(adx-b)\cosh(dx+c)+b\sinh(dx+c)}{d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] ((a*d*x - b)*cosh(d*x + c) + b*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c)**2,x)
```

```
[Out] Integral(a + b*sech(c + d*x)**2, x)
```

Giac [A]
time = 0.40, size = 23, normalized size = 1.53

$$ax - \frac{2b}{d(e^{(2dx+2c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c)^2,x, algorithm="giac")
```

```
[Out] a*x - 2*b/(d*(e^(2*d*x + 2*c) + 1))
```

Mupad [B]
time = 1.39, size = 23, normalized size = 1.53

$$a\,x - \frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/cosh(c + d*x)^2,x)
```

```
[Out] a*x - (2*b)/(d*(exp(2*c + 2*d*x) + 1))
```

3.107 $\int \coth(c+dx)\left(a+b\,\text{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$-\frac{b\log(\cosh(c+dx))}{d}+\frac{(a+b)\log(\sinh(c+dx))}{d}$$

```
[Out] -b*ln(cosh(d*x+c))/d+(a+b)*ln(sinh(d*x+c))/d
```

Rubi [A]
time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4223, 457, 78}

$$\frac{(a+b)\log(\sinh(c+dx))}{d}-\frac{b\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((b*Log[Cosh[c + d*x]])/d) + ((a + b)*Log[Sinh[c + d*x]])/d
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{b+ax^2}{x(1-x^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{b+ax}{(1-x)x}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(\frac{-a-b}{-1+x}+\frac{b}{x}\right)\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{b\log(\cosh(c+dx))}{d}+\frac{(a+b)\log(\sinh(c+dx))}{d}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 1.57

$$-\frac{b(\log(\cosh(c+dx))-\log(\sinh(c+dx)))}{d}+\frac{a(\log(\cosh(c+dx))+\log(\tanh(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Sech[c + d*x]^2),x]
[Out] -((b*(Log[Cosh[c + d*x]] - Log[Sinh[c + d*x]]))/d) + (a*(Log[Cosh[c + d*x]]
 + Log[Tanh[c + d*x]]))/d
```

Maple [A]
time = 1.46, size = 24, normalized size = 0.86

method	result	size
derivativedivides	$\frac{a\ln(\sinh(dx+c))+b\ln(\tanh(dx+c))}{d}$	24
default	$\frac{a\ln(\sinh(dx+c))+b\ln(\tanh(dx+c))}{d}$	24
risch	$-ax-\frac{2ac}{d}-\frac{b\ln(1+e^{2dx+2c})}{d}+\frac{\ln(e^{2dx+2c}-1)a}{d}+\frac{\ln(e^{2dx+2c}-1)b}{d}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*ln(sinh(d*x+c))+b*ln(tanh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(28) = 56.
time = 0.49, size = 65, normalized size = 2.32

$$b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}\right)+\frac{a\log\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c)
 + 1)/d) + a*log(sinh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(28) = 56$.
time = 0.38, size = 69, normalized size = 2.46

$$-\frac{adx + b\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right) - (a+b)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(a*d*x + b*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - (a + b)*
log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)\coth\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*coth(c + d*x), x)
```

Giac [A]
time = 0.39, size = 55, normalized size = 1.96

$$-\frac{b\log\left(e^{(2\,dx+2\,c)} + e^{(-2\,dx-2\,c)} + 2\right) - (a+b)\log\left(e^{(2\,dx+2\,c)} + e^{(-2\,dx-2\,c)} - 2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(b*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - (a + b)*log(e^(2*d*x
+ 2*c) + e^(-2*d*x - 2*c) - 2))/d
```

Mupad [B]
time = 0.17, size = 167, normalized size = 5.96

$$\frac{a\,\ln\left(4\,a^2\,e^{4\,c}\,e^{4\,d\,x} - 4\,a^2 - 16\,b^2 - 16\,a\,b + 16\,b^2\,e^{4\,c}\,e^{4\,d\,x} + 16\,a\,b\,e^{4\,c}\,e^{4\,d\,x}\right)}{2\,d} - a\,x - \frac{\mathrm{atan}\left(\frac{a\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2+4\,a\,b+4\,b^2}} + \frac{2\,b\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2+4\,a\,b+4\,b^2}}\right)\,\sqrt{a^2+4\,a\,b+4\,b^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (a*log(4*a^2*exp(4*c)*exp(4*d*x) - 4*a^2 - 16*b^2 - 16*a*b + 16*b^2*exp(4*c
)*exp(4*d*x) + 16*a*b*exp(4*c)*exp(4*d*x)))/(2*d) - a*x - (atan((a*exp(2*c)
*exp(2*d*x)*(-d^2)^(1/2))/(d*(4*a*b + a^2 + 4*b^2)^(1/2)) + (2*b*exp(2*c)*e
xp(2*d*x)*(-d^2)^(1/2))/(d*(4*a*b + a^2 + 4*b^2)^(1/2)))*(4*a*b + a^2 + 4*b
^2)^(1/2))/(-d^2)^(1/2)
```

3.108 $\int \coth^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=18

$$ax-\frac{(a+b)\coth(c+dx)}{d}$$

```
[Out] a*x-(a+b)*coth(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4226, 1816, 213}

$$ax-\frac{(a+b)\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] a*x - ((a + b)*Coth[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a+b(1-x^2)}{x^2(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^2} - \frac{a}{-1+x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{(a+b)\coth(c+dx)}{d} - \frac{a\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= ax - \frac{(a+b)\coth(c+dx)}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 41, normalized size = 2.28

$$-\frac{b\coth(c+dx)}{d} - \frac{a\coth(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -((b*Coth[c + d*x])/d) - (a*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, T
anh[c + d*x]^2])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(18) = 36$.
time = 1.76, size = 43, normalized size = 2.39

method	result	size
risch	$ax - \frac{2a}{d(e^{2dx+2c}-1)} - \frac{2b}{d(e^{2dx+2c}-1)}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] a*x-2/d/(exp(2*d*x+2*c)-1)*a-2/d/(exp(2*d*x+2*c)-1)*b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(18) = 36$.
time = 0.29, size = 47, normalized size = 2.61

$$a\left(x + \frac{c}{d} + \frac{2}{d(e^{(-2\,dx-2\,c)} - 1)}\right) + \frac{2\,b}{d(e^{(-2\,dx-2\,c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + 2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(18) = 36$.
time = 0.37, size = 39, normalized size = 2.17

$$-\frac{(a+b)\cosh(dx+c) - (adx+a+b)\sinh(dx+c)}{d\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*cosh(d*x + c) - (a*d*x + a + b)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c+dx)\right)\coth^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*coth(c + d*x)**2, x)
```

Giac [A]
time = 0.41, size = 30, normalized size = 1.67

$$\frac{(dx+c)a - \frac{2(a+b)}{e^{(2dx+2c)}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] ((d*x + c)*a - 2*(a + b)/(e^(2*d*x + 2*c) - 1))/d
```

Mupad [B]
time = 0.11, size = 25, normalized size = 1.39

$$a\,x - \frac{2\,(a+b)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] a*x - (2*(a + b))/(d*(exp(2*c + 2*d*x) - 1))
```

3.109 $\int \coth^3(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=31

$$-\frac{(a+b)\operatorname{csch}^2(c+dx)}{2d}+\frac{a\log(\sinh(c+dx))}{d}$$

```
[Out] -1/2*(a+b)*csch(d*x+c)^2/d+a*ln(sinh(d*x+c))/d
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 455, 45}

$$\frac{a\log(\sinh(c+dx))}{d}-\frac{(a+b)\operatorname{csch}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/2*((a + b)*Csch[c + d*x]^2)/d + (a*Log[Sinh[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{x(b+ax^2)}{(1-x^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{b+ax}{(1-x)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a+b}{(-1+x)^2}+\frac{a}{-1+x}\right)\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{(a+b)\text{csch}^2(c+dx)}{2d}+\frac{a\log(\sinh(c+dx))}{d}$$

Mathematica [**A**]
time = 0.12, size = 52, normalized size = 1.68

$$-\frac{b\text{csch}^2(c+dx)}{2d}-\frac{a\left(\coth^2(c+dx)-2\log(\cosh(c+dx))-2\log(\tanh(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/2*(b*Csch[c + d*x]^2)/d - (a*(Coth[c + d*x]^2 - 2*Log[Cosh[c + d*x]] - 2
*Log[Tanh[c + d*x]]))/(2*d)
```

Maple [**A**]
time = 2.37, size = 37, normalized size = 1.19

method	result	size
derivativedivides	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)-\frac{b}{2\sinh(dx+c)^2}}{d}$	37
default	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)-\frac{b}{2\sinh(dx+c)^2}}{d}$	37
risch	$-ax-\frac{2ac}{d}-\frac{2\,e^{2dx+2c}(a+b)}{d\left(e^{2dx+2c}-1\right)^2}+\frac{\ln\left(e^{2dx+2c}-1\right)a}{d}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2)-1/2*b/sinh(d*x+c)^2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(29) = 58.

time = 0.27, size = 108, normalized size = 3.48

$$a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)-\frac{2\,b}{d\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*
x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*b/(d*(e^(d*x
+ c) - e^(-d*x - c))^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(29) = 58.
time = 0.37, size = 378, normalized size = 12.19

$$-\frac{adx\cosh\left(dx+c\right)^4+4\,adx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+adx\sinh\left(dx+c\right)^4+adx-2\left(adx-a-b\right)\cosh\left(dx+c\right)^2+2\left(3\,adx\cosh\left(dx+c\right)^2-adx+a+b\right)\sinh\left(dx+c\right)^2-\left(a\cosh\left(dx+c\right)^4+4\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+a\sinh\left(dx+c\right)^4-2\,a\cosh\left(dx+c\right)^2+2\left(3\,a\cosh\left(dx+c\right)^2-a\right)\sinh\left(dx+c\right)^2+4\left(a\cosh\left(dx+c\right)^3-a\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+a\right)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)+4\left(adx\cosh\left(dx+c\right)^3-\left(adx-a-b\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{d\cosh\left(dx+c\right)^4+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+d\sinh\left(dx+c\right)^4-2\,d\cosh\left(dx+c\right)^2+2\left(3\,d\cosh\left(dx+c\right)^2-d\right)\sinh\left(dx+c\right)^2+4\left(d\cosh\left(dx+c\right)^3-d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(a*d*x*cosh(d*x + c)^4 + 4*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*x*sin
h(d*x + c)^4 + a*d*x - 2*(a*d*x - a - b)*cosh(d*x + c)^2 + 2*(3*a*d*x*cosh(
d*x + c)^2 - a*d*x + a + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^4 + 4*a*cosh
(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*a*cosh(d*x + c)^2 + 2*(3*
a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - a*cosh(d*x
+ c))*sinh(d*x + c) + a)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)
)) + 4*(a*d*x*cosh(d*x + c)^3 - (a*d*x - a - b)*cosh(d*x + c))*sinh(d*x + c
))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)
^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*
(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)\coth^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*coth(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(29) = 58.
time = 0.42, size = 84, normalized size = 2.71

$$-\frac{2\left(dx+c\right)a-2\,a\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{3\,ae^{(4\,dx+4\,c)}-2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+3\,a}{\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a - 2*a*log(abs(e^(2*d*x + 2*c) - 1)) + (3*a*e^(4*d*x + 4
*c) - 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + 3*a)/(e^(2*d*x + 2*c) - 1
)^2)/d
```

Mupad [B]
time = 1.41, size = 76, normalized size = 2.45

$$\frac{a\,\ln\left({\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}-1\right)}{d}-a\,x-\frac{2\,\left(a+b\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,\left(a+b\right)}{d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (a*log(exp(2*c)*exp(2*d*x) - 1))/d - a*x - (2*(a + b))/(d*(exp(2*c + 2*d*x)
 - 1)) - (2*(a + b))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.110 $\int \coth^4(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=34

$$ax-\frac{a\coth(c+dx)}{d}-\frac{(a+b)\coth^3(c+dx)}{3d}$$

```
[Out] a*x-a*coth(d*x+c)/d-1/3*(a+b)*coth(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4226, 1816, 213}

$$-\frac{(a+b)\coth^3(c+dx)}{3d}-\frac{a\coth(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] a*x - (a*Coth[c + d*x])/d - ((a + b)*Coth[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a+b(1-x^2)}{x^4(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^4}+\frac{a}{x^2}-\frac{a}{-1+x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d}\\ &= -\frac{a\coth(c+dx)}{d}-\frac{(a+b)\coth^3(c+dx)}{3d}-\frac{a\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx\right.}{d}\\ &= ax-\frac{a\coth(c+dx)}{d}-\frac{(a+b)\coth^3(c+dx)}{3d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 49, normalized size = 1.44

$$-\frac{b\coth^3(c+dx)}{3d}-\frac{a\coth^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/3*(b*Coth[c + d*x]^3)/d - (a*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1,
-1/2, Tanh[c + d*x]^2])/(3*d)
```

Maple [A]
time = 2.12, size = 64, normalized size = 1.88

method	result	size
risch	$ax-\frac{2\left(6a\,\mathrm{e}^{4dx+4c}+3b\,\mathrm{e}^{4dx+4c}-6a\,\mathrm{e}^{2dx+2c}+4a+b\right)}{3d\left(\mathrm{e}^{2dx+2c}-1\right)^3}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] a*x-2/3*(6*a*exp(4*d*x+4*c)+3*b*exp(4*d*x+4*c)-6*a*exp(2*d*x+2*c)+4*a+b)/d/
(exp(2*d*x+2*c)-1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(32) = 64.
time = 0.27, size = 170, normalized size = 5.00

$$\frac{1}{3}a\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)+\frac{2}{3}b\left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}+\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(3*
e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 2/3*b*(3*
e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x -
6*c) - 1)) + 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*
c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(32) = 64$.
time = 0.38, size = 140, normalized size = 4.12

$$-\frac{(4\,a+b)\cosh\left(dx+c\right)^{3}+3\,(4\,a+b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}-(3\,adx+4\,a+b)\sinh\left(dx+c\right)^{3}+3\,b\cosh\left(dx+c\right)+3\,\left(3\,adx-(3\,adx+4\,a+b)\cosh\left(dx+c\right)^{2}+4\,a+b\right)\sinh\left(dx+c\right)}{3\,\left(d\sinh\left(dx+c\right)^{3}+3\,\left(d\cosh\left(dx+c\right)^{2}-d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/3*((4*a + b)*cosh(d*x + c)^3 + 3*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^2
 - (3*a*d*x + 4*a + b)*sinh(d*x + c)^3 + 3*b*cosh(d*x + c) + 3*(3*a*d*x - (
3*a*d*x + 4*a + b)*cosh(d*x + c)^2 + 4*a + b)*sinh(d*x + c))/(d*sinh(d*x +
c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}^{2}\left(c+dx\right)\right)\coth^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)*coth(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.
time = 0.43, size = 70, normalized size = 2.06

$$\frac{3\,(dx+c)a-\frac{2\,\left(6\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}+4\,a+b\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a - 2*(6*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 6*a*e^(
2*d*x + 2*c) + 4*a + b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B]
time = 1.49, size = 161, normalized size = 4.74

$$a\,x-\frac{\frac{2\,b}{3\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,a+b\right)}{3\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{\frac{2\,\left(2\,a+b\right)}{3\,d}+\frac{4\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{3\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(2\,a+b\right)}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}-\frac{2\,\left(2\,a+b\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] a*x - ((2*b)/(3*d) + (2*exp(2*c + 2*d*x)*(2*a + b))/(3*d))/(exp(4*c + 4*d*x
) - 2*exp(2*c + 2*d*x) + 1) - ((2*(2*a + b))/(3*d) + (4*b*exp(2*c + 2*d*x))
/(3*d) + (2*exp(4*c + 4*d*x)*(2*a + b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(
4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (2*(2*a + b))/(3*d*(exp(2*c + 2*d*x)
 - 1))
```

3.111 $\int \coth^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)\,dx$

Optimal. Leaf size=51

$$-\frac{(2a+b)\operatorname{csch}^2(c+dx)}{2d}-\frac{(a+b)\operatorname{csch}^4(c+dx)}{4d}+\frac{a\log(\sinh(c+dx))}{d}$$

```
[Out] -1/2*(2*a+b)*csch(d*x+c)^2/d-1/4*(a+b)*csch(d*x+c)^4/d+a*ln(sinh(d*x+c))/d
```

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 457, 78}

$$-\frac{(a+b)\operatorname{csch}^4(c+dx)}{4d}-\frac{(2a+b)\operatorname{csch}^2(c+dx)}{2d}+\frac{a\log(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/2*((2*a + b)*Csch[c + d*x]^2)/d - ((a + b)*Csch[c + d*x]^4)/(4*d) + (a*L
og[Sinh[c + d*x]])/d
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)\,dx = -\frac{\text{Subst}\left(\int \frac{x^3(b+ax^2)}{(1-x^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{x(b+ax)}{(1-x)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{(-1+x)^3}+\frac{-2a-b}{(-1+x)^2}-\frac{a}{-1+x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{(2a+b)\text{csch}^2(c+dx)}{2d}-\frac{(a+b)\text{csch}^4(c+dx)}{4d}+\frac{a\log(\sinh(c}{d}$$

Mathematica [A]
time = 0.18, size = 62, normalized size = 1.22

$$-\frac{b\coth^4(c+dx)}{4d}-\frac{a\left(2\coth^2(c+dx)+\coth^4(c+dx)-4\log(\cosh(c+dx))-4\log(\tanh(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/4*(b*Coth[c + d*x]^4)/d - (a*(2*Coth[c + d*x]^2 + Coth[c + d*x]^4 - 4*Lo
g[Cosh[c + d*x]] - 4*Log[Tanh[c + d*x]]))/(4*d)
```

Maple [A]
time = 1.77, size = 67, normalized size = 1.31

method	result	size
derivativedivides	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}\right)+b\left(-\frac{\cosh^2(dx+c)}{2\sinh(dx+c)^4}+\frac{1}{4\sinh(dx+c)^4}\right)}{d}$	67
default	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}\right)+b\left(-\frac{\cosh^2(dx+c)}{2\sinh(dx+c)^4}+\frac{1}{4\sinh(dx+c)^4}\right)}{d}$	67
risch	$-ax-\frac{2ac}{d}-\frac{2\,e^{2dx+2c}\left(2a\,e^{4dx+4c}+b\,e^{4dx+4c}-2a\,e^{2dx+2c}+2a+b\right)}{d\left(e^{2dx+2c}-1\right)^4}+\frac{\ln\left(e^{2dx+2c}-1\right)a}{d}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4)+b*(-1/2/sinh(d
*x+c)^4*cosh(d*x+c)^2+1/4/sinh(d*x+c)^4))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(47) = 94$.
time = 0.26, size = 251, normalized size = 4.92

$$a\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)} + 1\right)}{d} + \frac{\log\left(e^{(-dx-c)} - 1\right)}{d} + \frac{4\left(e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)} - 6\,e^{(-4\,dx-4\,c)} + 4\,e^{(-6\,dx-6\,c)} - e^{(-8\,dx-8\,c)} - 1)}\right) + 2\,b\left(\frac{e^{(-2\,dx-2\,c)}}{d(4\,e^{(-2\,dx-2\,c)} - 6\,e^{(-4\,dx-4\,c)} + 4\,e^{(-6\,dx-6\,c)} - e^{(-8\,dx-8\,c)} - 1)} + \frac{e^{(-6\,dx-6\,c)}}{d(4\,e^{(-2\,dx-2\,c)} - 6\,e^{(-4\,dx-4\,c)} + 4\,e^{(-6\,dx-6\,c)} - e^{(-8\,dx-8\,c)} - 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6
*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 2*b*(e^(
-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6
*c) - e^(-8*d*x - 8*c) - 1)) + e^(-6*d*x - 6*c)/(d*(4*e^(-2*d*x - 2*c) - 6*
e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1099 vs. $2(47) = 94$.
time = 0.40, size = 1099, normalized size = 21.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(a*d*x*cosh(d*x + c)^8 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a*d*x*sin
h(d*x + c)^8 - 2*(2*a*d*x - 2*a - b)*cosh(d*x + c)^6 + 2*(14*a*d*x*cosh(d*x
 + c)^2 - 2*a*d*x + 2*a + b)*sinh(d*x + c)^6 + 4*(14*a*d*x*cosh(d*x + c)^3
- 3*(2*a*d*x - 2*a - b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*
cosh(d*x + c)^4 + 2*(35*a*d*x*cosh(d*x + c)^4 + 3*a*d*x - 15*(2*a*d*x - 2*a
 - b)*cosh(d*x + c)^2 - 2*a)*sinh(d*x + c)^4 + 8*(7*a*d*x*cosh(d*x + c)^5 -
 5*(2*a*d*x - 2*a - b)*cosh(d*x + c)^3 + (3*a*d*x - 2*a)*cosh(d*x + c))*sin
h(d*x + c)^3 + a*d*x - 2*(2*a*d*x - 2*a - b)*cosh(d*x + c)^2 + 2*(14*a*d*x*
cosh(d*x + c)^6 - 15*(2*a*d*x - 2*a - b)*cosh(d*x + c)^4 - 2*a*d*x + 6*(3*a
*d*x - 2*a)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 - (a*cosh(d*x + c)^8
 + 8*a*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 - 4*a*cosh(d*x + c
)^6 + 4*(7*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^6 + 8*(7*a*cosh(d*x + c)^3
- 3*a*cosh(d*x + c))*sinh(d*x + c)^5 + 6*a*cosh(d*x + c)^4 + 2*(35*a*cosh(d
*x + c)^4 - 30*a*cosh(d*x + c)^2 + 3*a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*x +
 c)^5 - 10*a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c))*sinh(d*x + c)^3 - 4*a*cos
h(d*x + c)^2 + 4*(7*a*cosh(d*x + c)^6 - 15*a*cosh(d*x + c)^4 + 9*a*cosh(d*x
 + c)^2 - a)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 - 3*a*cosh(d*x + c)^5 +
 3*a*cosh(d*x + c)^3 - a*cosh(d*x + c))*sinh(d*x + c) + a)*log(2*sinh(d*x +
 c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*a*d*x*cosh(d*x + c)^7 - 3*(2*a*
d*x - 2*a - b)*cosh(d*x + c)^5 + 2*(3*a*d*x - 2*a)*cosh(d*x + c)^3 - (2*a*d
```

```
*x - 2*a - b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d
*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*
cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*
x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 3
0*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*
cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2
+ 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*
sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x
 + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*sech(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(47) = 94.
time = 0.45, size = 120, normalized size = 2.35

$$-\frac{12\,(dx+c)a-12\,a\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{25\,ae^{(8\,dx+8\,c)}-52\,ae^{(6\,dx+6\,c)}+24\,be^{(6\,dx+6\,c)}+102\,ae^{(4\,dx+4\,c)}-52\,ae^{(2\,dx+2\,c)}+24\,be^{(2\,dx+2\,c)}+25\,a}{\left(e^{(2\,dx+2\,c)}-1\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/12*(12*(d*x + c)*a - 12*a*log(abs(e^(2*d*x + 2*c) - 1)) + (25*a*e^(8*d*x
 + 8*c) - 52*a*e^(6*d*x + 6*c) + 24*b*e^(6*d*x + 6*c) + 102*a*e^(4*d*x + 4*
c) - 52*a*e^(2*d*x + 2*c) + 24*b*e^(2*d*x + 2*c) + 25*a)/(e^(2*d*x + 2*c) -
 1)^4)/d
```

Mupad [**B**]
time = 0.11, size = 179, normalized size = 3.51

$$\frac{a\ln\left(\mathrm{e}^{2c}\,\mathrm{e}^{2dx}-1\right)}{d}-\frac{8\,(a+b)}{d\,\left(3\,\mathrm{e}^{2c+2dx}-3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}-1\right)}-\frac{2\,(2\,a+b)}{d\,\left(\mathrm{e}^{2c+2dx}-1\right)}-\frac{4\,(a+b)}{d\,\left(6\,\mathrm{e}^{4c+4dx}-4\,\mathrm{e}^{2c+2dx}-4\,\mathrm{e}^{6c+6dx}+\mathrm{e}^{8c+8dx}+1\right)}-a\,x-\frac{2\,(4\,a+3\,b)}{d\,\left(\mathrm{e}^{4c+4dx}-2\,\mathrm{e}^{2c+2dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^5*(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (a*log(exp(2*c)*exp(2*d*x) - 1))/d - (8*(a + b))/(d*(3*exp(2*c + 2*d*x) - 3
*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (2*(2*a + b))/(d*(exp(2*c + 2*
d*x) - 1)) - (4*(a + b))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*ex
p(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - a*x - (2*(4*a + 3*b))/(d*(exp(4*c
 + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.112 $\int \left(a+b\mathbf{sech}^2(c+dx)\right)^2 \tanh^4(c+dx)\,dx$

Optimal. Leaf size=77

$$a^2x-\frac{a^2\tanh(c+dx)}{d}-\frac{a^2\tanh^3(c+dx)}{3d}+\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

```
[Out] a^2*x-a^2*tanh(d*x+c)/d-1/3*a^2*tanh(d*x+c)^3/d+1/5*b*(2*a+b)*tanh(d*x+c)^5
/d-1/7*b^2*tanh(d*x+c)^7/d
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4226, 1816, 212}

$$-\frac{a^2\tanh^3(c+dx)}{3d}-\frac{a^2\tanh(c+dx)}{d}+a^2x+\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^4,x]
```

```
[Out] a^2*x - (a^2*Tanh[c + d*x])/d - (a^2*Tanh[c + d*x]^3)/(3*d) + (b*(2*a + b)*
Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\tanh^4(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1-x^2\right)\right)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^2-a^2x^2+b(2a+b)x^4-b^2x^6+\frac{a^2}{1-x^2}\right)\,dx, x, \tan\right.}{d}$$
$$= -\frac{a^2\tanh(c+dx)}{d}-\frac{a^2\tanh^3(c+dx)}{3d}+\frac{b(2a+b)\tanh^5(c+d}{5d}$$
$$= a^2x-\frac{a^2\tanh(c+dx)}{d}-\frac{a^2\tanh^3(c+dx)}{3d}+\frac{b(2a+b)\tanh^5(}{5d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 395 vs. 2(77) = 154.
time = 0.79, size = 395, normalized size = 5.13

$$\frac{\text{sech}(c)\text{sech}^7(c+dx)\left(3675a^2dx\cosh(dx)+3675a^2dx\cosh(2c+dx)+2205a^2dx\cosh(2c+3dx)+2205a^2dx\cosh(4c+3dx)+735a^2dx\cosh(4c+5dx)+735a^2dx\cosh(6c+5dx)+105a^2dx\cosh(6c+7dx)+105a^2dx\cosh(8c+7dx)-5320a^2\sinh(dx)+1680ab\sinh(dx)+840b^2\sinh(dx)+4480a^2\sinh(2c+dx)-1260ab\sinh(2c+dx)+420b^2\sinh(2c+dx)-3780a^2\sinh(2c+3dx)+924ab\sinh(2c+3dx)-168b^2\sinh(2c+3dx)+2100a^2\sinh(4c+3dx)-840ab\sinh(4c+3dx)-420b^2\sinh(4c+3dx)-1540a^2\sinh(4c+5dx)+168ab\sinh(4c+5dx)+84b^2\sinh(4c+5dx)+420a^2\sinh(6c+5dx)-420ab\sinh(6c+5dx)-280a^2\sinh(6c+7dx)+84ab\sinh(6c+7dx)+12b^2\sinh(6c+7dx)\right)}{13440d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^4,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^7*(3675*a^2*d*x*Cosh[d*x] + 3675*a^2*d*x*Cosh[2*c +
d*x] + 2205*a^2*d*x*Cosh[2*c + 3*d*x] + 2205*a^2*d*x*Cosh[4*c + 3*d*x] + 73
5*a^2*d*x*Cosh[4*c + 5*d*x] + 735*a^2*d*x*Cosh[6*c + 5*d*x] + 105*a^2*d*x*C
osh[6*c + 7*d*x] + 105*a^2*d*x*Cosh[8*c + 7*d*x] - 5320*a^2*Sinh[d*x] + 168
0*a*b*Sinh[d*x] + 840*b^2*Sinh[d*x] + 4480*a^2*Sinh[2*c + d*x] - 1260*a*b*S
inh[2*c + d*x] + 420*b^2*Sinh[2*c + d*x] - 3780*a^2*Sinh[2*c + 3*d*x] + 924
*a*b*Sinh[2*c + 3*d*x] - 168*b^2*Sinh[2*c + 3*d*x] + 2100*a^2*Sinh[4*c + 3*
d*x] - 840*a*b*Sinh[4*c + 3*d*x] - 420*b^2*Sinh[4*c + 3*d*x] - 1540*a^2*Sin
h[4*c + 5*d*x] + 168*a*b*Sinh[4*c + 5*d*x] + 84*b^2*Sinh[4*c + 5*d*x] + 420
*a^2*Sinh[6*c + 5*d*x] - 420*a*b*Sinh[6*c + 5*d*x] - 280*a^2*Sinh[6*c + 7*d
*x] + 84*a*b*Sinh[6*c + 7*d*x] + 12*b^2*Sinh[6*c + 7*d*x]))/(13440*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(71) = 142.
time = 2.44, size = 272, normalized size = 3.53

method	result
risch	$a^2x+\frac{4a^2e^{12dx+12c}-4ab\,e^{12dx+12c}+20a^2e^{10dx+10c}-8ab\,e^{10dx+10c}-4b^2e^{10dx+10c}+\frac{128a^2e^{8dx+8c}}{3}-12ab\,e^{8dx+8c}+4b^2e^{8dx+8c}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+4/105*(105*a^2*exp(12*d*x+12*c)-105*a*b*exp(12*d*x+12*c)+525*a^2*exp(
10*d*x+10*c)-210*a*b*exp(10*d*x+10*c)-105*b^2*exp(10*d*x+10*c)+1120*a^2*exp
(8*d*x+8*c)-315*a*b*exp(8*d*x+8*c)+105*b^2*exp(8*d*x+8*c)+1330*a^2*exp(6*d*
x+6*c)-420*a*b*exp(6*d*x+6*c)-210*b^2*exp(6*d*x+6*c)+945*a^2*exp(4*d*x+4*c)
-231*a*b*exp(4*d*x+4*c)+42*b^2*exp(4*d*x+4*c)+385*a^2*exp(2*d*x+2*c)-42*a*b
*exp(2*d*x+2*c)-21*b^2*exp(2*d*x+2*c)+70*a^2-21*a*b-3*b^2)/d/(1+exp(2*d*x+2
*c))^7
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(71) = 142$.
time = 0.28, size = 649, normalized size = 8.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 2/5*a*b*tanh(d*x + c)^5/d + 1/3*a^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) +
3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) + 1))) + 4/35*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) +
21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10
*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4*d
*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c
) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c) + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^
(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x
- 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x -
8*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 3
5*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*
d*x - 14*c) + 1)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4
*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 1
0*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x
- 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) +
21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(71) = 142$.
time = 0.40, size = 721, normalized size = 9.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/105*((105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^7 + 7*(105*a^
2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 - 2*(70*a^2
```

```
 - 21*a*b - 3*b^2)*sinh(d*x + c)^7 + 7*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*
b^2)*cosh(d*x + c)^5 - 14*(3*(70*a^2 - 21*a*b - 3*b^2)*cosh(d*x + c)^2 + 40
*a^2 + 9*a*b - 3*b^2)*sinh(d*x + c)^5 + 35*((105*a^2*d*x + 140*a^2 - 42*a*b
 - 6*b^2)*cosh(d*x + c)^3 + (105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d
*x + c))*sinh(d*x + c)^4 + 21*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh
(d*x + c)^3 - 14*(5*(70*a^2 - 21*a*b - 3*b^2)*cosh(d*x + c)^4 + 10*(40*a^2
+ 9*a*b - 3*b^2)*cosh(d*x + c)^2 + 60*a^2 - 3*a*b + 21*b^2)*sinh(d*x + c)^3
 + 7*(3*(105*a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^5 + 10*(105*
a^2*d*x + 140*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c)^3 + 9*(105*a^2*d*x + 140*
a^2 - 42*a*b - 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^2*d*x + 14
0*a^2 - 42*a*b - 6*b^2)*cosh(d*x + c) - 14*((70*a^2 - 21*a*b - 3*b^2)*cosh(
d*x + c)^6 + 5*(40*a^2 + 9*a*b - 3*b^2)*cosh(d*x + c)^4 + 9*(20*a^2 - a*b +
 7*b^2)*cosh(d*x + c)^2 + 30*a^2 - 15*a*b - 45*b^2)*sinh(d*x + c))/(d*cosh(
d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(
d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3
 + 7*(3*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(
d*x + c)^2 + 35*d*cosh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^{2}\left(c + dx\right)\right)^{2}\tanh^{4}\left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*tanh(c + d*x)**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(71) = 142.
time = 0.45, size = 278, normalized size = 3.61

$$\frac{105\,(dx+c)a^2 + \frac{4\,(105\,a^2e^{(12\,dx+12\,c)} - 105\,abe^{(12\,dx+12\,c)} + 525\,a^2e^{(10\,dx+10\,c)} - 210\,abe^{(10\,dx+10\,c)} - 105\,b^2e^{(10\,dx+10\,c)} + 1120\,a^2e^{(8\,dx+8\,c)} - 315\,abe^{(8\,dx+8\,c)} + 105\,b^2e^{(8\,dx+8\,c)} + 1330\,a^2e^{(6\,dx+6\,c)} - 420\,abe^{(6\,dx+6\,c)} - 210\,b^2e^{(6\,dx+6\,c)} + 945\,a^2e^{(4\,dx+4\,c)} - 231\,abe^{(4\,dx+4\,c)} + 42\,b^2e^{(4\,dx+4\,c)} + 385\,a^2e^{(2\,dx+2\,c)} - 42\,abe^{(2\,dx+2\,c)} - 21\,b^2e^{(2\,dx+2\,c)} + 70\,a^2 - 21\,ab - 3\,b^2)}{\left(e^{(2\,dx+2\,c)}+1\right)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/105*(105*(d*x + c)*a^2 + 4*(105*a^2*e^(12*d*x + 12*c) - 105*a*b*e^(12*d*x
 + 12*c) + 525*a^2*e^(10*d*x + 10*c) - 210*a*b*e^(10*d*x + 10*c) - 105*b^2*
e^(10*d*x + 10*c) + 1120*a^2*e^(8*d*x + 8*c) - 315*a*b*e^(8*d*x + 8*c) + 10
5*b^2*e^(8*d*x + 8*c) + 1330*a^2*e^(6*d*x + 6*c) - 420*a*b*e^(6*d*x + 6*c)
- 210*b^2*e^(6*d*x + 6*c) + 945*a^2*e^(4*d*x + 4*c) - 231*a*b*e^(4*d*x + 4*
c) + 42*b^2*e^(4*d*x + 4*c) + 385*a^2*e^(2*d*x + 2*c) - 42*a*b*e^(2*d*x + 2
*c) - 21*b^2*e^(2*d*x + 2*c) + 70*a^2 - 21*a*b - 3*b^2)/(e^(2*d*x + 2*c) +
1)^7)/d
```

Mupad [B]
time = 0.18, size = 1022, normalized size = 13.27

$$\frac{\frac{4\,\left(7\,a^2+a\,b+8\,b^2\right)}{105\,d}-\frac{4\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(a\,b-a^2\right)}{7\,d}-\frac{16\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(-2\,a^2+a\,b+3\,b^2\right)}{35\,d}+\frac{16\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(2\,a^2+a\,b-b^2\right)}{21\,d}+\frac{8\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(7\,a^2+a\,b+8\,b^2\right)}{35\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}-\frac{\frac{4\,\left(-2\,a^2+a\,b+3\,b^2\right)}{35\,d}+\frac{4\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a\,b-a^2\right)}{7\,d}-\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(2\,a^2+a\,b-b^2\right)}{7\,d}-\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(7\,a^2+a\,b+8\,b^2\right)}{35\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}+a^2\,x+\frac{\frac{4\,\left(7\,a^2+a\,b+8\,b^2\right)}{105\,d}-\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a\,b-a^2\right)}{7\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,a^2+a\,b-b^2\right)}{21\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}+\frac{\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,a^2+a\,b-b^2\right)}{7\,d}-\frac{4\,{\mathrm{e}}^{12\,c+12\,d\,x}\,\left(a\,b-a^2\right)}{7\,d}-\frac{4\,\left(a\,b-a^2\right)}{7\,d}-\frac{16\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(-2\,a^2+a\,b+3\,b^2\right)}{7\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(7\,a^2+a\,b+8\,b^2\right)}{7\,d}+\frac{8\,{\mathrm{e}}^{10\,c+10\,d\,x}\,\left(2\,a^2+a\,b-b^2\right)}{7\,d}+\frac{4\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(7\,a^2+a\,b+8\,b^2\right)}{7\,d}}{7\,{\mathrm{e}}^{2\,c+2\,d\,x}+21\,{\mathrm{e}}^{4\,c+4\,d\,x}+35\,{\mathrm{e}}^{6\,c+6\,d\,x}+35\,{\mathrm{e}}^{8\,c+8\,d\,x}+21\,{\mathrm{e}}^{10\,c+10\,d\,x}+7\,{\mathrm{e}}^{12\,c+12\,d\,x}+{\mathrm{e}}^{14\,c+14\,d\,x}+1}+\frac{\frac{4\,\left(2\,a^2+a\,b-b^2\right)}{21\,d}-\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a\,b-a^2\right)}{7\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}+\frac{\frac{4\,\left(2\,a^2+a\,b-b^2\right)}{21\,d}-\frac{4\,{\mathrm{e}}^{10\,c+10\,d\,x}\,\left(a\,b-a^2\right)}{7\,d}-\frac{8\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(-2\,a^2+a\,b+3\,b^2\right)}{7\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(7\,a^2+a\,b+8\,b^2\right)}{21\,d}+\frac{20\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(2\,a^2+a\,b-b^2\right)}{21\,d}+\frac{8\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(7\,a^2+a\,b+8\,b^2\right)}{21\,d}}{6\,{\mathrm{e}}^{2\,c+2\,d\,x}+15\,{\mathrm{e}}^{4\,c+4\,d\,x}+20\,{\mathrm{e}}^{6\,c+6\,d\,x}+15\,{\mathrm{e}}^{8\,c+8\,d\,x}+6\,{\mathrm{e}}^{10\,c+10\,d\,x}+{\mathrm{e}}^{12\,c+12\,d\,x}+1}-\frac{4\,\left(a\,b-a^2\right)}{7\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] ((4*(a*b + 7*a^2 + 8*b^2))/(105*d) - (4*exp(8*c + 8*d*x)*(a*b - a^2))/(7*d)
 - (16*exp(2*c + 2*d*x)*(a*b - 2*a^2 + 3*b^2))/(35*d) + (16*exp(6*c + 6*d*x
)*(a*b + 2*a^2 - b^2))/(21*d) + (8*exp(4*c + 4*d*x)*(a*b + 7*a^2 + 8*b^2))/
(35*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5
*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((4*(a*b - 2*a^2 + 3*b^2))/(3
5*d) + (4*exp(6*c + 6*d*x)*(a*b - a^2))/(7*d) - (4*exp(4*c + 4*d*x)*(a*b +
2*a^2 - b^2))/(7*d) - (4*exp(2*c + 2*d*x)*(a*b + 7*a^2 + 8*b^2))/(35*d))/(4
*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d
*x) + 1) + a^2*x + ((4*(a*b + 7*a^2 + 8*b^2))/(105*d) - (4*exp(4*c + 4*d*x)
*(a*b - a^2))/(7*d) + (8*exp(2*c + 2*d*x)*(a*b + 2*a^2 - b^2))/(21*d))/(3*e
xp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((8*exp(2*c
+ 2*d*x)*(a*b + 2*a^2 - b^2))/(7*d) - (4*exp(12*c + 12*d*x)*(a*b - a^2))/(7
*d) - (4*(a*b - a^2))/(7*d) - (16*exp(6*c + 6*d*x)*(a*b - 2*a^2 + 3*b^2))/(
7*d) + (4*exp(4*c + 4*d*x)*(a*b + 7*a^2 + 8*b^2))/(7*d) + (8*exp(10*c + 10*
d*x)*(a*b + 2*a^2 - b^2))/(7*d) + (4*exp(8*c + 8*d*x)*(a*b + 7*a^2 + 8*b^2)
)/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) +
35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14
*c + 14*d*x) + 1) + ((4*(a*b + 2*a^2 - b^2))/(21*d) - (4*exp(2*c + 2*d*x)*(
a*b - a^2))/(7*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((4*(a*b +
 2*a^2 - b^2))/(21*d) - (4*exp(10*c + 10*d*x)*(a*b - a^2))/(7*d) - (8*exp(4
*c + 4*d*x)*(a*b - 2*a^2 + 3*b^2))/(7*d) + (4*exp(2*c + 2*d*x)*(a*b + 7*a^2
 + 8*b^2))/(21*d) + (20*exp(8*c + 8*d*x)*(a*b + 2*a^2 - b^2))/(21*d) + (8*e
xp(6*c + 6*d*x)*(a*b + 7*a^2 + 8*b^2))/(21*d))/(6*exp(2*c + 2*d*x) + 15*exp
(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10
*d*x) + exp(12*c + 12*d*x) + 1) - (4*(a*b - a^2))/(7*d*(exp(2*c + 2*d*x) +
1))
```

3.113 $\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh^3(c+dx)\,dx$

Optimal. Leaf size=77

$$\frac{a^2\log(\cosh(c+dx))}{d}+\frac{a(a-2b)\text{sech}^2(c+dx)}{2d}+\frac{(2a-b)b\text{sech}^4(c+dx)}{4d}+\frac{b^2\text{sech}^6(c+dx)}{6d}$$

```
[Out] a^2*ln(cosh(d*x+c))/d+1/2*a*(a-2*b)*sech(d*x+c)^2/d+1/4*(2*a-b)*b*sech(d*x+
c)^4/d+1/6*b^2*sech(d*x+c)^6/d
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {4223, 457, 77}

$$\frac{a^2\log(\cosh(c+dx))}{d}+\frac{b(2a-b)\text{sech}^4(c+dx)}{4d}+\frac{a(a-2b)\text{sech}^2(c+dx)}{2d}+\frac{b^2\text{sech}^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^3,x]
```

```
[Out] (a^2*Log[Cosh[c + d*x]])/d + (a*(a - 2*b)*Sech[c + d*x]^2)/(2*d) + ((2*a -
b)*b*Sech[c + d*x]^4)/(4*d) + (b^2*Sech[c + d*x]^6)/(6*d)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2 \tanh^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)\left(b+ax^2\right)^2}{x^7}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)(b+ax)^2}{x^4}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^4}+\frac{(2a-b)b}{x^3}+\frac{a(a-2b)}{x^2}-\frac{a^2}{x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{a^2\log(\cosh(c+dx))}{d}+\frac{a(a-2b)\text{sech}^2(c+dx)}{2d}+\frac{(2a-b)b\text{sech}}{4d}$$

Mathematica [A]
time = 0.20, size = 107, normalized size = 1.39

$$\frac{\cosh^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\left(12a^2\log(\cosh(c+dx))+6a(a-2b)\text{sech}^2(c+dx)+3(2a-b)b\text{sech}^4(c+dx)+2b^2\text{sech}^6(c+dx)\right)}{3d(a+2b+a\cosh(2c+2dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^3,x]
```

```
[Out] (Cosh[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2*(12*a^2*Log[Cosh[c + d*x]] + 6*a
*(a - 2*b)*Sech[c + d*x]^2 + 3*(2*a - b)*b*Sech[c + d*x]^4 + 2*b^2*Sech[c +
 d*x]^6))/(3*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^2)
```

Maple [A]
time = 1.42, size = 94, normalized size = 1.22

method	result
derivativedivides	$\frac{a^2\left(\ln(\cosh(dx+c))-\frac{\left(\tanh^2(dx+c)\right)}{2}\right)+2ab\left(-\frac{\sinh^2(dx+c)}{2\cosh(dx+c)^4}-\frac{1}{4\cosh(dx+c)^4}\right)+b^2\left(-\frac{\sinh^2(dx+c)}{4\cosh(dx+c)^6}-\frac{1}{12\cosh(dx+c)^6}\right)}{d}$
default	$\frac{a^2\left(\ln(\cosh(dx+c))-\frac{\left(\tanh^2(dx+c)\right)}{2}\right)+2ab\left(-\frac{\sinh^2(dx+c)}{2\cosh(dx+c)^4}-\frac{1}{4\cosh(dx+c)^4}\right)+b^2\left(-\frac{\sinh^2(dx+c)}{4\cosh(dx+c)^6}-\frac{1}{12\cosh(dx+c)^6}\right)}{d}$
risch	$-a^2x-\frac{2a^2c}{d}+\frac{2\,e^{2dx+2c}\left(3a^2e^{8dx+8c}-6ab\,e^{8dx+8c}+12a^2e^{6dx+6c}-12ab\,e^{6dx+6c}-6b^2e^{6dx+6c}+18a^2e^{4dx+4c}-12ab\,e\right.}{3d\left(1+e^{2dx+2c}\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(ln(cosh(d*x+c))-1/2*tanh(d*x+c)^2)+2*a*b*(-1/2*sinh(d*x+c)^2/cosh
(d*x+c)^4-1/4/cosh(d*x+c)^4)+b^2*(-1/4*sinh(d*x+c)^2/cosh(d*x+c)^6-1/12/cos
h(d*x+c)^6))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(71) = 142$.
time = 0.47, size = 333, normalized size = 4.32

$$\frac{ab\tanh\left(dx+c\right)^4}{2\,d}+a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)-\frac{4}{3}\,b^2\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}-\frac{2\,e^{(-6\,dx-6\,c)}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}+\frac{3\,e^{(-8\,dx-8\,c)}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-10\,c)}+e^{(-12\,dx-12\,c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*b*tanh(d*x + c)^4/d + a^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*
e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 4/3*b^2
*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-
6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1)) - 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c)
 + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-1
2*d*x - 12*c) + 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*
c) + e^(-12*d*x - 12*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2591 vs. $2(71) = 142$.
time = 0.38, size = 2591, normalized size = 33.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*d*x*cosh(d*x + c)^12 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^1
1 + 3*a^2*d*x*sinh(d*x + c)^12 + 6*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^
10 + 6*(33*a^2*d*x*cosh(d*x + c)^2 + 3*a^2*d*x - a^2 + 2*a*b)*sinh(d*x + c)
^10 + 60*(11*a^2*d*x*cosh(d*x + c)^3 + (3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x +
 c))*sinh(d*x + c)^9 + 3*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)
^8 + 3*(495*a^2*d*x*cosh(d*x + c)^4 + 15*a^2*d*x + 90*(3*a^2*d*x - a^2 + 2*
a*b)*cosh(d*x + c)^2 - 8*a^2 + 8*a*b + 4*b^2)*sinh(d*x + c)^8 + 24*(99*a^2*
d*x*cosh(d*x + c)^5 + 30*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^3 + (15*a^
2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(15*a^2*d
*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^6 + 4*(693*a^2*d*x*cosh(d*x + c)^
6 + 315*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^4 + 15*a^2*d*x + 21*(15*a^2
*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^2 - 9*a^2 + 6*a*b - 2*b^2)*sinh
(d*x + c)^6 + 24*(99*a^2*d*x*cosh(d*x + c)^7 + 63*(3*a^2*d*x - a^2 + 2*a*b)
*cosh(d*x + c)^5 + 7*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^3 +
```

(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^4 + 3*(495*a^2*d*x*cosh(d*x + c)^8 + 420*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^6 + 70*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^4 + 15*a^2*d*x + 20*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^2 - 8*a^2 + 8*a*b + 4*b^2)*sinh(d*x + c)^4 + 3*a^2*d*x + 4*(165*a^2*d*x*cosh(d*x + c)^9 + 180*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^7 + 42*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^5 + 20*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^3 + 3*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^2 + 6*(33*a^2*d*x*cosh(d*x + c)^10 + 45*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^8 + 14*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^6 + 10*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^4 + 3*a^2*d*x + 3*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^2 - a^2 + 2*a*b)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^12 + 12*a^2*cosh(d*x + c)*sinh(d*x + c)^11 + a^2*sinh(d*x + c)^12 + 6*a^2*cosh(d*x + c)^10 + 6*(11*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^10 + 15*a^2*cosh(d*x + c)^8 + 20*(11*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(33*a^2*cosh(d*x + c)^4 + 18*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^8 + 20*a^2*cosh(d*x + c)^6 + 24*(33*a^2*cosh(d*x + c)^5 + 30*a^2*cosh(d*x + c)^3 + 5*a^2*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(231*a^2*cosh(d*x + c)^6 + 315*a^2*cosh(d*x + c)^4 + 105*a^2*cosh(d*x + c)^2 + 5*a^2)*sinh(d*x + c)^6 + 15*a^2*cosh(d*x + c)^4 + 24*(33*a^2*cosh(d*x + c)^7 + 63*a^2*cosh(d*x + c)^5 + 35*a^2*cosh(d*x + c)^3 + 5*a^2*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(33*a^2*cosh(d*x + c)^8 + 84*a^2*cosh(d*x + c)^6 + 70*a^2*cosh(d*x + c)^4 + 20*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + 20*(11*a^2*cosh(d*x + c)^9 + 36*a^2*cosh(d*x + c)^7 + 42*a^2*cosh(d*x + c)^5 + 20*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(11*a^2*cosh(d*x + c)^10 + 45*a^2*cosh(d*x + c)^8 + 70*a^2*cosh(d*x + c)^6 + 50*a^2*cosh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 12*(a^2*cosh(d*x + c)^11 + 5*a^2*cosh(d*x + c)^9 + 10*a^2*cosh(d*x + c)^7 + 10*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*a^2*d*x*cosh(d*x + c)^11 + 5*(3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c)^9 + 2*(15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^7 + 2*(15*a^2*d*x - 9*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^5 + (15*a^2*d*x - 8*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^3 + (3*a^2*d*x - a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 +

```
70*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*
d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh
(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)^2 + 6*
(11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 + 50*d
*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d*cosh(d
*x + c)^11 + 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(d*x + c
)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A]
time = 0.86, size = 129, normalized size = 1.68

$$\begin{cases} a^2x - \frac{a^2\log(\tanh(c+dx)+1)}{d} - \frac{a^2\tanh^2(c+dx)}{2d} - \frac{ab\tanh^2(c+dx)\operatorname{sech}^2(c+dx)}{2d} - \frac{ab\operatorname{sech}^2(c+dx)}{2d} - \frac{b^2\tanh^2(c+dx)\operatorname{sech}^4(c+dx)}{6d} - \frac{b^2\operatorname{sech}^4(c+dx)}{12d} & \text{for } d \neq 0 \\ x\left(a+b\operatorname{sech}^2(c)\right)^2\tanh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c)**3,x)
```

```
[Out] Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a**2*tanh(c + d*x)**2/(
2*d) - a*b*tanh(c + d*x)**2*sech(c + d*x)**2/(2*d) - a*b*sech(c + d*x)**2/(
2*d) - b**2*tanh(c + d*x)**2*sech(c + d*x)**4/(6*d) - b**2*sech(c + d*x)**4
/(12*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**2*tanh(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(71) = 142.
time = 0.45, size = 244, normalized size = 3.17

$$-\frac{60(dx+c)a^2 - 60a^2\log\left(e^{(2dx+2c)}+1\right) + \frac{147a^2e^{(12dx+12c)} + 762a^2e^{(10dx+10c)} + 240abe^{(10dx+10c)} + 1725a^2e^{(8dx+8c)} + 480abe^{(8dx+8c)} + 240b^2e^{(8dx+8c)} + 2220a^2e^{(6dx+6c)} + 480abe^{(6dx+6c)} - 160b^2e^{(6dx+6c)} + 1725a^2e^{(4dx+4c)} + 480abe^{(4dx+4c)} + 240b^2e^{(4dx+4c)} + 762a^2e^{(2dx+2c)} + 240abe^{(2dx+2c)} + 147a^2}{\left(e^{(2dx+2c)}+1\right)^6}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*(d*x + c)*a^2 - 60*a^2*log(e^(2*d*x + 2*c) + 1) + (147*a^2*e^(12*
d*x + 12*c) + 762*a^2*e^(10*d*x + 10*c) + 240*a*b*e^(10*d*x + 10*c) + 1725*
a^2*e^(8*d*x + 8*c) + 480*a*b*e^(8*d*x + 8*c) + 240*b^2*e^(8*d*x + 8*c) + 2
220*a^2*e^(6*d*x + 6*c) + 480*a*b*e^(6*d*x + 6*c) - 160*b^2*e^(6*d*x + 6*c)
 + 1725*a^2*e^(4*d*x + 4*c) + 480*a*b*e^(4*d*x + 4*c) + 240*b^2*e^(4*d*x +
4*c) + 762*a^2*e^(2*d*x + 2*c) + 240*a*b*e^(2*d*x + 2*c) + 147*a^2)/(e^(2*d
*x + 2*c) + 1)^6)/d
```

Mupad [B]
time = 1.52, size = 349, normalized size = 4.53

$$\frac{4\,(2\,a\,b-9\,b^2)}{d\,(4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1)} - \frac{32\,b^2}{3\,d\,(6\,e^{2c+2dx}+15\,e^{4c+4dx}+20\,e^{6c+6dx}+15\,e^{8c+8dx}+6\,e^{10c+10dx}+e^{12c+12dx}+1)} - \frac{2\,(a^2-6\,a\,b+2\,b^2)}{d\,(2\,e^{2c+2dx}+e^{4c+4dx}+1)} - \frac{2\,(2\,a\,b-a^2)}{d\,(e^{2c+2dx}+1)} - \frac{8\,(6\,a\,b-7\,b^2)}{3\,d\,(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1)} - a^2\,x + \frac{a^2\,\ln(e^{2c}\,e^{2dx}+1)}{d} + \frac{32\,b^2}{d\,(5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (4*(2*a*b - 9*b^2))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c
 + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*b^2)/(3*d*(6*exp(2*c + 2*d*x) + 15
*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c
+ 10*d*x) + exp(12*c + 12*d*x) + 1)) - (2*(a^2 - 6*a*b + 2*b^2))/(d*(2*exp(
2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*(2*a*b - a^2))/(d*(exp(2*c + 2*d
*x) + 1)) - (8*(6*a*b - 7*b^2))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*
x) + exp(6*c + 6*d*x) + 1)) - a^2*x + (a^2*log(exp(2*c)*exp(2*d*x) + 1))/d
+ (32*b^2)/(d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*
x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1))
```

3.114 $\int (a + b\mathbf{sech}^2(c + dx))^2 \tanh^2(c + dx)\, dx$

Optimal. Leaf size=59

$$a^2x - \frac{a^2\tanh(c+dx)}{d} + \frac{b(2a+b)\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^5(c+dx)}{5d}$$

```
[Out] a^2*x-a^2*tanh(d*x+c)/d+1/3*b*(2*a+b)*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5
/d
```

Rubi [A]
time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4226, 1816, 212}

$$-\frac{a^2\tanh(c+dx)}{d} + a^2x + \frac{b(2a+b)\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^2,x]
```

```
[Out] a^2*x - (a^2*Tanh[c + d*x])/d + (b*(2*a + b)*Tanh[c + d*x]^3)/(3*d) - (b^2*
Tanh[c + d*x]^5)/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\tanh^2(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1-x^2\right)\right)^2}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-a^2+b(2a+b)x^2-b^2x^4+\frac{a^2}{1-x^2}\right)\,dx, x, \tanh(c+dx$$

$$= -\frac{a^2\tanh(c+dx)}{d}+\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx}{5d}$$

$$= a^2x-\frac{a^2\tanh(c+dx)}{d}+\frac{b(2a+b)\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c}{5d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(59) = 118.
time = 0.59, size = 281, normalized size = 4.76

$$\frac{\text{sech}(c)\text{sech}^5(c+dx)\left(150a^2dx\cosh(dx)+150a^2dx\cosh(2c+dx)+75a^2dx\cosh(2c+3dx)+75a^2dx\cosh(4c+3dx)+15a^2dx\cosh(4c+5dx)+15a^2dx\cosh(6c+5dx)-180a^2\sinh(dx)+80ab\sinh(dx)-20b^2\sinh(dx)+120a^2\sinh(2c+dx)-120ab\sinh(2c+dx)-60b^2\sinh(2c+dx)-120a^2\sinh(2c+3dx)+40ab\sinh(2c+3dx)+20b^2\sinh(2c+3dx)+30a^2\sinh(4c+3dx)-60ab\sinh(4c+3dx)-30a^2\sinh(4c+5dx)+20ab\sinh(4c+5dx)+4b^2\sinh(4c+5dx)\right)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x]^2,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^5*(150*a^2*d*x*Cosh[d*x] + 150*a^2*d*x*Cosh[2*c + d*
x] + 75*a^2*d*x*Cosh[2*c + 3*d*x] + 75*a^2*d*x*Cosh[4*c + 3*d*x] + 15*a^2*d
*x*Cosh[4*c + 5*d*x] + 15*a^2*d*x*Cosh[6*c + 5*d*x] - 180*a^2*Sinh[d*x] + 8
0*a*b*Sinh[d*x] - 20*b^2*Sinh[d*x] + 120*a^2*Sinh[2*c + d*x] - 120*a*b*Sinh
[2*c + d*x] - 60*b^2*Sinh[2*c + d*x] - 120*a^2*Sinh[2*c + 3*d*x] + 40*a*b*S
inh[2*c + 3*d*x] + 20*b^2*Sinh[2*c + 3*d*x] + 30*a^2*Sinh[4*c + 3*d*x] - 60
*a*b*Sinh[4*c + 3*d*x] - 30*a^2*Sinh[4*c + 5*d*x] + 20*a*b*Sinh[4*c + 5*d*x
] + 4*b^2*Sinh[4*c + 5*d*x]))/(480*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(55) = 110.
time = 2.23, size = 190, normalized size = 3.22

method	result
risch	$a^2x+\frac{2a^2e^{8dx+8c}-4ab\,e^{8dx+8c}+8a^2e^{6dx+6c}-8ab\,e^{6dx+6c}-4b^2e^{6dx+6c}+12a^2e^{4dx+4c}-\frac{16ab\,e^{4dx+4c}}{3}+\frac{4b^2e^{4dx+4c}}{3}+8a^2e^{2dx+2c}-8a}{d\left(1+e^{2dx+2c}\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+2/15*(15*a^2*exp(8*d*x+8*c)-30*a*b*exp(8*d*x+8*c)+60*a^2*exp(6*d*x+6*
c)-60*a*b*exp(6*d*x+6*c)-30*b^2*exp(6*d*x+6*c)+90*a^2*exp(4*d*x+4*c)-40*a*b
```

```
*exp(4*d*x+4*c)+10*b^2*exp(4*d*x+4*c)+60*a^2*exp(2*d*x+2*c)-20*a*b*exp(2*d*
x+2*c)-10*b^2*exp(2*d*x+2*c)+15*a^2-10*a*b-2*b^2)/d/(1+exp(2*d*x+2*c))^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(55) = 110$.
time = 0.28, size = 325, normalized size = 5.51

$$\frac{2\,ab\tanh\left(dx+c\right)^{3}}{3\,d}+a^{2}\left(x+\frac{c}{d}-\frac{2}{d\left(e^{\left(-2\,dx-2\,c\right)}+1\right)}\right)+\frac{4}{15}\,b^{2}\left(\frac{5\,e^{\left(-2\,dx-2\,c\right)}}{d\left(5\,e^{\left(-2\,dx-2\,c\right)}+10\,e^{\left(-4\,dx-4\,c\right)}+10\,e^{\left(-6\,dx-6\,c\right)}+5\,e^{\left(-8\,dx-8\,c\right)}+e^{\left(-10\,dx-10\,c\right)}+1\right)}-\frac{5\,e^{\left(-4\,dx-4\,c\right)}}{d\left(5\,e^{\left(-2\,dx-2\,c\right)}+10\,e^{\left(-4\,dx-4\,c\right)}+10\,e^{\left(-6\,dx-6\,c\right)}+5\,e^{\left(-8\,dx-8\,c\right)}+e^{\left(-10\,dx-10\,c\right)}+1\right)}+\frac{15\,e^{\left(-6\,dx-6\,c\right)}}{d\left(5\,e^{\left(-2\,dx-2\,c\right)}+10\,e^{\left(-4\,dx-4\,c\right)}+10\,e^{\left(-6\,dx-6\,c\right)}+5\,e^{\left(-8\,dx-8\,c\right)}+e^{\left(-10\,dx-10\,c\right)}+1\right)}+\frac{1}{d\left(5\,e^{\left(-2\,dx-2\,c\right)}+10\,e^{\left(-4\,dx-4\,c\right)}+10\,e^{\left(-6\,dx-6\,c\right)}+5\,e^{\left(-8\,dx-8\,c\right)}+e^{\left(-10\,dx-10\,c\right)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 2/3*a*b*tanh(d*x + c)^3/d + a^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) +
4/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c
)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^
(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e
^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 1
0*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(55) = 110$.
time = 0.35, size = 435, normalized size = 7.37

$$\frac{\left(15\,a^{2}dx+15\,a^{2}-10\,ab-2\,b^{2}\right)\cosh\left(dx+c\right)^{5}+5\left(15\,a^{2}dx+15\,a^{2}-10\,ab-2\,b^{2}\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}-\left(15\,a^{2}-10\,ab-2\,b^{2}\right)\sinh\left(dx+c\right)^{5}+5\left(15\,a^{2}dx+15\,a^{2}-10\,ab-2\,b^{2}\right)\cosh\left(dx+c\right)^{3}-5\left(2\left(15\,a^{2}-10\,ab-2\,b^{2}\right)\cosh\left(dx+c\right)^{2}+9\,a^{2}+2\,ab-2\,b^{2}\right)\sinh\left(dx+c\right)^{3}+5\left(2\left(15\,a^{2}dx+15\,a^{2}-10\,ab-2\,b^{2}\right)\cosh\left(dx+c\right)^{3}+3\left(15\,a^{2}dx+15\,a^{2}-10\,ab-2\,b^{2}\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+10\left(15\,a^{2}dx+15\,a^{2}-10\,ab-2\,b^{2}\right)\cosh\left(dx+c\right)-5\left(\left(15\,a^{2}-10\,ab-2\,b^{2}\right)\cosh\left(dx+c\right)^{4}+3\left(9\,a^{2}+2\,ab-2\,b^{2}\right)\cosh\left(dx+c\right)^{2}+6\,a^{2}+4\,ab+8\,b^{2}\right)\sinh\left(dx+c\right)}{15\left(d\cosh\left(dx+c\right)^{5}+5\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}+5\,d\cosh\left(dx+c\right)^{3}+5\left(2\,d\cosh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+10\,d\cosh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/15*((15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^5 + 5*(15*a^2*d*
x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^2 - 10*a
*b - 2*b^2)*sinh(d*x + c)^5 + 5*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh
(d*x + c)^3 - 5*(2*(15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^2 + 9*a^2 + 2*a*
b - 2*b^2)*sinh(d*x + c)^3 + 5*(2*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*co
sh(d*x + c)^3 + 3*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c))*sin
h(d*x + c)^2 + 10*(15*a^2*d*x + 15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c) - 5*
((15*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c)^4 + 3*(9*a^2 + 2*a*b - 2*b^2)*cosh
(d*x + c)^2 + 6*a^2 + 4*a*b + 8*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*
d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c
)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{sech}^{2}\left(c+dx\right)\right)^{2}\tanh^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*tanh(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(55) = 110.

time = 0.43, size = 196, normalized size = 3.32

$$\frac{15\,(dx+c)a^2+\frac{2\,\left(15\,a^2e^{(8\,dx+8\,c)}-30\,abe^{(8\,dx+8\,c)}+60\,a^2e^{(6\,dx+6\,c)}-60\,abe^{(6\,dx+6\,c)}-30\,b^2e^{(6\,dx+6\,c)}+90\,a^2e^{(4\,dx+4\,c)}-40\,abe^{(4\,dx+4\,c)}+10\,b^2e^{(4\,dx+4\,c)}+60\,a^2e^{(2\,dx+2\,c)}-20\,abe^{(2\,dx+2\,c)}-10\,b^2e^{(2\,dx+2\,c)}+15\,a^2-10\,ab-2\,b^2\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/15*(15*(d*x + c)*a^2 + 2*(15*a^2*e^(8*d*x + 8*c) - 30*a*b*e^(8*d*x + 8*c)
 + 60*a^2*e^(6*d*x + 6*c) - 60*a*b*e^(6*d*x + 6*c) - 30*b^2*e^(6*d*x + 6*c)
 + 90*a^2*e^(4*d*x + 4*c) - 40*a*b*e^(4*d*x + 4*c) + 10*b^2*e^(4*d*x + 4*c)
 + 60*a^2*e^(2*d*x + 2*c) - 20*a*b*e^(2*d*x + 2*c) - 10*b^2*e^(2*d*x + 2*c)
 + 15*a^2 - 10*a*b - 2*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]

time = 0.14, size = 513, normalized size = 8.69

$$\frac{\frac{8\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(a^2-b^2\right)}{5\,d}-\frac{2\,\left(2\,a\,b-a^2\right)}{5\,d}+\frac{8\,\mathrm{e}^{6\,c+6\,d\,x}\,\left(a^2-b^2\right)}{5\,d}-\frac{2\,\mathrm{e}^{8\,c+8\,d\,x}\,\left(2\,a\,b-a^2\right)}{5\,d}+\frac{4\,\mathrm{e}^{4\,c+4\,d\,x}\,\left(3\,a^2+2\,a\,b+4\,b^2\right)}{5\,d}}{5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1}+\frac{\frac{2\,\left(3\,a^2+2\,a\,b+4\,b^2\right)}{15\,d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(a^2-b^2\right)}{5\,d}-\frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\,\left(2\,a\,b-a^2\right)}{5\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}+\frac{\frac{2\,\left(a^2-b^2\right)}{5\,d}-\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(2\,a\,b-a^2\right)}{5\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}+\frac{\frac{2\,\left(a^2-b^2\right)}{5\,d}+\frac{6\,\mathrm{e}^{4\,c+4\,d\,x}\,\left(a^2-b^2\right)}{5\,d}-\frac{2\,\mathrm{e}^{6\,c+6\,d\,x}\,\left(2\,a\,b-a^2\right)}{5\,d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(3\,a^2+2\,a\,b+4\,b^2\right)}{5\,d}}{4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1}+a^2\,x-\frac{2\,\left(2\,a\,b-a^2\right)}{5\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] ((8*exp(2*c + 2*d*x)*(a^2 - b^2))/(5*d) - (2*(2*a*b - a^2))/(5*d) + (8*exp(
6*c + 6*d*x)*(a^2 - b^2))/(5*d) - (2*exp(8*c + 8*d*x)*(2*a*b - a^2))/(5*d)
+ (4*exp(4*c + 4*d*x)*(2*a*b + 3*a^2 + 4*b^2))/(5*d))/(5*exp(2*c + 2*d*x) +
 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c
+ 10*d*x) + 1) + ((2*(2*a*b + 3*a^2 + 4*b^2))/(15*d) + (4*exp(2*c + 2*d*x)*
(a^2 - b^2))/(5*d) - (2*exp(4*c + 4*d*x)*(2*a*b - a^2))/(5*d))/(3*exp(2*c +
 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((2*(a^2 - b^2))/(5*
d) - (2*exp(2*c + 2*d*x)*(2*a*b - a^2))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*
c + 4*d*x) + 1) + ((2*(a^2 - b^2))/(5*d) + (6*exp(4*c + 4*d*x)*(a^2 - b^2))
/(5*d) - (2*exp(6*c + 6*d*x)*(2*a*b - a^2))/(5*d) + (2*exp(2*c + 2*d*x)*(2*
a*b + 3*a^2 + 4*b^2))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*e
xp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + a^2*x - (2*(2*a*b - a^2))/(5*d*(e
xp(2*c + 2*d*x) + 1))
```

3.115 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^2 \tanh(c+dx)\,dx$

Optimal. Leaf size=48

$$\frac{a^2\log(\cosh(c+dx))}{d}-\frac{ab\,\mathrm{sech}^2(c+dx)}{d}-\frac{b^2\mathrm{sech}^4(c+dx)}{4d}$$

```
[Out] a^2*ln(cosh(d*x+c))/d-a*b*sech(d*x+c)^2/d-1/4*b^2*sech(d*x+c)^4/d
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 272, 45}

$$\frac{a^2\log(\cosh(c+dx))}{d}-\frac{ab\,\mathrm{sech}^2(c+dx)}{d}-\frac{b^2\mathrm{sech}^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x],x]
```

```
[Out] (a^2*Log[Cosh[c + d*x]])/d - (a*b*Sech[c + d*x]^2)/d - (b^2*Sech[c + d*x]^4
)/(4*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^2\tanh(c+dx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{(b+ax^2)^2}{x^5}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{(b+ax)^2}{x^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{b^2}{x^3}+\frac{2ab}{x^2}+\frac{a^2}{x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{a^2\log(\cosh(c+dx))}{d} - \frac{ab\operatorname{sech}^2(c+dx)}{d} - \frac{b^2\operatorname{sech}^4(c+dx)}{4d}$$

Mathematica [A]
time = 0.09, size = 81, normalized size = 1.69

$$\frac{\left(b+a\cosh^2(c+dx)\right)^2\left(-b^2-4ab\cosh^2(c+dx)+4a^2\cosh^4(c+dx)\log(\cosh(c+dx))\right)\operatorname{sech}^4(c+dx)}{d(a+2b+a\cosh(2(c+dx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2*Tanh[c + d*x],x]
[Out] ((b + a*Cosh[c + d*x]^2)^2*(-b^2 - 4*a*b*Cosh[c + d*x]^2 + 4*a^2*Cosh[c + d
*x]^4*Log[Cosh[c + d*x]])*Sech[c + d*x]^4)/(d*(a + 2*b + a*Cosh[2*(c + d*x)
])^2)
```

Maple [A]
time = 0.67, size = 42, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{\frac{\operatorname{sech}(dx+c)^4 b^2}{4}+\operatorname{sech}(dx+c)^2 ab+a^2\ln(\operatorname{sech}(dx+c))}{d}$	42
default	$-\frac{\frac{\operatorname{sech}(dx+c)^4 b^2}{4}+\operatorname{sech}(dx+c)^2 ab+a^2\ln(\operatorname{sech}(dx+c))}{d}$	42
risch	$-a^2x-\frac{2a^2c}{d}-\frac{4b\,e^{2dx+2c}\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+b\,e^{2dx+2c}+a\right)}{d\left(1+e^{2dx+2c}\right)^4}+\frac{a^2\ln\left(1+e^{2dx+2c}\right)}{d}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x,method=_RETURNVERBOSE)
[Out] -1/d*(1/4*sech(d*x+c)^4*b^2+sech(d*x+c)^2*a*b+a^2*ln(sech(d*x+c)))
```

Maxima [A]
time = 0.27, size = 55, normalized size = 1.15

$$\frac{ab\tanh\left(dx+c\right)^2}{d}+\frac{a^2\log\left(\cosh\left(dx+c\right)\right)}{d}-\frac{4\,b^2}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*b*tanh(d*x + c)^2/d + a^2*log(cosh(d*x + c))/d - 4*b^2/(d*(e^(d*x + c) +
e^(-d*x - c))^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1180 vs. $2(46) = 92$.
time = 0.37, size = 1180, normalized size = 24.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*cosh(d*x + c)^8 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*d
*x*sinh(d*x + c)^8 + 4*(a^2*d*x + a*b)*cosh(d*x + c)^6 + 4*(7*a^2*d*x*cosh(
d*x + c)^2 + a^2*d*x + a*b)*sinh(d*x + c)^6 + 8*(7*a^2*d*x*cosh(d*x + c)^3
+ 3*(a^2*d*x + a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2*d*x + 4*a*b +
 2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*d*x*cosh(d*x + c)^4 + 3*a^2*d*x + 30*(a
^2*d*x + a*b)*cosh(d*x + c)^2 + 4*a*b + 2*b^2)*sinh(d*x + c)^4 + a^2*d*x +
8*(7*a^2*d*x*cosh(d*x + c)^5 + 10*(a^2*d*x + a*b)*cosh(d*x + c)^3 + (3*a^2*
d*x + 4*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2*d*x + a*b)*cos
h(d*x + c)^2 + 4*(7*a^2*d*x*cosh(d*x + c)^6 + 15*(a^2*d*x + a*b)*cosh(d*x +
 c)^4 + a^2*d*x + 3*(3*a^2*d*x + 4*a*b + 2*b^2)*cosh(d*x + c)^2 + a*b)*sinh
(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 +
a^2*sinh(d*x + c)^8 + 4*a^2*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^
2)*sinh(d*x + c)^6 + 6*a^2*cosh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^3 + 3*a
^2*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cosh(d*x + c)^4 + 30*a^2*cosh
(d*x + c)^2 + 3*a^2)*sinh(d*x + c)^4 + 4*a^2*cosh(d*x + c)^2 + 8*(7*a^2*cos
h(d*x + c)^5 + 10*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*(7*a^2*cosh(d*x + c)^6 + 15*a^2*cosh(d*x + c)^4 + 9*a^2*cosh(d*x + c)
^2 + a^2)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 + 3*a^2*cosh(d*x +
 c)^5 + 3*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*cos
h(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(a^2*d*x*cosh(d*x + c)^7 +
3*(a^2*d*x + a*b)*cosh(d*x + c)^5 + (3*a^2*d*x + 4*a*b + 2*b^2)*cosh(d*x +
c)^3 + (a^2*d*x + a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8
*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6
+ 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*
d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x +
 c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^
5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*
x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c
)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d
*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A]
time = 0.42, size = 63, normalized size = 1.31

$$\begin{cases} a^2x - \frac{a^2 \log(\tanh(c+dx)+1)}{d} - \frac{ab \operatorname{sech}^2(c+dx)}{d} - \frac{b^2 \operatorname{sech}^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x\left(a + b \operatorname{sech}^2(c)\right)^2 \tanh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2*tanh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a*b*sech(c + d*x)**2/d
- b**2*sech(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**2*tanh(c),
 True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(46) = 92.
time = 0.41, size = 162, normalized size = 3.38

$$-\frac{12(dx+c)a^2 - 12a^2\log\left(e^{(2dx+2c)}+1\right) + \frac{25a^2e^{(8dx+8c)}+100a^2e^{(6dx+6c)}+48abe^{(6dx+6c)}+150a^2e^{(4dx+4c)}+96abe^{(4dx+4c)}+48b^2e^{(4dx+4c)}+100a^2e^{(2dx+2c)}+48abe^{(2dx+2c)}+25a^2}{\left(e^{(2dx+2c)}+1\right)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] -1/12*(12*(d*x + c)*a^2 - 12*a^2*log(e^(2*d*x + 2*c) + 1) + (25*a^2*e^(8*d*
x + 8*c) + 100*a^2*e^(6*d*x + 6*c) + 48*a*b*e^(6*d*x + 6*c) + 150*a^2*e^(4*
d*x + 4*c) + 96*a*b*e^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) + 100*a^2*e^(2
*d*x + 2*c) + 48*a*b*e^(2*d*x + 2*c) + 25*a^2)/(e^(2*d*x + 2*c) + 1)^4)/d
```

Mupad [B]
time = 1.69, size = 182, normalized size = 3.79

$$\frac{4(ab-b^2)}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)} - a^2x + \frac{8b^2}{d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)} - \frac{4b^2}{d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)} + \frac{a^2\ln\left(e^{2c}e^{2dx}+1\right)}{d} - \frac{4ab}{d\left(e^{2c+2dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (4*(a*b - b^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - a^2*x + (
8*b^2)/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
 - (4*b^2)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x)
 + exp(8*c + 8*d*x) + 1)) + (a^2*log(exp(2*c)*exp(2*d*x) + 1))/d - (4*a*b)/
(d*(exp(2*c + 2*d*x) + 1))
```

3.116 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=40

$$a^2x+\frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

```
[Out] a^2*x+b*(2*a+b)*tanh(d*x+c)/d-1/3*b^2*tanh(d*x+c)^3/d
```

Rubi [A]

time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4213, 398, 212}

$$a^2x+\frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] a^2*x + (b*(2*a + b)*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^2}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b(2a+b)-b^2x^2+\frac{a^2}{1-x^2}\right)\,dx,x,\tanh(c+dx)\right)}{d}$$
$$= \frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}+\frac{a^2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\tanh(c+\right.}{d}$$
$$= a^2x+\frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 106 vs. 2(40) = 80.
time = 0.26, size = 106, normalized size = 2.65

$$\frac{4\left(b+a\cosh^2(c+dx)\right)^2\text{sech}^3(c+dx)\left(3a^2dx\cosh^3(c+dx)+b^2\text{sech}(c)\sinh(dx)+2b(3a+b)\cosh^2(c+dx)\text{sech}(c)\sinh(dx)+b^2\cosh(c+dx)\tanh(c)\right)}{3d(a+2b+a\cosh(2(c+dx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]^3*(3*a^2*d*x*Cosh[c + d*x]^3 + b
^2*Sech[c]*Sinh[d*x] + 2*b*(3*a + b)*Cosh[c + d*x]^2*Sech[c]*Sinh[d*x] + b^
2*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

Maple [A]
time = 1.53, size = 67, normalized size = 1.68

method	result	size
risch	$a^2x-\frac{4b\left(3a\,e^{4dx+4c}+6a\,e^{2dx+2c}+3b\,e^{2dx+2c}+3a+b\right)}{3d\left(1+e^{2dx+2c}\right)^3}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x-4/3*b*(3*a*exp(4*d*x+4*c)+6*a*exp(2*d*x+2*c)+3*b*exp(2*d*x+2*c)+3*a+b
)/d/(1+exp(2*d*x+2*c))^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(38) = 76.
time = 0.26, size = 120, normalized size = 3.00

$$a^2x+\frac{4}{3}b^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)+\frac{4\,ab}{d(e^{(-2\,dx-2\,c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 4/3*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*
c) + e^(-6*d*x - 6*c) + 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(38) = 76$.
time = 0.38, size = 176, normalized size = 4.40

$$\frac{(3\,a^2dx-6\,ab-2\,b^2)\cosh\left(dx+c\right)^3+3\,(3\,a^2dx-6\,ab-2\,b^2)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+2\,(3\,ab+b^2)\sinh\left(dx+c\right)^3+3\,(3\,a^2dx-6\,ab-2\,b^2)\cosh\left(dx+c\right)+6\,((3\,ab+b^2)\cosh\left(dx+c\right)^2+ab+b^2)\sinh\left(dx+c\right)}{3\,(d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+3\,d\cosh\left(dx+c\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*a^2*d*x - 6*a*b - 2*b^2)*cosh(d*x + c)^3 + 3*(3*a^2*d*x - 6*a*b - 2
*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(3*a*b + b^2)*sinh(d*x + c)^3 + 3*(
3*a^2*d*x - 6*a*b - 2*b^2)*cosh(d*x + c) + 6*((3*a*b + b^2)*cosh(d*x + c)^2
 + a*b + b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*
x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(38) = 76$.
time = 0.40, size = 79, normalized size = 1.98

$$\frac{3\,(dx+c)a^2-\frac{4\,\left(3\,abe^{(4\,dx+4\,c)}+6\,abe^{(2\,dx+2\,c)}+3\,b^2e^{(2\,dx+2\,c)}+3\,ab+b^2\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^2 - 4*(3*a*b*e^(4*d*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) + 3
*b^2*e^(2*d*x + 2*c) + 3*a*b + b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B]
time = 1.46, size = 163, normalized size = 4.08

$$a^2\,x-\frac{\frac{4\,\left(b^2+a\,b\right)}{3\,d}+\frac{4\,a\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{3\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(b^2+a\,b\right)}{3\,d}+\frac{4\,a\,b}{3\,d}+\frac{4\,a\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}-\frac{4\,a\,b}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] a^2*x - ((4*(a*b + b^2))/(3*d) + (4*a*b*exp(2*c + 2*d*x))/(3*d))/(2*exp(2*c
 + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((8*exp(2*c + 2*d*x)*(a*b + b^2))/(3*d)
 + (4*a*b)/(3*d) + (4*a*b*exp(4*c + 4*d*x))/(3*d))/(3*exp(2*c + 2*d*x) + 3*
exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (4*a*b)/(3*d*(exp(2*c + 2*d*x) +
 1))
```

3.117 $\int \coth(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=53

$$-\frac{b(2a+b)\log(\cosh(c+dx))}{d}+\frac{(a+b)^2\log(\sinh(c+dx))}{d}+\frac{b^2\mathrm{sech}^2(c+dx)}{2d}$$

```
[Out] -b*(2*a+b)*ln(cosh(d*x+c))/d+(a+b)^2*ln(sinh(d*x+c))/d+1/2*b^2*sech(d*x+c)^
2/d
```

Rubi [A]
time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4223, 457, 90}

$$\frac{(a+b)^2\log(\sinh(c+dx))}{d}-\frac{b(2a+b)\log(\cosh(c+dx))}{d}+\frac{b^2\mathrm{sech}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((b*(2*a + b)*Log[Cosh[c + d*x]])/d) + ((a + b)^2*Log[Sinh[c + d*x]])/d +
(b^2*Sech[c + d*x]^2)/(2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx = -\frac{\operatorname{Subst}\left(\int \frac{(b+ax^2)^2}{x^3(1-x^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{(b+ax)^2}{(1-x)x^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x}+\frac{b^2}{x^2}+\frac{b(2a+b)}{x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{b(2a+b)\log(\cosh(c+dx))}{d}+\frac{(a+b)^2\log(\sinh(c+dx))}{d}+\frac{b^2\text{s}}{}$$

Mathematica [A]
time = 0.17, size = 84, normalized size = 1.58

$$\frac{2\left(b^2+2\cosh^2(c+dx)\left(-b(2a+b)\log(\cosh(c+dx))+(a+b)^2\log(\sinh(c+dx))\right)\right)(a\cosh(c+dx)+b\operatorname{sech}(c+dx))^2}{d(a+2b+a\cosh(2(c+dx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (2*(b^2 + 2*Cosh[c + d*x]^2*(-(b*(2*a + b)*Log[Cosh[c + d*x]]) + (a + b)^2*
Log[Sinh[c + d*x]]))*(a*Cosh[c + d*x] + b*Sech[c + d*x])^2)/(d*(a + 2*b + a
*Cosh[2*(c + d*x)])^2)
```

Maple [A]
time = 1.58, size = 50, normalized size = 0.94

method	result
derivativedivides	$\frac{a^2\ln(\sinh(dx+c))+2ab\ln(\tanh(dx+c))+b^2\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)}{d}$
default	$\frac{a^2\ln(\sinh(dx+c))+2ab\ln(\tanh(dx+c))+b^2\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)}{d}$
risch	$-a^2x-\frac{2a^2c}{d}+\frac{2b^2e^{2dx+2c}}{d(1+e^{2dx+2c})^2}-\frac{2b\ln(1+e^{2dx+2c})a}{d}-\frac{b^2\ln(1+e^{2dx+2c})}{d}+\frac{\ln(e^{2dx+2c}-1)a^2}{d}+\frac{2\ln(e^{2dx+2c}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*ln(sinh(d*x+c))+2*a*b*ln(tanh(d*x+c))+b^2*(1/2/cosh(d*x+c)^2+ln(ta
nh(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(51) = 102.

time = 0.47, size = 161, normalized size = 3.04

$$b^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d\left(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1\right)}\right)+2ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}\right)+\frac{a^2\log\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] b^2*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*
c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) +
1))) + 2*a*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2
*d*x - 2*c) + 1)/d) + a^2*log(sinh(d*x + c))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(51) = 102.
time = 0.41, size = 665, normalized size = 12.55

$$-\frac{a^2dx\cosh\left(dx+c\right)^4+4a^2dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+a^2dx\sinh\left(dx+c\right)^4+a^2dx+2\left(a^2dx-b^2\right)\cosh\left(dx+c\right)^2+2\left(3a^2dx\cosh\left(dx+c\right)^2+a^2dx-b^2\right)\sinh\left(dx+c\right)^2+\left(\left(2ab+b^2\right)\cosh\left(dx+c\right)^4+4\left(2ab+b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+\left(2ab+b^2\right)\sinh\left(dx+c\right)^4+2\left(2ab+b^2\right)\cosh\left(dx+c\right)^2+2\left(3\left(2ab+b^2\right)\cosh\left(dx+c\right)^2+2ab+b^2\right)\sinh\left(dx+c\right)^2+2ab+b^2+4\left(\left(2ab+b^2\right)\cosh\left(dx+c\right)^3+\left(2ab+b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)\log\left(\frac{2\cosh\left(dx+c\right)}{\cosh\left(dx+c\right)-\sinh\left(dx+c\right)}\right)-\left(\left(a^2+2ab+b^2\right)\cosh\left(dx+c\right)^4+4\left(a^2+2ab+b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+\left(a^2+2ab+b^2\right)\sinh\left(dx+c\right)^4+2\left(a^2+2ab+b^2\right)\cosh\left(dx+c\right)^2+2\left(3\left(a^2+2ab+b^2\right)\cosh\left(dx+c\right)^2+a^2+2ab+b^2\right)\sinh\left(dx+c\right)^2+a^2+2ab+b^2+4\left(\left(a^2+2ab+b^2\right)\cosh\left(dx+c\right)^3+\left(a^2+2ab+b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)\log\left(\frac{2\sinh\left(dx+c\right)}{\cosh\left(dx+c\right)-\sinh\left(dx+c\right)}\right)+4\left(a^2dx\cosh\left(dx+c\right)^3+\left(a^2dx-b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{d\cosh\left(dx+c\right)^4+4d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+d\sinh\left(dx+c\right)^4+2d\cosh\left(dx+c\right)^2+2\left(3d\cosh\left(dx+c\right)^2+d\right)\sinh\left(dx+c\right)^2+4\left(d\cosh\left(dx+c\right)^3+d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*cosh(d*x + c)^4 + 4*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d
*x*sinh(d*x + c)^4 + a^2*d*x + 2*(a^2*d*x - b^2)*cosh(d*x + c)^2 + 2*(3*a^2
*d*x*cosh(d*x + c)^2 + a^2*d*x - b^2)*sinh(d*x + c)^2 + ((2*a*b + b^2)*cosh
(d*x + c)^4 + 4*(2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)
*sinh(d*x + c)^4 + 2*(2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cos
h(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2
)*cosh(d*x + c)^3 + (2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(
d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b
+ b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 +
 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)
*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*
x + c))) + 4*(a^2*d*x*cosh(d*x + c)^3 + (a^2*d*x - b^2)*cosh(d*x + c))*sinh
(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(
d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c
)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)^2\coth\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*coth(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(51) = 102$.

time = 0.42, size = 150, normalized size = 2.83

$$-\frac{(2\,ab+b^2)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)-(a^2+2\,ab+b^2)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)-\frac{2\,ab\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b^2\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+4\,ab+6\,b^2}{e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((2*a*b + b^2)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - (a^2 + 2*
a*b + b^2)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (2*a*b*(e^(2*d*x +
 2*c) + e^(-2*d*x - 2*c)) + b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 4*a*
b + 6*b^2)/(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2))/d
```

Mupad [B]

time = 0.30, size = 308, normalized size = 5.81

$$\frac{2\,b^2}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-a^2\,x+\frac{a^2\,\ln\left({\mathrm{e}}^{4\,c+4\,d\,x}-1\right)}{2\,d}-\frac{2\,b^2}{d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}-\frac{\mathrm{atan}\left(\frac{{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\,\left(a^4\,\sqrt{-d^2}+4\,b^4\,\sqrt{-d^2}+16\,a\,b^3\,\sqrt{-d^2}+8\,a^3\,b\,\sqrt{-d^2}+20\,a^2\,b^2\,\sqrt{-d^2}\right)}{a^2\,d\,\sqrt{a^4+8\,a^3\,b+20\,a^2\,b^2+16\,a\,b^3+4\,b^4}+2\,b^2\,d\,\sqrt{a^4+8\,a^3\,b+20\,a^2\,b^2+16\,a\,b^3+4\,b^4}+4\,a\,b\,d\,\sqrt{a^4+8\,a^3\,b+20\,a^2\,b^2+16\,a\,b^3+4\,b^4}}\right)\,\sqrt{a^4+8\,a^3\,b+20\,a^2\,b^2+16\,a\,b^3+4\,b^4}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) - a^2*x + (a^2*log(exp(4*c + 4*d*x) - 1)
)/(2*d) - (2*b^2)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (atan((
exp(2*c)*exp(2*d*x)*(a^4*(-d^2)^(1/2) + 4*b^4*(-d^2)^(1/2) + 16*a*b^3*(-d^2
)^(1/2) + 8*a^3*b*(-d^2)^(1/2) + 20*a^2*b^2*(-d^2)^(1/2)))/(a^2*d*(16*a*b^3
 + 8*a^3*b + a^4 + 4*b^4 + 20*a^2*b^2)^(1/2) + 2*b^2*d*(16*a*b^3 + 8*a^3*b
+ a^4 + 4*b^4 + 20*a^2*b^2)^(1/2) + 4*a*b*d*(16*a*b^3 + 8*a^3*b + a^4 + 4*b
^4 + 20*a^2*b^2)^(1/2)))*(16*a*b^3 + 8*a^3*b + a^4 + 4*b^4 + 20*a^2*b^2)^(1
/2))/(-d^2)^(1/2)
```

3.118 $\int \coth^2(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2 dx$

Optimal. Leaf size=36

$$a^2x-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

```
[Out] a^2*x-(a+b)^2*coth(d*x+c)/d-b^2*tanh(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {4226, 1816, 213}

$$a^2x-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] a^2*x - ((a + b)^2*Coth[c + d*x])/d - (b^2*Tanh[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && IntegerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+b(1-x^2))^2}{x^2(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b^2+\frac{(a+b)^2}{x^2}-\frac{a^2}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{(a+b)^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}-\frac{a^2\text{Subst}\left(\int \frac{1}{-1+x^2}\,c\right.}{}$$
$$= a^2x-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 82 vs. 2(36) = 72.
time = 0.52, size = 82, normalized size = 2.28

$$\frac{4\left(b+a\cosh^2(c+dx)\right)^2\text{sech}(c+dx)\left(a^2dx\cosh(c+dx)+\left((a+b)^2\coth(c+dx)\text{csch}(c)-b^2\text{sech}(c)\right)\sinh(dx)\right)}{d(a+2b+a\cosh(2(c+dx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cosh[c + d*x]^2)^2*Sech[c + d*x]*(a^2*d*x*Cosh[c + d*x] + ((a + b
)^2*Coth[c + d*x]*Csch[c] - b^2*Sech[c])*Sinh[d*x]))/(d*(a + 2*b + a*Cosh[2
*(c + d*x)])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(36) = 72.
time = 2.38, size = 77, normalized size = 2.14

method	result	size
risch	$a^2x-\frac{2\left(a^2\mathrm{e}^{2dx+2c}+2ab\,\mathrm{e}^{2dx+2c}+a^2+2ab+2b^2\right)}{d\left(\mathrm{e}^{2dx+2c}-1\right)\left(1+\mathrm{e}^{2dx+2c}\right)}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x-2*(a^2*exp(2*d*x+2*c)+2*a*b*exp(2*d*x+2*c)+a^2+2*a*b+2*b^2)/d/(exp(2*
d*x+2*c)-1)/(1+exp(2*d*x+2*c))
```

Maspan [A]
time = 0.28, size = 71, normalized size = 1.97

$$a^2\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)+\frac{4\,ab}{d(e^{(-2\,dx-2\,c)}-1)}+\frac{4\,b^2}{d(e^{(-4\,dx-4\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) -
 1)) + 4*b^2/(d*(e^(-4*d*x - 4*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(36) = 72$.
time = 0.46, size = 106, normalized size = 2.94

$$-\frac{(a^2+2\,ab+2\,b^2)\cosh\left(dx+c\right)^2-2\,(a^2dx+a^2+2\,ab+2\,b^2)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(a^2+2\,ab+2\,b^2)\sinh\left(dx+c\right)^2+a^2+2\,ab}{2\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^2 + 2*a*b + 2*b^2)*cosh(d*x + c)^2 - 2*(a^2*d*x + a^2 + 2*a*b + 2*
b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + 2*b^2)*sinh(d*x + c)^2 +
a^2 + 2*a*b)/(d*cosh(d*x + c)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{sech}^2\left(c+dx\right)\right)^2\coth^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**2*coth(c + d*x)**2, x)
```

Giac [A]
time = 0.41, size = 68, normalized size = 1.89

$$\frac{(dx+c)a^2-\frac{2\,\left(a^2e^{(2\,dx+2\,c)}+2\,abe^{(2\,dx+2\,c)}+a^2+2\,ab+2\,b^2\right)}{e^{(4\,dx+4\,c)}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^2 - 2*(a^2*e^(2*d*x + 2*c) + 2*a*b*e^(2*d*x + 2*c) + a^2 + 2*a
*b + 2*b^2)/(e^(4*d*x + 4*c) - 1))/d
```

Mupad [B]
time = 1.42, size = 60, normalized size = 1.67

$$a^2\,x-\frac{\frac{2\,\left(a^2+2\,a\,b+2\,b^2\right)}{d}+\frac{2\,a\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(a+2\,b\right)}{d}}{\mathrm{e}^{4\,c+4\,d\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] a^2*x - ((2*(2*a*b + a^2 + 2*b^2))/d + (2*a*exp(2*c + 2*d*x)*(a + 2*b))/d)/
(exp(4*c + 4*d*x) - 1)
```

3.119 $\int \coth^3(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=55

$$-\frac{(a+b)^2\mathrm{csch}^2(c+dx)}{2d}+\frac{b^2\log(\cosh(c+dx))}{d}+\frac{(a^2-b^2)\log(\sinh(c+dx))}{d}$$

```
[Out] -1/2*(a+b)^2*csch(d*x+c)^2/d+b^2*ln(cosh(d*x+c))/d+(a^2-b^2)*ln(sinh(d*x+c)
)/d
```

Rubi [A]
time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {4223, 457, 90}

$$\frac{(a^2-b^2)\log(\sinh(c+dx))}{d}-\frac{(a+b)^2\mathrm{csch}^2(c+dx)}{2d}+\frac{b^2\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((a + b)^2*Csch[c + d*x]^2)/d + (b^2*Log[Cosh[c + d*x]])/d + ((a^2 - b
^2)*Log[Sinh[c + d*x]])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x(1-x^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^2x}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{(-1+x)^2} + \frac{a^2-b^2}{-1+x} + \frac{b^2}{x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{(a+b)^2\text{csch}^2(c+dx)}{2d} + \frac{b^2\log(\cosh(c+dx))}{d} + \frac{(a^2-b^2)\log(}{}$$

Mathematica [A]
time = 0.14, size = 82, normalized size = 1.49

$$-\frac{2\left(b+a\cosh^2(c+dx)\right)^2\left((a+b)^2\text{csch}^2(c+dx)-2(b^2\log(\cosh(c+dx))+(a^2-b^2)\log(\sinh(c+dx)))\right)}{d(a+2b+a\cosh(2(c+dx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (-2*(b + a*Cosh[c + d*x]^2)^2*((a + b)^2*Csch[c + d*x]^2 - 2*(b^2*Log[Cosh[
c + d*x]] + (a^2 - b^2)*Log[Sinh[c + d*x]])))/(d*(a + 2*b + a*Cosh[2*(c + d
*x)])^2)
```

Maple [A]
time = 1.71, size = 64, normalized size = 1.16

method	result	size
derivativedivides	$\frac{a^2\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)-\frac{ab}{\sinh(dx+c)^2}+b^2\left(-\frac{1}{2\sinh(dx+c)^2}-\ln(\tanh(dx+c))\right)}{d}$	64
default	$\frac{a^2\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)-\frac{ab}{\sinh(dx+c)^2}+b^2\left(-\frac{1}{2\sinh(dx+c)^2}-\ln(\tanh(dx+c))\right)}{d}$	64
risch	$-a^2x-\frac{2a^2c}{d}-\frac{2\,e^{2dx+2c}(a^2+2ab+b^2)}{d(e^{2dx+2c}-1)^2}+\frac{\ln(e^{2dx+2c}-1)a^2}{d}-\frac{\ln(e^{2dx+2c}-1)b^2}{d}+\frac{b^2\ln(1+e^{2dx+2c})}{d}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2)-a*b/sinh(d*x+c)^2+b^2*(-1/2/si
nh(d*x+c)^2-ln(tanh(d*x+c))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(53) = 106$.
time = 0.48, size = 206, normalized size = 3.75

$$a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)-b^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)-\frac{4\,ab}{d(e^{(dx+c)}-e^{(-dx-c)})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - b^2*(log(e^(-
d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c) + 1)/d - 2
*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 4*a*b/
(d*(e^(d*x + c) - e^(-d*x - c))^2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(53) = 106$.
time = 0.43, size = 637, normalized size = 11.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*cosh(d*x + c)^4 + 4*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d
*x*sinh(d*x + c)^4 + a^2*d*x - 2*(a^2*d*x - a^2 - 2*a*b - b^2)*cosh(d*x + c
)^2 + 2*(3*a^2*d*x*cosh(d*x + c)^2 - a^2*d*x + a^2 + 2*a*b + b^2)*sinh(d*x
+ c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*s
inh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*si
nh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x
+ c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^2 - b^2)*c
osh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^2)*
sinh(d*x + c)^4 - 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(d*x
 + c)^2 - a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d*x
+ c)^3 - (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cos
h(d*x + c) - sinh(d*x + c))) + 4*(a^2*d*x*cosh(d*x + c)^3 - (a^2*d*x - a^2
- 2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(
d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d
*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x +
 c))*sinh(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(53) = 106.
time = 0.45, size = 148, normalized size = 2.69

$$\frac{b^2\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)+(a^2-b^2)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)-\frac{a^2\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)-b^2\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a^2+8\,ab+6\,b^2}{e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + (a^2 - b^2)*log(e^(2
*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (a^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2
*c)) - b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a^2 + 8*a*b + 6*b^2)/(e
^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2))/d
```

Mupad [B]
time = 0.22, size = 240, normalized size = 4.36

$$\frac{\ln\left(\mathrm{e}^{4\,c+4\,d\,x}-1\right)\,\left(d\,\left(a^2-b^2\right)+b^2\,d\right)}{2\,d^2}-a^2\,x-\frac{2\,\left(a^2+2\,a\,b+b^2\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\left(a^4\,\sqrt{-d^2}+4\,b^4\,\sqrt{-d^2}-4\,a^2\,b^2\,\sqrt{-d^2}\right)}{a^2\,d\,\sqrt{a^4-4\,a^2\,b^2+4\,b^4}-2\,b^2\,d\,\sqrt{a^4-4\,a^2\,b^2+4\,b^4}}\right)\,\sqrt{a^4-4\,a^2\,b^2+4\,b^4}}{\sqrt{-d^2}}-\frac{2\,\left(a^2+2\,a\,b+b^2\right)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (log(exp(4*c + 4*d*x) - 1)*(d*(a^2 - b^2) + b^2*d))/(2*d^2) - a^2*x - (2*(2
*a*b + a^2 + b^2))/(d*(exp(2*c + 2*d*x) - 1)) - (atan((exp(2*c)*exp(2*d*x)*
(a^4*(-d^2)^(1/2) + 4*b^4*(-d^2)^(1/2) - 4*a^2*b^2*(-d^2)^(1/2)))/(a^2*d*(a
^4 + 4*b^4 - 4*a^2*b^2)^(1/2) - 2*b^2*d*(a^4 + 4*b^4 - 4*a^2*b^2)^(1/2)))*(
a^4 + 4*b^4 - 4*a^2*b^2)^(1/2))/(-d^2)^(1/2) - (2*(2*a*b + a^2 + b^2))/(d*(
exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.120 $\int \coth^4(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=46

$$a^2x-\frac{(a^2-b^2)\coth(c+dx)}{d}-\frac{(a+b)^2\coth^3(c+dx)}{3d}$$

```
[Out] a^2*x-(a^2-b^2)*coth(d*x+c)/d-1/3*(a+b)^2*coth(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {4226, 1816, 213}

$$-\frac{(a^2-b^2)\coth(c+dx)}{d}+a^2x-\frac{(a+b)^2\coth^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] a^2*x - ((a^2 - b^2)*Coth[c + d*x])/d - ((a + b)^2*Coth[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+b(1-x^2))^2}{x^4(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{x^4}+\frac{a^2-b^2}{x^2}-\frac{a^2}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{(a^2-b^2)\coth(c+dx)}{d}-\frac{(a+b)^2\coth^3(c+dx)}{3d}-\frac{a^2\text{Subst}(\int$$
$$= a^2x-\frac{(a^2-b^2)\coth(c+dx)}{d}-\frac{(a+b)^2\coth^3(c+dx)}{3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(46) = 92.
time = 0.56, size = 160, normalized size = 3.48

$$\frac{\text{csch}(c)\text{csch}^3(c+dx)\left(9a^2dx\cosh(dx)-9a^2dx\cosh(2c+dx)-3a^2dx\cosh(2c+3dx)+3a^2dx\cosh(4c+3dx)-12a^2\sinh(dx)+12b^2\sinh(dx)-12a^2\sinh(2c+dx)-12ab\sinh(2c+dx)+8a^2\sinh(2c+3dx)+4ab\sinh(2c+3dx)-4b^2\sinh(2c+3dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (Csch[c]*Csch[c + d*x]^3*(9*a^2*d*x*Cosh[d*x] - 9*a^2*d*x*Cosh[2*c + d*x] -
 3*a^2*d*x*Cosh[2*c + 3*d*x] + 3*a^2*d*x*Cosh[4*c + 3*d*x] - 12*a^2*Sinh[d*
x] + 12*b^2*Sinh[d*x] - 12*a^2*Sinh[2*c + d*x] - 12*a*b*Sinh[2*c + d*x] + 8
*a^2*Sinh[2*c + 3*d*x] + 4*a*b*Sinh[2*c + 3*d*x] - 4*b^2*Sinh[2*c + 3*d*x])
)/(24*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(44) = 88.
time = 2.16, size = 94, normalized size = 2.04

method	result	size
risch	$a^2x-\frac{4\left(3a^2\mathrm{e}^{4dx+4c}+3ab\,\mathrm{e}^{4dx+4c}-3a^2\mathrm{e}^{2dx+2c}+3b^2\mathrm{e}^{2dx+2c}+2a^2+ab-b^2\right)}{3d\left(\mathrm{e}^{2dx+2c}-1\right)^3}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x-4/3*(3*a^2*exp(4*d*x+4*c)+3*a*b*exp(4*d*x+4*c)-3*a^2*exp(2*d*x+2*c)+3
*b^2*exp(2*d*x+2*c)+2*a^2+a*b-b^2)/d/(exp(2*d*x+2*c)-1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(44) = 88.
time = 0.27, size = 268, normalized size = 5.83

$$\frac{1}{3}a^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)+\frac{4}{3}b^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}-\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)+\frac{4}{3}ab\left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}+\frac{1}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 4/3*b^2
*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x
- 6*c) - 1))) + 4/3*a*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-
4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) + 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(44) = 88$.
time = 0.39, size = 201, normalized size = 4.37

$$-\frac{2\left(2a^2+ab-b^2\right)\cosh\left(dx+c\right)^3+6\left(2a^2+ab-b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2-\left(3a^2dx+4a^2+2ab-2b^2\right)\sinh\left(dx+c\right)^3+6\left(ab+b^2\right)\cosh\left(dx+c\right)+3\left(3a^2dx-\left(3a^2dx+4a^2+2ab-2b^2\right)\cosh\left(dx+c\right)^2+4a^2+2ab-2b^2\right)\sinh\left(dx+c\right)}{3\left(d\sinh\left(dx+c\right)^3+3\left(d\cosh\left(dx+c\right)^2-d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(2*a^2 + a*b - b^2)*cosh(d*x + c)^3 + 6*(2*a^2 + a*b - b^2)*cosh(d*
x + c)*sinh(d*x + c)^2 - (3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c)^
3 + 6*(a*b + b^2)*cosh(d*x + c) + 3*(3*a^2*d*x - (3*a^2*d*x + 4*a^2 + 2*a*b
 - 2*b^2)*cosh(d*x + c)^2 + 4*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c))/(d*sinh(d
*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.
time = 0.45, size = 100, normalized size = 2.17

$$\frac{3\,(dx+c)a^2-\frac{4\left(3\,a^2e^{(4\,dx+4\,c)}+3\,abe^{(4\,dx+4\,c)}-3\,a^2e^{(2\,dx+2\,c)}+3\,b^2e^{(2\,dx+2\,c)}+2\,a^2+ab-b^2\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^2 - 4*(3*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(4*d*x + 4*c) - 3
*a^2*e^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c) + 2*a^2 + a*b - b^2)/(e^(2*d*x
 + 2*c) - 1)^3)/d
```

Mupad [B]
time = 1.42, size = 183, normalized size = 3.98

$$a^2\,x-\frac{\frac{4\,\left(a^2+b\,a\right)}{3\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a^2+b\,a\right)}{3\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(b^2+a\,b\right)}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}-\frac{\frac{4\,\left(b^2+a\,b\right)}{3\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2+b\,a\right)}{3\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{4\,\left(a^2+b\,a\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] a^2*x - ((4*(a*b + a^2))/(3*d) + (4*exp(4*c + 4*d*x)*(a*b + a^2))/(3*d) + (
8*exp(2*c + 2*d*x)*(a*b + b^2))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*
d*x) + exp(6*c + 6*d*x) - 1) - ((4*(a*b + b^2))/(3*d) + (4*exp(2*c + 2*d*x)
*(a*b + a^2))/(3*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - (4*(a*b
+ a^2))/(3*d*(exp(2*c + 2*d*x) - 1))
```

3.121 $\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=52

$$-\frac{a(a+b)\text{csch}^2(c+dx)}{d}-\frac{(a+b)^2\text{csch}^4(c+dx)}{4d}+\frac{a^2\log(\sinh(c+dx))}{d}$$

```
[Out] -a*(a+b)*csch(d*x+c)^2/d-1/4*(a+b)^2*csch(d*x+c)^4/d+a^2*ln(sinh(d*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {4223, 455, 45}

$$\frac{a^2\log(\sinh(c+dx))}{d}-\frac{(a+b)^2\text{csch}^4(c+dx)}{4d}-\frac{a(a+b)\text{csch}^2(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((a*(a + b)*Csch[c + d*x]^2)/d) - ((a + b)^2*Csch[c + d*x]^4)/(4*d) + (a^2
*Log[Sinh[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = -\frac{\text{Subst}\left(\int \frac{x(b+ax^2)^2}{(1-x^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{(-1+x)^3} - \frac{2a(a+b)}{(-1+x)^2} - \frac{a^2}{-1+x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{a(a+b)\text{csch}^2(c+dx)}{d} - \frac{(a+b)^2\text{csch}^4(c+dx)}{4d} + \frac{a^2\log(\sinh(}{d}$$

Mathematica [A]
time = 0.16, size = 77, normalized size = 1.48

$$-\frac{\left(b+a\cosh^2(c+dx)\right)^2\left(4a(a+b)\text{csch}^2(c+dx)+(a+b)^2\text{csch}^4(c+dx)-4a^2\log(\sinh(c+dx))\right)}{d(a+2b+a\cosh(2(c+dx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -(((b + a*Cosh[c + d*x]^2)^2*(4*a*(a + b)*Csch[c + d*x]^2 + (a + b)^2*Csch[
c + d*x]^4 - 4*a^2*Log[Sinh[c + d*x]]))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^
2))
```

Maple [A]
time = 1.78, size = 84, normalized size = 1.62

method	result	size
derivativedivides	$\frac{a^2\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}\right)+2ab\left(-\frac{\cosh^2(dx+c)}{2\sinh(dx+c)^4}+\frac{1}{4\sinh(dx+c)^4}\right)-\frac{b^2}{4\sinh(dx+c)^4}}{d}$	84
default	$\frac{a^2\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}\right)+2ab\left(-\frac{\cosh^2(dx+c)}{2\sinh(dx+c)^4}+\frac{1}{4\sinh(dx+c)^4}\right)-\frac{b^2}{4\sinh(dx+c)^4}}{d}$	84
risch	$-a^2x-\frac{2a^2c}{d}-\frac{4\,e^{2dx+2c}\left(a^2e^{4dx+4c}+ab\,e^{4dx+4c}-a^2e^{2dx+2c}+b^2e^{2dx+2c}+a^2+ab\right)}{d\left(e^{2dx+2c}-1\right)^4}+\frac{\ln\left(e^{2dx+2c}-1\right)a^2}{d}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4)+2*a*b*(-1/2/
sinh(d*x+c)^4*cosh(d*x+c)^2+1/4/sinh(d*x+c)^4)-1/4*b^2/sinh(d*x+c)^4)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(50) = 100$.
time = 0.27, size = 282, normalized size = 5.42

$$a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1)}\right)+4\,ab\left(\frac{e^{(-2\,dx-2\,c)}}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1)}+\frac{e^{(-6\,dx-6\,c)}}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1)}\right)-\frac{4\,b^2}{d(e^{(dx+c)}-e^{(-dx-c)})^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2
*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 4*a*b*
(e^(-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x
 - 6*c) - e^(-8*d*x - 8*c) - 1)) + e^(-6*d*x - 6*c)/(d*(4*e^(-2*d*x - 2*c)
- 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) - 4*b^2
/(d*(e^(d*x + c) - e^(-d*x - c))^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1252 vs. $2(50) = 100$.
time = 0.70, size = 1252, normalized size = 24.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x*cosh(d*x + c)^8 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*d
*x*sinh(d*x + c)^8 - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 4*(7*a^2*d*x
*cosh(d*x + c)^2 - a^2*d*x + a^2 + a*b)*sinh(d*x + c)^6 + 8*(7*a^2*d*x*cosh
(d*x + c)^3 - 3*(a^2*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3
*a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*d*x*cosh(d*x + c)^4 +
 3*a^2*d*x - 30*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^2 - 2*a^2 + 2*b^2)*sinh
(d*x + c)^4 + a^2*d*x + 8*(7*a^2*d*x*cosh(d*x + c)^5 - 10*(a^2*d*x - a^2 -
a*b)*cosh(d*x + c)^3 + (3*a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^3 - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 4*(7*a^2*d*x*cosh(d*x +
c)^6 - 15*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^4 - a^2*d*x + 3*(3*a^2*d*x -
2*a^2 + 2*b^2)*cosh(d*x + c)^2 + a^2 + a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x
 + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 - 4*a^2
*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^6 + 6*a^2*
cosh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^3 - 3*a^2*cosh(d*x + c))*sinh(d*x
+ c)^5 + 2*(35*a^2*cosh(d*x + c)^4 - 30*a^2*cosh(d*x + c)^2 + 3*a^2)*sinh(d
*x + c)^4 - 4*a^2*cosh(d*x + c)^2 + 8*(7*a^2*cosh(d*x + c)^5 - 10*a^2*cosh(
d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^2*cosh(d*x + c)^
6 - 15*a^2*cosh(d*x + c)^4 + 9*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^2 +
 a^2 + 8*(a^2*cosh(d*x + c)^7 - 3*a^2*cosh(d*x + c)^5 + 3*a^2*cosh(d*x + c)
```

```
^3 - a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))) + 8*(a^2*d*x*cosh(d*x + c)^7 - 3*(a^2*d*x - a^2 - a*b)*cos
h(d*x + c)^5 + (3*a^2*d*x - 2*a^2 + 2*b^2)*cosh(d*x + c)^3 - (a^2*d*x - a^2
 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c
)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d
*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c)
)*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*co
sh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7
*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d
*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^
3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(50) = 100.
time = 0.46, size = 150, normalized size = 2.88

$$-\frac{12\,(dx+c)a^2-12\,a^2\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{25\,a^2e^{(8\,dx+8\,c)}-52\,a^2e^{(6\,dx+6\,c)}+48\,abe^{(6\,dx+6\,c)}+102\,a^2e^{(4\,dx+4\,c)}+48\,b^2e^{(4\,dx+4\,c)}-52\,a^2e^{(2\,dx+2\,c)}+48\,abe^{(2\,dx+2\,c)}+25\,a^2}{\left(e^{(2\,dx+2\,c)}-1\right)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/12*(12*(d*x + c)*a^2 - 12*a^2*log(abs(e^(2*d*x + 2*c) - 1)) + (25*a^2*e^
(8*d*x + 8*c) - 52*a^2*e^(6*d*x + 6*c) + 48*a*b*e^(6*d*x + 6*c) + 102*a^2*e
^(4*d*x + 4*c) + 48*b^2*e^(4*d*x + 4*c) - 52*a^2*e^(2*d*x + 2*c) + 48*a*b*e
^(2*d*x + 2*c) + 25*a^2)/(e^(2*d*x + 2*c) - 1)^4)/d
```

Mupad [**B**]
time = 1.47, size = 207, normalized size = 3.98

$$\frac{a^2\,\ln\left({\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}-1\right)}{d}-\frac{4\,\left(a^2+2\,a\,b+b^2\right)}{d\,\left(6\,{\mathrm{e}}^{4\,c+4\,d\,x}-4\,{\mathrm{e}}^{2\,c+2\,d\,x}-4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1\right)}-\frac{4\,\left(2\,a^2+3\,a\,b+b^2\right)}{d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-a^2\,x-\frac{4\,\left(a^2+b\,a\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{8\,\left(a^2+2\,a\,b+b^2\right)}{d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^5*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (a^2*log(exp(2*c)*exp(2*d*x) - 1))/d - (4*(2*a*b + a^2 + b^2))/(d*(6*exp(4*
c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1
)) - (4*(3*a*b + 2*a^2 + b^2))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) +
1)) - a^2*x - (4*(a*b + a^2))/(d*(exp(2*c + 2*d*x) - 1)) - (8*(2*a*b + a^2
+ b^2))/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)
)
```

3.122 $\int \coth^6(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=64

$$a^2x-\frac{a^2\coth(c+dx)}{d}-\frac{(a^2-b^2)\coth^3(c+dx)}{3d}-\frac{(a+b)^2\coth^5(c+dx)}{5d}$$

```
[Out] a^2*x-a^2*coth(d*x+c)/d-1/3*(a^2-b^2)*coth(d*x+c)^3/d-1/5*(a+b)^2*coth(d*x+
c)^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {4226, 1816, 213}

$$-\frac{(a^2-b^2)\coth^3(c+dx)}{3d}-\frac{a^2\coth(c+dx)}{d}+a^2x-\frac{(a+b)^2\coth^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] a^2*x - (a^2*Coth[c + d*x])/d - ((a^2 - b^2)*Coth[c + d*x]^3)/(3*d) - ((a +
 b)^2*Coth[c + d*x]^5)/(5*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \coth^6(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2\,dx &= \frac{\mathrm{Subst}\left(\int \frac{\left(a+b(1-x^2)\right)^2}{x^6(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\mathrm{Subst}\left(\int \left(\frac{(a+b)^2}{x^6}+\frac{a^2-b^2}{x^4}+\frac{a^2}{x^2}-\frac{a^2}{-1+x^2}\right)\,dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{a^2\coth(c+dx)}{d}-\frac{(a^2-b^2)\coth^3(c+dx)}{3d}-\frac{(a+b)^2\coth^5(}{5d} \\
&= a^2x-\frac{a^2\coth(c+dx)}{d}-\frac{(a^2-b^2)\coth^3(c+dx)}{3d}-\frac{(a+b)^2\mathrm{co}}{}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 256 vs. 2(64) = 128.

time = 0.75, size = 256, normalized size = 4.00

$$
\frac{\mathrm{csch}(c)\mathrm{csch}^5(c+dx)\left(-150a^2dx\cosh(dx)+150a^2dx\cosh(2c+dx)+75a^2dx\cosh(2c+3dx)-75a^2dx\cosh(4c+3dx)-15a^2dx\cosh(4c+5dx)+15a^2dx\cosh(6c+5dx)+280a^2\sinh(dx)+120ab\sinh(dx)+20b^2\sinh(dx)+180a^2\sinh(2c+dx)-60b^2\sinh(2c+dx)-140a^2\sinh(2c+3dx)+20b^2\sinh(2c+3dx)-90a^2\sinh(4c+3dx)-60ab\sinh(4c+3dx)+46a^2\sinh(4c+5dx)+12ab\sinh(4c+5dx)-4b^2\sinh(4c+5dx)\right)}{480d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (Csch[c]*Csch[c + d*x]^5*(-150*a^2*d*x*Cosh[d*x] + 150*a^2*d*x*Cosh[2*c + d
*x] + 75*a^2*d*x*Cosh[2*c + 3*d*x] - 75*a^2*d*x*Cosh[4*c + 3*d*x] - 15*a^2*
d*x*Cosh[4*c + 5*d*x] + 15*a^2*d*x*Cosh[6*c + 5*d*x] + 280*a^2*Sinh[d*x] +
120*a*b*Sinh[d*x] + 20*b^2*Sinh[d*x] + 180*a^2*Sinh[2*c + d*x] - 60*b^2*Sin
h[2*c + d*x] - 140*a^2*Sinh[2*c + 3*d*x] + 20*b^2*Sinh[2*c + 3*d*x] - 90*a^
2*Sinh[4*c + 3*d*x] - 60*a*b*Sinh[4*c + 3*d*x] + 46*a^2*Sinh[4*c + 5*d*x] +
 12*a*b*Sinh[4*c + 5*d*x] - 4*b^2*Sinh[4*c + 5*d*x]))/(480*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(60) = 120.

time = 2.57, size = 164, normalized size = 2.56

method	result
risch	$a^2x-\frac{2\left(45a^2\mathrm{e}^{8dx+8c}+30ab\,\mathrm{e}^{8dx+8c}-90a^2\mathrm{e}^{6dx+6c}+30b^2\mathrm{e}^{6dx+6c}+140a^2\mathrm{e}^{4dx+4c}+60ab\,\mathrm{e}^{4dx+4c}+10b^2\mathrm{e}^{4dx+4c}-70a^2\mathrm{e}^{2dx+2c}+10b\right.}{15d\left(\mathrm{e}^{2dx+2c}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x-2/15*(45*a^2*exp(8*d*x+8*c)+30*a*b*exp(8*d*x+8*c)-90*a^2*exp(6*d*x+6*
c)+30*b^2*exp(6*d*x+6*c)+140*a^2*exp(4*d*x+4*c)+60*a*b*exp(4*d*x+4*c)+10*b^
2*exp(4*d*x+4*c)-70*a^2*exp(2*d*x+2*c)+10*b^2*exp(2*d*x+2*c)+23*a^2+6*a*b-2
*b^2)/d/(exp(2*d*x+2*c)-1)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(60) = 120$.
time = 0.28, size = 613, normalized size = 9.58

$$\frac{1}{15}a^2\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}-140e^{(-4dx-4c)}+90e^{(-6dx-6c)}-45e^{(-8dx-8c)}-23\right)}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1\right)}\right)+\frac{4}{15}b^2\left(\frac{5e^{(-2dx-2c)}}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1\right)}+\frac{5e^{(-4dx-4c)}}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1\right)}+\frac{15e^{(-6dx-6c)}}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1\right)}-\frac{1}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1\right)}\right)+\frac{4}{5}ab\left(\frac{10e^{(-4dx-4c)}}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1\right)}+\frac{5e^{(-8dx-8c)}}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1\right)}+\frac{1}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*a^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1))) + 4/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) - 1)) + 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) + 15*
e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*
x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) +
 e^(-10*d*x - 10*c) - 1))) + 4/5*a*b*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x -
 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) - 1)) + 5*e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) - 1)) + 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(60) = 120$.
time = 0.70, size = 425, normalized size = 6.64

$$-\frac{(23a^2+6ab-2b^2)\cosh(dx+c)^5+5(23a^2+6ab-2b^2)\cosh(dx+c)\sinh(dx+c)^4-(15a^2dx+23a^2+6ab-2b^2)\sinh(dx+c)^5-5(5a^2-6ab-2b^2)\cosh(dx+c)^3+5\left(15a^2dx-2(15a^2dx+23a^2+6ab-2b^2)\cosh(dx+c)^2+23a^2+6ab-2b^2\right)\sinh(dx+c)^3+5\left(2(23a^2+6ab-2b^2)\cosh(dx+c)^3-3(5a^2-6ab-2b^2)\cosh(dx+c)\right)\sinh(dx+c)^2+10(5a^2+6ab+4b^2)\cosh(dx+c)-5\left((15a^2dx+23a^2+6ab-2b^2)\cosh(dx+c)^4+30a^2dx-3(15a^2dx+23a^2+6ab-2b^2)\cosh(dx+c)^2+46a^2+12ab-4b^2\right)\sinh(dx+c)}{15\left(d\sinh(dx+c)^5+5(2d\cosh(dx+c)^2-d)\sinh(dx+c)^3+5(d\cosh(dx+c)^4-3d\cosh(dx+c)^2+2d)\sinh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*((23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^5 + 5*(23*a^2 + 6*a*b - 2*b^2
)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*sin
h(d*x + c)^5 - 5*(5*a^2 - 6*a*b - 2*b^2)*cosh(d*x + c)^3 + 5*(15*a^2*d*x -
2*(15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^2 + 23*a^2 + 6*a*b -
2*b^2)*sinh(d*x + c)^3 + 5*(2*(23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)^3 - 3*
(5*a^2 - 6*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2 + 6*a*b
+ 4*b^2)*cosh(d*x + c) - 5*((15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*cosh(d*x
+ c)^4 + 30*a^2*d*x - 3*(15*a^2*d*x + 23*a^2 + 6*a*b - 2*b^2)*cosh(d*x + c)
^2 + 46*a^2 + 12*a*b - 4*b^2)*sinh(d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*co
sh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x +
c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**6*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(60) = 120.
time = 0.45, size = 170, normalized size = 2.66

$$\frac{15\,(dx+c)a^2-\frac{2\,\left(45\,a^2e^{(8\,dx+8\,c)}+30\,abe^{(8\,dx+8\,c)}-90\,a^2e^{(6\,dx+6\,c)}+30\,b^2e^{(6\,dx+6\,c)}+140\,a^2e^{(4\,dx+4\,c)}+60\,abe^{(4\,dx+4\,c)}+10\,b^2e^{(4\,dx+4\,c)}-70\,a^2e^{(2\,dx+2\,c)}+10\,b^2e^{(2\,dx+2\,c)}+23\,a^2+6\,ab-2\,b^2\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(15*(d*x + c)*a^2 - 2*(45*a^2*e^(8*d*x + 8*c) + 30*a*b*e^(8*d*x + 8*c)
 - 90*a^2*e^(6*d*x + 6*c) + 30*b^2*e^(6*d*x + 6*c) + 140*a^2*e^(4*d*x + 4*c
) + 60*a*b*e^(4*d*x + 4*c) + 10*b^2*e^(4*d*x + 4*c) - 70*a^2*e^(2*d*x + 2*c
) + 10*b^2*e^(2*d*x + 2*c) + 23*a^2 + 6*a*b - 2*b^2)/(e^(2*d*x + 2*c) - 1)^
5)/d
```

Mupad [**B**]
time = 1.44, size = 511, normalized size = 7.98

$$a^2\,x-\frac{\frac{2\,\left(5\,a^2+6\,a\,b+4\,b^2\right)}{15\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(b^2+2\,a\,b\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(3\,a^2+2\,b\,a\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}-\frac{\frac{2\,\left(b^2+2\,a\,b\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(3\,a^2+2\,b\,a\right)}{5\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{\frac{2\,\left(b^2+2\,a\,b\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(b^2+2\,a\,b\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(3\,a^2+2\,b\,a\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(5\,a^2+6\,a\,b+4\,b^2\right)}{5\,d}}{6\,{\mathrm{e}}^{4\,c+4\,d\,x}-4\,{\mathrm{e}}^{2\,c+2\,d\,x}-4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{\frac{2\,\left(3\,a^2+2\,b\,a\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(b^2+2\,a\,b\right)}{5\,d}+\frac{8\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(b^2+2\,a\,b\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(3\,a^2+2\,b\,a\right)}{5\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(5\,a^2+6\,a\,b+4\,b^2\right)}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}-10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}-5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}-1}-\frac{2\,\left(3\,a^2+2\,b\,a\right)}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^6*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] a^2*x - ((2*(6*a*b + 5*a^2 + 4*b^2))/(15*d) + (4*exp(2*c + 2*d*x)*(2*a*b +
b^2))/(5*d) + (2*exp(4*c + 4*d*x)*(2*a*b + 3*a^2))/(5*d))/(3*exp(2*c + 2*d*
x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((2*(2*a*b + b^2))/(5*d)
+ (2*exp(2*c + 2*d*x)*(2*a*b + 3*a^2))/(5*d))/(exp(4*c + 4*d*x) - 2*exp(2*c
 + 2*d*x) + 1) - ((2*(2*a*b + b^2))/(5*d) + (6*exp(4*c + 4*d*x)*(2*a*b + b^
2))/(5*d) + (2*exp(6*c + 6*d*x)*(2*a*b + 3*a^2))/(5*d) + (2*exp(2*c + 2*d*x
)*(6*a*b + 5*a^2 + 4*b^2))/(5*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x)
- 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(2*a*b + 3*a^2))/(5*d) +
 (8*exp(2*c + 2*d*x)*(2*a*b + b^2))/(5*d) + (8*exp(6*c + 6*d*x)*(2*a*b + b^
2))/(5*d) + (2*exp(8*c + 8*d*x)*(2*a*b + 3*a^2))/(5*d) + (4*exp(4*c + 4*d*x
)*(6*a*b + 5*a^2 + 4*b^2))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x)
 + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - (2*
(2*a*b + 3*a^2))/(5*d*(exp(2*c + 2*d*x) - 1))
```

3.123 $\int \coth^7(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=86

$$-\frac{a(a+b)\operatorname{csch}^2(c+dx)}{d}-\frac{(a+b)^2\operatorname{csch}^4(c+dx)}{4d}-\frac{\left(b+a\cosh^2(c+dx)\right)^3\operatorname{csch}^6(c+dx)}{6(a+b)d}+\frac{a^2\log(\sinh(c+dx))}{d}$$

```
[Out] -a*(a+b)*csch(d*x+c)^2/d-1/4*(a+b)^2*csch(d*x+c)^4/d-1/6*(b+a*cosh(d*x+c)^2
)^3*csch(d*x+c)^6/(a+b)/d+a^2*ln(sinh(d*x+c))/d
```

Rubi [A]
time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4223, 457, 79, 45}

$$\frac{a^2\log(\sinh(c+dx))}{d}-\frac{(a+b)^2\operatorname{csch}^4(c+dx)}{4d}-\frac{a(a+b)\operatorname{csch}^2(c+dx)}{d}-\frac{\operatorname{csch}^6(c+dx)\left(a\cosh^2(c+dx)+b\right)^3}{6d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -((a*(a + b)*Csch[c + d*x]^2)/d) - ((a + b)^2*Csch[c + d*x]^4)/(4*d) - ((b
+ a*Cosh[c + d*x]^2)^3*Csch[c + d*x]^6)/(6*(a + b)*d) + (a^2*Log[Sinh[c + d
*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth^7(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^3(b+ax^2)^2}{(1-x^2)^4}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x(b+ax)^2}{(1-x)^4}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= -\frac{\left(b+a\cosh^2(c+dx)\right)^3\text{csch}^6(c+dx)}{6(a+b)d} - \frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^3}\,dx, \right.}{2d}$$

$$= -\frac{\left(b+a\cosh^2(c+dx)\right)^3\text{csch}^6(c+dx)}{6(a+b)d} - \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{(-1+x)^3}\right.\right.}{}$$

$$= -\frac{a(a+b)\text{csch}^2(c+dx)}{d} - \frac{(a+b)^2\text{csch}^4(c+dx)}{4d} - \frac{(b+a\cos}{}$$

Mathematica [A]
time = 0.36, size = 107, normalized size = 1.24

$$-\frac{\left(b+a\cosh^2(c+dx)\right)^2\left(6a(3a+2b)\text{csch}^2(c+dx)+3(3a^2+4ab+b^2)\text{csch}^4(c+dx)+2(a+b)^2\text{csch}^6(c+dx)-12a^2\log(\sinh(c+dx))\right)}{3d(a+2b+a\cosh(2(c+dx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] -1/3*((b + a*Cosh[c + d*x]^2)^2*(6*a*(3*a + 2*b)*Csch[c + d*x]^2 + 3*(3*a^2
 + 4*a*b + b^2)*Csch[c + d*x]^4 + 2*(a + b)^2*Csch[c + d*x]^6 - 12*a^2*Log[
Sinh[c + d*x]]))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

Maple [A]
time = 1.81, size = 132, normalized size = 1.53

method	result
derivativedivides	$\frac{a^2\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}-\frac{\left(\coth^6(dx+c)\right)}{6}\right)+2ab\left(-\frac{\cosh^4(dx+c)}{2\sinh(dx+c)^6}+\frac{\cosh^2(dx+c)}{2\sinh(dx+c)^6}-\frac{1}{6\sinh(dx+c)^6}\right.}{d}$
default	$\frac{a^2\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}-\frac{\left(\coth^6(dx+c)\right)}{6}\right)+2ab\left(-\frac{\cosh^4(dx+c)}{2\sinh(dx+c)^6}+\frac{\cosh^2(dx+c)}{2\sinh(dx+c)^6}-\frac{1}{6\sinh(dx+c)^6}\right.}{d}$
risch	$-a^2x-\frac{2a^2c}{d}-\frac{2\,e^{2dx+2c}(9a^2e^{8dx+8c}+6ab\,e^{8dx+8c}-18a^2e^{6dx+6c}+6b^2e^{6dx+6c}+34a^2e^{4dx+4c}+20ab\,e^{4dx+4c}+4b^2e^{4}}{3d\left(e^{2dx+2c}-1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4-1/6*coth(d*x+
c)^6)+2*a*b*(-1/2/sinh(d*x+c)^6*cosh(d*x+c)^4+1/2/sinh(d*x+c)^6*cosh(d*x+c)
^2-1/6/sinh(d*x+c)^6)+b^2*(-1/4/sinh(d*x+c)^6*cosh(d*x+c)^2+1/12/sinh(d*x+c
)^6))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(82) = 164$.
time = 0.28, size = 696, normalized size = 8.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)/
d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 18*
e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*
c) - e^(-12*d*x - 12*c) - 1))) + 4/3*a*b*(3*e^(-2*d*x - 2*c)/(d*(6*e^(-2*d*
x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c)
+ 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 10*e^(-6*d*x - 6*c)/(d*
(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*
d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 3*e^(-10*d*x
 - 10*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c)
 - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) +
 4/3*b^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) +
 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*
d*x - 12*c) - 1)) + 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*
x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c)
 - e^(-12*d*x - 12*c) - 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) - 1
5*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d
*x - 10*c) - e^(-12*d*x - 12*c) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2548 vs. $2(82) = 164$.
time = 0.48, size = 2548, normalized size = 29.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*d*x*cosh(d*x + c)^12 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^1
1 + 3*a^2*d*x*sinh(d*x + c)^12 - 6*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c
)^10 + 6*(33*a^2*d*x*cosh(d*x + c)^2 - 3*a^2*d*x + 3*a^2 + 2*a*b)*sinh(d*x
+ c)^10 + 60*(11*a^2*d*x*cosh(d*x + c)^3 - (3*a^2*d*x - 3*a^2 - 2*a*b)*cosh
(d*x + c))*sinh(d*x + c)^9 + 3*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^
8 + 3*(495*a^2*d*x*cosh(d*x + c)^4 + 15*a^2*d*x - 90*(3*a^2*d*x - 3*a^2 - 2
*a*b)*cosh(d*x + c)^2 - 12*a^2 + 4*b^2)*sinh(d*x + c)^8 + 24*(99*a^2*d*x*co
sh(d*x + c)^5 - 30*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^3 + (15*a^2*d*
x - 12*a^2 + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*a^2*d*x - 17*a^2
 - 10*a*b - 2*b^2)*cosh(d*x + c)^6 + 4*(693*a^2*d*x*cosh(d*x + c)^6 - 315*(
3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^4 - 15*a^2*d*x + 21*(15*a^2*d*x -
12*a^2 + 4*b^2)*cosh(d*x + c)^2 + 17*a^2 + 10*a*b + 2*b^2)*sinh(d*x + c)^6
+ 24*(99*a^2*d*x*cosh(d*x + c)^7 - 63*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x
+ c)^5 + 7*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^3 - (15*a^2*d*x - 17
*a^2 - 10*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(15*a^2*d*x - 12*
a^2 + 4*b^2)*cosh(d*x + c)^4 + 3*(495*a^2*d*x*cosh(d*x + c)^8 - 420*(3*a^2*
d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^6 + 70*(15*a^2*d*x - 12*a^2 + 4*b^2)*cos
h(d*x + c)^4 + 15*a^2*d*x - 20*(15*a^2*d*x - 17*a^2 - 10*a*b - 2*b^2)*cosh(
d*x + c)^2 - 12*a^2 + 4*b^2)*sinh(d*x + c)^4 + 3*a^2*d*x + 4*(165*a^2*d*x*c
osh(d*x + c)^9 - 180*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^7 + 42*(15*a
^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^5 - 20*(15*a^2*d*x - 17*a^2 - 10*a*b
 - 2*b^2)*cosh(d*x + c)^3 + 3*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c))*
sinh(d*x + c)^3 - 6*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^2 + 6*(33*a^2
*d*x*cosh(d*x + c)^10 - 45*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^8 + 14
*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^6 - 10*(15*a^2*d*x - 17*a^2 -
10*a*b - 2*b^2)*cosh(d*x + c)^4 - 3*a^2*d*x + 3*(15*a^2*d*x - 12*a^2 + 4*b^
2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b)*sinh(d*x + c)^2 - 3*(a^2*cosh(d*x + c)^
12 + 12*a^2*cosh(d*x + c)*sinh(d*x + c)^11 + a^2*sinh(d*x + c)^12 - 6*a^2*c
osh(d*x + c)^10 + 6*(11*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^10 + 15*a^
2*cosh(d*x + c)^8 + 20*(11*a^2*cosh(d*x + c)^3 - 3*a^2*cosh(d*x + c))*sinh(
d*x + c)^9 + 15*(33*a^2*cosh(d*x + c)^4 - 18*a^2*cosh(d*x + c)^2 + a^2)*sin
h(d*x + c)^8 - 20*a^2*cosh(d*x + c)^6 + 24*(33*a^2*cosh(d*x + c)^5 - 30*a^2
*cosh(d*x + c)^3 + 5*a^2*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(231*a^2*cosh(d
*x + c)^6 - 315*a^2*cosh(d*x + c)^4 + 105*a^2*cosh(d*x + c)^2 - 5*a^2)*sinh
(d*x + c)^6 + 15*a^2*cosh(d*x + c)^4 + 24*(33*a^2*cosh(d*x + c)^7 - 63*a^2*
cosh(d*x + c)^5 + 35*a^2*cosh(d*x + c)^3 - 5*a^2*cosh(d*x + c))*sinh(d*x +
```

```
c)^5 + 15*(33*a^2*cosh(d*x + c)^8 - 84*a^2*cosh(d*x + c)^6 + 70*a^2*cosh(d*
x + c)^4 - 20*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 - 6*a^2*cosh(d*x +
 c)^2 + 20*(11*a^2*cosh(d*x + c)^9 - 36*a^2*cosh(d*x + c)^7 + 42*a^2*cosh(d
*x + c)^5 - 20*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 +
 6*(11*a^2*cosh(d*x + c)^10 - 45*a^2*cosh(d*x + c)^8 + 70*a^2*cosh(d*x + c)
^6 - 50*a^2*cosh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^2
 + a^2 + 12*(a^2*cosh(d*x + c)^11 - 5*a^2*cosh(d*x + c)^9 + 10*a^2*cosh(d*x
 + c)^7 - 10*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c
))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12
*(3*a^2*d*x*cosh(d*x + c)^11 - 5*(3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c)^
9 + 2*(15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*x + c)^7 - 2*(15*a^2*d*x - 17*a^
2 - 10*a*b - 2*b^2)*cosh(d*x + c)^5 + (15*a^2*d*x - 12*a^2 + 4*b^2)*cosh(d*
x + c)^3 - (3*a^2*d*x - 3*a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cos
h(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 -
6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^10 + 20*(
11*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x +
 c)^8 + 15*(33*d*cosh(d*x + c)^4 - 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^
8 + 24*(33*d*cosh(d*x + c)^5 - 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*si
nh(d*x + c)^7 - 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 - 315*d*cos
h(d*x + c)^4 + 105*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh
(d*x + c)^7 - 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 - 5*d*cosh(d*x +
c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 - 84*
d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d
*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 - 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x
 + c)^5 - 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 6*d*c
osh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 - 45*d*cosh(d*x + c)^8 + 70*d*cos
h(d*x + c)^6 - 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x +
c)^2 + 12*(d*cosh(d*x + c)^11 - 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7
- 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x +
c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**7*(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(82) = 164.
time = 0.49, size = 219, normalized size = 2.55

$$-\frac{60\,(dx+c)a^2 - 60\,a^2\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right) + \frac{147\,a^2e^{(12\,dx+12\,c)}-522\,a^2e^{(10\,dx+10\,c)}+240\,abe^{(10\,dx+10\,c)}+1485\,a^2e^{(8\,dx+8\,c)}+240\,b^2e^{(8\,dx+8\,c)}-1580\,a^2e^{(6\,dx+6\,c)}+800\,abe^{(6\,dx+6\,c)}+160\,b^2e^{(6\,dx+6\,c)}+1485\,a^2e^{(4\,dx+4\,c)}+240\,b^2e^{(4\,dx+4\,c)}-522\,a^2e^{(2\,dx+2\,c)}+240\,abe^{(2\,dx+2\,c)}+147\,a^2}{\left(e^{(2\,dx+2\,c)}-1\right)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/60*(60*(d*x + c)*a^2 - 60*a^2*log(abs(e^(2*d*x + 2*c) - 1)) + (147*a^2*e
^(12*d*x + 12*c) - 522*a^2*e^(10*d*x + 10*c) + 240*a*b*e^(10*d*x + 10*c) +
1485*a^2*e^(8*d*x + 8*c) + 240*b^2*e^(8*d*x + 8*c) - 1580*a^2*e^(6*d*x + 6*
c) + 800*a*b*e^(6*d*x + 6*c) + 160*b^2*e^(6*d*x + 6*c) + 1485*a^2*e^(4*d*x
+ 4*c) + 240*b^2*e^(4*d*x + 4*c) - 522*a^2*e^(2*d*x + 2*c) + 240*a*b*e^(2*d
*x + 2*c) + 147*a^2)/(e^(2*d*x + 2*c) - 1)^6)/d
```

Mupad [B]
time = 1.53, size = 377, normalized size = 4.38

$$\frac{a^2\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)}{d}-\frac{32\,\left(a^2+2\,a\,b+b^2\right)}{d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1\right)}-\frac{2\,\left(3\,a^2+2\,b\,a\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{32\,\left(a^2+2\,a\,b+b^2\right)}{3\,d\,\left(15\,\mathrm{e}^{4\,c+4\,d\,x}-6\,\mathrm{e}^{2\,c+2\,d\,x}-20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}-6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1\right)}-\frac{2\,\left(9\,a^2+10\,a\,b+2\,b^2\right)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{8\,\left(13\,a^2+20\,a\,b+7\,b^2\right)}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{4\,\left(11\,a^2+20\,a\,b+9\,b^2\right)}{d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}-a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^7*(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] (a^2*log(exp(2*c)*exp(2*d*x) - 1))/d - (32*(2*a*b + a^2 + b^2))/(d*(5*exp(2
*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x
) + exp(10*c + 10*d*x) - 1)) - (2*(2*a*b + 3*a^2))/(d*(exp(2*c + 2*d*x) - 1
)) - (32*(2*a*b + a^2 + b^2))/(3*d*(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x
) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(
12*c + 12*d*x) + 1)) - (2*(10*a*b + 9*a^2 + 2*b^2))/(d*(exp(4*c + 4*d*x) -
2*exp(2*c + 2*d*x) + 1)) - (8*(20*a*b + 13*a^2 + 7*b^2))/(3*d*(3*exp(2*c +
2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*(20*a*b + 11*a^2
+ 9*b^2))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x)
+ exp(8*c + 8*d*x) + 1)) - a^2*x
```

3.124 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^3 \tanh^4(c+dx)\,dx$

Optimal. Leaf size=110

$$a^3x-\frac{a^3\tanh(c+dx)}{d}-\frac{a^3\tanh^3(c+dx)}{3d}+\frac{b(3a^2+3ab+b^2)\tanh^5(c+dx)}{5d}-\frac{b^2(3a+2b)\tanh^7(c+dx)}{7d}+\frac{b^3\tanh^9(c+dx)}{9d}$$

```
[Out] a^3*x-a^3*tanh(d*x+c)/d-1/3*a^3*tanh(d*x+c)^3/d+1/5*b*(3*a^2+3*a*b+b^2)*tan
h(d*x+c)^5/d-1/7*b^2*(3*a+2*b)*tanh(d*x+c)^7/d+1/9*b^3*tanh(d*x+c)^9/d
```

Rubi [A]
time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$,
Rules used = {4226, 1816, 212}

$$-\frac{a^3\tanh^3(c+dx)}{3d}-\frac{a^3\tanh(c+dx)}{d}+a^3x+\frac{b(3a^2+3ab+b^2)\tanh^5(c+dx)}{5d}-\frac{b^2(3a+2b)\tanh^7(c+dx)}{7d}+\frac{b^3\tanh^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^4,x]
```

```
[Out] a^3*x - (a^3*Tanh[c + d*x])/d - (a^3*Tanh[c + d*x]^3)/(3*d) + (b*(3*a^2 + 3
*a*b + b^2)*Tanh[c + d*x]^5)/(5*d) - (b^2*(3*a + 2*b)*Tanh[c + d*x]^7)/(7*d
) + (b^3*Tanh[c + d*x]^9)/(9*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^3 \tanh^4(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1-x^2\right)\right)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-a^3-a^3x^2+b\left(3a^2+3ab+b^2\right)x^4-b^2(3a+2b)x^6\right.\right.}{d}$$

$$= -\frac{a^3\tanh(c+dx)}{d} - \frac{a^3\tanh^3(c+dx)}{3d} + \frac{b\left(3a^2+3ab+b^2\right)\tan}{5d}$$

$$= a^3x - \frac{a^3\tanh(c+dx)}{d} - \frac{a^3\tanh^3(c+dx)}{3d} + \frac{b\left(3a^2+3ab+b^2\right.}{5d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 301 vs. 2(110) = 220.
time = 3.84, size = 301, normalized size = 2.74

$$\frac{8\left(b+a\cosh^2(c+dx)\right)^3\text{sech}^9(c+dx)\left(315a^3dx\cosh^9(c+dx)+35b^3\text{sech}(c)\sinh(dx)+5(27a-10b)b^2\cosh^2(c+dx)\text{sech}(c)\sinh(dx)+3b\left(63a^2-72ab+b^2\right)\cosh^4(c+dx)\text{sech}(c)\sinh(dx)+\left(105a^3-378a^2b+27ab^2+4b^3\right)\cosh^6(c+dx)\text{sech}(c)\sinh(dx)-\left(420a^3-189a^2b-54ab^2-8b^3\right)\cosh^8(c+dx)\text{sech}(c)\sinh(dx)+35b^3\cosh(c+dx)\tanh(c)+5(27a-10b)b^2\cosh^3(c+dx)\tanh(c)+3b\left(63a^2-72ab+b^2\right)\cosh^5(c+dx)\tanh(c)+\left(105a^3-378a^2b+27ab^2+4b^3\right)\cosh^7(c+dx)\tanh(c)\right)}{315d\left(a+2b+a\cosh(2(c+dx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^4,x]
```

```
[Out] (8*(b + a*Cosh[c + d*x]^2)^3*Sech[c + d*x]^9*(315*a^3*d*x*Cosh[c + d*x]^9 +
 35*b^3*Sech[c]*Sinh[d*x] + 5*(27*a - 10*b)*b^2*Cosh[c + d*x]^2*Sech[c]*Sin
h[d*x] + 3*b*(63*a^2 - 72*a*b + b^2)*Cosh[c + d*x]^4*Sech[c]*Sinh[d*x] + (1
05*a^3 - 378*a^2*b + 27*a*b^2 + 4*b^3)*Cosh[c + d*x]^6*Sech[c]*Sinh[d*x] -
(420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*Cosh[c + d*x]^8*Sech[c]*Sinh[d*x]
+ 35*b^3*Cosh[c + d*x]*Tanh[c] + 5*(27*a - 10*b)*b^2*Cosh[c + d*x]^3*Tanh[c
] + 3*b*(63*a^2 - 72*a*b + b^2)*Cosh[c + d*x]^5*Tanh[c] + (105*a^3 - 378*a^
2*b + 27*a*b^2 + 4*b^3)*Cosh[c + d*x]^7*Tanh[c]))/(315*d*(a + 2*b + a*Cosh[
2*(c + d*x)])^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 468 vs. 2(102) = 204.
time = 2.72, size = 469, normalized size = 4.26

method	result
risch	$a^3x+\frac{-\frac{12a\,b^2}{35}-12a\,b^2\text{e}^{14dx+14c}-48a^2b\,\text{e}^{12dx+12c}-12a\,b^2\text{e}^{12dx+12c}-72a^2b\,\text{e}^{10dx+10c}-12a\,b^2\text{e}^{10dx+10c}-6a^2b\,\text{e}^{16dx+16c}-24a^2b\,\text{e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x+2/315*(-54*a*b^2-1890*a*b^2*exp(14*d*x+14*c)-7560*a^2*b*exp(12*d*x+12
*c)-1890*a*b^2*exp(12*d*x+12*c)-11340*a^2*b*exp(10*d*x+10*c)-1890*a*b^2*exp
```

```
(10*d*x+10*c)-945*a^2*b*exp(16*d*x+16*c)-3780*a^2*b*exp(14*d*x+14*c)-3024*a
^2*b*exp(4*d*x+4*c)-756*a^2*b*exp(2*d*x+2*c)-189*a^2*b+420*a^3-8*b^3-4914*a
*b^2*exp(8*d*x+8*c)-2646*a*b^2*exp(6*d*x+6*c)-54*a*b^2*exp(4*d*x+4*c)-12474
*a^2*b*exp(8*d*x+8*c)-486*a*b^2*exp(2*d*x+2*c)-8316*a^2*b*exp(6*d*x+6*c)+44
10*a^3*exp(14*d*x+14*c)+3150*a^3*exp(2*d*x+2*c)+13650*a^3*exp(12*d*x+12*c)-
1680*b^3*exp(12*d*x+12*c)+24570*a^3*exp(10*d*x+10*c)-72*b^3*exp(2*d*x+2*c)+
630*a^3*exp(16*d*x+16*c)-3528*b^3*exp(8*d*x+8*c)+10710*a^3*exp(4*d*x+4*c)-2
88*b^3*exp(4*d*x+4*c)+2520*b^3*exp(10*d*x+10*c)+28350*a^3*exp(8*d*x+8*c)+21
630*a^3*exp(6*d*x+6*c)+1008*b^3*exp(6*d*x+6*c))/d/(1+exp(2*d*x+2*c))^9
```

<u>**Maxima**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1453 vs. $2(102) = 204$.
time = 0.28, size = 1453, normalized size = 13.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 3/5*a^2*b*tanh(d*x + c)^5/d + 1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c)
+ 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) + 16/315*b^3*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c
) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*
e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-
16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2
*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8
*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c
) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) - 126*e^(-6*d*x - 6*c)/
(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^
(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14
*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 441*e^(-8*
d*x - 8*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*
c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c)
+ 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) -
 315*e^(-10*d*x - 10*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e
^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12
*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x -
18*c) + 1)) + 210*e^(-12*d*x - 12*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x
- 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c
) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) +
e^(-18*d*x - 18*c) + 1)) + 1/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) +
 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e
^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d
*x - 18*c) + 1))) + 12/35*a*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c)
+ 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-
```

```
10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4
*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6
*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) +
e^(-14*d*x - 14*c) + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*
e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*
x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x
- 8*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) +
 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-1
4*d*x - 14*c) + 1)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(
-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x -
 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*
x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c)
+ 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1323 vs. $2(102) = 204$.
time = 0.38, size = 1323, normalized size = 12.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/315*((315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)
^9 + 9*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)
*sinh(d*x + c)^8 - (420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*sinh(d*x + c)^9
 + 9*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^7
 - 9*(280*a^3 + 21*a^2*b - 54*a*b^2 - 8*b^3 + 4*(420*a^3 - 189*a^2*b - 54*a
*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 21*(4*(315*a^3*d*x + 420*a
^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 3*(315*a^3*d*x + 420*a
^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 36*(315
*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^5 - 9*(14*
(420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 700*a^3 + 84*a^2
*b + 204*a*b^2 - 32*b^3 + 21*(280*a^3 + 21*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^5 + 9*(14*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*
a*b^2 - 8*b^3)*cosh(d*x + c)^5 + 35*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54
*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 20*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 5
4*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 84*(315*a^3*d*x + 420*a^3
 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^3 - 3*(28*(420*a^3 - 189*a^2
*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 105*(280*a^3 + 21*a^2*b - 54*a*b^2
 - 8*b^3)*cosh(d*x + c)^4 + 2660*a^3 - 252*a^2*b - 252*a*b^2 + 896*b^3 + 12
0*(175*a^3 + 21*a^2*b + 51*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3
+ 9*(4*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)
^7 + 21*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c
)^5 + 40*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x +
```

```
c)^3 + 28*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^2 + 126*(315*a^3*d*x + 420*a^3 - 189*a^2*b - 54*a*b^2 -
8*b^3)*cosh(d*x + c) - 9*((420*a^3 - 189*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x
 + c)^8 + 7*(280*a^3 + 21*a^2*b - 54*a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 20*(1
75*a^3 + 21*a^2*b + 51*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 420*a^3 - 126*a^2*b
 - 336*a*b^2 - 672*b^3 + 28*(95*a^3 - 9*a^2*b - 9*a*b^2 + 32*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)
^8 + 9*d*cosh(d*x + c)^7 + 21*(4*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sin
h(d*x + c)^6 + 36*d*cosh(d*x + c)^5 + 9*(14*d*cosh(d*x + c)^5 + 35*d*cosh(d
*x + c)^3 + 20*d*cosh(d*x + c))*sinh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 + 9*
(4*d*cosh(d*x + c)^7 + 21*d*cosh(d*x + c)^5 + 40*d*cosh(d*x + c)^3 + 28*d*c
osh(d*x + c))*sinh(d*x + c)^2 + 126*d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2\left(c + dx\right)\right)^3 \tanh^4\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*tanh(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(102) = 204.
time = 0.45, size = 475, normalized size = 4.32

$$\frac{315\,(dx+c)a^3 + \frac{2\left(630\,a^3e^{(16\,dx+16\,c)} - 945\,a^2be^{(16\,dx+16\,c)} + 4410\,a^3e^{(14\,dx+14\,c)} - 3780\,a^2be^{(14\,dx+14\,c)} - 1890\,ab^2e^{(14\,dx+14\,c)} + 13650\,a^3e^{(12\,dx+12\,c)} - 7560\,a^2be^{(12\,dx+12\,c)} - 1890\,ab^2e^{(12\,dx+12\,c)} - 1680\,b^3e^{(12\,dx+12\,c)} + 24570\,a^3e^{(10\,dx+10\,c)} - 11340\,a^2be^{(10\,dx+10\,c)} - 1890\,ab^2e^{(10\,dx+10\,c)} + 2520\,b^3e^{(10\,dx+10\,c)} + 28350\,a^3e^{(8\,dx+8\,c)} - 12474\,a^2be^{(8\,dx+8\,c)} - 4914\,ab^2e^{(8\,dx+8\,c)} - 3528\,b^3e^{(8\,dx+8\,c)} + 21630\,a^3e^{(6\,dx+6\,c)} - 8316\,a^2be^{(6\,dx+6\,c)} - 2646\,ab^2e^{(6\,dx+6\,c)} + 1008\,b^3e^{(6\,dx+6\,c)} + 10710\,a^3e^{(4\,dx+4\,c)} - 3024\,a^2be^{(4\,dx+4\,c)} - 54\,ab^2e^{(4\,dx+4\,c)} - 288\,b^3e^{(4\,dx+4\,c)} + 3150\,a^3e^{(2\,dx+2\,c)} - 756\,a^2be^{(2\,dx+2\,c)} - 486\,ab^2e^{(2\,dx+2\,c)} - 72\,b^3e^{(2\,dx+2\,c)} + 420\,a^3 - 189\,a^2b - 54\,ab^2 - 8\,b^3\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^9}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/315*(315*(d*x + c)*a^3 + 2*(630*a^3*e^(16*d*x + 16*c) - 945*a^2*b*e^(16*d
*x + 16*c) + 4410*a^3*e^(14*d*x + 14*c) - 3780*a^2*b*e^(14*d*x + 14*c) - 18
90*a*b^2*e^(14*d*x + 14*c) + 13650*a^3*e^(12*d*x + 12*c) - 7560*a^2*b*e^(12
*d*x + 12*c) - 1890*a*b^2*e^(12*d*x + 12*c) - 1680*b^3*e^(12*d*x + 12*c) +
24570*a^3*e^(10*d*x + 10*c) - 11340*a^2*b*e^(10*d*x + 10*c) - 1890*a*b^2*e^
(10*d*x + 10*c) + 2520*b^3*e^(10*d*x + 10*c) + 28350*a^3*e^(8*d*x + 8*c) -
12474*a^2*b*e^(8*d*x + 8*c) - 4914*a*b^2*e^(8*d*x + 8*c) - 3528*b^3*e^(8*d*
x + 8*c) + 21630*a^3*e^(6*d*x + 6*c) - 8316*a^2*b*e^(6*d*x + 6*c) - 2646*a*
b^2*e^(6*d*x + 6*c) + 1008*b^3*e^(6*d*x + 6*c) + 10710*a^3*e^(4*d*x + 4*c)
 - 3024*a^2*b*e^(4*d*x + 4*c) - 54*a*b^2*e^(4*d*x + 4*c) - 288*b^3*e^(4*d*x
 + 4*c) + 3150*a^3*e^(2*d*x + 2*c) - 756*a^2*b*e^(2*d*x + 2*c) - 486*a*b^2*e
^(2*d*x + 2*c) - 72*b^3*e^(2*d*x + 2*c) + 420*a^3 - 189*a^2*b - 54*a*b^2 -
8*b^3)/(e^(2*d*x + 2*c) + 1)^9)/d
```

Mupad [B]
time = 1.62, size = 1834, normalized size = 16.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3,x)
[Out] ((3*a*b^2 + 13*a^3 + 16*b^3)/(63*d) + (10*exp(4*c + 4*d*x)*(3*a*b^2 + 13*a^
3 + 16*b^3))/(63*d) + (20*exp(6*c + 6*d*x)*(6*a*b^2 + 3*a^2*b + 8*a^3 - 4*b
^3))/(63*d) - (2*exp(2*c + 2*d*x)*(8*a*b^2 + 3*a^2*b - 10*a^3 + 16*b^3))/(2
1*d) - (5*exp(8*c + 8*d*x)*(a*b^2 - a^3))/(3*d) - (2*exp(10*c + 10*d*x)*(3*
a^2*b - 2*a^3))/(9*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6
*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*
x) + 1) - ((2*exp(2*c + 2*d*x)*(a*b^2 - a^3))/(3*d) - (2*(6*a*b^2 + 3*a^2*b
 + 8*a^3 - 4*b^3))/(63*d) + (2*exp(4*c + 4*d*x)*(3*a^2*b - 2*a^3))/(9*d))/(
3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((3*a*b^2
 + 13*a^3 + 16*b^3)/(63*d) + (2*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 8*a^3
 - 4*b^3))/(21*d) - (exp(4*c + 4*d*x)*(a*b^2 - a^3))/d - (2*exp(6*c + 6*d*x
)*(3*a^2*b - 2*a^3))/(9*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*ex
p(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((a*b^2 - a^3)/(3*d) + (2*exp(2*c
+ 2*d*x)*(3*a^2*b - 2*a^3))/(9*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) +
 1) + a^3*x + ((2*(6*a*b^2 + 3*a^2*b + 8*a^3 - 4*b^3))/(63*d) + (2*exp(2*c
+ 2*d*x)*(3*a*b^2 + 13*a^3 + 16*b^3))/(21*d) + (20*exp(6*c + 6*d*x)*(3*a*b^
2 + 13*a^3 + 16*b^3))/(63*d) + (10*exp(8*c + 8*d*x)*(6*a*b^2 + 3*a^2*b + 8*
a^3 - 4*b^3))/(21*d) - (2*exp(4*c + 4*d*x)*(8*a*b^2 + 3*a^2*b - 10*a^3 + 16
*b^3))/(7*d) - (2*exp(10*c + 10*d*x)*(a*b^2 - a^3))/d - (2*exp(12*c + 12*d*
x)*(3*a^2*b - 2*a^3))/(9*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35
*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*
c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((a*b^2 - a^3)/(3*d) - (exp(4*c + 4
*d*x)*(3*a*b^2 + 13*a^3 + 16*b^3))/(3*d) - (5*exp(8*c + 8*d*x)*(3*a*b^2 + 1
3*a^3 + 16*b^3))/(9*d) - (2*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 8*a^3 - 4
*b^3))/(9*d) - (2*exp(10*c + 10*d*x)*(6*a*b^2 + 3*a^2*b + 8*a^3 - 4*b^3))/(
3*d) + (2*exp(6*c + 6*d*x)*(8*a*b^2 + 3*a^2*b - 10*a^3 + 16*b^3))/(3*d) + (
7*exp(12*c + 12*d*x)*(a*b^2 - a^3))/(3*d) + (2*exp(14*c + 14*d*x)*(3*a^2*b
- 2*a^3))/(9*d))/(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6
*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x)
 + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1) - ((2*(3*a^2*b - 2*a^3))/
(9*d) - (8*exp(6*c + 6*d*x)*(3*a*b^2 + 13*a^3 + 16*b^3))/(9*d) - (8*exp(10*
c + 10*d*x)*(3*a*b^2 + 13*a^3 + 16*b^3))/(9*d) - (8*exp(4*c + 4*d*x)*(6*a*b
^2 + 3*a^2*b + 8*a^3 - 4*b^3))/(9*d) - (8*exp(12*c + 12*d*x)*(6*a*b^2 + 3*a
^2*b + 8*a^3 - 4*b^3))/(9*d) + (4*exp(8*c + 8*d*x)*(8*a*b^2 + 3*a^2*b - 10*
a^3 + 16*b^3))/(3*d) + (8*exp(2*c + 2*d*x)*(a*b^2 - a^3))/(3*d) + (8*exp(14
*c + 14*d*x)*(a*b^2 - a^3))/(3*d) + (2*exp(16*c + 16*d*x)*(3*a^2*b - 2*a^3)
)/(9*d))/(9*exp(2*c + 2*d*x) + 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) +
```

```
126*exp(8*c + 8*d*x) + 126*exp(10*c + 10*d*x) + 84*exp(12*c + 12*d*x) + 36*
exp(14*c + 14*d*x) + 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) + 1) - ((2*(
8*a*b^2 + 3*a^2*b - 10*a^3 + 16*b^3))/(105*d) - (4*exp(2*c + 2*d*x)*(3*a*b^
2 + 13*a^3 + 16*b^3))/(63*d) - (4*exp(4*c + 4*d*x)*(6*a*b^2 + 3*a^2*b + 8*a
^3 - 4*b^3))/(21*d) + (4*exp(6*c + 6*d*x)*(a*b^2 - a^3))/(3*d) + (2*exp(8*c
 + 8*d*x)*(3*a^2*b - 2*a^3))/(9*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*
x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (
2*(3*a^2*b - 2*a^3))/(9*d*(exp(2*c + 2*d*x) + 1))
```

3.125 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^3 \tanh^3(c+dx)\,dx$

Optimal. Leaf size=103

$$\frac{a^3\log(\cosh(c+dx))}{d}-\frac{3a^2b\,\mathrm{sech}^2(c+dx)}{2d}-\frac{3ab^2\mathrm{sech}^4(c+dx)}{4d}-\frac{b^3\mathrm{sech}^6(c+dx)}{6d}+\frac{\left(b+a\cosh^2(c+dx)\right)^4\mathrm{sech}^8(c+dx)}{8bd}$$

```
[Out] a^3*ln(cosh(d*x+c))/d-3/2*a^2*b*sech(d*x+c)^2/d-3/4*a*b^2*sech(d*x+c)^4/d-1
/6*b^3*sech(d*x+c)^6/d+1/8*(b+a*cosh(d*x+c)^2)^4*sech(d*x+c)^8/b/d
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4223, 457, 79, 45}

$$\frac{a^3\log(\cosh(c+dx))}{d}-\frac{3a^2b\,\mathrm{sech}^2(c+dx)}{2d}-\frac{3ab^2\mathrm{sech}^4(c+dx)}{4d}+\frac{\mathrm{sech}^8(c+dx)\left(a\cosh^2(c+dx)+b\right)^4}{8bd}-\frac{b^3\mathrm{sech}^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^3,x]
```

```
[Out] (a^3*Log[Cosh[c + d*x]])/d - (3*a^2*b*Sech[c + d*x]^2)/(2*d) - (3*a*b^2*Sec
h[c + d*x]^4)/(4*d) - (b^3*Sech[c + d*x]^6)/(6*d) + ((b + a*Cosh[c + d*x]^2
)^4*Sech[c + d*x]^8)/(8*b*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)^3\tanh^3(c+dx)\,dx=-\frac{\mathrm{Subst}\left(\int\frac{(1-x^2)\left(b+ax^2\right)^3}{x^9}\,dx,x,\cosh(c+dx)\right)}{d}$$
$$=-\frac{\mathrm{Subst}\left(\int\frac{(1-x)(b+ax)^3}{x^5}\,dx,x,\cosh^2(c+dx)\right)}{2d}$$
$$=\frac{\left(b+a\cosh^2(c+dx)\right)^4\mathrm{sech}^8(c+dx)}{8bd}+\frac{\mathrm{Subst}\left(\int\frac{(b+ax)^3}{x^4}\,dx,x,\mathrm{c}\right.}{2d}$$
$$=\frac{\left(b+a\cosh^2(c+dx)\right)^4\mathrm{sech}^8(c+dx)}{8bd}+\frac{\mathrm{Subst}\left(\int\left(\frac{b^3}{x^4}+\frac{3ab^2}{x^3}+3\right.\right.}{}$$
$$=\frac{a^3\log(\cosh(c+dx))}{d}-\frac{3a^2b\mathrm{sech}^2(c+dx)}{2d}-\frac{3ab^2\mathrm{sech}^4(c+dx)}{4d}$$

Mathematica [A]
time = 0.51, size = 128, normalized size = 1.24

$$\frac{\cosh^6(c+dx)\left(a+b\mathrm{sech}^2(c+dx)\right)^3\left(24a^3\log(\cosh(c+dx))+12a^2(a-3b)\mathrm{sech}^2(c+dx)+18a(a-b)b\mathrm{sech}^4(c+dx)+4(3a-b)b^2\mathrm{sech}^6(c+dx)+3b^3\mathrm{sech}^8(c+dx)\right)}{3d(a+2b+a\cosh(2c+2dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^3,x]
```

```
[Out] (Cosh[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3*(24*a^3*Log[Cosh[c + d*x]] + 12*
a^2*(a - 3*b)*Sech[c + d*x]^2 + 18*a*(a - b)*b*Sech[c + d*x]^4 + 4*(3*a - b
)*b^2*Sech[c + d*x]^6 + 3*b^3*Sech[c + d*x]^8))/(3*d*(a + 2*b + a*Cosh[2*c
+ 2*d*x])^3)
```

Maple [A]
time = 1.72, size = 131, normalized size = 1.27

method	result
derivativedivides	$\frac{a^3\left(\ln(\cosh(dx+c))-\frac{\left(\tanh^2(dx+c)\right)}{2}\right)+3a^2b\left(-\frac{\sinh^2(dx+c)}{2\cosh(dx+c)^4}-\frac{1}{4\cosh(dx+c)^4}\right)+3a\,b^2\left(-\frac{\sinh^2(dx+c)}{4\cosh(dx+c)^6}-\frac{1}{12\cosh(dx+c)^6}\right.}{d}$
default	$\frac{a^3\left(\ln(\cosh(dx+c))-\frac{\left(\tanh^2(dx+c)\right)}{2}\right)+3a^2b\left(-\frac{\sinh^2(dx+c)}{2\cosh(dx+c)^4}-\frac{1}{4\cosh(dx+c)^4}\right)+3a\,b^2\left(-\frac{\sinh^2(dx+c)}{4\cosh(dx+c)^6}-\frac{1}{12\cosh(dx+c)^6}\right.}{d}$
risch	$-a^3x-\frac{2a^3c}{d}+\frac{2\,e^{2dx+2c}\left(3a^3e^{12dx+12c}-9a^2b\,e^{12dx+12c}+18a^3e^{10dx+10c}-36a^2b\,e^{10dx+10c}-18a\,b^2e^{10dx+10c}+45a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(ln(cosh(d*x+c))-1/2*tanh(d*x+c)^2)+3*a^2*b*(-1/2*sinh(d*x+c)^2/co
sh(d*x+c)^4-1/4/cosh(d*x+c)^4)+3*a*b^2*(-1/4*sinh(d*x+c)^2/cosh(d*x+c)^6-1/
12/cosh(d*x+c)^6)+b^3*(-1/6*sinh(d*x+c)^2/cosh(d*x+c)^8-1/24/cosh(d*x+c)^8)
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(95) = 190$.
time = 0.48, size = 652, normalized size = 6.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 3/4*a^2*b*tanh(d*x + c)^4/d + a^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d +
2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 4*a*b
^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^
(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x -
12*c) + 1)) - 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*
c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(
-12*d*x - 12*c) + 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-
4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 1
0*c) + e^(-12*d*x - 12*c) + 1))) - 32/3*b^3*(e^(-6*d*x - 6*c)/(d*(8*e^(-2*d
*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c)
 + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e
^(-16*d*x - 16*c) + 1)) - e^(-8*d*x - 8*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-
4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x -
10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) +
 1)) + e^(-10*d*x - 10*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56
*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12
*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4658 vs. $2(95) = 190$.
time = 0.43, size = 4658, normalized size = 45.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^3*d*x*cosh(d*x + c)^16 + 48*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^1
5 + 3*a^3*d*x*sinh(d*x + c)^16 + 6*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c
)^14 + 6*(60*a^3*d*x*cosh(d*x + c)^2 + 4*a^3*d*x - a^3 + 3*a^2*b)*sinh(d*x
+ c)^14 + 84*(20*a^3*d*x*cosh(d*x + c)^3 + (4*a^3*d*x - a^3 + 3*a^2*b)*cosh
(d*x + c))*sinh(d*x + c)^13 + 12*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*co
sh(d*x + c)^12 + 6*(910*a^3*d*x*cosh(d*x + c)^4 + 14*a^3*d*x - 6*a^3 + 12*a
^2*b + 6*a*b^2 + 91*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^2)*sinh(d*x +
 c)^12 + 24*(546*a^3*d*x*cosh(d*x + c)^5 + 91*(4*a^3*d*x - a^3 + 3*a^2*b)*c
osh(d*x + c)^3 + 6*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c))*s
inh(d*x + c)^11 + 2*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*co
sh(d*x + c)^10 + 2*(12012*a^3*d*x*cosh(d*x + c)^6 + 84*a^3*d*x + 3003*(4*a^
3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^4 - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*
b^3 + 396*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x
 + c)^10 + 4*(8580*a^3*d*x*cosh(d*x + c)^7 + 3003*(4*a^3*d*x - a^3 + 3*a^2*
b)*cosh(d*x + c)^5 + 660*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x +
 c)^3 + 5*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c
))*sinh(d*x + c)^9 + 2*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3
)*cosh(d*x + c)^8 + 2*(19305*a^3*d*x*cosh(d*x + c)^8 + 9009*(4*a^3*d*x - a^
3 + 3*a^2*b)*cosh(d*x + c)^6 + 105*a^3*d*x + 2970*(7*a^3*d*x - 3*a^3 + 6*a^
2*b + 3*a*b^2)*cosh(d*x + c)^4 - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3 + 45
*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^8 + 16*(2145*a^3*d*x*cosh(d*x + c)^9 + 1287*(4*a^3*d*x - a^3 + 3*
a^2*b)*cosh(d*x + c)^7 + 594*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d
*x + c)^5 + 15*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*
x + c)^3 + (105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^7 + 2*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^
3)*cosh(d*x + c)^6 + 2*(12012*a^3*d*x*cosh(d*x + c)^10 + 9009*(4*a^3*d*x -
a^3 + 3*a^2*b)*cosh(d*x + c)^8 + 5544*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^
2)*cosh(d*x + c)^6 + 84*a^3*d*x + 210*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*
a*b^2 + 16*b^3)*cosh(d*x + c)^4 - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3 + 2
8*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^6 + 4*(3276*a^3*d*x*cosh(d*x + c)^11 + 3003*(4*a^3*d*x - a^3 +
3*a^2*b)*cosh(d*x + c)^9 + 2376*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cos
h(d*x + c)^7 + 126*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cos
h(d*x + c)^5 + 28*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*cos
h(d*x + c)^3 + 3*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(
```

```
d*x + c))*sinh(d*x + c)^5 + 3*a^3*d*x + 12*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3
*a*b^2)*cosh(d*x + c)^4 + 2*(2730*a^3*d*x*cosh(d*x + c)^12 + 3003*(4*a^3*d*
x - a^3 + 3*a^2*b)*cosh(d*x + c)^10 + 2970*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3
*a*b^2)*cosh(d*x + c)^8 + 210*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 +
16*b^3)*cosh(d*x + c)^6 + 42*a^3*d*x + 70*(105*a^3*d*x - 60*a^3 + 72*a^2*b
+ 12*a*b^2 - 16*b^3)*cosh(d*x + c)^4 - 18*a^3 + 36*a^2*b + 18*a*b^2 + 15*(8
4*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 8*(210*a^3*d*x*cosh(d*x + c)^13 + 273*(4*a^3*d*x - a^3 + 3*a^2*b
)*cosh(d*x + c)^11 + 330*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x +
 c)^9 + 30*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x +
c)^7 + 14*(105*a^3*d*x - 60*a^3 + 72*a^2*b + 12*a*b^2 - 16*b^3)*cosh(d*x +
c)^5 + 5*(84*a^3*d*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)
^3 + 6*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c)
^3 + 6*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^2 + 2*(180*a^3*d*x*cosh(d*
x + c)^14 + 273*(4*a^3*d*x - a^3 + 3*a^2*b)*cosh(d*x + c)^12 + 396*(7*a^3*d
*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^10 + 45*(84*a^3*d*x - 45*a^3
+ 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)^8 + 28*(105*a^3*d*x - 60*a^3
+ 72*a^2*b + 12*a*b^2 - 16*b^3)*cosh(d*x + c)^6 + 12*a^3*d*x + 15*(84*a^3*d
*x - 45*a^3 + 63*a^2*b + 24*a*b^2 + 16*b^3)*cosh(d*x + c)^4 - 3*a^3 + 9*a^2
*b + 36*(7*a^3*d*x - 3*a^3 + 6*a^2*b + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x +
 c)^2 - 3*(a^3*cosh(d*x + c)^16 + 16*a^3*cosh(d*x + c)*sinh(d*x + c)^15 + a
^3*sinh(d*x + c)^16 + 8*a^3*cosh(d*x + c)^14 + 28*a^3*cosh(d*x + c)^12 + 8*
(15*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^14 + 112*(5*a^3*cosh(d*x + c)^
3 + a^3*cosh(d*x + c))*sinh(d*x + c)^13 + 56*a^3*cosh(d*x + c)^10 + 28*(65*
a^3*cosh(d*x + c)^4 + 26*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^12 + 112*
(39*a^3*cosh(d*x + c)^5 + 26*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sin
h(d*x + c)^11 + 70*a^3*cosh(d*x + c)^8 + 56*(143*a^3*cosh(d*x + c)^6 + 143*
a^3*cosh(d*x + c)^4 + 33*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^10 + 16*(
715*a^3*cosh(d*x + c)^7 + 1001*a^3*cosh(d*x + c)^5 + 385*a^3*cosh(d*x + c)^
3 + 35*a^3*cosh(d*x + c))*sinh(d*x + c)^9 + 56*a^3*cosh(d*x + c)^6 + 2*(643
5*a^3*cosh(d*x + c)^8 + 12012*a^3*cosh(d*x + c)...
```

Sympy [A]

time = 1.72, size = 178, normalized size = 1.73

$$\begin{cases} a^3x - \frac{a^3\log(\tanh(c+dx)+1)}{d} - \frac{a^3\tanh^2(c+dx)}{2d} - \frac{3a^2b\tanh^2(c+dx)\operatorname{sech}^2(c+dx)}{4d} - \frac{3a^2b\operatorname{sech}^2(c+dx)}{4d} - \frac{ab^2\tanh^2(c+dx)\operatorname{sech}^4(c+dx)}{2d} - \frac{ab^2\operatorname{sech}^4(c+dx)}{4d} - \frac{b^3\tanh^2(c+dx)\operatorname{sech}^6(c+dx)}{8d} - \frac{b^3\operatorname{sech}^6(c+dx)}{24d} & \text{for } d \neq 0 \\ x\left(a + b\operatorname{sech}^2(c)\right)^3\tanh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c)**3,x)
```

```
[Out] Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d - a**3*tanh(c + d*x)**2/(
2*d) - 3*a**2*b*tanh(c + d*x)**2*sech(c + d*x)**2/(4*d) - 3*a**2*b*sech(c +
 d*x)**2/(4*d) - a*b**2*tanh(c + d*x)**2*sech(c + d*x)**4/(2*d) - a*b**2*se
ch(c + d*x)**4/(4*d) - b**3*tanh(c + d*x)**2*sech(c + d*x)**6/(8*d) - b**3*
```

```
sech(c + d*x)**6/(24*d), Ne(d, 0)), (x*(a + b*sech(c)**2)**3*tanh(c)**3, Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(95) = 190.
time = 0.48, size = 387, normalized size = 3.76

$$-\frac{840\,(dx+c)a^3 - 840\,a^3\log\left(e^{(2\,dx+2\,c)}+1\right) + \frac{2283\,a^3e^{(16\,dx+16\,c)} + 16584\,a^3e^{(14\,dx+14\,c)} + 5040\,a^2be^{(14\,dx+14\,c)} + 53844\,a^3e^{(12\,dx+12\,c)} + 20160\,a^2be^{(12\,dx+12\,c)} + 10080\,ab^2e^{(12\,dx+12\,c)} + 102648\,a^3e^{(10\,dx+10\,c)} + 35280\,a^2be^{(10\,dx+10\,c)} + 13440\,ab^2e^{(10\,dx+10\,c)} + 8960\,b^3e^{(10\,dx+10\,c)} + 126210\,a^3e^{(8\,dx+8\,c)} + 40320\,a^2be^{(8\,dx+8\,c)} + 6720\,ab^2e^{(8\,dx+8\,c)} - 8960\,b^3e^{(8\,dx+8\,c)} + 102648\,a^3e^{(6\,dx+6\,c)} + 35280\,a^2be^{(6\,dx+6\,c)} + 13440\,ab^2e^{(6\,dx+6\,c)} + 8960\,b^3e^{(6\,dx+6\,c)} + 53844\,a^3e^{(4\,dx+4\,c)} + 20160\,a^2be^{(4\,dx+4\,c)} + 10080\,ab^2e^{(4\,dx+4\,c)} + 16584\,a^3e^{(2\,dx+2\,c)} + 5040\,a^2be^{(2\,dx+2\,c)} + 2283\,a^3}{\left(e^{(2\,dx+2\,c)}+1\right)^8}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/840*(840*(d*x + c)*a^3 - 840*a^3*log(e^(2*d*x + 2*c) + 1) + (2283*a^3*e^
(16*d*x + 16*c) + 16584*a^3*e^(14*d*x + 14*c) + 5040*a^2*b*e^(14*d*x + 14*c
) + 53844*a^3*e^(12*d*x + 12*c) + 20160*a^2*b*e^(12*d*x + 12*c) + 10080*a*b
^2*e^(12*d*x + 12*c) + 102648*a^3*e^(10*d*x + 10*c) + 35280*a^2*b*e^(10*d*x
 + 10*c) + 13440*a*b^2*e^(10*d*x + 10*c) + 8960*b^3*e^(10*d*x + 10*c) + 126
210*a^3*e^(8*d*x + 8*c) + 40320*a^2*b*e^(8*d*x + 8*c) + 6720*a*b^2*e^(8*d*x
 + 8*c) - 8960*b^3*e^(8*d*x + 8*c) + 102648*a^3*e^(6*d*x + 6*c) + 35280*a^2
*b*e^(6*d*x + 6*c) + 13440*a*b^2*e^(6*d*x + 6*c) + 8960*b^3*e^(6*d*x + 6*c)
 + 53844*a^3*e^(4*d*x + 4*c) + 20160*a^2*b*e^(4*d*x + 4*c) + 10080*a*b^2*e^
(4*d*x + 4*c) + 16584*a^3*e^(2*d*x + 2*c) + 5040*a^2*b*e^(2*d*x + 2*c) + 22
83*a^3)/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B]
time = 1.64, size = 573, normalized size = 5.56

$$\frac{32\,(3\,a\,b^2-5\,b^3)}{d\,(5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1)} - a^3\,x - \frac{128\,b^3}{d\,(7\,e^{2c+2dx}+21\,e^{4c+4dx}+35\,e^{6c+6dx}+35\,e^{8c+8dx}+21\,e^{10c+10dx}+7\,e^{12c+12dx}+e^{14c+14dx}+1)} - \frac{32\,(3\,a\,b^2-19\,b^3)}{3\,d\,(6\,e^{2c+2dx}+15\,e^{4c+4dx}+20\,e^{6c+6dx}+15\,e^{8c+8dx}+6\,e^{10c+10dx}+e^{12c+12dx}+1)} + \frac{32\,b^3}{d\,(8\,e^{2c+2dx}+28\,e^{4c+4dx}+56\,e^{6c+6dx}+70\,e^{8c+8dx}+56\,e^{10c+10dx}+28\,e^{12c+12dx}+8\,e^{14c+14dx}+e^{16c+16dx}+1)} - \frac{8\,(9\,a^2\,b-21\,a\,b^2+4\,b^3)}{3\,d\,(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1)} + \frac{4\,(3\,a^2\,b-27\,a\,b^2+16\,b^3)}{d\,(4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1)} - \frac{2\,(3\,a^2\,b-a^3)}{d\,(e^{2c+2dx}+1)} + \frac{a^3\ln(e^{2c}\,e^{2dx}+1)}{d} - \frac{2\,(a^3-9\,a^2\,b+6\,a\,b^2)}{d\,(2\,e^{2c+2dx}+e^{4c+4dx}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (32*(3*a*b^2 - 5*b^3))/(d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*ex
p(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - a^3*x - (1
28*b^3)/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x)
+ 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(
14*c + 14*d*x) + 1)) - (32*(3*a*b^2 - 19*b^3))/(3*d*(6*exp(2*c + 2*d*x) + 1
5*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c
 + 10*d*x) + exp(12*c + 12*d*x) + 1)) + (32*b^3)/(d*(8*exp(2*c + 2*d*x) + 2
8*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*
c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*
d*x) + 1)) - (8*(9*a^2*b - 21*a*b^2 + 4*b^3))/(3*d*(3*exp(2*c + 2*d*x) + 3*
exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (4*(3*a^2*b - 27*a*b^2 + 16*b^3
))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8
*c + 8*d*x) + 1)) - (2*(3*a^2*b - a^3))/(d*(exp(2*c + 2*d*x) + 1)) + (a^3*l
og(exp(2*c)*exp(2*d*x) + 1))/d - (2*(6*a*b^2 - 9*a^2*b + a^3))/(d*(2*exp(2*
c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.126 $\int \left(a+b\,\mathrm{sech}^2(c+dx)\right)^3 \tanh^2(c+dx)\,dx$

Optimal. Leaf size=92

$$a^3x-\frac{a^3\tanh(c+dx)}{d}+\frac{b(3a^2+3ab+b^2)\tanh^3(c+dx)}{3d}-\frac{b^2(3a+2b)\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^7(c+dx)}{7d}$$

```
[Out] a^3*x-a^3*tanh(d*x+c)/d+1/3*b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)^3/d-1/5*b^2*(3*
a+2*b)*tanh(d*x+c)^5/d+1/7*b^3*tanh(d*x+c)^7/d
```

Rubi [A]
time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4226, 1816, 212}

$$-\frac{a^3\tanh(c+dx)}{d}+a^3x+\frac{b(3a^2+3ab+b^2)\tanh^3(c+dx)}{3d}-\frac{b^2(3a+2b)\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^2,x]
```

```
[Out] a^3*x - (a^3*Tanh[c + d*x])/d + (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^3)/(
3*d) - (b^2*(3*a + 2*b)*Tanh[c + d*x]^5)/(5*d) + (b^3*Tanh[c + d*x]^7)/(7*d
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\mathrm{sech}^2(c+dx)\right)^3\tanh^2(c+dx)\,dx = \frac{\mathrm{Subst}\left(\int \frac{x^2\left(a+b(1-x^2)\right)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\mathrm{Subst}\left(\int \left(-a^3+b(3a^2+3ab+b^2)\,x^2-b^2(3a+2b)x^4+b^3x^6+\right.\right.}{d}$$
$$= -\frac{a^3\tanh(c+dx)}{d} + \frac{b(3a^2+3ab+b^2)\tanh^3(c+dx)}{3d} - \frac{b^2(3a+}{}$$
$$= a^3x - \frac{a^3\tanh(c+dx)}{d} + \frac{b(3a^2+3ab+b^2)\tanh^3(c+dx)}{3d} - \frac{b^2(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 479 vs. 2(92) = 184.
time = 1.21, size = 479, normalized size = 5.21

$$\frac{\mathrm{sech}(c)\mathrm{sech}^7(c+dx)\left(3675a^3dx\cosh(dx)+3675a^3dx\cosh(2c+dx)+2205a^3dx\cosh(2c+3dx)+2205a^3dx\cosh(4c+3dx)+735a^3dx\cosh(4c+5dx)+735a^3dx\cosh(6c+5dx)+105a^3dx\cosh(6c+7dx)+105a^3dx\cosh(8c+7dx)-4200a^3\sinh(dx)+3360a^2b\sinh(dx)+840ab^2\sinh(dx)-560b^3\sinh(dx)+3150a^3\sinh(2c+dx)-3990a^2b\sinh(2c+dx)-2100ab^2\sinh(2c+dx)-1120b^3\sinh(2c+dx)-3150a^3\sinh(2c+3dx)+1890a^2b\sinh(2c+3dx)+504ab^2\sinh(2c+3dx)+336b^3\sinh(2c+3dx)+1260a^3\sinh(4c+3dx)-2520a^2b\sinh(4c+3dx)-1260ab^2\sinh(4c+3dx)-1260a^3\sinh(4c+5dx)+840a^2b\sinh(4c+5dx)+588ab^2\sinh(4c+5dx)+112b^3\sinh(4c+5dx)+210a^3\sinh(6c+5dx)-630a^2b\sinh(6c+5dx)-210a^3\sinh(6c+7dx)+210a^2b\sinh(6c+7dx)+84ab^2\sinh(6c+7dx)+16b^3\sinh(6c+7dx)\right)}{13440d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x]^2,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^7*(3675*a^3*d*x*Cosh[d*x] + 3675*a^3*d*x*Cosh[2*c +
d*x] + 2205*a^3*d*x*Cosh[2*c + 3*d*x] + 2205*a^3*d*x*Cosh[4*c + 3*d*x] + 73
5*a^3*d*x*Cosh[4*c + 5*d*x] + 735*a^3*d*x*Cosh[6*c + 5*d*x] + 105*a^3*d*x*C
osh[6*c + 7*d*x] + 105*a^3*d*x*Cosh[8*c + 7*d*x] - 4200*a^3*Sinh[d*x] + 336
0*a^2*b*Sinh[d*x] + 840*a*b^2*Sinh[d*x] - 560*b^3*Sinh[d*x] + 3150*a^3*Sinh
[2*c + d*x] - 3990*a^2*b*Sinh[2*c + d*x] - 2100*a*b^2*Sinh[2*c + d*x] - 112
0*b^3*Sinh[2*c + d*x] - 3150*a^3*Sinh[2*c + 3*d*x] + 1890*a^2*b*Sinh[2*c +
3*d*x] + 504*a*b^2*Sinh[2*c + 3*d*x] + 336*b^3*Sinh[2*c + 3*d*x] + 1260*a^3
*Sinh[4*c + 3*d*x] - 2520*a^2*b*Sinh[4*c + 3*d*x] - 1260*a*b^2*Sinh[4*c + 3
*d*x] - 1260*a^3*Sinh[4*c + 5*d*x] + 840*a^2*b*Sinh[4*c + 5*d*x] + 588*a*b^
2*Sinh[4*c + 5*d*x] + 112*b^3*Sinh[4*c + 5*d*x] + 210*a^3*Sinh[6*c + 5*d*x]
 - 630*a^2*b*Sinh[6*c + 5*d*x] - 210*a^3*Sinh[6*c + 7*d*x] + 210*a^2*b*Sinh
[6*c + 7*d*x] + 84*a*b^2*Sinh[6*c + 7*d*x] + 16*b^3*Sinh[6*c + 7*d*x]))/(13
440*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(86) = 172.
time = 2.40, size = 353, normalized size = 3.84

method	result
risch	$a^3x + \frac{-\frac{4ab^2}{5}-6a^2b\,e^{12dx+12c}-24a^2b\,e^{10dx+10c}-12a\,b^2e^{10dx+10c}-18a^2b\,e^{4dx+4c}-8a^2b\,e^{2dx+2c}-2a^2b+2a^3-\frac{16b^3}{105}-20a\,b^2e^{8dx+8c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x+2/105*(-42*a*b^2-315*a^2*b*exp(12*d*x+12*c)-1260*a^2*b*exp(10*d*x+10*
c)-630*a*b^2*exp(10*d*x+10*c)-945*a^2*b*exp(4*d*x+4*c)-420*a^2*b*exp(2*d*x+
2*c)-105*a^2*b+105*a^3-8*b^3-1050*a*b^2*exp(8*d*x+8*c)-420*a*b^2*exp(6*d*x+
6*c)-252*a*b^2*exp(4*d*x+4*c)-1995*a^2*b*exp(8*d*x+8*c)-294*a*b^2*exp(2*d*x
+2*c)-1680*a^2*b*exp(6*d*x+6*c)+630*a^3*exp(2*d*x+2*c)+105*a^3*exp(12*d*x+1
2*c)+630*a^3*exp(10*d*x+10*c)-56*b^3*exp(2*d*x+2*c)-560*b^3*exp(8*d*x+8*c)+
1575*a^3*exp(4*d*x+4*c)-168*b^3*exp(4*d*x+4*c)+1575*a^3*exp(8*d*x+8*c)+2100
*a^3*exp(6*d*x+6*c)+280*b^3*exp(6*d*x+6*c))/d/(1+exp(2*d*x+2*c))^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(86) = 172$.
time = 0.28, size = 788, normalized size = 8.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] a^2*b*tanh(d*x + c)^3/d + a^3*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 16
/105*b^3*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) +
 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-
12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2
*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*
c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)
) - 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e
^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*
x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 70*e^(-8*d*x - 8*c)/(d*(7*e^(-2*d*x
- 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) +
21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1
/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^
(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x
- 14*c) + 1))) + 4/5*a*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*
c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) +
15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6
*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2
*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c
) + e^(-10*d*x - 10*c) + 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 881 vs. $2(86) = 172$.
time = 0.38, size = 881, normalized size = 9.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/105*((105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)
^7 + 7*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)
*sinh(d*x + c)^6 - (105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*sinh(d*x + c)^7
 + 7*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^5
 - 7*(75*a^3 - 15*a^2*b - 42*a*b^2 - 8*b^3 + 3*(105*a^3 - 105*a^2*b - 42*a*
b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 35*((105*a^3*d*x + 105*a^3
- 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + (105*a^3*d*x + 105*a^3 -
105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*a^3*
d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^3 - 7*(5*(105*a
^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^4 + 135*a^3 + 45*a^2*b + 5
4*a*b^2 - 24*b^3 + 10*(75*a^3 - 15*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^3 + 7*(3*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8
*b^3)*cosh(d*x + c)^5 + 10*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 -
8*b^3)*cosh(d*x + c)^3 + 9*(105*a^3*d*x + 105*a^3 - 105*a^2*b - 42*a*b^2 -
8*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^3*d*x + 105*a^3 - 105*a^2
*b - 42*a*b^2 - 8*b^3)*cosh(d*x + c) - 7*((105*a^3 - 105*a^2*b - 42*a*b^2 -
 8*b^3)*cosh(d*x + c)^6 + 5*(75*a^3 - 15*a^2*b - 42*a*b^2 - 8*b^3)*cosh(d*x
 + c)^4 + 75*a^3 + 45*a^2*b + 90*a*b^2 + 120*b^3 + 9*(45*a^3 + 15*a^2*b + 1
8*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*c
osh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3
+ d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x
 + c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*
cosh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^{2}\left(c + dx\right)\right)^{3}\tanh^{2}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3*tanh(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(86) = 172$.

time = 0.43, size = 359, normalized size = 3.90

$$\frac{105\,(dx+c)a^3+\frac{2\,\left(105\,a^3e^{(12\,dx+12\,c)}-315\,a^2be^{(12\,dx+12\,c)}+630\,a^3e^{(10\,dx+10\,c)}-1260\,a^2be^{(10\,dx+10\,c)}-630\,ab^2e^{(10\,dx+10\,c)}+1575\,a^3e^{(8\,dx+8\,c)}-1995\,a^2be^{(8\,dx+8\,c)}-1050\,ab^2e^{(8\,dx+8\,c)}-560\,b^3e^{(8\,dx+8\,c)}+2100\,a^3e^{(6\,dx+6\,c)}-1680\,a^2be^{(6\,dx+6\,c)}-420\,ab^2e^{(6\,dx+6\,c)}+280\,b^3e^{(6\,dx+6\,c)}+1575\,a^3e^{(4\,dx+4\,c)}-945\,a^2be^{(4\,dx+4\,c)}-252\,ab^2e^{(4\,dx+4\,c)}-168\,b^3e^{(4\,dx+4\,c)}+630\,a^3e^{(2\,dx+2\,c)}-420\,a^2be^{(2\,dx+2\,c)}-294\,ab^2e^{(2\,dx+2\,c)}-56\,b^3e^{(2\,dx+2\,c)}+105\,a^3-105\,a^2b-42\,ab^2-8\,b^3\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/105*(105*(d*x + c)*a^3 + 2*(105*a^3*e^(12*d*x + 12*c) - 315*a^2*b*e^(12*d
*x + 12*c) + 630*a^3*e^(10*d*x + 10*c) - 1260*a^2*b*e^(10*d*x + 10*c) - 630
*a*b^2*e^(10*d*x + 10*c) + 1575*a^3*e^(8*d*x + 8*c) - 1995*a^2*b*e^(8*d*x +
 8*c) - 1050*a*b^2*e^(8*d*x + 8*c) - 560*b^3*e^(8*d*x + 8*c) + 2100*a^3*e^(
6*d*x + 6*c) - 1680*a^2*b*e^(6*d*x + 6*c) - 420*a*b^2*e^(6*d*x + 6*c) + 280
*b^3*e^(6*d*x + 6*c) + 1575*a^3*e^(4*d*x + 4*c) - 945*a^2*b*e^(4*d*x + 4*c)
 - 252*a*b^2*e^(4*d*x + 4*c) - 168*b^3*e^(4*d*x + 4*c) + 630*a^3*e^(2*d*x +
 2*c) - 420*a^2*b*e^(2*d*x + 2*c) - 294*a*b^2*e^(2*d*x + 2*c) - 56*b^3*e^(2
*d*x + 2*c) + 105*a^3 - 105*a^2*b - 42*a*b^2 - 8*b^3)/(e^(2*d*x + 2*c) + 1)
^7)/d
```

Mupad [B]
time = 0.21, size = 1133, normalized size = 12.32

$$a^3x-\frac{\frac{2\,\left(-a^3+a^2b+2ab^2\right)}{7d}-\frac{2e^{2c+2dx}\left(15a^3+3a^2b-16b^3\right)}{21d}-\frac{4e^{6c+6dx}\left(15a^3+3a^2b-16b^3\right)}{21d}+\frac{10e^{8c+8dx}\left(-a^3+a^2b+2ab^2\right)}{7d}-\frac{4e^{4c+4dx}\left(5a^3+3a^2b+6ab^2+8b^3\right)}{7d}+\frac{2e^{10c+10dx}\left(3a^2b-a^3\right)}{7d}}{6\,e^{2c+2dx}+15\,e^{4c+4dx}+20\,e^{6c+6dx}+15\,e^{8c+8dx}+6\,e^{10c+10dx}+e^{12c+12dx}+1}-\frac{\frac{2\left(3a^2b-a^3\right)}{7d}-\frac{2e^{4c+4dx}\left(15a^3+3a^2b-16b^3\right)}{7d}-\frac{2e^{8c+8dx}\left(15a^3+3a^2b-16b^3\right)}{7d}+\frac{12e^{2c+2dx}\left(-a^3+a^2b+2ab^2\right)}{7d}+\frac{12e^{10c+10dx}\left(-a^3+a^2b+2ab^2\right)}{7d}-\frac{8e^{6c+6dx}\left(5a^3+3a^2b+6ab^2+8b^3\right)}{7d}+\frac{2e^{12c+12dx}\left(3a^2b-a^3\right)}{7d}}{7\,e^{2c+2dx}+21\,e^{4c+4dx}+35\,e^{6c+6dx}+35\,e^{8c+8dx}+21\,e^{10c+10dx}+7\,e^{12c+12dx}+e^{14c+14dx}+1}-\frac{\frac{4e^{2c+2dx}\left(-a^3+a^2b+2ab^2\right)}{7d}-\frac{2\left(15a^3+3a^2b-16b^3\right)}{105d}+\frac{2e^{4c+4dx}\left(3a^2b-a^3\right)}{7d}}{3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1}+\frac{\frac{2\left(5a^3+3a^2b+6ab^2+8b^3\right)}{35d}+\frac{2e^{2c+2dx}\left(15a^3+3a^2b-16b^3\right)}{35d}-\frac{6e^{4c+4dx}\left(-a^3+a^2b+2ab^2\right)}{7d}-\frac{2e^{6c+6dx}\left(3a^2b-a^3\right)}{7d}}{4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1}+\frac{\frac{2\left(15a^3+3a^2b-16b^3\right)}{105d}+\frac{4e^{4c+4dx}\left(15a^3+3a^2b-16b^3\right)}{35d}-\frac{8e^{6c+6dx}\left(-a^3+a^2b+2ab^2\right)}{7d}+\frac{8e^{2c+2dx}\left(5a^3+3a^2b+6ab^2+8b^3\right)}{35d}-\frac{2e^{8c+8dx}\left(3a^2b-a^3\right)}{7d}}{5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1}-\frac{\frac{2\left(-a^3+a^2b+2ab^2\right)}{7d}+\frac{2e^{2c+2dx}\left(3a^2b-a^3\right)}{7d}}{2\,e^{2c+2dx}+e^{4c+4dx}+1}-\frac{2\left(3a^2b-a^3\right)}{7d\left(e^{2c+2dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] a^3*x - ((2*(2*a*b^2 + a^2*b - a^3))/(7*d) - (2*exp(2*c + 2*d*x)*(3*a^2*b +
 15*a^3 - 16*b^3))/(21*d) - (4*exp(6*c + 6*d*x)*(3*a^2*b + 15*a^3 - 16*b^3)
)/(21*d) + (10*exp(8*c + 8*d*x)*(2*a*b^2 + a^2*b - a^3))/(7*d) - (4*exp(4*c
 + 4*d*x)*(6*a*b^2 + 3*a^2*b + 5*a^3 + 8*b^3))/(7*d) + (2*exp(10*c + 10*d*x
)*(3*a^2*b - a^3))/(7*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*ex
p(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12
*d*x) + 1) - ((2*(3*a^2*b - a^3))/(7*d) - (2*exp(4*c + 4*d*x)*(3*a^2*b + 15
*a^3 - 16*b^3))/(7*d) - (2*exp(8*c + 8*d*x)*(3*a^2*b + 15*a^3 - 16*b^3))/(7
*d) + (12*exp(2*c + 2*d*x)*(2*a*b^2 + a^2*b - a^3))/(7*d) + (12*exp(10*c +
10*d*x)*(2*a*b^2 + a^2*b - a^3))/(7*d) - (8*exp(6*c + 6*d*x)*(6*a*b^2 + 3*a
^2*b + 5*a^3 + 8*b^3))/(7*d) + (2*exp(12*c + 12*d*x)*(3*a^2*b - a^3))/(7*d)
)/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(
8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14
*d*x) + 1) - ((4*exp(2*c + 2*d*x)*(2*a*b^2 + a^2*b - a^3))/(7*d) - (2*(3*a^
2*b + 15*a^3 - 16*b^3))/(105*d) + (2*exp(4*c + 4*d*x)*(3*a^2*b - a^3))/(7*d
))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((2*(
6*a*b^2 + 3*a^2*b + 5*a^3 + 8*b^3))/(35*d) + (2*exp(2*c + 2*d*x)*(3*a^2*b +
 15*a^3 - 16*b^3))/(35*d) - (6*exp(4*c + 4*d*x)*(2*a*b^2 + a^2*b - a^3))/(7
*d) - (2*exp(6*c + 6*d*x)*(3*a^2*b - a^3))/(7*d))/(4*exp(2*c + 2*d*x) + 6*e
```

```
xp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + ((2*(3*a^2*b
 + 15*a^3 - 16*b^3))/(105*d) + (4*exp(4*c + 4*d*x)*(3*a^2*b + 15*a^3 - 16*b
^3))/(35*d) - (8*exp(6*c + 6*d*x)*(2*a*b^2 + a^2*b - a^3))/(7*d) + (8*exp(2
*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 5*a^3 + 8*b^3))/(35*d) - (2*exp(8*c + 8*d*
x)*(3*a^2*b - a^3))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(2*a*b
^2 + a^2*b - a^3))/(7*d) + (2*exp(2*c + 2*d*x)*(3*a^2*b - a^3))/(7*d))/(2*e
xp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (2*(3*a^2*b - a^3))/(7*d*(exp(2*c
 + 2*d*x) + 1))
```

3.127 $\int (a+b\text{sech}^2(c+dx))^3 \tanh(c+dx)\,dx$

Optimal. Leaf size=71

$$\frac{a^3\log(\cosh(c+dx))}{d}-\frac{3a^2b\text{sech}^2(c+dx)}{2d}-\frac{3ab^2\text{sech}^4(c+dx)}{4d}-\frac{b^3\text{sech}^6(c+dx)}{6d}$$

```
[Out] a^3*ln(cosh(d*x+c))/d-3/2*a^2*b*sech(d*x+c)^2/d-3/4*a*b^2*sech(d*x+c)^4/d-1
/6*b^3*sech(d*x+c)^6/d
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 272, 45}

$$\frac{a^3\log(\cosh(c+dx))}{d}-\frac{3a^2b\text{sech}^2(c+dx)}{2d}-\frac{3ab^2\text{sech}^4(c+dx)}{4d}-\frac{b^3\text{sech}^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x],x]
```

```
[Out] (a^3*Log[Cosh[c + d*x]])/d - (3*a^2*b*Sech[c + d*x]^2)/(2*d) - (3*a*b^2*Sec
h[c + d*x]^4)/(4*d) - (b^3*Sech[c + d*x]^6)/(6*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int (a+b\text{sech}^2(c+dx))^3\tanh(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{(b+ax^2)^3}{x^7}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b+ax)^3}{x^4}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^3}{x^4}+\frac{3ab^2}{x^3}+\frac{3a^2b}{x^2}+\frac{a^3}{x}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{a^3\log(\cosh(c+dx))}{d} - \frac{3a^2b\text{sech}^2(c+dx)}{2d} - \frac{3ab^2\text{sech}^4(c+dx)}{4d}$$

Mathematica [A]
time = 0.20, size = 100, normalized size = 1.41

$$\frac{2(b+a\cosh^2(c+dx))^3\left(-2b^3-9ab^2\cosh^2(c+dx)-18a^2b\cosh^4(c+dx)+12a^3\cosh^6(c+dx)\log(\cosh(c+dx))\right)\text{sech}^6(c+dx)}{3d(a+2b+a\cosh(2(c+dx)))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3*Tanh[c + d*x],x]
[Out] (2*(b + a*Cosh[c + d*x]^2)^3*(-2*b^3 - 9*a*b^2*Cosh[c + d*x]^2 - 18*a^2*b*C
osh[c + d*x]^4 + 12*a^3*Cosh[c + d*x]^6*Log[Cosh[c + d*x]])*Sech[c + d*x]^6
)/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [A]
time = 0.95, size = 59, normalized size = 0.83

method	result
derivativedivides	$-\frac{\frac{\text{sech}(dx+c)^6 b^3}{6}+\frac{3\text{sech}(dx+c)^4 a\,b^2}{4}+\frac{3\text{sech}(dx+c)^2 a^2 b}{2}+a^3\ln(\text{sech}(dx+c))}{d}$
default	$-\frac{\frac{\text{sech}(dx+c)^6 b^3}{6}+\frac{3\text{sech}(dx+c)^4 a\,b^2}{4}+\frac{3\text{sech}(dx+c)^2 a^2 b}{2}+a^3\ln(\text{sech}(dx+c))}{d}$
risch	$-a^3x-\frac{2a^3c}{d}-\frac{2b\,e^{2dx+2c}(9a^2e^{8dx+8c}+36a^2e^{6dx+6c}+18ab\,e^{6dx+6c}+54a^2e^{4dx+4c}+36ab\,e^{4dx+4c}+16b^2e^{4dx+4c}+36}{3d(1+e^{2dx+2c})^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x,method=_RETURNVERBOSE)
[Out] -1/d*(1/6*sech(d*x+c)^6*b^3+3/4*sech(d*x+c)^4*a*b^2+3/2*sech(d*x+c)^2*a^2*b
+a^3*ln(sech(d*x+c)))
```

Maxima [A]
time = 0.26, size = 85, normalized size = 1.20

$$\frac{3\,a^2b\tanh(dx+c)^2}{2\,d}+\frac{a^3\log(\cosh(dx+c))}{d}-\frac{12\,ab^2}{d(e^{(dx+c)}+e^{(-dx-c)})^4}-\frac{32\,b^3}{3\,d(e^{(dx+c)}+e^{(-dx-c)})^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] 3/2*a^2*b*tanh(d*x + c)^2/d + a^3*log(cosh(d*x + c))/d - 12*a*b^2/(d*(e^(d*
x + c) + e^(-d*x - c))^4) - 32/3*b^3/(d*(e^(d*x + c) + e^(-d*x - c))^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2519 vs. $2(65) = 130$.

time = 0.37, size = 2519, normalized size = 35.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^3*d*x*cosh(d*x + c)^12 + 36*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^1
1 + 3*a^3*d*x*sinh(d*x + c)^12 + 18*(a^3*d*x + a^2*b)*cosh(d*x + c)^10 + 18
*(11*a^3*d*x*cosh(d*x + c)^2 + a^3*d*x + a^2*b)*sinh(d*x + c)^10 + 60*(11*a
^3*d*x*cosh(d*x + c)^3 + 3*(a^3*d*x + a^2*b)*cosh(d*x + c))*sinh(d*x + c)^9
 + 9*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^8 + 9*(165*a^3*d*x*cosh(
d*x + c)^4 + 5*a^3*d*x + 8*a^2*b + 4*a*b^2 + 90*(a^3*d*x + a^2*b)*cosh(d*x
+ c)^2)*sinh(d*x + c)^8 + 72*(33*a^3*d*x*cosh(d*x + c)^5 + 30*(a^3*d*x + a^
2*b)*cosh(d*x + c)^3 + (5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c))*sinh(
d*x + c)^7 + 4*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^6 +
 4*(693*a^3*d*x*cosh(d*x + c)^6 + 15*a^3*d*x + 945*(a^3*d*x + a^2*b)*cosh(d
*x + c)^4 + 27*a^2*b + 18*a*b^2 + 8*b^3 + 63*(5*a^3*d*x + 8*a^2*b + 4*a*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(99*a^3*d*x*cosh(d*x + c)^7 + 189*(
a^3*d*x + a^2*b)*cosh(d*x + c)^5 + 21*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(
d*x + c)^3 + (15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh
(d*x + c)^5 + 3*a^3*d*x + 9*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^4
 + 3*(495*a^3*d*x*cosh(d*x + c)^8 + 1260*(a^3*d*x + a^2*b)*cosh(d*x + c)^6
+ 15*a^3*d*x + 210*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^4 + 24*a^2
*b + 12*a*b^2 + 20*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 + 4*(165*a^3*d*x*cosh(d*x + c)^9 + 540*(a^3*d*x + a^2*b
)*cosh(d*x + c)^7 + 126*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^5 + 2
0*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^3 + 9*(5*a^3*d*x
 + 8*a^2*b + 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 18*(a^3*d*x + a^2*b)
*cosh(d*x + c)^2 + 6*(33*a^3*d*x*cosh(d*x + c)^10 + 135*(a^3*d*x + a^2*b)*c
osh(d*x + c)^8 + 42*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^6 + 3*a^3
*d*x + 10*(15*a^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 3*a^
2*b + 9*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 -
3*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(d*x + c)^11 + a^3*sinh(
d*x + c)^12 + 6*a^3*cosh(d*x + c)^10 + 15*a^3*cosh(d*x + c)^8 + 6*(11*a^3*c
osh(d*x + c)^2 + a^3)*sinh(d*x + c)^10 + 20*(11*a^3*cosh(d*x + c)^3 + 3*a^3
*cosh(d*x + c))*sinh(d*x + c)^9 + 20*a^3*cosh(d*x + c)^6 + 15*(33*a^3*cosh(
```

```
d*x + c)^4 + 18*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^8 + 24*(33*a^3*cos
h(d*x + c)^5 + 30*a^3*cosh(d*x + c)^3 + 5*a^3*cosh(d*x + c))*sinh(d*x + c)^
7 + 15*a^3*cosh(d*x + c)^4 + 4*(231*a^3*cosh(d*x + c)^6 + 315*a^3*cosh(d*x
+ c)^4 + 105*a^3*cosh(d*x + c)^2 + 5*a^3)*sinh(d*x + c)^6 + 24*(33*a^3*cosh
(d*x + c)^7 + 63*a^3*cosh(d*x + c)^5 + 35*a^3*cosh(d*x + c)^3 + 5*a^3*cosh(
d*x + c))*sinh(d*x + c)^5 + 6*a^3*cosh(d*x + c)^2 + 15*(33*a^3*cosh(d*x + c
)^8 + 84*a^3*cosh(d*x + c)^6 + 70*a^3*cosh(d*x + c)^4 + 20*a^3*cosh(d*x + c
)^2 + a^3)*sinh(d*x + c)^4 + 20*(11*a^3*cosh(d*x + c)^9 + 36*a^3*cosh(d*x +
 c)^7 + 42*a^3*cosh(d*x + c)^5 + 20*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x +
c))*sinh(d*x + c)^3 + a^3 + 6*(11*a^3*cosh(d*x + c)^10 + 45*a^3*cosh(d*x +
c)^8 + 70*a^3*cosh(d*x + c)^6 + 50*a^3*cosh(d*x + c)^4 + 15*a^3*cosh(d*x +
c)^2 + a^3)*sinh(d*x + c)^2 + 12*(a^3*cosh(d*x + c)^11 + 5*a^3*cosh(d*x + c
)^9 + 10*a^3*cosh(d*x + c)^7 + 10*a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)
^3 + a^3*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))) + 12*(3*a^3*d*x*cosh(d*x + c)^11 + 15*(a^3*d*x + a^2*b)*co
sh(d*x + c)^9 + 6*(5*a^3*d*x + 8*a^2*b + 4*a*b^2)*cosh(d*x + c)^7 + 2*(15*a
^3*d*x + 27*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^5 + 3*(5*a^3*d*x + 8*a^
2*b + 4*a*b^2)*cosh(d*x + c)^3 + 3*(a^3*d*x + a^2*b)*cosh(d*x + c))*sinh(d*
x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(
d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 1
5*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)*
sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*cos
h(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)
^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6 +
 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 5
*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*
x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)
^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c)^7 +
 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x
+ c)^3 + 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c
)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 +
d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 + 10*d*co
sh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c
))*sinh(d*x + c) + d)
```

Sympy [A]
time = 0.87, size = 87, normalized size = 1.23

$$\begin{cases} a^3x - \frac{a^3\log(\tanh(c+dx)+1)}{d} - \frac{3a^2b\operatorname{sech}^2(c+dx)}{2d} - \frac{3ab^2\operatorname{sech}^4(c+dx)}{4d} - \frac{b^3\operatorname{sech}^6(c+dx)}{6d} & \text{for } d \neq 0 \\ x\left(a+b\operatorname{sech}^2(c)\right)^3\tanh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3*tanh(d*x+c),x)
```

```
[Out] Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d - 3*a**2*b*sech(c + d*x)*
*2/(2*d) - 3*a*b**2*sech(c + d*x)**4/(4*d) - b**3*sech(c + d*x)**6/(6*d), N
e(d, 0)), (x*(a + b*sech(c)**2)**3*tanh(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(65) = 130$.
time = 0.43, size = 271, normalized size = 3.82

$$-\frac{60\,(dx+c)a^3-60\,a^3\log\left(e^{(2\,dx+2\,c)}+1\right)+\frac{147\,a^3e^{(12\,dx+12\,c)}+882\,a^3e^{(10\,dx+10\,c)}+360\,a^2be^{(10\,dx+10\,c)}+2205\,a^3e^{(8\,dx+8\,c)}+1440\,a^2be^{(8\,dx+8\,c)}+720\,ab^2e^{(8\,dx+8\,c)}+2940\,a^3e^{(6\,dx+6\,c)}+2160\,a^2be^{(6\,dx+6\,c)}+1440\,ab^2e^{(6\,dx+6\,c)}+640\,b^3e^{(6\,dx+6\,c)}+2205\,a^3e^{(4\,dx+4\,c)}+1440\,a^2be^{(4\,dx+4\,c)}+720\,ab^2e^{(4\,dx+4\,c)}+882\,a^3e^{(2\,dx+2\,c)}+360\,a^2be^{(2\,dx+2\,c)}+147\,a^3}{\left(e^{(2\,dx+2\,c)}+1\right)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] -1/60*(60*(d*x + c)*a^3 - 60*a^3*log(e^(2*d*x + 2*c) + 1) + (147*a^3*e^(12*
d*x + 12*c) + 882*a^3*e^(10*d*x + 10*c) + 360*a^2*b*e^(10*d*x + 10*c) + 220
5*a^3*e^(8*d*x + 8*c) + 1440*a^2*b*e^(8*d*x + 8*c) + 720*a*b^2*e^(8*d*x + 8
*c) + 2940*a^3*e^(6*d*x + 6*c) + 2160*a^2*b*e^(6*d*x + 6*c) + 1440*a*b^2*e^
(6*d*x + 6*c) + 640*b^3*e^(6*d*x + 6*c) + 2205*a^3*e^(4*d*x + 4*c) + 1440*a
^2*b*e^(4*d*x + 4*c) + 720*a*b^2*e^(4*d*x + 4*c) + 882*a^3*e^(2*d*x + 2*c)
+ 360*a^2*b*e^(2*d*x + 2*c) + 147*a^3)/(e^(2*d*x + 2*c) + 1)^6)/d
```

Mupad [B]
time = 0.21, size = 347, normalized size = 4.89

$$\frac{32\,b^3}{3\,d\,\left(6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1\right)}-\frac{4\,\left(3\,a\,b^2-8\,b^3\right)}{d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}-a^3\,x-\frac{6\,\left(2\,a\,b^2-a^2\,b\right)}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}+\frac{a^3\,\ln\left(e^{2\,c}\,e^{2\,d\,x}+1\right)}{d}+\frac{8\,\left(9\,a\,b^2-4\,b^3\right)}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}-\frac{32\,b^3}{d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}-\frac{6\,a^2\,b}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (32*b^3)/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*
x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1))
- (4*(3*a*b^2 - 8*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp
(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - a^3*x - (6*(2*a*b^2 - a^2*b))/(d*(
2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (a^3*log(exp(2*c)*exp(2*d*x)
+ 1))/d + (8*(9*a*b^2 - 4*b^3))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*
x) + exp(6*c + 6*d*x) + 1)) - (32*b^3)/(d*(5*exp(2*c + 2*d*x) + 10*exp(4*c
+ 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) +
1)) - (6*a^2*b)/(d*(exp(2*c + 2*d*x) + 1))
```

3.128 $\int \left(a+b\text{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=73

$$a^3x+\frac{b(3a^2+3ab+b^2)\tanh(c+dx)}{d}-\frac{b^2(3a+2b)\tanh^3(c+dx)}{3d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

```
[Out] a^3*x+b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(3*a+2*b)*tanh(d*x+c)^3/d+1
/5*b^3*tanh(d*x+c)^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {4213, 398, 212}

$$a^3x+\frac{b(3a^2+3ab+b^2)\tanh(c+dx)}{d}-\frac{b^2(3a+2b)\tanh^3(c+dx)}{3d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] a^3*x + (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(3*a + 2*b)*Tanh[c
 + d*x]^3)/(3*d) + (b^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b(3a^2+3ab+b^2)-b^2(3a+2b)x^2+b^3x^4+\frac{a^3}{1-x^2}\right)\,dx, x, \tanh(c+\right.}{d}$$
$$= \frac{b(3a^2+3ab+b^2)\tanh(c+dx)}{d} - \frac{b^2(3a+2b)\tanh^3(c+dx)}{3d} + \frac{b^3\tanh^5(c+}{5d}$$
$$= a^3x + \frac{b(3a^2+3ab+b^2)\tanh(c+dx)}{d} - \frac{b^2(3a+2b)\tanh^3(c+dx)}{3d} + \frac{b^3\tanh}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 268 vs. 2(73) = 146.
time = 0.62, size = 268, normalized size = 3.67

$$\frac{\operatorname{sech}(c)\operatorname{sech}^5(c+dx)\left(150a^3dx\cosh(dx)+150a^3dx\cosh(2c+dx)+75a^3dx\cosh(2c+3dx)+75a^3dx\cosh(4c+3dx)+15a^3dx\cosh(4c+5dx)+15a^3dx\cosh(6c+5dx)+540a^2b\sinh(dx)+420ab^2\sinh(dx)+160b^3\sinh(dx)-360a^2b\sinh(2c+dx)-180ab^2\sinh(2c+dx)+360a^2b\sinh(2c+3dx)+300ab^2\sinh(2c+3dx)+80b^3\sinh(2c+3dx)-90a^2b\sinh(4c+3dx)+90a^2b\sinh(4c+5dx)+60ab^2\sinh(4c+5dx)+16b^3\sinh(4c+5dx)\right)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^5*(150*a^3*d*x*Cosh[d*x] + 150*a^3*d*x*Cosh[2*c + d*
x] + 75*a^3*d*x*Cosh[2*c + 3*d*x] + 75*a^3*d*x*Cosh[4*c + 3*d*x] + 15*a^3*d
*x*Cosh[4*c + 5*d*x] + 15*a^3*d*x*Cosh[6*c + 5*d*x] + 540*a^2*b*Sinh[d*x] +
 420*a*b^2*Sinh[d*x] + 160*b^3*Sinh[d*x] - 360*a^2*b*Sinh[2*c + d*x] - 180*
a*b^2*Sinh[2*c + d*x] + 360*a^2*b*Sinh[2*c + 3*d*x] + 300*a*b^2*Sinh[2*c +
3*d*x] + 80*b^3*Sinh[2*c + 3*d*x] - 90*a^2*b*Sinh[4*c + 3*d*x] + 90*a^2*b*S
inh[4*c + 5*d*x] + 60*a*b^2*Sinh[4*c + 5*d*x] + 16*b^3*Sinh[4*c + 5*d*x]))/
(480*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(69) = 138.
time = 1.70, size = 164, normalized size = 2.25

method	result
risch	$a^3x - \frac{2b\left(45a^2 e^{8dx+8c}+180a^2 e^{6dx+6c}+90ab\, e^{6dx+6c}+270a^2 e^{4dx+4c}+210ab\, e^{4dx+4c}+80b^2 e^{4dx+4c}+180a^2 e^{2dx+2c}+150ab\, e^{2dx+2}\right.}{15d\left(1+e^{2dx+2c}\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x-2/15*b*(45*a^2*exp(8*d*x+8*c)+180*a^2*exp(6*d*x+6*c)+90*a*b*exp(6*d*x
+6*c)+270*a^2*exp(4*d*x+4*c)+210*a*b*exp(4*d*x+4*c)+80*b^2*exp(4*d*x+4*c)+1
```

80*a^2*exp(2*d*x+2*c)+150*a*b*exp(2*d*x+2*c)+40*b^2*exp(2*d*x+2*c)+45*a^2+3
0*a*b+8*b^2)/d/(1+exp(2*d*x+2*c))^5

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(69) = 138$.
time = 0.28, size = 332, normalized size = 4.55

$$a^3x + \frac{16}{15}b^3\left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} + \frac{10e^{(-4dx-4c)}}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} + \frac{1}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)}\right) + 4ab^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} + \frac{1}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}\right) + \frac{6a^2b}{d(e^{(-2dx-2c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] a^3*x + 16/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1
)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*
e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*
e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x
- 8*c) + e^(-10*d*x - 10*c) + 1))) + 4*a*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-
2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*
d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 6*a^2*b/(d*(e^(
-2*d*x - 2*c) + 1))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(69) = 138$.
time = 0.41, size = 470, normalized size = 6.44

$$\frac{(15a^3dx - 45a^2b - 30ab^2 - 8b^3)\cosh(dx+c)^5 + 5(15a^3dx - 45a^2b - 30ab^2 - 8b^3)\cosh(dx+c)\sinh(dx+c)^4 + (45a^2b + 30ab^2 + 8b^3)\sinh(dx+c)^5 + 5(15a^3dx - 45a^2b - 30ab^2 - 8b^3)\cosh(dx+c)^3 + 5(27a^2b + 30ab^2 + 8b^3 + 2(45a^2b + 30ab^2 + 8b^3)\cosh(dx+c)^2)\sinh(dx+c)^3 + 5(2(15a^3dx - 45a^2b - 30ab^2 - 8b^3)\cosh(dx+c)^3 + 3(15a^3dx - 45a^2b - 30ab^2 - 8b^3)\cosh(dx+c))\sinh(dx+c)^2 + 10(15a^3dx - 45a^2b - 30ab^2 - 8b^3)\cosh(dx+c) + 5((45a^2b + 30ab^2 + 8b^3)\cosh(dx+c)^4 + 18a^2b + 24ab^2 + 16b^3 + 3(27a^2b + 30ab^2 + 8b^3)\cosh(dx+c)^2)\sinh(dx+c)}{15(d\cosh(dx+c)^5 + 5d\cosh(dx+c)\sinh(dx+c)^4 + 5d\cosh(dx+c)^3 + 5(2d\cosh(dx+c)^3 + 3d\cosh(dx+c))\sinh(dx+c)^2 + 10d\cosh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/15*((15*a^3*d*x - 45*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)^5 + 5*(15*a^
3*d*x - 45*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (45*a^
2*b + 30*a*b^2 + 8*b^3)*sinh(d*x + c)^5 + 5*(15*a^3*d*x - 45*a^2*b - 30*a*b
^2 - 8*b^3)*cosh(d*x + c)^3 + 5*(27*a^2*b + 30*a*b^2 + 8*b^3 + 2*(45*a^2*b
+ 30*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 5*(2*(15*a^3*d*x - 4
5*a^2*b - 30*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 3*(15*a^3*d*x - 45*a^2*b - 30
*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(15*a^3*d*x - 45*a^2*b
- 30*a*b^2 - 8*b^3)*cosh(d*x + c) + 5*((45*a^2*b + 30*a*b^2 + 8*b^3)*cosh(d
*x + c)^4 + 18*a^2*b + 24*a*b^2 + 16*b^3 + 3*(27*a^2*b + 30*a*b^2 + 8*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh
(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x +
 c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(c + dx)\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(69) = 138.
time = 0.41, size = 182, normalized size = 2.49

$$\frac{15\,(dx+c)a^3-\frac{2\,\left(45\,a^2be^{(8\,dx+8\,c)}+180\,a^2be^{(6\,dx+6\,c)}+90\,ab^2e^{(6\,dx+6\,c)}+270\,a^2be^{(4\,dx+4\,c)}+210\,ab^2e^{(4\,dx+4\,c)}+80\,b^3e^{(4\,dx+4\,c)}+180\,a^2be^{(2\,dx+2\,c)}+150\,ab^2e^{(2\,dx+2\,c)}+40\,b^3e^{(2\,dx+2\,c)}+45\,a^2b+30\,ab^2+8\,b^3\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(d*x + c)*a^3 - 2*(45*a^2*b*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x +
 6*c) + 90*a*b^2*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) + 210*a*b^2*e^
(4*d*x + 4*c) + 80*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) + 150*a*
b^2*e^(2*d*x + 2*c) + 40*b^3*e^(2*d*x + 2*c) + 45*a^2*b + 30*a*b^2 + 8*b^3)
/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B]
time = 1.44, size = 502, normalized size = 6.88

$$a^3\,x-\frac{\frac{2\,\left(9\,a^2\,b+12\,a\,b^2+8\,b^3\right)}{15\,d}+\frac{12\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{6\,a^2\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}-\frac{\frac{6\,a^2\,b}{5\,d}+\frac{24\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{24\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(9\,a^2\,b+12\,a\,b^2+8\,b^3\right)}{5\,d}+\frac{6\,a^2\,b\,{\mathrm{e}}^{8\,c+8\,d\,x}}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}+5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}+1}-\frac{\frac{6\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{18\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(9\,a^2\,b+12\,a\,b^2+8\,b^3\right)}{5\,d}+\frac{6\,a^2\,b\,{\mathrm{e}}^{6\,c+6\,d\,x}}{5\,d}}{4\,{\mathrm{e}}^{2\,c+2\,d\,x}+6\,{\mathrm{e}}^{4\,c+4\,d\,x}+4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{\frac{6\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{6\,a^2\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{6\,a^2\,b}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] a^3*x - ((2*(12*a*b^2 + 9*a^2*b + 8*b^3))/(15*d) + (12*exp(2*c + 2*d*x)*(a*
b^2 + a^2*b))/(5*d) + (6*a^2*b*exp(4*c + 4*d*x))/(5*d))/(3*exp(2*c + 2*d*x)
 + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((6*a^2*b)/(5*d) + (24*exp(
2*c + 2*d*x)*(a*b^2 + a^2*b))/(5*d) + (24*exp(6*c + 6*d*x)*(a*b^2 + a^2*b))
/(5*d) + (4*exp(4*c + 4*d*x)*(12*a*b^2 + 9*a^2*b + 8*b^3))/(5*d) + (6*a^2*b
*exp(8*c + 8*d*x))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*ex
p(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((6*(a*b^2
+ a^2*b))/(5*d) + (18*exp(4*c + 4*d*x)*(a*b^2 + a^2*b))/(5*d) + (2*exp(2*c
+ 2*d*x)*(12*a*b^2 + 9*a^2*b + 8*b^3))/(5*d) + (6*a^2*b*exp(6*c + 6*d*x))/(
5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8
*c + 8*d*x) + 1) - ((6*(a*b^2 + a^2*b))/(5*d) + (6*a^2*b*exp(2*c + 2*d*x))/
(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (6*a^2*b)/(5*d*(exp(2*
c + 2*d*x) + 1))
```

3.129 $\int \coth(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=84

$$-\frac{b(3a^2+3ab+b^2)\log(\cosh(c+dx))}{d}+\frac{(a+b)^3\log(\sinh(c+dx))}{d}+\frac{b^2(3a+b)\mathrm{sech}^2(c+dx)}{2d}+\frac{b^3\mathrm{sech}^4(c+dx)}{4d}$$

```
[Out] -b*(3*a^2+3*a*b+b^2)*ln(cosh(d*x+c))/d+(a+b)^3*ln(sinh(d*x+c))/d+1/2*b^2*(3
*a+b)*sech(d*x+c)^2/d+1/4*b^3*sech(d*x+c)^4/d
```

Rubi [A]
time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 457, 90}

$$-\frac{b(3a^2+3ab+b^2)\log(\cosh(c+dx))}{d}+\frac{b^2(3a+b)\mathrm{sech}^2(c+dx)}{2d}+\frac{(a+b)^3\log(\sinh(c+dx))}{d}+\frac{b^3\mathrm{sech}^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -((b*(3*a^2 + 3*a*b + b^2)*Log[Cosh[c + d*x]])/d) + ((a + b)^3*Log[Sinh[c +
 d*x]])/d + (b^2*(3*a + b)*Sech[c + d*x]^2)/(2*d) + (b^3*Sech[c + d*x]^4)/(
4*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^3}{x^5(1-x^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+ax)^3}{(1-x)x^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^3}{-1+x}+\frac{b^3}{x^3}+\frac{b^2(3a+b)}{x^2}+\frac{b(3a^2+3ab+b^2)}{x}\right)\,dx, x, \cosh\right.}{2d}$$
$$= -\frac{b(3a^2+3ab+b^2)\log(\cosh(c+dx))}{d}+\frac{(a+b)^3\log(\sinh(c+d}{d}$$

Mathematica [A]

time = 0.44, size = 114, normalized size = 1.36

$$-\frac{2\cosh^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\left(4b(3a^2+3ab+b^2)\log(\cosh(c+dx))-4(a+b)^3\log(\sinh(c+dx))-2b^2(3a+b)\text{sech}^2(c+dx)-b^3\text{sech}^4(c+dx)\right)}{d(a+2b+a\cosh(2c+2dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-2*Cosh[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3*(4*b*(3*a^2 + 3*a*b + b^2)*Lo
g[Cosh[c + d*x]] - 4*(a + b)^3*Log[Sinh[c + d*x]] - 2*b^2*(3*a + b)*Sech[c
+ d*x]^2 - b^3*Sech[c + d*x]^4))/(d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)
```

Maple [A]

time = 1.83, size = 86, normalized size = 1.02

method	result
derivativedivides	$\frac{a^3\ln(\sinh(dx+c))+3a^2b\ln(\tanh(dx+c))+3ab^2\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+b^3\left(\frac{1}{4\cosh(dx+c)^4}+\frac{1}{2\cosh(dx+c)^2}+\ln\right.}{d}$
default	$\frac{a^3\ln(\sinh(dx+c))+3a^2b\ln(\tanh(dx+c))+3ab^2\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+b^3\left(\frac{1}{4\cosh(dx+c)^4}+\frac{1}{2\cosh(dx+c)^2}+\ln\right.}{d}$
risch	$-a^3x-\frac{2a^3c}{d}+\frac{2b^2e^{2dx+2c}\left(3a\,e^{4dx+4c}+b\,e^{4dx+4c}+6a\,e^{2dx+2c}+4b\,e^{2dx+2c}+3a+b\right)}{d\left(1+e^{2dx+2c}\right)^4}+\frac{\ln\left(e^{2dx+2c}-1\right)a^3}{d}+\frac{3\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*ln(sinh(d*x+c))+3*a^2*b*ln(tanh(d*x+c))+3*a*b^2*(1/2/cosh(d*x+c)^2
+ln(tanh(d*x+c)))+b^3*(1/4/cosh(d*x+c)^4+1/2/cosh(d*x+c)^2+ln(tanh(d*x+c)))
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(80) = 160$.
time = 0.49, size = 300, normalized size = 3.57

$$b^{3}\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\left(e^{(-2\,dx-2\,c)}+4\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)+3\,ab^{2}\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+3\,a^{2}b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}\right)+\frac{a^{3}\log\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] b^3*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*
c) + 1)/d + 2*(e^(-2*d*x - 2*c) + 4*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d
*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x
- 8*c) + 1))) + 3*a*b^2*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d
- log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) +
 e^(-4*d*x - 4*c) + 1))) + 3*a^2*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x -
 c) - 1)/d - log(e^(-2*d*x - 2*c) + 1)/d) + a^3*log(sinh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2376 vs. $2(80) = 160$.
time = 0.38, size = 2376, normalized size = 28.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -(a^3*d*x*cosh(d*x + c)^8 + 8*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*d
*x*sinh(d*x + c)^8 + 2*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c)^6 + 2*(14*
a^3*d*x*cosh(d*x + c)^2 + 2*a^3*d*x - 3*a*b^2 - b^3)*sinh(d*x + c)^6 + 4*(1
4*a^3*d*x*cosh(d*x + c)^3 + 3*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c))*si
nh(d*x + c)^5 + a^3*d*x + 2*(3*a^3*d*x - 6*a*b^2 - 4*b^3)*cosh(d*x + c)^4 +
 2*(35*a^3*d*x*cosh(d*x + c)^4 + 3*a^3*d*x - 6*a*b^2 - 4*b^3 + 15*(2*a^3*d*
x - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^3*d*x*cosh(d*x
 + c)^5 + 5*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + (3*a^3*d*x - 6*a*
b^2 - 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(2*a^3*d*x - 3*a*b^2 - b^3)
*cosh(d*x + c)^2 + 2*(14*a^3*d*x*cosh(d*x + c)^6 + 2*a^3*d*x + 15*(2*a^3*d*
x - 3*a*b^2 - b^3)*cosh(d*x + c)^4 - 3*a*b^2 - b^3 + 6*(3*a^3*d*x - 6*a*b^2
 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^8 + 8*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^7 +
(3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^8 + 4*(3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^6 + 4*(3*a^2*b + 3*a*b^2 + b^3 + 7*(3*a^2*b + 3*a*b^2 + b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c
)^3 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(3*a^2*b + 3*a*b^2 + b^3)*cosh(
d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(
```

```
d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^
5 + 10*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(3*a^2*b + 3*a*b^2 + b
^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^2*b + 3*a*b^2 + b^3 + 4*(3*a^2*b +
 3*a*b^2 + b^3)*cosh(d*x + c)^2 + 4*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x +
 c)^6 + 15*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 3*a^2*b + 3*a*b^2 +
b^3 + 9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*
a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d
*x + c)^5 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (3*a^2*b + 3*a*b^
2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))) - ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 8*(a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*sinh(d*x + c)^8 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(
d*x + c)^6 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 7*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh
(d*x + c)^5 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(a^
3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 3*a^3 + 9*a^2*b + 9*a*b^2 +
3*b^3 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4
 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 10*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(
d*x + c))*sinh(d*x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 4*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*cosh(d*x + c)^2 + 4*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^6 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 +
 3*a^2*b + 3*a*b^2 + b^3 + 9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 8*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 3
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 3*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*
sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*
a^3*d*x*cosh(d*x + c)^7 + 3*(2*a^3*d*x - 3*a*b^2 - b^3)*cosh(d*x + c)^5 + 2
*(3*a^3*d*x - 6*a*b^2 - 4*b^3)*cosh(d*x + c)^3 + (2*a^3*d*x - 3*a*b^2 - b^3
)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh
(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c
)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh
(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x
 + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c
)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cos
h(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c
)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*
cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(80) = 160.
time = 0.42, size = 283, normalized size = 3.37

$$-\frac{2\,(3\,a^2b+3\,ab^2+b^3)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)-2\,(a^3+3\,a^2b+3\,ab^2+b^3)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)-\frac{9\,a^2b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+9\,ab^2\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+3\,b^3\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+36\,a^2b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+60\,ab^2\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+20\,b^3\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+36\,a^2b+84\,ab^2+44\,b^3}{\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*(3*a^2*b + 3*a*b^2 + b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) +
2) - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*
c) - 2) - (9*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 9*a*b^2*(e^(2*d
*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)
)^2 + 36*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 60*a*b^2*(e^(2*d*x +
2*c) + e^(-2*d*x - 2*c)) + 20*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 36
*a^2*b + 84*a*b^2 + 44*b^3)/(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2)^2)/d
```

Mupad [B]
time = 1.64, size = 360, normalized size = 4.29

$$\frac{2\,(b^3+3\,a\,b^2)}{d\,(e^{2c+2dx}+1)}-a^3\,x-\frac{\mathrm{atan}\left(\frac{e^{2c}\,e^{2dx}\,\left(a^3\,\sqrt{-d^2}+2\,b^3\,\sqrt{-d^2}+6\,a\,b^2\,\sqrt{-d^2}+6\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6+12\,a^5\,b+48\,a^4\,b^2+76\,a^3\,b^3+60\,a^2\,b^4+24\,a\,b^5+4\,b^6}}\right)\,\sqrt{a^6+12\,a^5\,b+48\,a^4\,b^2+76\,a^3\,b^3+60\,a^2\,b^4+24\,a\,b^5+4\,b^6}}{\sqrt{-d^2}}+\frac{a^3\,\ln\left(e^{4c+4dx}-1\right)}{2\,d}-\frac{8\,b^3}{d\,(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1)}-\frac{2\,(3\,a\,b^2-b^3)}{d\,(2\,e^{2c+2dx}+e^{4c+4dx}+1)}+\frac{4\,b^3}{d\,(4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (2*(3*a*b^2 + b^3))/(d*(exp(2*c + 2*d*x) + 1)) - a^3*x - (atan((exp(2*c)*ex
p(2*d*x)*(a^3*(-d^2)^(1/2) + 2*b^3*(-d^2)^(1/2) + 6*a*b^2*(-d^2)^(1/2) + 6*
a^2*b*(-d^2)^(1/2)))/(d*(24*a*b^5 + 12*a^5*b + a^6 + 4*b^6 + 60*a^2*b^4 + 7
6*a^3*b^3 + 48*a^4*b^2)^(1/2)))*(24*a*b^5 + 12*a^5*b + a^6 + 4*b^6 + 60*a^2
*b^4 + 76*a^3*b^3 + 48*a^4*b^2)^(1/2))/(-d^2)^(1/2) + (a^3*log(exp(4*c + 4*
d*x) - 1))/(2*d) - (8*b^3)/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + ex
p(6*c + 6*d*x) + 1)) - (2*(3*a*b^2 - b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c
 + 4*d*x) + 1)) + (4*b^3)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*e
xp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

3.130 $\int \coth^2(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=61

$$a^3x-\frac{(a+b)^3\coth(c+dx)}{d}-\frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{b^3\tanh^3(c+dx)}{3d}$$

```
[Out] a^3*x-(a+b)^3*coth(d*x+c)/d-b^2*(3*a+2*b)*tanh(d*x+c)/d+1/3*b^3*tanh(d*x+c)
^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {4226, 1816, 213}

$$a^3x-\frac{b^2(3a+2b)\tanh(c+dx)}{d}-\frac{(a+b)^3\coth(c+dx)}{d}+\frac{b^3\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] a^3*x - ((a + b)^3*Coth[c + d*x])/d - (b^2*(3*a + 2*b)*Tanh[c + d*x])/d + (
b^3*Tanh[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^2(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b(1-x^2)\right)^3}{x^2(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(-b^2(3a+2b)+\frac{(a+b)^3}{x^2}+b^3x^2-\frac{a^3}{-1+x^2}\right)\,dx, x, \tanh(c$$

$$= -\frac{(a+b)^3\coth(c+dx)}{d}-\frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{b^3\tanh^3(}{3c}$$

$$= a^3x-\frac{(a+b)^3\coth(c+dx)}{d}-\frac{b^2(3a+2b)\tanh(c+dx)}{d}+\frac{b^3\,\mathrm{ta}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 126 vs. 2(61) = 122.
time = 1.21, size = 126, normalized size = 2.07

$$\frac{8(a\cosh(c+dx)+b\operatorname{sech}(c+dx))^3\left(3a^3dx\cosh^3(c+dx)-b^3\operatorname{sech}(c)\sinh(dx)+\cosh^2(c+dx)\left(3(a+b)^3\coth(c+dx)\operatorname{csch}(c)-b^2(9a+5b)\operatorname{sech}(c)\right)\sinh(dx)-b^3\cosh(c+dx)\tanh(c)\right)}{3d(a+2b+a\cosh(2(c+dx)))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (8*(a*Cosh[c + d*x] + b*Sech[c + d*x])^3*(3*a^3*d*x*Cosh[c + d*x]^3 - b^3*S
ech[c]*Sinh[d*x] + Cosh[c + d*x]^2*(3*(a + b)^3*Coth[c + d*x]*Csch[c] - b^2
*(9*a + 5*b)*Sech[c])*Sinh[d*x] - b^3*Cosh[c + d*x]*Tanh[c]))/(3*d*(a + 2*b
 + a*Cosh[2*(c + d*x)])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(59) = 118.
time = 2.67, size = 192, normalized size = 3.15

method	result
risch	$a^3x-\frac{2\left(3a^3e^{6dx+6c}+9a^2b\,e^{6dx+6c}+9a^3e^{4dx+4c}+27a^2b\,e^{4dx+4c}+18a\,b^2e^{4dx+4c}+9a^3e^{2dx+2c}+27a^2b\,e^{2dx+2c}+36a\,b^2e^{2dx+2c}+16b^3\right.}{3d\left(1+e^{2dx+2c}\right)^3\left(e^{2dx+2c}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x-2/3*(3*a^3*exp(6*d*x+6*c)+9*a^2*b*exp(6*d*x+6*c)+9*a^3*exp(4*d*x+4*c)
+27*a^2*b*exp(4*d*x+4*c)+18*a*b^2*exp(4*d*x+4*c)+9*a^3*exp(2*d*x+2*c)+27*a^
2*b*exp(2*d*x+2*c)+36*a*b^2*exp(2*d*x+2*c)+16*b^3*exp(2*d*x+2*c)+3*a^3+9*a^
2*b+18*a*b^2+8*b^3)/d/(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(59) = 118.

time = 0.27, size = 172, normalized size = 2.82

$$a^3\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)-\frac{16}{3}\,b^3\left(\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}+1)}+\frac{1}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}+1)}\right)+\frac{6\,a^2b}{d(e^{(-2\,dx-2\,c)}-1)}+\frac{12\,ab^2}{d(e^{(-4\,dx-4\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) - 16/3*b^3*(2*e^(-2*d*x - 2*c)
/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1)) + 1/(
d*(2*e^(-2*d*x - 2*c) - 2*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) + 1))) + 6*a^
2*b/(d*(e^(-2*d*x - 2*c) - 1)) + 12*a*b^2/(d*(e^(-4*d*x - 4*c) - 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(59) = 118$.
time = 0.49, size = 359, normalized size = 5.89

$$-\frac{(3\,a^3+9\,a^2b+18\,ab^2+8\,b^3)\cosh(dx+c)^4-4\,(3\,a^3dx+3\,a^3+9\,a^2b+18\,ab^2+8\,b^3)\cosh(dx+c)\sinh(dx+c)^3+(3\,a^3+9\,a^2b+18\,ab^2+8\,b^3)\sinh(dx+c)^4+9\,a^3+27\,a^2b+18\,ab^2+4\,(3\,a^3+9\,a^2b+9\,ab^2+4\,b^3)\cosh(dx+c)^2+2\,(6\,a^3+18\,a^2b+18\,ab^2+8\,b^3+3\,(3\,a^3+9\,a^2b+18\,ab^2+8\,b^3)\cosh(dx+c)^2)\sinh(dx+c)^2-4\,((3\,a^3dx+3\,a^3+9\,a^2b+18\,ab^2+8\,b^3)\cosh(dx+c)^3+(3\,a^3dx+3\,a^3+9\,a^2b+18\,ab^2+8\,b^3)\cosh(dx+c))\sinh(dx+c)}{12\,(d\cosh(dx+c)\sinh(dx+c)^3+(d\cosh(dx+c)^3+d\cosh(dx+c))\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*((3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)^4 - 4*(3*a^3*d*x
+ 3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^
3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*sinh(d*x + c)^4 + 9*a^3 + 27*a^2*b + 18*a*b
^2 + 4*(3*a^3 + 9*a^2*b + 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 2*(6*a^3 + 18*
a^2*b + 18*a*b^2 + 8*b^3 + 3*(3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 - 4*((3*a^3*d*x + 3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^
3)*cosh(d*x + c)^3 + (3*a^3*d*x + 3*a^3 + 9*a^2*b + 18*a*b^2 + 8*b^3)*cosh(
d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c
)^3 + d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(59) = 118$.
time = 0.43, size = 135, normalized size = 2.21

$$\frac{3\,(dx+c)a^3-\frac{6\,(a^3+3\,a^2b+3\,ab^2+b^3)}{e^{(2\,dx+2\,c)}-1}+\frac{2\,(9\,ab^2e^{(4\,dx+4\,c)}+3\,b^3e^{(4\,dx+4\,c)}+18\,ab^2e^{(2\,dx+2\,c)}+12\,b^3e^{(2\,dx+2\,c)}+9\,ab^2+5\,b^3)}{(e^{(2\,dx+2\,c)}+1)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^3 - 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)/(e^(2*d*x + 2*c) -
 1) + 2*(9*a*b^2*e^(4*d*x + 4*c) + 3*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*d*
x + 2*c) + 12*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 5*b^3)/(e^(2*d*x + 2*c) + 1)^
3)/d
```

Mupad [B]
time = 0.14, size = 234, normalized size = 3.84

$$\frac{\frac{2\,\left(b^3+3\,a\,b^2\right)}{3\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(b^3+a\,b^2\right)}{d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(b^3+3\,a\,b^2\right)}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}+a^3\,x+\frac{\frac{2\,\left(b^3+a\,b^2\right)}{d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(b^3+3\,a\,b^2\right)}{3\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}+\frac{2\,\left(b^3+3\,a\,b^2\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] ((2*(3*a*b^2 + b^3))/(3*d) + (4*exp(2*c + 2*d*x)*(a*b^2 + b^3))/d + (2*exp(
4*c + 4*d*x)*(3*a*b^2 + b^3))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*
x) + exp(6*c + 6*d*x) + 1) + a^3*x + ((2*(a*b^2 + b^3))/d + (2*exp(2*c + 2*
d*x)*(3*a*b^2 + b^3))/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) +
(2*(3*a*b^2 + b^3))/(3*d*(exp(2*c + 2*d*x) + 1)) - (2*(3*a*b^2 + 3*a^2*b +
a^3 + b^3))/(d*(exp(2*c + 2*d*x) - 1))
```

3.131 $\int \coth^3(c+dx)\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=81

$$-\frac{(a+b)^3\mathrm{csch}^2(c+dx)}{2d}+\frac{b^2(3a+2b)\log(\cosh(c+dx))}{d}+\frac{(a-2b)(a+b)^2\log(\sinh(c+dx))}{d}-\frac{b^3\mathrm{sech}^2(c+dx)}{2d}$$

```
[Out] -1/2*(a+b)^3*csch(d*x+c)^2/d+b^2*(3*a+2*b)*ln(cosh(d*x+c))/d+(a-2*b)*(a+b)^
2*ln(sinh(d*x+c))/d-1/2*b^3*sech(d*x+c)^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 457, 90}

$$\frac{b^2(3a+2b)\log(\cosh(c+dx))}{d}-\frac{(a+b)^3\mathrm{csch}^2(c+dx)}{2d}+\frac{(a-2b)(a+b)^2\log(\sinh(c+dx))}{d}-\frac{b^3\mathrm{sech}^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/2*((a + b)^3*Csch[c + d*x]^2)/d + (b^2*(3*a + 2*b)*Log[Cosh[c + d*x]])/d
 + ((a - 2*b)*(a + b)^2*Log[Sinh[c + d*x]])/d - (b^3*Sech[c + d*x]^2)/(2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth^3(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(b+ax^2)^3}{x^3(1-x^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b+ax)^3}{(1-x)^2x^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^3}{(-1+x)^2}+\frac{(a-2b)(a+b)^2}{-1+x}+\frac{b^3}{x^2}+\frac{b^2(3a+2b)}{x}\right)\,dx, x, \cosh^2(c\right.}{2d}$$
$$= -\frac{(a+b)^3\text{csch}^2(c+dx)}{2d}+\frac{b^2(3a+2b)\log(\cosh(c+dx))}{d}+\frac{(a-}{}$$

Mathematica [A]
time = 0.85, size = 110, normalized size = 1.36

$$-\frac{4\cosh^6(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\left((a+b)^3\text{csch}^2(c+dx)-2b^2(3a+2b)\log(\cosh(c+dx))-2(a-2b)(a+b)^2\log(\sinh(c+dx))+b^3\text{sech}^2(c+dx)\right)}{d(a+2b+a\cosh(2c+2dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-4*Cosh[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3*((a + b)^3*Csch[c + d*x]^2 -
2*b^2*(3*a + 2*b)*Log[Cosh[c + d*x]] - 2*(a - 2*b)*(a + b)^2*Log[Sinh[c + d
*x]] + b^3*Sech[c + d*x]^2))/(d*(a + 2*b + a*Cosh[2*c + 2*d*x])^3)
```

Maple [A]
time = 2.14, size = 110, normalized size = 1.36

method	result
derivativedivides	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)-\frac{3a^2b}{2\sinh(dx+c)^2}+3a\,b^2\left(-\frac{1}{2\sinh(dx+c)^2}-\ln(\tanh(dx+c))\right)+b^3\left(-\frac{1}{2\sinh(dx+c)^2\cosh(d}\right.}{d}$
default	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}\right)-\frac{3a^2b}{2\sinh(dx+c)^2}+3a\,b^2\left(-\frac{1}{2\sinh(dx+c)^2}-\ln(\tanh(dx+c))\right)+b^3\left(-\frac{1}{2\sinh(dx+c)^2\cosh(d}\right.}{d}$
risch	$-a^3x-\frac{2a^3c}{d}-\frac{2\,\mathrm{e}^{2dx+2c}\left(a^3\mathrm{e}^{4dx+4c}+3a^2b\,\mathrm{e}^{4dx+4c}+3a\,b^2\mathrm{e}^{4dx+4c}+2b^3\mathrm{e}^{4dx+4c}+2a^3\mathrm{e}^{2dx+2c}+6a^2b\,\mathrm{e}^{2dx+2c}+6a\,b^2\mathrm{e}^{2}\right.}{d\left(1+\mathrm{e}^{2dx+2c}\right)^2\left(\mathrm{e}^{2dx+2c}-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2)-3/2*a^2*b/sinh(d*x+c)^2+3*a*b^
2*(-1/2/sinh(d*x+c)^2-ln(tanh(d*x+c)))+b^3*(-1/2/sinh(d*x+c)^2/cosh(d*x+c)^
2-1/cosh(d*x+c)^2-2*ln(tanh(d*x+c))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(77) = 154$.
time = 0.49, size = 314, normalized size = 3.88

$$a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)-2\,b^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{2\left(e^{(-2\,dx-2\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(2\,e^{(-4\,dx-4\,c)}-e^{(-8\,dx-8\,c)}-1)}\right)-3\,ab^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)-\frac{6\,a^2b}{d(e^{(dx+c)}-e^{(-dx-c)})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*b^3*(log(e^
(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c) + 1)/d -
 2*(e^(-2*d*x - 2*c) + e^(-6*d*x - 6*c))/(d*(2*e^(-4*d*x - 4*c) - e^(-8*d*x
 - 8*c) - 1))) - 3*a*b^2*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d
 - log(e^(-2*d*x - 2*c) + 1)/d - 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c)
- e^(-4*d*x - 4*c) - 1))) - 6*a^2*b/(d*(e^(d*x + c) - e^(-d*x - c))^2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1701 vs. $2(77) = 154$.
time = 0.44, size = 1701, normalized size = 21.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -(a^3*d*x*cosh(d*x + c)^8 + 8*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*d
*x*sinh(d*x + c)^8 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^6 +
2*(14*a^3*d*x*cosh(d*x + c)^2 + a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*sinh(d*x +
 c)^6 + 4*(14*a^3*d*x*cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)
*cosh(d*x + c))*sinh(d*x + c)^5 + a^3*d*x - 2*(a^3*d*x - 2*a^3 - 6*a^2*b -
6*a*b^2)*cosh(d*x + c)^4 + 2*(35*a^3*d*x*cosh(d*x + c)^4 - a^3*d*x + 2*a^3
+ 6*a^2*b + 6*a*b^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^4 + 8*(7*a^3*d*x*cosh(d*x + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^
2 + 2*b^3)*cosh(d*x + c)^3 - (a^3*d*x - 2*a^3 - 6*a^2*b - 6*a*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^3 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^
2 + 2*(14*a^3*d*x*cosh(d*x + c)^6 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*co
sh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3 - 6*(a^3*d*x - 2*a^3 - 6*a^
2*b - 6*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a*b^2 + 2*b^3)*cosh(d
*x + c)^8 + 56*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(3*a*
b^2 + 2*b^3)*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(3*a*b^2 + 2*b^3)*cosh(d*x
 + c)*sinh(d*x + c)^7 + (3*a*b^2 + 2*b^3)*sinh(d*x + c)^8 - 2*(3*a*b^2 + 2*
b^3)*cosh(d*x + c)^4 + 2*(35*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^4 - 3*a*b^2 -
2*b^3)*sinh(d*x + c)^4 + 8*(7*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^5 - (3*a*b^2
+ 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a*b^2 + 2*b^3 + 4*(7*(3*a*b^2 +
```

```
 2*b^3)*cosh(d*x + c)^6 - 3*(3*a*b^2 + 2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 8*((3*a*b^2 + 2*b^3)*cosh(d*x + c)^7 - (3*a*b^2 + 2*b^3)*cosh(d*x + c
)^3)*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) -
((a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^8 + 56*(a^3 - 3*a*b^2 - 2*b^3)*cosh(
d*x + c)^3*sinh(d*x + c)^5 + 28*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^2*sin
h(d*x + c)^6 + 8*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a
^3 - 3*a*b^2 - 2*b^3)*sinh(d*x + c)^8 - 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x
+ c)^4 + 2*(35*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^4 - a^3 + 3*a*b^2 + 2*
b^3)*sinh(d*x + c)^4 + 8*(7*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^5 - (a^3
- 3*a*b^2 - 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 - 3*a*b^2 - 2*b^3 +
 4*(7*(a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^6 - 3*(a^3 - 3*a*b^2 - 2*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^
7 - (a^3 - 3*a*b^2 - 2*b^3)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*sinh(d*x
+ c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*a^3*d*x*cosh(d*x + c)^7 + 3*(a
^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x + c)^5 - 2*(a^3*d*x - 2*a^3 - 6*a^
2*b - 6*a*b^2)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*cosh(d*x
 + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^3*sinh(d*x +
c)^5 + 28*d*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x +
c)^7 + d*sinh(d*x + c)^8 - 2*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 -
d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - d*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(7*d*cosh(d*x + c)^6 - 3*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*
cosh(d*x + c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(77) = 154.
time = 0.46, size = 247, normalized size = 3.05

$$\frac{2\left(3ab^2+2b^3\right)\log\left(e^{(2dx+2c)}+e^{(-2dx-2c)}+2\right)+2\left(a^3-3ab^2-2b^3\right)\log\left(e^{(2dx+2c)}+e^{(-2dx-2c)}-2\right)-\frac{a^3\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)^2+8a^3\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)+24a^2b\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)+24ab^2\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)+16b^3\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)+12a^3+48a^2b+48ab^2}{\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)^2-4}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*a*b^2 + 2*b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + 2*(a
^3 - 3*a*b^2 - 2*b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (a^3*(e
^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 8*a^3*(e^(2*d*x + 2*c) + e^(-2*d*x -
 2*c)) + 24*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 24*a*b^2*(e^(2*d*x
```

```
 + 2*c) + e^(-2*d*x - 2*c)) + 16*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) +
 12*a^3 + 48*a^2*b + 48*a*b^2)/((e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 - 4)
)/d
```

Mupad [B]
time = 1.72, size = 324, normalized size = 4.00

$$\frac{\operatorname{atan}\left(\frac{e^{2c}\,e^{2dx}\,\left(4\,b^3\,\sqrt{-d^2}-a^3\,\sqrt{-d^2}+6\,a\,b^2\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6-12\,a^4\,b^2-8\,a^3\,b^3+36\,a^2\,b^4+48\,a\,b^5+16\,b^6}}\right)\,\sqrt{a^6-12\,a^4\,b^2-8\,a^3\,b^3+36\,a^2\,b^4+48\,a\,b^5+16\,b^6}}{\sqrt{-d^2}}-\frac{\frac{4\,\left(a^3+3\,a^2\,b+3\,a\,b^2\right)}{d}+\frac{2\,e^{2c+2dx}\,\left(a^3+3\,a^2\,b+3\,a\,b^2+2\,b^3\right)}{d}}{e^{4c+4dx}-1}-\frac{\frac{4\,\left(a^3+3\,a^2\,b+3\,a\,b^2\right)}{d}+\frac{4\,e^{2c+2dx}\,\left(a^3+3\,a^2\,b+3\,a\,b^2+2\,b^3\right)}{d}}{e^{8c+8dx}-2\,e^{4c+4dx}+1}-a^3\,x+\frac{a^3\,\ln\left(e^{4c+4dx}-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (atan((exp(2*c)*exp(2*d*x)*(4*b^3*(-d^2)^(1/2) - a^3*(-d^2)^(1/2) + 6*a*b^2
*(-d^2)^(1/2)))/(d*(48*a*b^5 + a^6 + 16*b^6 + 36*a^2*b^4 - 8*a^3*b^3 - 12*a
^4*b^2)^(1/2)))*(48*a*b^5 + a^6 + 16*b^6 + 36*a^2*b^4 - 8*a^3*b^3 - 12*a^4*
b^2)^(1/2))/(-d^2)^(1/2) - ((4*(3*a*b^2 + 3*a^2*b + a^3))/d + (2*exp(2*c +
2*d*x)*(3*a*b^2 + 3*a^2*b + a^3 + 2*b^3))/d)/(exp(4*c + 4*d*x) - 1) - ((4*(
3*a*b^2 + 3*a^2*b + a^3))/d + (4*exp(2*c + 2*d*x)*(3*a*b^2 + 3*a^2*b + a^3
+ 2*b^3))/d)/(exp(8*c + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - a^3*x + (a^3*log
(exp(4*c + 4*d*x) - 1))/(2*d)
```

3.132 $\int \coth^4(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=60

$$a^3x-\frac{(a-2b)(a+b)^2\coth(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{b^3\tanh(c+dx)}{d}$$

```
[Out] a^3*x-(a-2*b)*(a+b)^2*coth(d*x+c)/d-1/3*(a+b)^3*coth(d*x+c)^3/d+b^3*tanh(d*
x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4226, 1816, 213}

$$a^3x-\frac{(a+b)^3\coth^3(c+dx)}{3d}-\frac{(a-2b)(a+b)^2\coth(c+dx)}{d}+\frac{b^3\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] a^3*x - ((a - 2*b)*(a + b)^2*Coth[c + d*x])/d - ((a + b)^3*Coth[c + d*x]^3)
/(3*d) + (b^3*Tanh[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^4(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b(1-x^2)\right)^3}{x^4(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b^3+\frac{(a+b)^3}{x^4}+\frac{(a-2b)(a+b)^2}{x^2}-\frac{a^3}{-1+x^2}\right)dx, x, \tanh(c+$$
$$= -\frac{(a-2b)(a+b)^2\coth(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}+\frac{b^3}{}$$
$$= a^3x-\frac{(a-2b)(a+b)^2\coth(c+dx)}{d}-\frac{(a+b)^3\coth^3(c+dx)}{3d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 343 vs. $2(60) = 120$.
time = 1.15, size = 343, normalized size = 5.72

$$\frac{\text{csch}(c)\text{csch}^3(c+dx)\text{sech}(c)\text{sech}(c+dx)\left(6a^3dx\cosh(2dx)-3a^3dx\cosh(2(c+2dx))-6a^3dx\cosh(4c+2dx)+3a^3dx\cosh(6c+4dx)-18a^2b\sinh(2c)-36ab^2\sinh(2c)-4a^3\sinh(2dx)+6a^2b\sinh(2dx)+24ab^2\sinh(2dx)+32b^3\sinh(2dx)-16a^3\sinh(2(c+dx))-12a^2b\sinh(2(c+dx))+24ab^2\sinh(2(c+dx))+8b^3\sinh(2(c+dx))+8a^3\sinh(4(c+dx))+6a^2b\sinh(4(c+dx))-12ab^2\sinh(4(c+dx))-4b^3\sinh(4(c+dx))+8a^3\sinh(2(c+2dx))+6a^2b\sinh(2(c+2dx))-12ab^2\sinh(2(c+2dx))-16b^3\sinh(2(c+2dx))-12a^3\sinh(4c+2dx)-18a^2b\sinh(4c+2dx)\right)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c]*Csch[c + d*x]^3*Sech[c]*Sech[c + d*x]*(6*a^3*d*x*Cosh[2*d*x] - 3*a
^3*d*x*Cosh[2*(c + 2*d*x)] - 6*a^3*d*x*Cosh[4*c + 2*d*x] + 3*a^3*d*x*Cosh[6
*c + 4*d*x] - 18*a^2*b*Sinh[2*c] - 36*a*b^2*Sinh[2*c] - 4*a^3*Sinh[2*d*x] +
 6*a^2*b*Sinh[2*d*x] + 24*a*b^2*Sinh[2*d*x] + 32*b^3*Sinh[2*d*x] - 16*a^3*S
inh[2*(c + d*x)] - 12*a^2*b*Sinh[2*(c + d*x)] + 24*a*b^2*Sinh[2*(c + d*x)]
+ 8*b^3*Sinh[2*(c + d*x)] + 8*a^3*Sinh[4*(c + d*x)] + 6*a^2*b*Sinh[4*(c + d
*x)] - 12*a*b^2*Sinh[4*(c + d*x)] - 4*b^3*Sinh[4*(c + d*x)] + 8*a^3*Sinh[2*
(c + 2*d*x)] + 6*a^2*b*Sinh[2*(c + 2*d*x)] - 12*a*b^2*Sinh[2*(c + 2*d*x)] -
 16*b^3*Sinh[2*(c + 2*d*x)] - 12*a^3*Sinh[4*c + 2*d*x] - 18*a^2*b*Sinh[4*c
+ 2*d*x]))/(96*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(58) = 116$.
time = 2.65, size = 178, normalized size = 2.97

method	result
risch	$a^3x-\frac{2\left(6a^3e^{6dx+6c}+9a^2b\,e^{6dx+6c}+9a^2b\,e^{4dx+4c}+18a\,b^2e^{4dx+4c}-2a^3e^{2dx+2c}+3a^2b\,e^{2dx+2c}+12a\,b^2e^{2dx+2c}+16b^3e^{2dx+2c}+4a^3\right.}{3d\left(1+e^{2dx+2c}\right)\left(e^{2dx+2c}-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x-2/3*(6*a^3*exp(6*d*x+6*c)+9*a^2*b*exp(6*d*x+6*c)+9*a^2*b*exp(4*d*x+4*
c)+18*a*b^2*exp(4*d*x+4*c)-2*a^3*exp(2*d*x+2*c)+3*a^2*b*exp(2*d*x+2*c)+12*a
*b^2*exp(2*d*x+2*c)+16*b^3*exp(2*d*x+2*c)+4*a^3+3*a^2*b-6*a*b^2-8*b^3)/d/(1
+exp(2*d*x+2*c))/(exp(2*d*x+2*c)-1)^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(58) = 116$.
time = 0.28, size = 366, normalized size = 6.10

$$\frac{1}{3}\,a^3\left(3\,x+\frac{3\,c}{d}-\frac{4\,\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}-2\right)}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)+4\,ab^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)+\frac{16}{3}\,b^3\left(\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}-1)}-\frac{1}{d(2\,e^{(-2\,dx-2\,c)}-2\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}-1)}\right)+2\,a^2b\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 4*a*b^2
*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x
- 6*c) - 1))) + 16/3*b^3*(2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - 2*e^(
-6*d*x - 6*c) + e^(-8*d*x - 8*c) - 1)) - 1/(d*(2*e^(-2*d*x - 2*c) - 2*e^(-6
*d*x - 6*c) + e^(-8*d*x - 8*c) - 1))) + 2*a^2*b*(3*e^(-4*d*x - 4*c)/(d*(3*e
^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) + 1/(d*(3*e^(
-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(58) = 116$.
time = 0.36, size = 354, normalized size = 5.90

$$-\frac{(4\,a^3+3\,a^2b-6\,ab^2-8\,b^3)\cosh\left(dx+c\right)^4-4\,(3\,a^3dx+4\,a^3+3\,a^2b-6\,ab^2-8\,b^3)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+(4\,a^3+3\,a^2b-6\,ab^2-8\,b^3)\sinh\left(dx+c\right)^4+9\,a^2b+18\,ab^2+4\,(a^3+3\,a^2b+3\,ab^2+4\,b^3)\cosh\left(dx+c\right)^2+2\,\left(2\,a^3+6\,a^2b+6\,ab^2+8\,b^3+3\,(4\,a^3+3\,a^2b-6\,ab^2-8\,b^3)\cosh\left(dx+c\right)^2\right)\sinh\left(dx+c\right)^2-4\,\left((3\,a^3dx+4\,a^3+3\,a^2b-6\,ab^2-8\,b^3)\cosh\left(dx+c\right)^3-(3\,a^3dx+4\,a^3+3\,a^2b-6\,ab^2-8\,b^3)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{12\,\left(d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+\left(d\cosh\left(dx+c\right)^3-d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*((4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^4 - 4*(3*a^3*d*x +
 4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a^3
+ 3*a^2*b - 6*a*b^2 - 8*b^3)*sinh(d*x + c)^4 + 9*a^2*b + 18*a*b^2 + 4*(a^3
+ 3*a^2*b + 3*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 2*(2*a^3 + 6*a^2*b + 6*a*b^2
 + 8*b^3 + 3*(4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 - 4*((3*a^3*d*x + 4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^3
 - (3*a^3*d*x + 4*a^3 + 3*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x
+ c))/(d*cosh(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 - d*cosh(d*x +
c))*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(58) = 116.
time = 0.46, size = 158, normalized size = 2.63

$$\frac{3\,(dx+c)a^3-\frac{6\,b^3}{e^{(2\,dx+2\,c)}+1}-\frac{2\,\left(6\,a^3e^{(4\,dx+4\,c)}+9\,a^2be^{(4\,dx+4\,c)}-3\,b^3e^{(4\,dx+4\,c)}-6\,a^3e^{(2\,dx+2\,c)}+18\,ab^2e^{(2\,dx+2\,c)}+12\,b^3e^{(2\,dx+2\,c)}+4\,a^3+3\,a^2b-6\,ab^2-5\,b^3\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^3 - 6*b^3/(e^(2*d*x + 2*c) + 1) - 2*(6*a^3*e^(4*d*x + 4*
c) + 9*a^2*b*e^(4*d*x + 4*c) - 3*b^3*e^(4*d*x + 4*c) - 6*a^3*e^(2*d*x + 2*c
) + 18*a*b^2*e^(2*d*x + 2*c) + 12*b^3*e^(2*d*x + 2*c) + 4*a^3 + 3*a^2*b - 6
*a*b^2 - 5*b^3)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B]
time = 1.48, size = 260, normalized size = 4.33

$$a^3\,x-\frac{\frac{2\,\left(a^2\,b+2\,a\,b^2+b^3\right)}{d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(2\,a^3+3\,a^2\,b-b^3\right)}{3\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{\frac{2\,\left(2\,a^3+3\,a^2\,b-b^3\right)}{3\,d}+\frac{2\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(2\,a^3+3\,a^2\,b-b^3\right)}{3\,d}+\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2\,b+2\,a\,b^2+b^3\right)}{d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}-\frac{2\,b^3}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,\left(2\,a^3+3\,a^2\,b-b^3\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] a^3*x - ((2*(2*a*b^2 + a^2*b + b^3))/d + (2*exp(2*c + 2*d*x)*(3*a^2*b + 2*a
^3 - b^3))/(3*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - ((2*(3*a^2*
b + 2*a^3 - b^3))/(3*d) + (2*exp(4*c + 4*d*x)*(3*a^2*b + 2*a^3 - b^3))/(3*d
) + (4*exp(2*c + 2*d*x)*(2*a*b^2 + a^2*b + b^3))/d)/(3*exp(2*c + 2*d*x) - 3
*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (2*b^3)/(d*(exp(2*c + 2*d*x) +
1)) - (2*(3*a^2*b + 2*a^3 - b^3))/(3*d*(exp(2*c + 2*d*x) - 1))
```

3.133 $\int \coth^5(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=81

$$-\frac{(2a-b)(a+b)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{(a+b)^3\operatorname{csch}^4(c+dx)}{4d}-\frac{b^3\log(\cosh(c+dx))}{d}+\frac{(a^3+b^3)\log(\sinh(c+dx))}{d}$$

```
[Out] -1/2*(2*a-b)*(a+b)^2*csch(d*x+c)^2/d-1/4*(a+b)^3*csch(d*x+c)^4/d-b^3*ln(cos
h(d*x+c))/d+(a^3+b^3)*ln(sinh(d*x+c))/d
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 90}

$$\frac{(a^3+b^3)\log(\sinh(c+dx))}{d}-\frac{(a+b)^3\operatorname{csch}^4(c+dx)}{4d}-\frac{(2a-b)(a+b)^2\operatorname{csch}^2(c+dx)}{2d}-\frac{b^3\log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/2*((2*a - b)*(a + b)^2*Csch[c + d*x]^2)/d - ((a + b)^3*Csch[c + d*x]^4)/
(4*d) - (b^3*Log[Cosh[c + d*x]])/d + ((a^3 + b^3)*Log[Sinh[c + d*x]])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth^5(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^3}{x(1-x^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+ax)^3}{(1-x)^3x}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^3}{(-1+x)^3} - \frac{(2a-b)(a+b)^2}{(-1+x)^2} + \frac{-a^3-b^3}{-1+x} + \frac{b^3}{x}\right)\,dx, x, \cosh\right.}{2d}$$
$$= -\frac{(2a-b)(a+b)^2\text{csch}^2(c+dx)}{2d} - \frac{(a+b)^3\text{csch}^4(c+dx)}{4d} - \frac{b^3}{}$$

Mathematica [A]
time = 0.61, size = 101, normalized size = 1.25

$$-\frac{2\left(b+a\cosh^2(c+dx)\right)^3\left(2(2a-b)(a+b)^2\text{csch}^2(c+dx)+(a+b)^3\text{csch}^4(c+dx)+4b^3\log(\cosh(c+dx))-4(a^3+b^3)\log(\sinh(c+dx))\right)}{d(a+2b+a\cosh(2(c+dx)))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^5*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-2*(b + a*Cosh[c + d*x]^2)^3*(2*(2*a - b)*(a + b)^2*Csch[c + d*x]^2 + (a +
 b)^3*Csch[c + d*x]^4 + 4*b^3*Log[Cosh[c + d*x]] - 4*(a^3 + b^3)*Log[Sinh[c
 + d*x]]))/(d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [A]
time = 1.98, size = 119, normalized size = 1.47

method	result
derivativedivides	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}\right)+3a^2b\left(-\frac{\cosh^2(dx+c)}{2\sinh(dx+c)^4}+\frac{1}{4\sinh(dx+c)^4}\right)-\frac{3a\,b^2}{4\sinh(dx+c)^4}+b^3\left(-\frac{}{4\sin}\right.}{d}$
default	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}\right)+3a^2b\left(-\frac{\cosh^2(dx+c)}{2\sinh(dx+c)^4}+\frac{1}{4\sinh(dx+c)^4}\right)-\frac{3a\,b^2}{4\sinh(dx+c)^4}+b^3\left(-\frac{}{4\sin}\right.}{d}$
risch	$-a^3x-\frac{2a^3c}{d}-\frac{2\,\mathrm{e}^{2dx+2c}\left(2a^3\mathrm{e}^{4dx+4c}+3a^2b\,\mathrm{e}^{4dx+4c}-b^3\mathrm{e}^{4dx+4c}-2a^3\mathrm{e}^{2dx+2c}+6a\,b^2\mathrm{e}^{2dx+2c}+4b^3\mathrm{e}^{2dx+2c}+2a^3+3\right.}{d\left(\mathrm{e}^{2dx+2c}-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4)+3*a^2*b*(-1/
2/sinh(d*x+c)^4*cosh(d*x+c)^2+1/4/sinh(d*x+c)^4)-3/4*a*b^2/sinh(d*x+c)^4+b^
3*(-1/4/sinh(d*x+c)^4+1/2/sinh(d*x+c)^2+ln(tanh(d*x+c))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(77) = 154$.
time = 0.49, size = 422, normalized size = 5.21

$$a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}-e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d\left(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e^{(-8dx-8c)}-1\right)}\right)+b^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}-\frac{2\left(e^{(-2dx-2c)}-4e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d\left(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e^{(-8dx-8c)}-1\right)}\right)+6a^2b\left(\frac{e^{(-2dx-2c)}}{d\left(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e^{(-8dx-8c)}-1\right)}+\frac{e^{(-6dx-6c)}}{d\left(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e^{(-8dx-8c)}-1\right)}\right)-\frac{12ab^2}{d\left(e^{(dx+c)}-e^{(-dx-c)}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2
*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + b^3*(l
og(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c) + 1
)/d - 2*(e^(-2*d*x - 2*c) - 4*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^
(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c)
 - 1))) + 6*a^2*b*(e^(-2*d*x - 2*c)/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x -
4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1)) + e^(-6*d*x - 6*c)/(d*(4
*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8
*c) - 1))) - 12*a*b^2/(d*(e^(d*x + c) - e^(-d*x - c))^4)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1830 vs. $2(77) = 154$.
time = 0.47, size = 1830, normalized size = 22.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -(a^3*d*x*cosh(d*x + c)^8 + 8*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*d
*x*sinh(d*x + c)^8 - 2*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^6
+ 2*(14*a^3*d*x*cosh(d*x + c)^2 - 2*a^3*d*x + 2*a^3 + 3*a^2*b - b^3)*sinh(d
*x + c)^6 + 4*(14*a^3*d*x*cosh(d*x + c)^3 - 3*(2*a^3*d*x - 2*a^3 - 3*a^2*b
+ b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + a^3*d*x + 2*(3*a^3*d*x - 2*a^3 + 6*
a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 2*(35*a^3*d*x*cosh(d*x + c)^4 + 3*a^3*d*x
- 2*a^3 + 6*a*b^2 + 4*b^3 - 15*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 8*(7*a^3*d*x*cosh(d*x + c)^5 - 5*(2*a^3*d*x - 2*
a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^3 + (3*a^3*d*x - 2*a^3 + 6*a*b^2 + 4*b^3
)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*co
sh(d*x + c)^2 + 2*(14*a^3*d*x*cosh(d*x + c)^6 - 2*a^3*d*x - 15*(2*a^3*d*x -
 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c)^4 + 2*a^3 + 3*a^2*b - b^3 + 6*(3*a^3*
d*x - 2*a^3 + 6*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + (b^3*cosh
(d*x + c)^8 + 8*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + b^3*sinh(d*x + c)^8 - 4
*b^3*cosh(d*x + c)^6 + 6*b^3*cosh(d*x + c)^4 + 4*(7*b^3*cosh(d*x + c)^2 - b
^3)*sinh(d*x + c)^6 + 8*(7*b^3*cosh(d*x + c)^3 - 3*b^3*cosh(d*x + c))*sinh(
```

```
d*x + c)^5 - 4*b^3*cosh(d*x + c)^2 + 2*(35*b^3*cosh(d*x + c)^4 - 30*b^3*cos
h(d*x + c)^2 + 3*b^3)*sinh(d*x + c)^4 + 8*(7*b^3*cosh(d*x + c)^5 - 10*b^3*c
osh(d*x + c)^3 + 3*b^3*cosh(d*x + c))*sinh(d*x + c)^3 + b^3 + 4*(7*b^3*cosh
(d*x + c)^6 - 15*b^3*cosh(d*x + c)^4 + 9*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*
x + c)^2 + 8*(b^3*cosh(d*x + c)^7 - 3*b^3*cosh(d*x + c)^5 + 3*b^3*cosh(d*x
+ c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x +
c) - sinh(d*x + c))) - ((a^3 + b^3)*cosh(d*x + c)^8 + 8*(a^3 + b^3)*cosh(d*
x + c)*sinh(d*x + c)^7 + (a^3 + b^3)*sinh(d*x + c)^8 - 4*(a^3 + b^3)*cosh(d
*x + c)^6 - 4*(a^3 + b^3 - 7*(a^3 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 +
 8*(7*(a^3 + b^3)*cosh(d*x + c)^3 - 3*(a^3 + b^3)*cosh(d*x + c))*sinh(d*x +
 c)^5 + 6*(a^3 + b^3)*cosh(d*x + c)^4 + 2*(35*(a^3 + b^3)*cosh(d*x + c)^4 +
 3*a^3 + 3*b^3 - 30*(a^3 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^
3 + b^3)*cosh(d*x + c)^5 - 10*(a^3 + b^3)*cosh(d*x + c)^3 + 3*(a^3 + b^3)*c
osh(d*x + c))*sinh(d*x + c)^3 + a^3 + b^3 - 4*(a^3 + b^3)*cosh(d*x + c)^2 +
 4*(7*(a^3 + b^3)*cosh(d*x + c)^6 - 15*(a^3 + b^3)*cosh(d*x + c)^4 - a^3 -
b^3 + 9*(a^3 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^3 + b^3)*cosh(
d*x + c)^7 - 3*(a^3 + b^3)*cosh(d*x + c)^5 + 3*(a^3 + b^3)*cosh(d*x + c)^3
- (a^3 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x +
 c) - sinh(d*x + c))) + 4*(2*a^3*d*x*cosh(d*x + c)^7 - 3*(2*a^3*d*x - 2*a^3
 - 3*a^2*b + b^3)*cosh(d*x + c)^5 + 2*(3*a^3*d*x - 2*a^3 + 6*a*b^2 + 4*b^3)
*cosh(d*x + c)^3 - (2*a^3*d*x - 2*a^3 - 3*a^2*b + b^3)*cosh(d*x + c))*sinh(
d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d
*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)
^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh
(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*
x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c
))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*co
sh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x +
c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*
x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(77) = 154.
time = 0.52, size = 227, normalized size = 2.80

$$-\frac{2\,b^3\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)-2\,(a^3+b^3)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)+\frac{3\,a^3\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+3\,b^3\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+4\,a^3\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+24\,a^2b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)-20\,b^3\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)-4\,a^3+48\,ab^2+44\,b^3}{\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^5*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*b^3*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - 2*(a^3 + b^3)*log
(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) + (3*a^3*(e^(2*d*x + 2*c) + e^(-2*
d*x - 2*c))^2 + 3*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 4*a^3*(e^(2*
d*x + 2*c) + e^(-2*d*x - 2*c)) + 24*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*
c)) - 20*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 4*a^3 + 48*a*b^2 + 44*b
^3)/(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2)^2)/d
```

Mupad [B]
time = 1.64, size = 384, normalized size = 4.74

$$-a^3\,x-\frac{2\,\left(4\,a^3+9\,a^2\,b+6\,a\,b^2+b^3\right)}{d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{\ln\left({\mathrm{e}}^{4\,c+4\,d\,x}-1\right)\,\left(b^3\,d-d\,\left(a^3+b^3\right)\right)}{2\,d^2}-\frac{\mathrm{atan}\left(\frac{{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\,\left(a^6\,\sqrt{-d^2}+4\,b^6\,\sqrt{-d^2}+4\,a^3\,b^3\,\sqrt{-d^2}\right)}{a^3\,d\,\sqrt{a^6+4\,a^3\,b^3+4\,b^6}+2\,b^3\,d\,\sqrt{a^6+4\,a^3\,b^3+4\,b^6}}\right)\,\sqrt{a^6+4\,a^3\,b^3+4\,b^6}}{\sqrt{-d^2}}-\frac{2\,\left(2\,a^3+3\,a^2\,b-b^3\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{8\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}-\frac{4\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{d\,\left(6\,{\mathrm{e}}^{4\,c+4\,d\,x}-4\,{\mathrm{e}}^{2\,c+2\,d\,x}-4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^5*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] - a^3*x - (2*(6*a*b^2 + 9*a^2*b + 4*a^3 + b^3))/(d*(exp(4*c + 4*d*x) - 2*ex
p(2*c + 2*d*x) + 1)) - (log(exp(4*c + 4*d*x) - 1)*(b^3*d - d*(a^3 + b^3)))/
(2*d^2) - (atan((exp(2*c)*exp(2*d*x)*(a^6*(-d^2)^(1/2) + 4*b^6*(-d^2)^(1/2)
 + 4*a^3*b^3*(-d^2)^(1/2)))/(a^3*d*(a^6 + 4*b^6 + 4*a^3*b^3)^(1/2) + 2*b^3*
d*(a^6 + 4*b^6 + 4*a^3*b^3)^(1/2)))*(a^6 + 4*b^6 + 4*a^3*b^3)^(1/2))/(-d^2)
^(1/2) - (2*(3*a^2*b + 2*a^3 - b^3))/(d*(exp(2*c + 2*d*x) - 1)) - (8*(3*a*b
^2 + 3*a^2*b + a^3 + b^3))/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + ex
p(6*c + 6*d*x) - 1)) - (4*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(d*(6*exp(4*c +
4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

3.134 $\int \coth^6(c+dx)\left(a+b\,\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=69

$$a^3x-\frac{(a^3+b^3)\coth(c+dx)}{d}-\frac{(a-2b)(a+b)^2\coth^3(c+dx)}{3d}-\frac{(a+b)^3\coth^5(c+dx)}{5d}$$

```
[Out] a^3*x-(a^3+b^3)*coth(d*x+c)/d-1/3*(a-2*b)*(a+b)^2*coth(d*x+c)^3/d-1/5*(a+b)
^3*coth(d*x+c)^5/d
```

Rubi [A]
time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4226, 1816, 213}

$$-\frac{(a^3+b^3)\coth(c+dx)}{d}+a^3x-\frac{(a+b)^3\coth^5(c+dx)}{5d}-\frac{(a-2b)(a+b)^2\coth^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] a^3*x - ((a^3 + b^3)*Coth[c + d*x])/d - ((a - 2*b)*(a + b)^2*Coth[c + d*x]^
3)/(3*d) - ((a + b)^3*Coth[c + d*x]^5)/(5*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^6(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b(1-x^2)\right)^3}{x^6(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{(a+b)^3}{x^6} + \frac{(a-2b)(a+b)^2}{x^4} + \frac{a^3+b^3}{x^2} - \frac{a^3}{-1+x^2}\right) dx, x, \tanh(c+\right.}{d}$$

$$= -\frac{(a^3+b^3)\coth(c+dx)}{d} - \frac{(a-2b)(a+b)^2\coth^3(c+dx)}{3d} - \frac{(a}{}$$

$$= a^3x - \frac{(a^3+b^3)\coth(c+dx)}{d} - \frac{(a-2b)(a+b)^2\coth^3(c+dx)}{3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 303 vs. 2(69) = 138.
time = 0.72, size = 303, normalized size = 4.39

$$\frac{\operatorname{csch}(c)\operatorname{csch}^5(c+dx)\left(-150a^3dx\cosh(dx)+150a^3dx\cosh(2c+dx)+75a^3dx\cosh(2c+3dx)-75a^3dx\cosh(4c+3dx)-15a^3dx\cosh(4c+5dx)+15a^3dx\cosh(6c+5dx)+280a^3\sinh(dx)+180a^2b\sinh(dx)+60ab^2\sinh(dx)+160b^3\sinh(dx)+180a^3\sinh(2c+dx)-180ab^2\sinh(2c+dx)-140a^3\sinh(2c+3dx)+60ab^2\sinh(2c+3dx)-80b^3\sinh(2c+3dx)-90a^3\sinh(4c+3dx)-90a^2b\sinh(4c+3dx)+46a^3\sinh(4c+5dx)+18a^2b\sinh(4c+5dx)-12ab^2\sinh(4c+5dx)+16b^3\sinh(4c+5dx)\right)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^6*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c]*Csch[c + d*x]^5*(-150*a^3*d*x*Cosh[d*x] + 150*a^3*d*x*Cosh[2*c + d
*x] + 75*a^3*d*x*Cosh[2*c + 3*d*x] - 75*a^3*d*x*Cosh[4*c + 3*d*x] - 15*a^3*
d*x*Cosh[4*c + 5*d*x] + 15*a^3*d*x*Cosh[6*c + 5*d*x] + 280*a^3*Sinh[d*x] +
180*a^2*b*Sinh[d*x] + 60*a*b^2*Sinh[d*x] + 160*b^3*Sinh[d*x] + 180*a^3*Sinh
[2*c + d*x] - 180*a*b^2*Sinh[2*c + d*x] - 140*a^3*Sinh[2*c + 3*d*x] + 60*a*
b^2*Sinh[2*c + 3*d*x] - 80*b^3*Sinh[2*c + 3*d*x] - 90*a^3*Sinh[4*c + 3*d*x]
 - 90*a^2*b*Sinh[4*c + 3*d*x] + 46*a^3*Sinh[4*c + 5*d*x] + 18*a^2*b*Sinh[4*
c + 5*d*x] - 12*a*b^2*Sinh[4*c + 5*d*x] + 16*b^3*Sinh[4*c + 5*d*x]))/(480*d
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(65) = 130.
time = 2.83, size = 207, normalized size = 3.00

method	result
risch	$a^3x - \frac{2\left(45a^3e^{8dx+8c}+45a^2b\,e^{8dx+8c}-90a^3e^{6dx+6c}+90a\,b^2e^{6dx+6c}+140a^3e^{4dx+4c}+90a^2b\,e^{4dx+4c}+30a\,b^2e^{4dx+4c}+80b^3e^{4dx+4c}-\right.}{15d\left(e^{2dx+2c}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x-2/15*(45*a^3*exp(8*d*x+8*c)+45*a^2*b*exp(8*d*x+8*c)-90*a^3*exp(6*d*x+
6*c)+90*a*b^2*exp(6*d*x+6*c)+140*a^3*exp(4*d*x+4*c)+90*a^2*b*exp(4*d*x+4*c)
```

```
+30*a*b^2*exp(4*d*x+4*c)+80*b^3*exp(4*d*x+4*c)-70*a^3*exp(2*d*x+2*c)+30*a*b
^2*exp(2*d*x+2*c)-40*b^3*exp(2*d*x+2*c)+23*a^3+9*a^2*b-6*a*b^2+8*b^3)/d/(ex
p(2*d*x+2*c)-1)^5
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(65) = 130$.
time = 0.28, size = 826, normalized size = 11.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*a^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1))) + 4/5*a*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) - 1)) + 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c)
 + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) + 15
*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d
*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c)
+ e^(-10*d*x - 10*c) - 1))) - 16/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x
 - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) +
e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1))) + 6/5*a^2*b*(10*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) + 5*e^(-8*d*x - 8*c)
/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) + 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) - 1)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(65) = 130$.
time = 0.38, size = 521, normalized size = 7.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*((23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)^5 + 5*(23*a^3 + 9
*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^3*d*x + 23*
a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*sinh(d*x + c)^5 - 5*(5*a^3 - 9*a^2*b - 6*a
*b^2 + 8*b^3)*cosh(d*x + c)^3 + 5*(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2
+ 8*b^3 - 2*(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)
^2)*sinh(d*x + c)^3 + 5*(2*(23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x +
c)^3 - 3*(5*a^3 - 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^2
 + 10*(5*a^3 + 9*a^2*b + 12*a*b^2 + 8*b^3)*cosh(d*x + c) - 5*(30*a^3*d*x +
(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + 46*a^3
+ 18*a^2*b - 12*a*b^2 + 16*b^3 - 3*(15*a^3*d*x + 23*a^3 + 9*a^2*b - 6*a*b^2
 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(
d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^
2 + 2*d)*sinh(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**6*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(65) = 130.
time = 0.48, size = 213, normalized size = 3.09

$$\frac{15\,(dx+c)a^3-\frac{2\,\left(45\,a^3e^{(8\,dx+8\,c)}+45\,a^2be^{(8\,dx+8\,c)}-90\,a^3e^{(6\,dx+6\,c)}+90\,ab^2e^{(6\,dx+6\,c)}+140\,a^3e^{(4\,dx+4\,c)}+90\,a^2be^{(4\,dx+4\,c)}+30\,ab^2e^{(4\,dx+4\,c)}+80\,b^3e^{(4\,dx+4\,c)}-70\,a^3e^{(2\,dx+2\,c)}+30\,ab^2e^{(2\,dx+2\,c)}-40\,b^3e^{(2\,dx+2\,c)}+23\,a^3+9\,a^2b-6\,ab^2+8\,b^3\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^6*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(d*x + c)*a^3 - 2*(45*a^3*e^(8*d*x + 8*c) + 45*a^2*b*e^(8*d*x + 8*
c) - 90*a^3*e^(6*d*x + 6*c) + 90*a*b^2*e^(6*d*x + 6*c) + 140*a^3*e^(4*d*x +
 4*c) + 90*a^2*b*e^(4*d*x + 4*c) + 30*a*b^2*e^(4*d*x + 4*c) + 80*b^3*e^(4*d
*x + 4*c) - 70*a^3*e^(2*d*x + 2*c) + 30*a*b^2*e^(2*d*x + 2*c) - 40*b^3*e^(2
*d*x + 2*c) + 23*a^3 + 9*a^2*b - 6*a*b^2 + 8*b^3)/(e^(2*d*x + 2*c) - 1)^5)/
d
```

Mupad [B]
time = 1.60, size = 547, normalized size = 7.93

$$a^3\,x-\frac{\frac{6\,\left(a^3+b\,a^2\right)}{5\,d}+\frac{24\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{24\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{8\,c+8\,d\,x}\,\left(a^3+b\,a^2\right)}{5\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(5\,a^3+9\,a^2\,b+12\,a\,b^2+8\,b^3\right)}{5\,d}}{5\,{\mathrm{e}}^{2\,c+2\,d\,x}-10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}-5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}-1}-\frac{\frac{6\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^3+b\,a^2\right)}{5\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{\frac{6\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{18\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{6\,c+6\,d\,x}\,\left(a^3+b\,a^2\right)}{5\,d}+\frac{2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(5\,a^3+9\,a^2\,b+12\,a\,b^2+8\,b^3\right)}{5\,d}}{6\,{\mathrm{e}}^{4\,c+4\,d\,x}-4\,{\mathrm{e}}^{2\,c+2\,d\,x}-4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{\frac{2\,\left(5\,a^3+9\,a^2\,b+12\,a\,b^2+8\,b^3\right)}{15\,d}+\frac{12\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a^2\,b+a\,b^2\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a^3+b\,a^2\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}-\frac{6\,\left(a^3+b\,a^2\right)}{5\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^6*(a + b/cosh(c + d*x)^2)^3,x)

[Out] a^3*x - ((6*(a^2*b + a^3))/(5*d) + (24*exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/(5
*d) + (24*exp(6*c + 6*d*x)*(a*b^2 + a^2*b))/(5*d) + (6*exp(8*c + 8*d*x)*(a^
2*b + a^3))/(5*d) + (4*exp(4*c + 4*d*x)*(12*a*b^2 + 9*a^2*b + 5*a^3 + 8*b^3
))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) -
 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - ((6*(a*b^2 + a^2*b))/(5*d)
+ (6*exp(2*c + 2*d*x)*(a^2*b + a^3))/(5*d))/(exp(4*c + 4*d*x) - 2*exp(2*c +
 2*d*x) + 1) - ((6*(a*b^2 + a^2*b))/(5*d) + (18*exp(4*c + 4*d*x)*(a*b^2 + a
^2*b))/(5*d) + (6*exp(6*c + 6*d*x)*(a^2*b + a^3))/(5*d) + (2*exp(2*c + 2*d*
x)*(12*a*b^2 + 9*a^2*b + 5*a^3 + 8*b^3))/(5*d))/(6*exp(4*c + 4*d*x) - 4*exp
(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(12*a*b^2
+ 9*a^2*b + 5*a^3 + 8*b^3))/(15*d) + (12*exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/
(5*d) + (6*exp(4*c + 4*d*x)*(a^2*b + a^3))/(5*d))/(3*exp(2*c + 2*d*x) - 3*e
xp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (6*(a^2*b + a^3))/(5*d*(exp(2*c +
 2*d*x) - 1))
```

3.135 $\int \coth^7(c+dx)\left(a+b\mathbf{sech}^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=77

$$-\frac{3a^2(a+b)\text{csch}^2(c+dx)}{2d}-\frac{3a(a+b)^2\text{csch}^4(c+dx)}{4d}-\frac{(a+b)^3\text{csch}^6(c+dx)}{6d}+\frac{a^3\log(\sinh(c+dx))}{d}$$

```
[Out] -3/2*a^2*(a+b)*csch(d*x+c)^2/d-3/4*a*(a+b)^2*csch(d*x+c)^4/d-1/6*(a+b)^3*cs
ch(d*x+c)^6/d+a^3*ln(sinh(d*x+c))/d
```

Rubi [A]
time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 455, 45}

$$\frac{a^3\log(\sinh(c+dx))}{d}-\frac{3a^2(a+b)\text{csch}^2(c+dx)}{2d}-\frac{(a+b)^3\text{csch}^6(c+dx)}{6d}-\frac{3a(a+b)^2\text{csch}^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-3*a^2*(a + b)*Csch[c + d*x]^2)/(2*d) - (3*a*(a + b)^2*Csch[c + d*x]^4)/(4
*d) - ((a + b)^3*Csch[c + d*x]^6)/(6*d) + (a^3*Log[Sinh[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \coth^7(c+dx)\left(a+b\text{sech}^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x(b+ax^2)^3}{(1-x^2)^4}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b+ax)^3}{(1-x)^4}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^3}{(-1+x)^4} + \frac{3a(a+b)^2}{(-1+x)^3} + \frac{3a^2(a+b)}{(-1+x)^2} + \frac{a^3}{-1+x}\right)\,dx, x, \cosh^2(c\right.}{2d}$$
$$= -\frac{3a^2(a+b)\text{csch}^2(c+dx)}{2d} - \frac{3a(a+b)^2\text{csch}^4(c+dx)}{4d} - \frac{(a+}{}$$

Mathematica [A]
time = 0.53, size = 98, normalized size = 1.27

$$-\frac{2\left(b+a\cosh^2(c+dx)\right)^3\left(18a^2(a+b)\text{csch}^2(c+dx)+9a(a+b)^2\text{csch}^4(c+dx)+2(a+b)^3\text{csch}^6(c+dx)-12a^3\log(\sinh(c+dx))\right)}{3d(a+2b+a\cosh(2(c+dx)))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^7*(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (-2*(b + a*Cosh[c + d*x]^2)^3*(18*a^2*(a + b)*Csch[c + d*x]^2 + 9*a*(a + b)
^2*Csch[c + d*x]^4 + 2*(a + b)^3*Csch[c + d*x]^6 - 12*a^3*Log[Sinh[c + d*x]
]))/(3*d*(a + 2*b + a*Cosh[2*(c + d*x)])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(71) = 142.
time = 2.13, size = 149, normalized size = 1.94

method	result
derivativedivides	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}-\frac{\left(\coth^6(dx+c)\right)}{6}\right)+3a^2b\left(-\frac{\cosh^4(dx+c)}{2\sinh(dx+c)^6}+\frac{\cosh^2(dx+c)}{2\sinh(dx+c)^6}-\frac{1}{6\sinh(dx+}\right.}{d}$
default	$\frac{a^3\left(\ln(\sinh(dx+c))-\frac{\left(\coth^2(dx+c)\right)}{2}-\frac{\left(\coth^4(dx+c)\right)}{4}-\frac{\left(\coth^6(dx+c)\right)}{6}\right)+3a^2b\left(-\frac{\cosh^4(dx+c)}{2\sinh(dx+c)^6}+\frac{\cosh^2(dx+c)}{2\sinh(dx+c)^6}-\frac{1}{6\sinh(dx+}\right.}{d}$
risch	$-a^3x-\frac{2a^3c}{d}-\frac{2\,e^{2dx+2c}\left(9a^3e^{8dx+8c}+9a^2b\,e^{8dx+8c}-18a^3e^{6dx+6c}+18a\,b^2e^{6dx+6c}+34a^3e^{4dx+4c}+30a^2b\,e^{4dx+4c}\right.}{3d\left(e^{2dx+2c}-1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4-1/6*coth(d*x+
c)^6)+3*a^2*b*(-1/2/sinh(d*x+c)^6*cosh(d*x+c)^4+1/2/sinh(d*x+c)^6*cosh(d*x+
```

```
c)^2-1/6/sinh(d*x+c)^6)+3*a*b^2*(-1/4/sinh(d*x+c)^6*cosh(d*x+c)^2+1/12/sinh
(d*x+c)^6)-1/6*b^3/sinh(d*x+c)^6)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(71) = 142$.
time = 0.27, size = 727, normalized size = 9.44

$$\frac{1}{3}a^3\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}-18e^{(-4dx-4c)}+34e^{(-6dx-6c)}-18e^{(-8dx-8c)}+9e^{(-10dx-10c)}\right)}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e^{(-8dx-8c)}+6e^{(-10dx-10c)}-e^{(-12dx-12c)}-1)}\right)+2a^2b\left(\frac{3e^{(-2dx-2c)}}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e^{(-8dx-8c)}+6e^{(-10dx-10c)}-e^{(-12dx-12c)}-1)}+\frac{10e^{(-6dx-6c)}}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e^{(-8dx-8c)}+6e^{(-10dx-10c)}-e^{(-12dx-12c)}-1)}+\frac{3e^{(-10dx-10c)}}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e^{(-8dx-8c)}+6e^{(-10dx-10c)}-e^{(-12dx-12c)}-1)}\right)+4ab^2\left(\frac{3e^{(-4dx-4c)}}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e^{(-8dx-8c)}+6e^{(-10dx-10c)}-e^{(-12dx-12c)}-1)}+\frac{2e^{(-6dx-6c)}}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e^{(-8dx-8c)}+6e^{(-10dx-10c)}-e^{(-12dx-12c)}-1)}+\frac{3e^{(-8dx-8c)}}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e^{(-8dx-8c)}+6e^{(-10dx-10c)}-e^{(-12dx-12c)}-1)}\right)-\frac{32b^3}{3d\left(e^{(dx+c)}-e^{(-dx-c)}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)/
d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 18*
e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*
d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*
c) - e^(-12*d*x - 12*c) - 1))) + 2*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(6*e^(-2*d*
x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c)
+ 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 10*e^(-6*d*x - 6*c)/(d*
(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*
d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1)) + 3*e^(-10*d*x
 - 10*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c)
 - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) +
 4*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) +
 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*
d*x - 12*c) - 1)) + 2*e^(-6*d*x - 6*c)/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*
x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c)
 - e^(-12*d*x - 12*c) - 1)) + 3*e^(-8*d*x - 8*c)/(d*(6*e^(-2*d*x - 2*c) - 1
5*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d
*x - 10*c) - e^(-12*d*x - 12*c) - 1))) - 32/3*b^3/(d*(e^(d*x + c) - e^(-d*x
 - c))^6)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2632 vs. $2(71) = 142$.
time = 0.48, size = 2632, normalized size = 34.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^3*d*x*cosh(d*x + c)^12 + 36*a^3*d*x*cosh(d*x + c)*sinh(d*x + c)^1
1 + 3*a^3*d*x*sinh(d*x + c)^12 - 18*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^1
0 + 18*(11*a^3*d*x*cosh(d*x + c)^2 - a^3*d*x + a^3 + a^2*b)*sinh(d*x + c)^1
0 + 60*(11*a^3*d*x*cosh(d*x + c)^3 - 3*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c
))*sinh(d*x + c)^9 + 9*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^8 + 9*(1
```

65*a^3*d*x*cosh(d*x + c)^4 + 5*a^3*d*x - 4*a^3 + 4*a*b^2 - 90*(a^3*d*x - a^
3 - a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 72*(33*a^3*d*x*cosh(d*x + c)^
5 - 30*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^3 + (5*a^3*d*x - 4*a^3 + 4*a*b
^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*
a*b^2 - 8*b^3)*cosh(d*x + c)^6 + 4*(693*a^3*d*x*cosh(d*x + c)^6 - 15*a^3*d*
x - 945*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^4 + 17*a^3 + 15*a^2*b + 6*a*b
^2 + 8*b^3 + 63*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c
)^6 + 24*(99*a^3*d*x*cosh(d*x + c)^7 - 189*(a^3*d*x - a^3 - a^2*b)*cosh(d*x
 + c)^5 + 21*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^3 - (15*a^3*d*x -
17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*a^3
*d*x + 9*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^4 + 3*(495*a^3*d*x*cos
h(d*x + c)^8 - 1260*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^6 + 15*a^3*d*x +
210*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^4 - 12*a^3 + 12*a*b^2 - 20*
(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 4*(165*a^3*d*x*cosh(d*x + c)^9 - 540*(a^3*d*x - a^3 - a^2*b)*cos
h(d*x + c)^7 + 126*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^5 - 20*(15*a
^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(d*x + c)^3 + 9*(5*a^3*d*
x - 4*a^3 + 4*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 18*(a^3*d*x - a^3 - a
^2*b)*cosh(d*x + c)^2 + 6*(33*a^3*d*x*cosh(d*x + c)^10 - 135*(a^3*d*x - a^3
 - a^2*b)*cosh(d*x + c)^8 + 42*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^
6 - 3*a^3*d*x - 10*(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*cosh(
d*x + c)^4 + 3*a^3 + 3*a^2*b + 9*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c
)^2)*sinh(d*x + c)^2 - 3*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(
d*x + c)^11 + a^3*sinh(d*x + c)^12 - 6*a^3*cosh(d*x + c)^10 + 15*a^3*cosh(d
x + c)^8 + 6(11*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^10 + 20*(11*a^3*
cosh(d*x + c)^3 - 3*a^3*cosh(d*x + c))*sinh(d*x + c)^9 - 20*a^3*cosh(d*x +
c)^6 + 15*(33*a^3*cosh(d*x + c)^4 - 18*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x
+ c)^8 + 24*(33*a^3*cosh(d*x + c)^5 - 30*a^3*cosh(d*x + c)^3 + 5*a^3*cosh(d
*x + c))*sinh(d*x + c)^7 + 15*a^3*cosh(d*x + c)^4 + 4*(231*a^3*cosh(d*x + c
)^6 - 315*a^3*cosh(d*x + c)^4 + 105*a^3*cosh(d*x + c)^2 - 5*a^3)*sinh(d*x +
 c)^6 + 24*(33*a^3*cosh(d*x + c)^7 - 63*a^3*cosh(d*x + c)^5 + 35*a^3*cosh(d
*x + c)^3 - 5*a^3*cosh(d*x + c))*sinh(d*x + c)^5 - 6*a^3*cosh(d*x + c)^2 +
15*(33*a^3*cosh(d*x + c)^8 - 84*a^3*cosh(d*x + c)^6 + 70*a^3*cosh(d*x + c)^
4 - 20*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 20*(11*a^3*cosh(d*x + c
)^9 - 36*a^3*cosh(d*x + c)^7 + 42*a^3*cosh(d*x + c)^5 - 20*a^3*cosh(d*x + c
)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^3 + a^3 + 6*(11*a^3*cosh(d*x + c)^
10 - 45*a^3*cosh(d*x + c)^8 + 70*a^3*cosh(d*x + c)^6 - 50*a^3*cosh(d*x + c)
^4 + 15*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^2 + 12*(a^3*cosh(d*x + c)^
11 - 5*a^3*cosh(d*x + c)^9 + 10*a^3*cosh(d*x + c)^7 - 10*a^3*cosh(d*x + c)^
5 + 5*a^3*cosh(d*x + c)^3 - a^3*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*
x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 12*(3*a^3*d*x*cosh(d*x + c)^11 -
15*(a^3*d*x - a^3 - a^2*b)*cosh(d*x + c)^9 + 6*(5*a^3*d*x - 4*a^3 + 4*a*b^2
)*cosh(d*x + c)^7 - 2*(15*a^3*d*x - 17*a^3 - 15*a^2*b - 6*a*b^2 - 8*b^3)*co
sh(d*x + c)^5 + 3*(5*a^3*d*x - 4*a^3 + 4*a*b^2)*cosh(d*x + c)^3 - 3*(a^3*d*
x - a^3 - a^2*b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*c

```
osh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 - 6*d*cosh(d*x + c)^10 +
 6*(11*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 -
 3*d*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(
d*x + c)^4 - 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x
+ c)^5 - 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 20*d*c
osh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 - 315*d*cosh(d*x + c)^4 + 105*d*c
osh(d*x + c)^2 - 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 - 63*d*cos
h(d*x + c)^5 + 35*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^5 +
15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 70
*d*cosh(d*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*c
osh(d*x + c)^9 - 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 - 20*d*cosh(d*
x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 6*d*cosh(d*x + c)^2 + 6*(11
*d*cosh(d*x + c)^10 - 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 - 50*d*co
sh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x
+ c)^11 - 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 - 10*d*cosh(d*x + c)^5
 + 5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**7*(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(71) = 142.
time = 0.55, size = 242, normalized size = 3.14

$$-\frac{60\,(dx+c)a^3 - 60\,a^3\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right) + \frac{147\,a^3e^{(12\,dx+12\,c)}-522\,a^3e^{(10\,dx+10\,c)}+360\,a^2be^{(10\,dx+10\,c)}+1485\,a^3e^{(8\,dx+8\,c)}+720\,ab^2e^{(8\,dx+8\,c)}-1580\,a^3e^{(6\,dx+6\,c)}+1200\,a^2be^{(6\,dx+6\,c)}+480\,ab^2e^{(6\,dx+6\,c)}+640\,b^3e^{(6\,dx+6\,c)}+1485\,a^3e^{(4\,dx+4\,c)}+720\,ab^2e^{(4\,dx+4\,c)}-522\,a^3e^{(2\,dx+2\,c)}+360\,a^2be^{(2\,dx+2\,c)}+147\,a^3}{\left(e^{(2\,dx+2\,c)}-1\right)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^7*(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*(d*x + c)*a^3 - 60*a^3*log(abs(e^(2*d*x + 2*c) - 1)) + (147*a^3*e
^(12*d*x + 12*c) - 522*a^3*e^(10*d*x + 10*c) + 360*a^2*b*e^(10*d*x + 10*c)
+ 1485*a^3*e^(8*d*x + 8*c) + 720*a*b^2*e^(8*d*x + 8*c) - 1580*a^3*e^(6*d*x
+ 6*c) + 1200*a^2*b*e^(6*d*x + 6*c) + 480*a*b^2*e^(6*d*x + 6*c) + 640*b^3*e
^(6*d*x + 6*c) + 1485*a^3*e^(4*d*x + 4*c) + 720*a*b^2*e^(4*d*x + 4*c) - 522
*a^3*e^(2*d*x + 2*c) + 360*a^2*b*e^(2*d*x + 2*c) + 147*a^3)/(e^(2*d*x + 2*c
) - 1)^6)/d
```

Mupad [**B**]
time = 1.61, size = 411, normalized size = 5.34

$$\frac{a^3\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)}{d}-\frac{32\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}-10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}-5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}-1\right)}-\frac{32\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{3\,d\,\left(15\,\mathrm{e}^{4\,c+4\,d\,x}-6\,\mathrm{e}^{2\,c+2\,d\,x}-20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}-6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1\right)}-\frac{6\,\left(a^3+b\,a^2\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{6\,\left(3\,a^3+5\,a^2\,b+2\,a\,b^2\right)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{8\,\left(13\,a^3+30\,a^2\,b+21\,a\,b^2+4\,b^3\right)}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{4\,\left(11\,a^3+30\,a^2\,b+27\,a\,b^2+8\,b^3\right)}{d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}-a^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^7*(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] (a^3*log(exp(2*c)*exp(2*d*x) - 1))/d - (32*(3*a*b^2 + 3*a^2*b + a^3 + b^3))
/(d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp
(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)) - (32*(3*a*b^2 + 3*a^2*b + a^3 + b
^3))/(3*d*(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) +
 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (6
*(a^2*b + a^3))/(d*(exp(2*c + 2*d*x) - 1)) - (6*(2*a*b^2 + 5*a^2*b + 3*a^3)
)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*(21*a*b^2 + 30*a^2*b
 + 13*a^3 + 4*b^3))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) - 1)) - (4*(27*a*b^2 + 30*a^2*b + 11*a^3 + 8*b^3))/(d*(6*exp(4*c
+ 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
 - a^3*x
```

3.136 $\int \left(a+b\mathbf{sech}^2(c+dx)\right)^4 \, dx$

Optimal. Leaf size=111

$$a^4x+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}-\frac{b^2\left(6a^2+8ab+3b^2\right)\tanh^3(c+dx)}{3d}+\frac{b^3(4a+3b)\tanh^5(c+dx}{5d}$$

```
[Out] a^4*x+b*(2*a+b)*(2*a^2+2*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(6*a^2+8*a*b+3*b^2)
*tanh(d*x+c)^3/d+1/5*b^3*(4*a+3*b)*tanh(d*x+c)^5/d-1/7*b^4*tanh(d*x+c)^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4213, 398, 212}

$$a^4x-\frac{b^2\left(6a^2+8ab+3b^2\right)\tanh^3(c+dx)}{3d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d}+\frac{b^3(4a+3b)\tanh^5(c+dx)}{5d}-\frac{b^4\tanh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^4,x]
```

```
[Out] a^4*x + (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(6*a^2 +
 8*a*b + 3*b^2)*Tanh[c + d*x]^3)/(3*d) + (b^3*(4*a + 3*b)*Tanh[c + d*x]^5)/
(5*d) - (b^4*Tanh[c + d*x]^7)/(7*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^4\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a+b-bx^2)^4}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(b(2a+b)\left(2a^2+2ab+b^2\right)-b^2\left(6a^2+8ab+3b^2\right)x^2+b^3(4a+3b)\right.\right.}{d}$$

$$= \frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d} - \frac{b^2\left(6a^2+8ab+3b^2\right)\tanh^3(c+dx}{3d}$$

$$= a^4x + \frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d} - \frac{b^2\left(6a^2+8ab+3b^2\right)\tanh^3(}{3d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 455 vs. $2(111) = 222$.

time = 1.11, size = 455, normalized size = 4.10

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^4,x]
```

```
[Out] (Sech[c]*Sech[c + d*x]^7*(3675*a^4*d*x*Cosh[d*x] + 3675*a^4*d*x*Cosh[2*c +
d*x] + 2205*a^4*d*x*Cosh[2*c + 3*d*x] + 2205*a^4*d*x*Cosh[4*c + 3*d*x] + 73
5*a^4*d*x*Cosh[4*c + 5*d*x] + 735*a^4*d*x*Cosh[6*c + 5*d*x] + 105*a^4*d*x*C
osh[6*c + 7*d*x] + 105*a^4*d*x*Cosh[8*c + 7*d*x] + 16800*a^3*b*Sinh[d*x] +
18480*a^2*b^2*Sinh[d*x] + 11200*a*b^3*Sinh[d*x] + 3360*b^4*Sinh[d*x] - 1260
0*a^3*b*Sinh[2*c + d*x] - 10920*a^2*b^2*Sinh[2*c + d*x] - 4480*a*b^3*Sinh[2
*c + d*x] + 12600*a^3*b*Sinh[2*c + 3*d*x] + 15120*a^2*b^2*Sinh[2*c + 3*d*x]
 + 9408*a*b^3*Sinh[2*c + 3*d*x] + 2016*b^4*Sinh[2*c + 3*d*x] - 5040*a^3*b*S
inh[4*c + 3*d*x] - 2520*a^2*b^2*Sinh[4*c + 3*d*x] + 5040*a^3*b*Sinh[4*c + 5
*d*x] + 5880*a^2*b^2*Sinh[4*c + 5*d*x] + 3136*a*b^3*Sinh[4*c + 5*d*x] + 672
*b^4*Sinh[4*c + 5*d*x] - 840*a^3*b*Sinh[6*c + 5*d*x] + 840*a^3*b*Sinh[6*c +
 7*d*x] + 840*a^2*b^2*Sinh[6*c + 7*d*x] + 448*a*b^3*Sinh[6*c + 7*d*x] + 96*
b^4*Sinh[6*c + 7*d*x]))/(13440*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(105) = 210$.

time = 1.86, size = 310, normalized size = 2.79

method	result
risch	$x\,a^4 - \frac{8b\left(105a^3 e^{12dx+12c}+630a^3 e^{10dx+10c}+315a^2 b\, e^{10dx+10c}+1575a^3 e^{8dx+8c}+1365a^2 b\, e^{8dx+8c}+560a\,b^2 e^{8dx+8c}+2100a^3 e^{6dx+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] x*a^4-8/105*b*(105*a^3*exp(12*d*x+12*c)+630*a^3*exp(10*d*x+10*c)+315*a^2*b*
exp(10*d*x+10*c)+1575*a^3*exp(8*d*x+8*c)+1365*a^2*b*exp(8*d*x+8*c)+560*a*b^
2*exp(8*d*x+8*c)+2100*a^3*exp(6*d*x+6*c)+2310*a^2*b*exp(6*d*x+6*c)+1400*a*b
^2*exp(6*d*x+6*c)+420*b^3*exp(6*d*x+6*c)+1575*a^3*exp(4*d*x+4*c)+1890*a^2*b
*exp(4*d*x+4*c)+1176*a*b^2*exp(4*d*x+4*c)+252*b^3*exp(4*d*x+4*c)+630*a^3*ex
p(2*d*x+2*c)+735*a^2*b*exp(2*d*x+2*c)+392*a*b^2*exp(2*d*x+2*c)+84*b^3*exp(2
*d*x+2*c)+105*a^3+105*a^2*b+56*a*b^2+12*b^3)/d/(1+exp(2*d*x+2*c))^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(105) = 210$.
time = 0.29, size = 703, normalized size = 6.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 32/35*b^4*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x
 - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
 + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d
*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8
*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 1
4*c) + 1)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*
c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*
e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) +
21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10
*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 64/15*a*b
^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^
(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d
*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c
) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c
) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10
*d*x - 10*c) + 1))) + 8*a^2*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c)
+ 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) +
3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 8*a^3*b/(d*(e^(-2*d*x - 2*c)
 + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 941 vs. $2(105) = 210$.
time = 0.39, size = 941, normalized size = 8.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*((105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*
x + c)^7 + 7*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*c
osh(d*x + c)*sinh(d*x + c)^6 + 4*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b
^4)*sinh(d*x + c)^7 + 7*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3
- 48*b^4)*cosh(d*x + c)^5 + 28*(75*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4
+ 3*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^2)*sinh(d*x
 + c)^5 + 35*((105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*
cosh(d*x + c)^3 + (105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b
^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*a^4*d*x - 420*a^3*b - 420*a^2*
b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c)^3 + 28*(5*(105*a^3*b + 105*a^2*b^2
+ 56*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 135*a^3*b + 225*a^2*b^2 + 168*a*b^3
+ 36*b^4 + 10*(75*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^2)
*sinh(d*x + c)^3 + 7*(3*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224*a*b^3
- 48*b^4)*cosh(d*x + c)^5 + 10*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2 - 224
*a*b^3 - 48*b^4)*cosh(d*x + c)^3 + 9*(105*a^4*d*x - 420*a^3*b - 420*a^2*b^2
 - 224*a*b^3 - 48*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^4*d*x - 4
20*a^3*b - 420*a^2*b^2 - 224*a*b^3 - 48*b^4)*cosh(d*x + c) + 28*((105*a^3*b
 + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^6 + 5*(75*a^3*b + 105*a^2
*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 75*a^3*b + 135*a^2*b^2 + 120*a*
b^3 + 60*b^4 + 9*(45*a^3*b + 75*a^2*b^2 + 56*a*b^3 + 12*b^4)*cosh(d*x + c)^
2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 +
7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c
)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3
+ 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*cosh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(105) = 210.
time = 0.42, size = 334, normalized size = 3.01

$$\frac{105\,(dx+c)a^4 - \frac{8\,\left(105\,a^3be^{(12\,dx+12\,c)}+630\,a^3be^{(10\,dx+10\,c)}+315\,a^2b^2e^{(10\,dx+10\,c)}+1575\,a^3be^{(8\,dx+8\,c)}+1365\,a^2b^2e^{(8\,dx+8\,c)}+560\,ab^3e^{(8\,dx+8\,c)}+2100\,a^3be^{(6\,dx+6\,c)}+2310\,a^2b^2e^{(6\,dx+6\,c)}+1400\,ab^3e^{(6\,dx+6\,c)}+420\,b^4e^{(6\,dx+6\,c)}+1575\,a^3be^{(4\,dx+4\,c)}+1890\,a^2b^2e^{(4\,dx+4\,c)}+1176\,ab^3e^{(4\,dx+4\,c)}+252\,b^4e^{(4\,dx+4\,c)}+630\,a^3be^{(2\,dx+2\,c)}+735\,a^2b^2e^{(2\,dx+2\,c)}+392\,ab^3e^{(2\,dx+2\,c)}+84\,b^4e^{(2\,dx+2\,c)}+105\,a^3b+105\,a^2b^2+56\,ab^3+12\,b^4\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/105*(105*(d*x + c)*a^4 - 8*(105*a^3*b*e^(12*d*x + 12*c) + 630*a^3*b*e^(10
*d*x + 10*c) + 315*a^2*b^2*e^(10*d*x + 10*c) + 1575*a^3*b*e^(8*d*x + 8*c) +
 1365*a^2*b^2*e^(8*d*x + 8*c) + 560*a*b^3*e^(8*d*x + 8*c) + 2100*a^3*b*e^(6
*d*x + 6*c) + 2310*a^2*b^2*e^(6*d*x + 6*c) + 1400*a*b^3*e^(6*d*x + 6*c) + 4
20*b^4*e^(6*d*x + 6*c) + 1575*a^3*b*e^(4*d*x + 4*c) + 1890*a^2*b^2*e^(4*d*x
 + 4*c) + 1176*a*b^3*e^(4*d*x + 4*c) + 252*b^4*e^(4*d*x + 4*c) + 630*a^3*b*
e^(2*d*x + 2*c) + 735*a^2*b^2*e^(2*d*x + 2*c) + 392*a*b^3*e^(2*d*x + 2*c) +
 84*b^4*e^(2*d*x + 2*c) + 105*a^3*b + 105*a^2*b^2 + 56*a*b^3 + 12*b^4)/(e^(
2*d*x + 2*c) + 1)^7)/d
```

Mupad [B]
time = 0.20, size = 1083, normalized size = 9.76

$$a^4x-\frac{\frac{8\left(a^3b+a^2b^2\right)}{7d}+\frac{8e^{2c+2dx}\left(15a^3b+24a^2b^2+16ab^3\right)}{21d}+\frac{16e^{6c+6dx}\left(15a^3b+24a^2b^2+16ab^3\right)}{21d}+\frac{16e^{4c+4dx}\left(5a^3b+9a^2b^2+8ab^3+4b^4\right)}{7d}+\frac{40e^{8c+8dx}\left(a^3b+a^2b^2\right)}{7d}+\frac{8a^3be^{10c+10dx}}{7d}}{6e^{2c+2dx}+15e^{4c+4dx}+20e^{6c+6dx}+15e^{8c+8dx}+6e^{10c+10dx}+e^{12c+12dx}+1}-\frac{\frac{8e^{4c+4dx}\left(15a^3b+24a^2b^2+16ab^3\right)}{7d}+\frac{8e^{8c+8dx}\left(15a^3b+24a^2b^2+16ab^3\right)}{7d}+\frac{8a^3b}{7d}+\frac{32e^{6c+6dx}\left(5a^3b+9a^2b^2+8ab^3+4b^4\right)}{7d}+\frac{48e^{2c+2dx}\left(a^3b+a^2b^2\right)}{7d}+\frac{48e^{10c+10dx}\left(a^3b+a^2b^2\right)}{7d}+\frac{8a^3be^{12c+12dx}}{7d}}{7e^{2c+2dx}+21e^{4c+4dx}+35e^{6c+6dx}+35e^{8c+8dx}+21e^{10c+10dx}+7e^{12c+12dx}+e^{14c+14dx}+1}-\frac{\frac{8\left(a^3b+a^2b^2\right)}{7d}+\frac{8a^3be^{2c+2dx}}{7d}}{2e^{2c+2dx}+e^{4c+4dx}+1}-\frac{\frac{8\left(15a^3b+24a^2b^2+16ab^3\right)}{105d}+\frac{16e^{4c+4dx}\left(15a^3b+24a^2b^2+16ab^3\right)}{35d}+\frac{32e^{2c+2dx}\left(5a^3b+9a^2b^2+8ab^3+4b^4\right)}{35d}+\frac{32e^{6c+6dx}\left(a^3b+a^2b^2\right)}{7d}+\frac{8a^3be^{8c+8dx}}{7d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}-\frac{\frac{8\left(5a^3b+9a^2b^2+8ab^3+4b^4\right)}{35d}+\frac{8e^{2c+2dx}\left(15a^3b+24a^2b^2+16ab^3\right)}{35d}+\frac{24e^{4c+4dx}\left(a^3b+a^2b^2\right)}{7d}+\frac{8a^3be^{6c+6dx}}{7d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}-\frac{\frac{8\left(15a^3b+24a^2b^2+16ab^3\right)}{105d}+\frac{16e^{2c+2dx}\left(a^3b+a^2b^2\right)}{7d}+\frac{8a^3be^{4c+4dx}}{7d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}-\frac{8a^3b}{7d\left(e^{2c+2dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^4,x)
```

```
[Out] a^4*x - ((8*(a^3*b + a^2*b^2))/(7*d) + (8*exp(2*c + 2*d*x)*(16*a*b^3 + 15*a
^3*b + 24*a^2*b^2))/(21*d) + (16*exp(6*c + 6*d*x)*(16*a*b^3 + 15*a^3*b + 24
*a^2*b^2))/(21*d) + (16*exp(4*c + 4*d*x)*(8*a*b^3 + 5*a^3*b + 4*b^4 + 9*a^2
*b^2))/(7*d) + (40*exp(8*c + 8*d*x)*(a^3*b + a^2*b^2))/(7*d) + (8*a^3*b*exp
(10*c + 10*d*x))/(7*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(
6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d
*x) + 1) - ((8*exp(4*c + 4*d*x)*(16*a*b^3 + 15*a^3*b + 24*a^2*b^2))/(7*d) +
 (8*exp(8*c + 8*d*x)*(16*a*b^3 + 15*a^3*b + 24*a^2*b^2))/(7*d) + (8*a^3*b)/
(7*d) + (32*exp(6*c + 6*d*x)*(8*a*b^3 + 5*a^3*b + 4*b^4 + 9*a^2*b^2))/(7*d)
 + (48*exp(2*c + 2*d*x)*(a^3*b + a^2*b^2))/(7*d) + (48*exp(10*c + 10*d*x)*(
a^3*b + a^2*b^2))/(7*d) + (8*a^3*b*exp(12*c + 12*d*x))/(7*d))/(7*exp(2*c +
2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) +
21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((
8*(a^3*b + a^2*b^2))/(7*d) + (8*a^3*b*exp(2*c + 2*d*x))/(7*d))/(2*exp(2*c +
 2*d*x) + exp(4*c + 4*d*x) + 1) - ((8*(16*a*b^3 + 15*a^3*b + 24*a^2*b^2))/(
105*d) + (16*exp(4*c + 4*d*x)*(16*a*b^3 + 15*a^3*b + 24*a^2*b^2))/(35*d) +
(32*exp(2*c + 2*d*x)*(8*a*b^3 + 5*a^3*b + 4*b^4 + 9*a^2*b^2))/(35*d) + (32*
exp(6*c + 6*d*x)*(a^3*b + a^2*b^2))/(7*d) + (8*a^3*b*exp(8*c + 8*d*x))/(7*d
))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(
8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((8*(8*a*b^3 + 5*a^3*b + 4*b^4 + 9
*a^2*b^2))/(35*d) + (8*exp(2*c + 2*d*x)*(16*a*b^3 + 15*a^3*b + 24*a^2*b^2))
/(35*d) + (24*exp(4*c + 4*d*x)*(a^3*b + a^2*b^2))/(7*d) + (8*a^3*b*exp(6*c
+ 6*d*x))/(7*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d
*x) + exp(8*c + 8*d*x) + 1) - ((8*(16*a*b^3 + 15*a^3*b + 24*a^2*b^2))/(105*
d) + (16*exp(2*c + 2*d*x)*(a^3*b + a^2*b^2))/(7*d) + (8*a^3*b*exp(4*c + 4*d
*x))/(7*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1
) - (8*a^3*b)/(7*d*(exp(2*c + 2*d*x) + 1))
```

3.137 $\int \left(a+b\text{sech}^2(c+dx)\right)^5\,dx$

Optimal. Leaf size=163

$$a^5x+\frac{b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)\tanh(c+dx)}{d}-\frac{b^2(10a^3+20a^2b+15ab^2+4b^3)\tanh^3(c+dx)}{3d}+\frac{b^3}{}$$

```
[Out] a^5*x+b*(5*a^4+10*a^3*b+10*a^2*b^2+5*a*b^3+b^4)*tanh(d*x+c)/d-1/3*b^2*(10*a
^3+20*a^2*b+15*a*b^2+4*b^3)*tanh(d*x+c)^3/d+1/5*b^3*(10*a^2+15*a*b+6*b^2)*t
anh(d*x+c)^5/d-1/7*b^4*(5*a+4*b)*tanh(d*x+c)^7/d+1/9*b^5*tanh(d*x+c)^9/d
```

Rubi [A]
time = 0.07, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4213, 398, 212}

$$a^5x+\frac{b^3(10a^2+15ab+6b^2)\tanh^5(c+dx)}{5d}-\frac{b^2(10a^3+20a^2b+15ab^2+4b^3)\tanh^3(c+dx)}{3d}+\frac{b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)\tanh(c+dx)}{d}-\frac{b^4(5a+4b)\tanh^7(c+dx)}{7d}+\frac{b^5\tanh^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^5,x]
```

```
[Out] a^5*x + (b*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a*b^3 + b^4)*Tanh[c + d*x])/d
 - (b^2*(10*a^3 + 20*a^2*b + 15*a*b^2 + 4*b^3)*Tanh[c + d*x]^3)/(3*d) + (b^
3*(10*a^2 + 15*a*b + 6*b^2)*Tanh[c + d*x]^5)/(5*d) - (b^4*(5*a + 4*b)*Tanh[
c + d*x]^7)/(7*d) + (b^5*Tanh[c + d*x]^9)/(9*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\text{sech}^2(c+dx)\right)^5\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^5}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(b\left(5a^4+10a^3b+10a^2b^2+5ab^3+b^4\right)-b^2\left(10a^3+20a^2b+15ab^2+\right.\right.\right.}{}$$

$$= \frac{b\left(5a^4+10a^3b+10a^2b^2+5ab^3+b^4\right)\tanh(c+dx)}{d} - \frac{b^2\left(10a^3+20a^2b+15ab^2\right.}{3d}$$

$$= a^5x + \frac{b\left(5a^4+10a^3b+10a^2b^2+5ab^3+b^4\right)\tanh(c+dx)}{d} - \frac{b^2\left(10a^3+20a^2b+\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 724 vs. 2(163) = 326.
time = 6.38, size = 724, normalized size = 4.44

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^5,x]
```

```
[Out] (32*a^5*x*Cosh[c + d*x]^10*(a + b*Sech[c + d*x]^2)^5)/(a + 2*b + a*Cosh[2*c
 + 2*d*x])^5 + (32*Cosh[c + d*x]^4*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(45*a*
b^4*Sinh[c] + 8*b^5*Sinh[c]))/(63*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (6
4*Cosh[c + d*x]^6*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(105*a^2*b^3*Sinh[c] +
45*a*b^4*Sinh[c] + 8*b^5*Sinh[c]))/(105*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5
) + (64*Cosh[c + d*x]^8*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(525*a^3*b^2*Sinh
[c] + 420*a^2*b^3*Sinh[c] + 180*a*b^4*Sinh[c] + 32*b^5*Sinh[c]))/(315*d*(a
+ 2*b + a*Cosh[2*c + 2*d*x])^5) + (32*b^5*Cosh[c + d*x]*Sech[c]*(a + b*Sech
[c + d*x]^2)^5*Sinh[d*x])/(9*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (32*Cos
h[c + d*x]^3*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(45*a*b^4*Sinh[d*x] + 8*b^5*
Sinh[d*x]))/(63*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (64*Cosh[c + d*x]^5*
Sech[c]*(a + b*Sech[c + d*x]^2)^5*(105*a^2*b^3*Sinh[d*x] + 45*a*b^4*Sinh[d*
x] + 8*b^5*Sinh[d*x]))/(105*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5) + (64*Cosh
[c + d*x]^7*Sech[c]*(a + b*Sech[c + d*x]^2)^5*(525*a^3*b^2*Sinh[d*x] + 420*
a^2*b^3*Sinh[d*x] + 180*a*b^4*Sinh[d*x] + 32*b^5*Sinh[d*x]))/(315*d*(a + 2*
b + a*Cosh[2*c + 2*d*x])^5) + (32*Cosh[c + d*x]^9*Sech[c]*(a + b*Sech[c + d
*x]^2)^5*(1575*a^4*b*Sinh[d*x] + 2100*a^3*b^2*Sinh[d*x] + 1680*a^2*b^3*Sinh
[d*x] + 720*a*b^4*Sinh[d*x] + 128*b^5*Sinh[d*x]))/(315*d*(a + 2*b + a*Cosh[
2*c + 2*d*x])^5) + (32*b^5*Cosh[c + d*x]^2*(a + b*Sech[c + d*x]^2)^5*Tanh[c
])/(9*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(155) = 310$.
time = 1.96, size = 507, normalized size = 3.11

method	result
risch	$a^5x - \frac{2b(136080a^2b^2e^{8dx+8c}+88200a^4e^{6dx+6c}+60480a^2b^2e^{4dx+4c}+25920a\,b^3e^{4dx+4c}+75600a^2b^2e^{10dx+10c}+1575a^4+107100a^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*x-2/315*b*(136080*a^2*b^2*exp(8*d*x+8*c)+88200*a^4*exp(6*d*x+6*c)+60480
*a^2*b^2*exp(4*d*x+4*c)+25920*a*b^3*exp(4*d*x+4*c)+75600*a^2*b^2*exp(10*d*x
+10*c)+1575*a^4+107100*a^3*b*exp(10*d*x+10*c)+6480*a*b^3*exp(2*d*x+2*c)+720
*a*b^3+128*b^4+1680*a^2*b^2+25200*a*b^3*exp(10*d*x+10*c)+157500*a^3*b*exp(8
*d*x+8*c)+6300*a^3*b*exp(14*d*x+14*c)+39900*a^3*b*exp(12*d*x+12*c)+16800*a^
2*b^2*exp(12*d*x+12*c)+2100*a^3*b+18900*a^3*b*exp(2*d*x+2*c)+15120*a^2*b^2*
exp(2*d*x+2*c)+69300*a^3*b*exp(4*d*x+4*c)+4608*b^4*exp(4*d*x+4*c)+10752*b^4
*exp(6*d*x+6*c)+12600*a^4*exp(14*d*x+14*c)+44100*a^4*exp(12*d*x+12*c)+11025
0*a^4*exp(8*d*x+8*c)+88200*a^4*exp(10*d*x+10*c)+16128*b^4*exp(8*d*x+8*c)+12
600*a^4*exp(2*d*x+2*c)+65520*a*b^3*exp(8*d*x+8*c)+136500*a^3*b*exp(6*d*x+6*
c)+124320*a^2*b^2*exp(6*d*x+6*c)+60480*a*b^3*exp(6*d*x+6*c)+44100*a^4*exp(4
*d*x+4*c)+1575*a^4*exp(16*d*x+16*c)+1152*b^4*exp(2*d*x+2*c))/d/(1+exp(2*d*x
+2*c))^9
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1277 vs. $2(155) = 310$.
time = 0.29, size = 1277, normalized size = 7.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^5,x, algorithm="maxima")
```

```
[Out] a^5*x + 256/315*b^5*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d
*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 1
0*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c)
 + e^(-18*d*x - 18*c) + 1)) + 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) +
36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-
10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d
*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x
 - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c)
+ 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) +
9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 126*e^(-8*d*x - 8*c)/(d*(
9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*
```

```
d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x
 - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 1/(d*(9*e^(-2*
d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*
c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c)
 + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1))) + 32/7*a*b^4*(7*e^(-2*d
*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c
) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^
(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x
 - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 35*e^(-6*d*x -
6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 3
5*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*
d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(
-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x
 - 12*c) + e^(-14*d*x - 14*c) + 1))) + 32/3*a^2*b^3*(5*e^(-2*d*x - 2*c)/(d*(
5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*
x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 40/3*
a^3*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1))) + 10*a^4*b/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1652 vs. $2(155) = 310$.
time = 0.40, size = 1652, normalized size = 10.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^5,x, algorithm="fricas")
```

```
[Out] 1/315*((315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4
- 128*b^5)*cosh(d*x + c)^9 + 9*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1
680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)*sinh(d*x + c)^8 + (1575*a^
4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*sinh(d*x + c)^9 +
9*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128
*b^5)*cosh(d*x + c)^7 + 9*(1225*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a
*b^4 + 128*b^5 + 4*(1575*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 +
128*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 21*(4*(315*a^5*d*x - 1575*a^4*b
 - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^3 + 3*(
315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^
5)*cosh(d*x + c))*sinh(d*x + c)^6 + 36*(315*a^5*d*x - 1575*a^4*b - 2100*a^3
*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^5 + 9*(3500*a^4*b
```

```
+ 7000*a^3*b^2 + 6720*a^2*b^3 + 2880*a*b^4 + 512*b^5 + 14*(1575*a^4*b + 210
0*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^4 + 21*(1225*
a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^2)
*sinh(d*x + c)^5 + 9*(14*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^
2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^5 + 35*(315*a^5*d*x - 1575*a^4*b
 - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^3 + 20*
(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b
^5)*cosh(d*x + c))*sinh(d*x + c)^4 + 84*(315*a^5*d*x - 1575*a^4*b - 2100*a^
3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^3 + 3*(28*(1575*a
^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^6 +
 14700*a^4*b + 32200*a^3*b^2 + 35840*a^2*b^3 + 20160*a*b^4 + 3584*b^5 + 105
*(1225*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x
+ c)^4 + 120*(875*a^4*b + 1750*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5
)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 9*(4*(315*a^5*d*x - 1575*a^4*b - 2100*
a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c)^7 + 21*(315*a^5
*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh
(d*x + c)^5 + 40*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 -
720*a*b^4 - 128*b^5)*cosh(d*x + c)^3 + 28*(315*a^5*d*x - 1575*a^4*b - 2100*
a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4 - 128*b^5)*cosh(d*x + c))*sinh(d*x + c)^
2 + 126*(315*a^5*d*x - 1575*a^4*b - 2100*a^3*b^2 - 1680*a^2*b^3 - 720*a*b^4
 - 128*b^5)*cosh(d*x + c) + 9*((1575*a^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 +
720*a*b^4 + 128*b^5)*cosh(d*x + c)^8 + 7*(1225*a^4*b + 2100*a^3*b^2 + 1680*
a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^6 + 2450*a^4*b + 5600*a^3*b^2
+ 6720*a^2*b^3 + 4480*a*b^4 + 1792*b^5 + 20*(875*a^4*b + 1750*a^3*b^2 + 168
0*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^4 + 28*(525*a^4*b + 1150*a^3
*b^2 + 1280*a^2*b^3 + 720*a*b^4 + 128*b^5)*cosh(d*x + c)^2)*sinh(d*x + c))/
(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + 9*d*cosh(d*x + c)^
7 + 21*(4*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^6 + 36*d*cos
h(d*x + c)^5 + 9*(14*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 20*d*cosh(d
*x + c))*sinh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 + 9*(4*d*cosh(d*x + c)^7 +
21*d*cosh(d*x + c)^5 + 40*d*cosh(d*x + c)^3 + 28*d*cosh(d*x + c))*sinh(d*x
+ c)^2 + 126*d*cosh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2\left(c + dx\right)\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**5,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**5, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. 2(155) = 310.

time = 0.44, size = 537, normalized size = 3.29

$$\frac{315\,(dx+c)a^5 - \frac{2\,(1575\,a^4be^{(16\,dx+16\,c)}+12600\,a^4be^{(14\,dx+14\,c)}+6300\,a^3b^2e^{(14\,dx+14\,c)}+44100\,a^4be^{(12\,dx+12\,c)}+39900\,a^3b^2e^{(12\,dx+12\,c)}+16800\,a^2b^3e^{(12\,dx+12\,c)}+88200\,a^4be^{(10\,dx+10\,c)}+107100\,a^3b^2e^{(10\,dx+10\,c)}+75600\,a^2b^3e^{(10\,dx+10\,c)}+25200\,ab^4e^{(10\,dx+10\,c)}+110250\,a^4be^{(8\,dx+8\,c)}+157500\,a^3b^2e^{(8\,dx+8\,c)}+136080\,a^2b^3e^{(8\,dx+8\,c)}+65520\,ab^4e^{(8\,dx+8\,c)}+16128\,b^5e^{(8\,dx+8\,c)}+88200\,a^4be^{(6\,dx+6\,c)}+136500\,a^3b^2e^{(6\,dx+6\,c)}+124320\,a^2b^3e^{(6\,dx+6\,c)}+60480\,ab^4e^{(6\,dx+6\,c)}+10752\,b^5e^{(6\,dx+6\,c)}+44100\,a^4be^{(4\,dx+4\,c)}+69300\,a^3b^2e^{(4\,dx+4\,c)}+60480\,a^2b^3e^{(4\,dx+4\,c)}+25920\,ab^4e^{(4\,dx+4\,c)}+4608\,b^5e^{(4\,dx+4\,c)}+12600\,a^4be^{(2\,dx+2\,c)}+18900\,a^3b^2e^{(2\,dx+2\,c)}+15120\,a^2b^3e^{(2\,dx+2\,c)}+6480\,ab^4e^{(2\,dx+2\,c)}+1152\,b^5e^{(2\,dx+2\,c)}+1575\,a^4b+2100\,a^3b^2+1680\,a^2b^3+720\,ab^4+128\,b^5)}{(e^{(2\,dx+2\,c)}+1)^9}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^5,x, algorithm="giac")
```

```
[Out] 1/315*(315*(d*x + c)*a^5 - 2*(1575*a^4*b*e^(16*d*x + 16*c) + 12600*a^4*b*e^
(14*d*x + 14*c) + 6300*a^3*b^2*e^(14*d*x + 14*c) + 44100*a^4*b*e^(12*d*x +
12*c) + 39900*a^3*b^2*e^(12*d*x + 12*c) + 16800*a^2*b^3*e^(12*d*x + 12*c) +
 88200*a^4*b*e^(10*d*x + 10*c) + 107100*a^3*b^2*e^(10*d*x + 10*c) + 75600*a
^2*b^3*e^(10*d*x + 10*c) + 25200*a*b^4*e^(10*d*x + 10*c) + 110250*a^4*b*e^(
8*d*x + 8*c) + 157500*a^3*b^2*e^(8*d*x + 8*c) + 136080*a^2*b^3*e^(8*d*x + 8
*c) + 65520*a*b^4*e^(8*d*x + 8*c) + 16128*b^5*e^(8*d*x + 8*c) + 88200*a^4*b
*e^(6*d*x + 6*c) + 136500*a^3*b^2*e^(6*d*x + 6*c) + 124320*a^2*b^3*e^(6*d*x
 + 6*c) + 60480*a*b^4*e^(6*d*x + 6*c) + 10752*b^5*e^(6*d*x + 6*c) + 44100*a
^4*b*e^(4*d*x + 4*c) + 69300*a^3*b^2*e^(4*d*x + 4*c) + 60480*a^2*b^3*e^(4*d
*x + 4*c) + 25920*a*b^4*e^(4*d*x + 4*c) + 4608*b^5*e^(4*d*x + 4*c) + 12600*
a^4*b*e^(2*d*x + 2*c) + 18900*a^3*b^2*e^(2*d*x + 2*c) + 15120*a^2*b^3*e^(2*
d*x + 2*c) + 6480*a*b^4*e^(2*d*x + 2*c) + 1152*b^5*e^(2*d*x + 2*c) + 1575*a
^4*b + 2100*a^3*b^2 + 1680*a^2*b^3 + 720*a*b^4 + 128*b^5)/(e^(2*d*x + 2*c)
+ 1)^9)/d
```

Mupad [B]
time = 0.32, size = 1952, normalized size = 11.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^5,x)
```

```
[Out] a^5*x - ((10*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2))/(63*d) + (10*ex
p(2*c + 2*d*x)*(7*a^4*b + 8*a^2*b^3 + 12*a^3*b^2))/(21*d) + (10*exp(4*c + 4
*d*x)*(a^4*b + a^3*b^2))/(3*d) + (10*a^4*b*exp(6*c + 6*d*x))/(9*d))/(4*exp(
2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) +
 1) - ((80*exp(6*c + 6*d*x)*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2))/
(9*d) + (80*exp(10*c + 10*d*x)*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2
))/(9*d) + (4*exp(8*c + 8*d*x)*(320*a*b^4 + 175*a^4*b + 128*b^5 + 480*a^2*b
^3 + 400*a^3*b^2))/(9*d) + (10*a^4*b)/(9*d) + (40*exp(4*c + 4*d*x)*(7*a^4*b
 + 8*a^2*b^3 + 12*a^3*b^2))/(9*d) + (40*exp(12*c + 12*d*x)*(7*a^4*b + 8*a^2
*b^3 + 12*a^3*b^2))/(9*d) + (80*exp(2*c + 2*d*x)*(a^4*b + a^3*b^2))/(9*d) +
 (80*exp(14*c + 14*d*x)*(a^4*b + a^3*b^2))/(9*d) + (10*a^4*b*exp(16*c + 16*
d*x))/(9*d))/(9*exp(2*c + 2*d*x) + 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x
) + 126*exp(8*c + 8*d*x) + 126*exp(10*c + 10*d*x) + 84*exp(12*c + 12*d*x) +
 36*exp(14*c + 14*d*x) + 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) + 1) - (
```

```
(10*(a^4*b + a^3*b^2))/(9*d) + (10*a^4*b*exp(2*c + 2*d*x))/(9*d))/(2*exp(2*
c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((10*(7*a^4*b + 8*a^2*b^3 + 12*a^3*b^2
))/(63*d) + (20*exp(2*c + 2*d*x)*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b
^2))/(21*d) + (200*exp(6*c + 6*d*x)*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^
3*b^2))/(63*d) + (2*exp(4*c + 4*d*x)*(320*a*b^4 + 175*a^4*b + 128*b^5 + 480
*a^2*b^3 + 400*a^3*b^2))/(21*d) + (50*exp(8*c + 8*d*x)*(7*a^4*b + 8*a^2*b^3
 + 12*a^3*b^2))/(21*d) + (20*exp(10*c + 10*d*x)*(a^4*b + a^3*b^2))/(3*d) +
(10*a^4*b*exp(12*c + 12*d*x))/(9*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d
*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7
*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((2*(320*a*b^4 + 175*a^4*b
+ 128*b^5 + 480*a^2*b^3 + 400*a^3*b^2))/(315*d) + (40*exp(2*c + 2*d*x)*(8*a
*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2))/(63*d) + (20*exp(4*c + 4*d*x)*(7
*a^4*b + 8*a^2*b^3 + 12*a^3*b^2))/(21*d) + (40*exp(6*c + 6*d*x)*(a^4*b + a^
3*b^2))/(9*d) + (10*a^4*b*exp(8*c + 8*d*x))/(9*d))/(5*exp(2*c + 2*d*x) + 10
*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 1
0*d*x) + 1) - ((10*(a^4*b + a^3*b^2))/(9*d) + (10*exp(4*c + 4*d*x)*(8*a*b^4
 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2))/(3*d) + (50*exp(8*c + 8*d*x)*(8*a*b^
4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2))/(9*d) + (2*exp(6*c + 6*d*x)*(320*a*
b^4 + 175*a^4*b + 128*b^5 + 480*a^2*b^3 + 400*a^3*b^2))/(9*d) + (10*exp(2*c
 + 2*d*x)*(7*a^4*b + 8*a^2*b^3 + 12*a^3*b^2))/(9*d) + (10*exp(10*c + 10*d*x
)*(7*a^4*b + 8*a^2*b^3 + 12*a^3*b^2))/(3*d) + (70*exp(12*c + 12*d*x)*(a^4*b
 + a^3*b^2))/(9*d) + (10*a^4*b*exp(14*c + 14*d*x))/(9*d))/(8*exp(2*c + 2*d*
x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*e
xp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c
 + 16*d*x) + 1) - ((10*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2))/(63*d
) + (100*exp(4*c + 4*d*x)*(8*a*b^4 + 7*a^4*b + 16*a^2*b^3 + 15*a^3*b^2))/(6
3*d) + (2*exp(2*c + 2*d*x)*(320*a*b^4 + 175*a^4*b + 128*b^5 + 480*a^2*b^3 +
 400*a^3*b^2))/(63*d) + (100*exp(6*c + 6*d*x)*(7*a^4*b + 8*a^2*b^3 + 12*a^3
*b^2))/(63*d) + (50*exp(8*c + 8*d*x)*(a^4*b + a^3*b^2))/(9*d) + (10*a^4*b*e
xp(10*c + 10*d*x))/(9*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*ex
p(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12
*d*x) + 1) - ((10*(7*a^4*b + 8*a^2*b^3 + 12*a^3*b^2))/(63*d) + (20*exp(2*c
+ 2*d*x)*(a^4*b + a^3*b^2))/(9*d) + (10*a^4*b*exp(4*c + 4*d*x))/(9*d))/(3*e
xp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (10*a^4*b)/(
9*d*(exp(2*c + 2*d*x) + 1))
```

3.138 $\int \frac{\tanh^5(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=70

$$-\frac{(a+2b)\log(\cosh(c+dx))}{b^2d}+\frac{(a+b)^2\log\left(b+a\cosh^2(c+dx)\right)}{2ab^2d}-\frac{\text{sech}^2(c+dx)}{2bd}$$

```
[Out] -(a+2*b)*ln(cosh(d*x+c))/b^2/d+1/2*(a+b)^2*ln(b+a*cosh(d*x+c)^2)/a/b^2/d-1/
2*sech(d*x+c)^2/b/d
```

Rubi [A]
time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 457, 90}

$$\frac{(a+b)^2\log\left(a\cosh^2(c+dx)+b\right)}{2ab^2d}-\frac{(a+2b)\log(\cosh(c+dx))}{b^2d}-\frac{\text{sech}^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -(((a + 2*b)*Log[Cosh[c + d*x]])/(b^2*d)) + ((a + b)^2*Log[b + a*Cosh[c + d
*x]^2])/(2*a*b^2*d) - Sech[c + d*x]^2/(2*b*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^3(b+ax^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2(b+ax)}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^2}+\frac{-a-2b}{b^2x}+\frac{(a+b)^2}{b^2(b+ax)}\right)\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{(a+2b)\log(\cosh(c+dx))}{b^2d}+\frac{(a+b)^2\log\left(b+a\cosh^2(c+dx)\right)}{2ab^2d}-\frac{\text{sech}^2(c+d}{2bd}$$

Mathematica [A]
time = 0.22, size = 98, normalized size = 1.40

$$-\frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^2(c+dx)\left(2a(a+2b)\log(\cosh(c+dx))-(a+b)^2\log\left(a+b+a\sinh^2(c+dx)\right)+ab\text{sech}^2(c+dx)\right)}{4ab^2d\left(a+b\text{sech}^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(2*a*(a + 2*b)*Log[Co
sh[c + d*x]] - (a + b)^2*Log[a + b + a*Sinh[c + d*x]^2] + a*b*Sech[c + d*x]
^2))/(a*b^2*d*(a + b*Sech[c + d*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(66) = 132.
time = 1.91, size = 183, normalized size = 2.61

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2\left(\frac{1}{4}a^2+\frac{1}{2}ab+\frac{1}{4}b^2\right)\ln\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{b^2a}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2\left(\frac{1}{4}a^2+\frac{1}{2}ab+\frac{1}{4}b^2\right)\ln\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{b^2a}}{d}$
risch	$-\frac{x}{a}-\frac{2c}{ad}-\frac{2\,e^{2dx+2c}}{bd\left(1+e^{2dx+2c}\right)^2}+\frac{a\ln\left(e^{4dx+4c}+\frac{2(2b+a)e^{2dx+2c}}{a}+1\right)}{2b^2d}+\frac{\ln\left(e^{4dx+4c}+\frac{2(2b+a)e^{2dx+2c}}{a}+1\right)}{bd}+\frac{\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a*ln(tanh(1/2*d*x+1/2*c)+1)-1/a*ln(tanh(1/2*d*x+1/2*c)-1)+2/b^2/a*(
1/4*a^2+1/2*a*b+1/4*b^2)*ln(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4
+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)-1/b^2*((2*b+a)*ln
(tanh(1/2*d*x+1/2*c)^2+1)-2*b/(tanh(1/2*d*x+1/2*c)^2+1)+2*b/(tanh(1/2*d*x+1
/2*c)^2+1)^2))
```

Maxima [A]
time = 0.48, size = 131, normalized size = 1.87

$$\frac{dx+c}{ad} - \frac{2\,e^{(-2\,dx-2\,c)}}{\left(2\,be^{(-2\,dx-2\,c)} + be^{(-4\,dx-4\,c)} + b\right)d} - \frac{(a+2\,b)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^2d} + \frac{(a^2+2\,ab+b^2)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)} + ae^{(-4\,dx-4\,c)} + a\right)}{2\,ab^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) - 2*e^(-2*d*x - 2*c)/((2*b*e^(-2*d*x - 2*c) + b*e^(-4*d*x -
 4*c) + b)*d) - (a + 2*b)*log(e^(-2*d*x - 2*c) + 1)/(b^2*d) + 1/2*(a^2 + 2*
a*b + b^2)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a*b^
2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(66) = 132$.
time = 0.44, size = 736, normalized size = 10.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*d*x*cosh(d*x + c)^4 + 8*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 +
 2*b^2*d*x*sinh(d*x + c)^4 + 2*b^2*d*x + 4*(b^2*d*x + a*b)*cosh(d*x + c)^2
+ 4*(3*b^2*d*x*cosh(d*x + c)^2 + b^2*d*x + a*b)*sinh(d*x + c)^2 - ((a^2 + 2
*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b + b^2)*cosh(d
*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*s
inh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3
 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)
^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2)) + 2*((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2
*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^
2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 2*a
*b)*sinh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2
 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c)
- sinh(d*x + c))) + 8*(b^2*d*x*cosh(d*x + c)^3 + (b^2*d*x + a*b)*cosh(d*x +
 c))*sinh(d*x + c))/(a*b^2*d*cosh(d*x + c)^4 + 4*a*b^2*d*cosh(d*x + c)*sinh
(d*x + c)^3 + a*b^2*d*sinh(d*x + c)^4 + 2*a*b^2*d*cosh(d*x + c)^2 + a*b^2*d
```

```
 + 2*(3*a*b^2*d*cosh(d*x + c)^2 + a*b^2*d)*sinh(d*x + c)^2 + 4*(a*b^2*d*cos
h(d*x + c)^3 + a*b^2*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(tanh(c + d*x)**5/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.81, size = 421, normalized size = 6.01

$$\frac{2}{b\,d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}-\frac{2}{b\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{x}{a}-\frac{\ln\left(39\,a\,b^7+243\,a^7\,b+27\,a^8+2\,b^8+289\,a^2\,b^6+1017\,a^3\,b^5+1791\,a^4\,b^4+1701\,a^5\,b^3+891\,a^6\,b^2+27\,a^8\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+2\,b^8\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+39\,a\,b^7\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+243\,a^7\,b\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+289\,a^2\,b^6\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+1017\,a^3\,b^5\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+1791\,a^4\,b^4\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+1701\,a^5\,b^3\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+891\,a^6\,b^2\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\right)\,\left(a+2\,b\right)}{b^2\,d}+\frac{\ln\left(a\,b^2+6\,a^2\,b+3\,a^3+6\,a^3\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+3\,a^3\,{\mathrm{e}}^{4\,c}\,{\mathrm{e}}^{4\,d\,x}+4\,b^3\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+26\,a\,b^2\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+24\,a^2\,b\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+a\,b^2\,{\mathrm{e}}^{4\,c}\,{\mathrm{e}}^{4\,d\,x}+6\,a^2\,b\,{\mathrm{e}}^{4\,c}\,{\mathrm{e}}^{4\,d\,x}\right)\,\left(a^2+2\,a\,b+b^2\right)}{2\,a\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] 2/(b*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - 2/(b*d*(exp(2*c + 2*d
*x) + 1)) - x/a - (log(39*a*b^7 + 243*a^7*b + 27*a^8 + 2*b^8 + 289*a^2*b^6
+ 1017*a^3*b^5 + 1791*a^4*b^4 + 1701*a^5*b^3 + 891*a^6*b^2 + 27*a^8*exp(2*c
)*exp(2*d*x) + 2*b^8*exp(2*c)*exp(2*d*x) + 39*a*b^7*exp(2*c)*exp(2*d*x) + 2
43*a^7*b*exp(2*c)*exp(2*d*x) + 289*a^2*b^6*exp(2*c)*exp(2*d*x) + 1017*a^3*b
^5*exp(2*c)*exp(2*d*x) + 1791*a^4*b^4*exp(2*c)*exp(2*d*x) + 1701*a^5*b^3*ex
p(2*c)*exp(2*d*x) + 891*a^6*b^2*exp(2*c)*exp(2*d*x))*(a + 2*b))/(b^2*d) + (
log(a*b^2 + 6*a^2*b + 3*a^3 + 6*a^3*exp(2*c)*exp(2*d*x) + 3*a^3*exp(4*c)*ex
p(4*d*x) + 4*b^3*exp(2*c)*exp(2*d*x) + 26*a*b^2*exp(2*c)*exp(2*d*x) + 24*a^
2*b*exp(2*c)*exp(2*d*x) + a*b^2*exp(4*c)*exp(4*d*x) + 6*a^2*b*exp(4*c)*exp(
4*d*x))*(2*a*b + a^2 + b^2))/(2*a*b^2*d)
```

3.139 $\int \frac{\tanh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{x}{a}-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ab^{3/2}d}+\frac{\tanh(c+dx)}{bd}$$

```
[Out] x/a-(a+b)^(3/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a/b^(3/2)/d+tanh(d
*x+c)/b/d
```

Rubi [A]
time = 0.12, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4226, 2000, 490, 536, 212, 214}

$$-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ab^{3/2}d}+\frac{x}{a}+\frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] x/a - ((a + b)^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*b^(3/
2)*d) + Tanh[c + d*x]/(b*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^4(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+b(1-x^2))}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\
&= \frac{\tanh(c+dx)}{bd} - \frac{\text{Subst}\left(\int \frac{a+b+(-a-2b)x^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{bd}\\
&= \frac{\tanh(c+dx)}{bd} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{ad} - \frac{(a+b)^2\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,d\right.}{abd}\\
&= \frac{x}{a} - \frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{ab^{3/2}d} + \frac{\tanh(c+dx)}{bd}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 196 vs. 2(59) = 118.

time = 0.79, size = 196, normalized size = 3.32

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^2(c+dx)\left((a+b)^2\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\,\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(-\cosh(2c)+\sinh(2c))+\sqrt{a+b}\,\sqrt{b(\cosh(c)-\sinh(c))^4}\,(bdx+a\operatorname{sech}(c)\operatorname{sech}(c+dx)\sinh(dx))\right)}{2ab\sqrt{a+b}\,d\,(a+b\operatorname{sech}^2(c+dx))\,\sqrt{b(\cosh(c)-\sinh(c))^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*((a + b)^2*ArcTanh[(Sech[d
*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*S
qrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + Sqrt[
a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(b*d*x + a*Sech[c]*Sech[c + d*x]*Sinh[
d*x])))/(2*a*b*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh
[c])^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(51) = 102$.
time = 2.34, size = 184, normalized size = 3.12

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}+\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2\left(a^2+2ab+b^2\right)\left(-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh(\right.}{4\sqrt{b}\,\sqrt{a+}}\right.}{}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}+\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2\left(a^2+2ab+b^2\right)\left(-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh(\right.}{4\sqrt{b}\,\sqrt{a+}}\right.}{}}{d}$
risch	$\frac{x}{a}-\frac{2}{bd\left(1+e^{2dx+2c}\right)}+\frac{\sqrt{b\,(a+b)}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{b\,(a+b)}+a+2b}{a}\right)}{2b^2d}+\frac{\sqrt{b\,(a+b)}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{}}{}\right.}{2bda}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(tanh(1/2*d*x+1/2*c)+1)+2/b*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/
2*c)^2+1)-1/a*ln(tanh(1/2*d*x+1/2*c)-1)+2/a/b*(a^2+2*a*b+b^2)*(-1/4/b^(1/2)
/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(
1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c
)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(51) = 102$.
time = 0.57, size = 637, normalized size = 10.80

$$-\frac{(a+2b)\log\left(\frac{ae^{(2dx+2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(2dx+2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}\,bd}+\frac{(a+2b)\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}\,bd}+\frac{3a\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{16\sqrt{(a+b)b}\,bd}+\frac{(a+2b)\log\left(ae^{(4dx+4c)}+2(a+2b)e^{(2dx+2c)}+a\right)}{8abd}+\frac{\log\left(ae^{(4dx+4c)}+2(a+2b)e^{(2dx+2c)}+a\right)}{4bd}-\frac{(a+2b)\log\left(2(a+2b)e^{(-2dx-2c)}+ae^{(-4dx-4c)}+a\right)}{8abd}-\frac{\log\left(2(a+2b)e^{(-2dx-2c)}+ae^{(-4dx-4c)}+a\right)}{4bd}-\frac{3\log\left(e^{(2dx+2c)}+1\right)}{4bd}+\frac{3\log\left(e^{(-2dx-2c)}+1\right)}{4bd}-\frac{(a^2+8ab+8b^2)\log\left(\frac{ae^{(2dx+2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(2dx+2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{32\sqrt{(a+b)b}\,abd}+\frac{(a^2+8ab+8b^2)\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{32\sqrt{(a+b)b}\,abd}-\frac{5}{8\left(be^{(2dx+2c)}+b\right)d}+\frac{11}{8\left(be^{(-2dx-2c)}+b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(a + 2*b)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(
2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*b*d) + 1/8*(a
 + 2*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
 - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*b*d) + 3/16*a*log(
(a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a
+ 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*b*d) + 1/8*(a + 2*b)*log(a*e^(
4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a*b*d) + 1/4*log(a*e^(4*d*
x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(b*d) - 1/8*(a + 2*b)*log(2*(a
+ 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a*b*d) - 1/4*log(2*(a +
2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(b*d) - 3/4*log(e^(2*d*x +
2*c) + 1)/(b*d) + 3/4*log(e^(-2*d*x - 2*c) + 1)/(b*d) - 1/32*(a^2 + 8*a*b +
 8*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x +
 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*b*d) + 1/32*(a^2 +
 8*a*b + 8*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e
^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*b*d) - 5
/8/((b*e^(2*d*x + 2*c) + b)*d) + 11/8/((b*e^(-2*d*x - 2*c) + b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(51) = 102$.
time = 0.39, size = 683, normalized size = 11.58

$$\left[\frac{2\,bdx\cosh\left(dx+c\right)^{2}+4\,bdx\cosh\left(dx+c\right)\sinh\left(dx+c\right)+2\,bdx\sinh\left(dx+c\right)^{2}+2\,bdx+\left(\left(a+b\right)\cosh\left(dx+c\right)^{2}+2\,\left(a+b\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(a+b\right)\sinh\left(dx+c\right)^{2}+a+b\right)\sqrt{\frac{a+b}{b}}\log\left(\frac{a^{2}\cosh(dx+c)^{4}+4a^{2}\cosh(dx+c)\sinh(dx+c)^{3}+a^{2}\sinh(dx+c)^{4}+2\left(a^{2}+2ab\right)\cosh(dx+c)^{2}+2\left(3a^{2}\cosh(dx+c)^{2}+a^{2}+2ab\right)\sinh(dx+c)^{2}+a^{2}+8ab+8b^{2}+4\left(a^{2}\cosh(dx+c)^{3}+\left(a^{2}+2ab\right)\cosh(dx+c)\right)\sinh(dx+c)+4\left(ab\cosh(dx+c)^{2}+2ab\cosh(dx+c)\sinh(dx+c)+ab\sinh(dx+c)^{2}+ab+2b^{2}\right)\sqrt{\frac{a+b}{b}}}{a\cosh(dx+c)^{4}+4a\cosh(dx+c)\sinh(dx+c)^{3}+a\sinh(dx+c)^{4}+2\left(a+2b\right)\cosh(dx+c)^{2}+2\left(3a\cosh(dx+c)^{2}+a+2b\right)\sinh(dx+c)^{2}+4\left(a\cosh(dx+c)^{3}+\left(a+2b\right)\cosh(dx+c)\right)\sinh(dx+c)+a}\right)-4\,a}{2\,\left(abd\cosh\left(dx+c\right)^{2}+2\,abd\cosh\left(dx+c\right)\sinh\left(dx+c\right)+abd\sinh\left(dx+c\right)^{2}+abd\right)},\ \frac{bdx\cosh\left(dx+c\right)^{2}+2\,bdx\cosh\left(dx+c\right)\sinh\left(dx+c\right)+bdx\sinh\left(dx+c\right)^{2}+bdx-\left(\left(a+b\right)\cosh\left(dx+c\right)^{2}+2\,\left(a+b\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(a+b\right)\sinh\left(dx+c\right)^{2}+a+b\right)\sqrt{-\frac{a+b}{b}}\arctan\left(\frac{\left(a\cosh(dx+c)^{2}+2a\cosh(dx+c)\sinh(dx+c)+a\sinh(dx+c)^{2}+a+2b\right)\sqrt{-\frac{a+b}{b}}}{2\,(a+b)}\right)-2\,a}{abd\cosh\left(dx+c\right)^{2}+2\,abd\cosh\left(dx+c\right)\sinh\left(dx+c\right)+abd\sinh\left(dx+c\right)^{2}+abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*d*x*cosh(d*x + c)^2 + 4*b*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*b*d
*x*sinh(d*x + c)^2 + 2*b*d*x + ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*
x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a + b)*sqrt((a + b)/b)*log
((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x
+ c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 +
 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a
^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*c
osh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b + 2*b^2)*sqrt((a + b
)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x +
 c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sin
h(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c
) + a)) - 4*a)/(a*b*d*cosh(d*x + c)^2 + 2*a*b*d*cosh(d*x + c)*sinh(d*x + c)
 + a*b*d*sinh(d*x + c)^2 + a*b*d), (b*d*x*cosh(d*x + c)^2 + 2*b*d*x*cosh(d*
x + c)*sinh(d*x + c) + b*d*x*sinh(d*x + c)^2 + b*d*x - ((a + b)*cosh(d*x +
c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a
+ b)*sqrt(-(a + b)/b)*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sin
```

```
h(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-(a + b)/b)/(a + b)) - 2*a)/
(a*b*d*cosh(d*x + c)^2 + 2*a*b*d*cosh(d*x + c)*sinh(d*x + c) + a*b*d*sinh(d
*x + c)^2 + a*b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

Giac [A]
time = 1.06, size = 94, normalized size = 1.59

$$\frac{\frac{dx+c}{a} - \frac{(a^2+2\,ab+b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,ab} - \frac{2}{b\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a - (a^2 + 2*a*b + b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)
/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a*b) - 2/(b*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B]
time = 1.87, size = 183, normalized size = 3.10

$$\frac{x}{a} - \frac{2}{b\,d\,\left(e^{2\,c+2\,d\,x}+1\right)} + \frac{\ln\left(\frac{4\,e^{2\,c+2\,d\,x}\,(a+b)^2}{a^2\,b} - \frac{2\,(a+b)^{3/2}\,(a+a\,e^{2\,c+2\,d\,x}+2\,b\,e^{2\,c+2\,d\,x})}{a^2\,b^{3/2}}\right)\,(a+b)^{3/2}}{2\,a\,b^{3/2}\,d} - \frac{\ln\left(\frac{4\,e^{2\,c+2\,d\,x}\,(a+b)^2}{a^2\,b} + \frac{2\,(a+b)^{3/2}\,(a+a\,e^{2\,c+2\,d\,x}+2\,b\,e^{2\,c+2\,d\,x})}{a^2\,b^{3/2}}\right)\,(a+b)^{3/2}}{2\,a\,b^{3/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] x/a - 2/(b*d*(exp(2*c + 2*d*x) + 1)) + (log((4*exp(2*c + 2*d*x)*(a + b)^2)/
(a^2*b) - (2*(a + b)^(3/2)*(a + a*exp(2*c + 2*d*x) + 2*b*exp(2*c + 2*d*x)))
/(a^2*b^(3/2)))*(a + b)^(3/2))/(2*a*b^(3/2)*d) - (log((4*exp(2*c + 2*d*x)*(
a + b)^2)/(a^2*b) + (2*(a + b)^(3/2)*(a + a*exp(2*c + 2*d*x) + 2*b*exp(2*c
+ 2*d*x)))/(a^2*b^(3/2)))*(a + b)^(3/2))/(2*a*b^(3/2)*d)
```

3.140 $\int \frac{\tanh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=45

$$-\frac{\log(\cosh(c+dx))}{bd}+\frac{(a+b)\log\left(b+a\cosh^2(c+dx)\right)}{2abd}$$

```
[Out] -ln(cosh(d*x+c))/b/d+1/2*(a+b)*ln(b+a*cosh(d*x+c)^2)/a/b/d
```

Rubi [A]
time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 78}

$$\frac{(a+b)\log\left(a\cosh^2(c+dx)+b\right)}{2abd}-\frac{\log(\cosh(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -(Log[Cosh[c + d*x]]/(b*d)) + ((a + b)*Log[b + a*Cosh[c + d*x]^2])/(2*a*b*d
)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x(b+ax^2)}\,dx,x,\cosh(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{1-x}{x(b+ax)}\,dx,x,\cosh^2(c+dx)\right)}{2d}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{bx}+\frac{-a-b}{b(b+ax)}\right)\,dx,x,\cosh^2(c+dx)\right)}{2d}\\
&= -\frac{\log(\cosh(c+dx))}{bd}+\frac{(a+b)\log\left(b+a\cosh^2(c+dx)\right)}{2abd}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 41, normalized size = 0.91

$$\frac{-2a\log(\cosh(c+dx))+(a+b)\log\left(b+a\cosh^2(c+dx)\right)}{2abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
[Out] (-2*a*Log[Cosh[c + d*x]] + (a + b)*Log[b + a*Cosh[c + d*x]^2])/(2*a*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(43) = 86$.
time = 2.15, size = 132, normalized size = 2.93

method	result
risch	$-\frac{x}{a}-\frac{2c}{ad}-\frac{\ln\left(1+e^{2dx+2c}\right)}{bd}+\frac{\ln\left(e^{4dx+4c}+\frac{2(2b+a)e^{2dx+2c}}{a}+1\right)}{2bd}+\frac{\ln\left(e^{4dx+4c}+\frac{2(2b+a)e^{2dx+2c}}{a}+1\right)}{2ad}$
derivativedivides	$\frac{-\frac{\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}+\frac{2\left(\frac{a}{4}+\frac{b}{4}\right)\ln\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b(\tan}{ab}}{d}$
default	$\frac{-\frac{\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}+\frac{2\left(\frac{a}{4}+\frac{b}{4}\right)\ln\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b(\tan}{ab}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b*ln(tanh(1/2*d*x+1/2*c)^2+1)-1/a*ln(tanh(1/2*d*x+1/2*c)+1)+2/a/b*(
1/4*a+1/4*b)*ln(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/
2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)-1/a*ln(tanh(1/2*d*x+1/2*c)-1)
)
```

Maxima [A]
time = 0.47, size = 77, normalized size = 1.71

$$\frac{dx+c}{ad}+\frac{(a+b)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,abd}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) + 1/2*(a + b)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*
x - 4*c) + a)/(a*b*d) - log(e^(-2*d*x - 2*c) + 1)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(43) = 86.
time = 0.43, size = 112, normalized size = 2.49

$$-\frac{2\,bdx-(a+b)\log\left(\frac{2\left(a\cosh(dx+c)^2+a\sinh(dx+c)^2+a+2\,b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)+2\,a\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\,abd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x - (a + b)*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a +
2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) +
 2*a*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/(a*b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(tanh(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.65, size = 238, normalized size = 5.29

$$\frac{\ln\left(a\,b+3\,a^2+6\,a^2\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+3\,a^2\,{\mathrm{e}}^{4\,c}\,{\mathrm{e}}^{4\,d\,x}+4\,b^2\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+14\,a\,b\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+a\,b\,{\mathrm{e}}^{4\,c}\,{\mathrm{e}}^{4\,d\,x}\right)\,\left(a+b\right)}{2\,a\,b\,d}-\frac{\ln\left(21\,a\,b^4+108\,a^4\,b+27\,a^5+2\,b^5+82\,a^2\,b^3+144\,a^3\,b^2+27\,a^5\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+2\,b^5\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+21\,a\,b^4\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+108\,a^4\,b\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+82\,a^2\,b^3\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+144\,a^3\,b^2\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\right)}{b\,d}-\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (log(a*b + 3*a^2 + 6*a^2*exp(2*c)*exp(2*d*x) + 3*a^2*exp(4*c)*exp(4*d*x) +
4*b^2*exp(2*c)*exp(2*d*x) + 14*a*b*exp(2*c)*exp(2*d*x) + a*b*exp(4*c)*exp(4
*d*x))*(a + b))/(2*a*b*d) - log(21*a*b^4 + 108*a^4*b + 27*a^5 + 2*b^5 + 82*
a^2*b^3 + 144*a^3*b^2 + 27*a^5*exp(2*c)*exp(2*d*x) + 2*b^5*exp(2*c)*exp(2*d
*x) + 21*a*b^4*exp(2*c)*exp(2*d*x) + 108*a^4*b*exp(2*c)*exp(2*d*x) + 82*a^2
*b^3*exp(2*c)*exp(2*d*x) + 144*a^3*b^2*exp(2*c)*exp(2*d*x))/(b*d) - x/a
```

3.141 $\int \frac{\tanh^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{x}{a}-\frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\,d}$$

```
[Out] x/a-arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*(a+b)^(1/2)/a/d/b^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4226, 2000, 492, 212, 214}

$$\frac{x}{a}-\frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] x/a - (Sqrt[a + b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[b]
*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b(1-x^2))}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\mathrm{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{ad} - \frac{(a+b)\mathrm{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{ad}\\ &= \frac{x}{a} - \frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\,d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 174 vs. 2(46) = 92.
time = 0.24, size = 174, normalized size = 3.78

$$\frac{(a+2b+a\cosh(2(c+dx)))\mathrm{sech}^2(c+dx)\left(\sqrt{a+b}\,dx\sqrt{b(\cosh(c)-\sinh(c))^4}+(a+b)\tanh^{-1}\left(\frac{\mathrm{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\,\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(-\cosh(2c)+\sinh(2c))\right)}{2a\sqrt{a+b}\,d\left(a+b\mathrm{sech}^2(c+dx)\right)\sqrt{b(\cosh(c)-\sinh(c))^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[a + b]*d*x*Sqrt[b*(C
osh[c] - Sinh[c])^4] + (a + b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*(
(a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] -
Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c])))/(2*a*Sqrt[a + b]*d*(a + b*Sech[c +
 d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(38) = 76$.
time = 2.45, size = 144, normalized size = 3.13

method	result
risch	$\frac{x}{a} + \frac{\sqrt{b(a+b)}\, \ln\left(\mathrm{e}^{2dx+2c} + \frac{2\sqrt{b(a+b)}+a+2b}{a}\right)}{2bda} - \frac{\sqrt{b(a+b)}\, \ln\left(\mathrm{e}^{2dx+2c} - \frac{2\sqrt{b(a+b)}-a-}{a}\right.}{2bda}$
derivativedivides	$\frac{\frac{2(a+b)\left(-\frac{\ln\left(\sqrt{a+b}\,(\tanh^2(\frac{dx}{2}+\frac{c}{2}))+2\tanh(\frac{dx}{2}+\frac{c}{2})\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}} + \frac{\ln\left(\sqrt{a+b}\,(\tanh^2(\frac{dx}{2}+\frac{c}{2}))-2\tanh(\frac{dx}{2}\right.}{4\sqrt{b}\,\sqrt{a+b}}\right.}{a}}{d}$
default	$\frac{\frac{2(a+b)\left(-\frac{\ln\left(\sqrt{a+b}\,(\tanh^2(\frac{dx}{2}+\frac{c}{2}))+2\tanh(\frac{dx}{2}+\frac{c}{2})\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}} + \frac{\ln\left(\sqrt{a+b}\,(\tanh^2(\frac{dx}{2}+\frac{c}{2}))-2\tanh(\frac{dx}{2}\right.}{4\sqrt{b}\,\sqrt{a+b}}\right.}{a}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a*(a+b)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)
^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a
+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)))
-1/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/a*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(38) = 76$.
time = 0.50, size = 291, normalized size = 6.33

$$-\frac{(a+2b)\log\left(\frac{ae^{(2dx+2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(2dx+2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}\,ad} + \frac{\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{4\sqrt{(a+b)b}\,d} + \frac{(a+2b)\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}\,ad} + \frac{\log\left(ae^{(4dx+4c)}+2(a+2b)e^{(2dx+2c)}+a\right)}{4ad} - \frac{\log\left(2(a+2b)e^{(-2dx-2c)}+ae^{(-4dx-4c)}+a\right)}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(a + 2*b)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(
2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + 1/4*lo
g((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) +
a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*d) + 1/8*(a + 2*b)*log((a*e^
(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b
 + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + 1/4*log(a*e^(4*d*x + 4*c) +
2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a*d) - 1/4*log(2*(a + 2*b)*e^(-2*d*x - 2*
c) + a*e^(-4*d*x - 4*c) + a)/(a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(38) = 76$.

time = 0.44, size = 419, normalized size = 9.11

$$\left[\frac{2\,dx+\sqrt{\frac{a+b}{b}}\log\left(\frac{a^{2}\cosh(dx+c)^{4}+4\,a^{2}\cosh(dx+c)\sinh(dx+c)^{3}+a^{2}\sinh(dx+c)^{4}+2\,(a^{2}+2\,ab)\cosh(dx+c)^{2}+2\,\left(3\,a^{2}\cosh(dx+c)^{2}+a^{2}+2\,ab\right)\sinh(dx+c)^{2}+a^{2}+8\,ab+8\,b^{2}+4\,\left(a^{2}\cosh(dx+c)^{3}+(a^{2}+2\,ab)\cosh(dx+c)\right)\sinh(dx+c)+4\,\left(ab\cosh(dx+c)^{2}+2\,ab\cosh(dx+c)\sinh(dx+c)+ab\sinh(dx+c)^{2}+ab+2\,b^{2}\right)\sqrt{\frac{a+b}{b}}}{a\cosh(dx+c)^{4}+4\,a\cosh(dx+c)\sinh(dx+c)^{3}+a\sinh(dx+c)^{4}+2\,(a+2\,b)\cosh(dx+c)^{2}+2\,\left(3\,a\cosh(dx+c)^{2}+a+2\,b\right)\sinh(dx+c)^{2}+4\,\left(a\cosh(dx+c)^{3}+(a+2\,b)\cosh(dx+c)\right)\sinh(dx+c)+a}\right)}{2\,ad},\ \frac{dx-\sqrt{-\frac{a+b}{b}}\arctan\left(\frac{\left(a\cosh(dx+c)^{2}+2\,a\cosh(dx+c)\sinh(dx+c)+a\sinh(dx+c)^{2}+a+2\,b\right)\sqrt{-\frac{a+b}{b}}}{2\,(a+b)}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt((a + b)/b)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c
)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 +
 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*
b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) +
 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x
+ c)^2 + a*b + 2*b^2)*sqrt((a + b)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3
*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a +
 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))/(a*d), (d*x - sqrt(-(a + b)/b)*ar
ctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x
+ c)^2 + a + 2*b)*sqrt(-(a + b)/b)/(a + b)))/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{2}(c+dx)}{a+b\,\mathrm{sech}^{2}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

Giac [A]
time = 1.16, size = 66, normalized size = 1.43

$$-\frac{\frac{(a+b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^{2}}}\right)}{\sqrt{-ab-b^{2}}\,a}-\frac{dx+c}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((a + b)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(
-a*b - b^2)*a) - (d*x + c)/a)/d
```

Mupad [B]
time = 0.43, size = 105, normalized size = 2.28

$$\frac{x}{a}+\frac{\mathrm{atan}\left(\frac{\sqrt{-a^2\,b\,d^2}}{a\,d\,\sqrt{a+b}}+\frac{\sqrt{-a^2\,b\,d^2}}{2\,b\,d\,\sqrt{a+b}}+\frac{{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\,\sqrt{-a^2\,b\,d^2}}{2\,b\,d\,\sqrt{a+b}}\right)\,\sqrt{a+b}}{\sqrt{-a^2\,b\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] x/a + (atan((-a^2*b*d^2)^(1/2)/(a*d*(a + b)^(1/2)) + (-a^2*b*d^2)^(1/2)/(2*
b*d*(a + b)^(1/2)) + (exp(2*c)*exp(2*d*x)*(-a^2*b*d^2)^(1/2))/(2*b*d*(a + b
)^(1/2)))*(a + b)^(1/2))/(-a^2*b*d^2)^(1/2)
```

3.142 $\int \frac{\tanh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=23

$$\frac{\log\left(b+a\cosh^2(c+dx)\right)}{2ad}$$

```
[Out] 1/2*ln(b+a*cosh(d*x+c)^2)/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4223, 266}

$$\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] Log[b + a*Cosh[c + d*x]^2]/(2*a*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(c+dx)}{a+b\text{sech}^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{b+ax^2}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= \frac{\log\left(b+a\cosh^2(c+dx)\right)}{2ad}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 26, normalized size = 1.13

$$\frac{\log(a+2b+a\cosh(2(c+dx)))}{2ad}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2),x]

[Out] Log[a + 2*b + a*Cosh[2*(c + d*x)]]/(2*a*d)

Maple [A]
time = 1.05, size = 36, normalized size = 1.57

method	result	size
derivativedivides	$-\frac{-\frac{\ln\left(a+b\,\mathrm{sech}(dx+c)^{2}\right)}{2a}+\frac{\ln(\mathrm{sech}(dx+c))}{a}}{d}$	36
default	$-\frac{-\frac{\ln\left(a+b\,\mathrm{sech}(dx+c)^{2}\right)}{2a}+\frac{\ln(\mathrm{sech}(dx+c))}{a}}{d}$	36
risch	$-\frac{x}{a}-\frac{2c}{ad}+\frac{\ln\left(\mathrm{e}^{4dx+4c}+\frac{2(2b+a)\mathrm{e}^{2dx+2c}}{a}+1\right)}{2ad}$	56

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] -1/d*(-1/2/a*ln(a+b*sech(d*x+c)^2)+1/a*ln(sech(d*x+c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(21) = 42$.
time = 0.27, size = 51, normalized size = 2.22

$$\frac{dx+c}{ad}+\frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")

[Out] (d*x + c)/(a*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a*d)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(21) = 42$.
time = 0.38, size = 76, normalized size = 3.30

$$-\frac{2\,dx-\log\left(\frac{2\,\left(a\cosh(dx+c)^{2}+a\sinh(dx+c)^{2}+a+2\,b\right)}{\cosh(dx+c)^{2}-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^{2}}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")

```
[Out] -1/2*(2*d*x - log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh
(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(17) = 34$.
time = 2.62, size = 114, normalized size = 4.96

$$\begin{cases} \frac{\tilde{\infty} x \tanh(c)}{\operatorname{sech}^2(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x \tanh(c)}{a + b \operatorname{sech}^2(c)} & \text{for } d = 0 \\ \frac{1}{2bd \operatorname{sech}^2(c + dx)} & \text{for } a = 0 \\ \frac{x - \frac{\log(\tanh(c+dx)+1)}{d}}{a} & \text{for } b = 0 \\ \frac{x}{a} + \frac{\log\left(-\sqrt{-\frac{a}{b}} + \operatorname{sech}(c+dx)\right)}{2ad} + \frac{\log\left(\sqrt{-\frac{a}{b}} + \operatorname{sech}(c+dx)\right)}{2ad} - \frac{\log(\tanh(c+dx)+1)}{ad} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*tanh(c)/sech(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x*ta
nh(c)/(a + b*sech(c)**2), Eq(d, 0)), (1/(2*b*d*sech(c + d*x)**2), Eq(a, 0))
, ((x - log(tanh(c + d*x) + 1)/d)/a, Eq(b, 0)), (x/a + log(-sqrt(-a/b) + se
ch(c + d*x))/(2*a*d) + log(sqrt(-a/b) + sech(c + d*x))/(2*a*d) - log(tanh(c
 + d*x) + 1)/(a*d), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 0.35, size = 51, normalized size = 2.22

$$\frac{\ln\left(a + 2\,a\,e^{2\,c}\,e^{2\,d\,x} + a\,e^{4\,c}\,e^{4\,d\,x} + 4\,b\,e^{2\,c}\,e^{2\,d\,x}\right) - 2\,d\,x}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (log(a + 2*a*exp(2*c)*exp(2*d*x) + a*exp(4*c)*exp(4*d*x) + 4*b*exp(2*c)*exp
(2*d*x)) - 2*d*x)/(2*a*d)
```

3.143 $\int \frac{1}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{x}{a}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}\,d}$$

```
[Out] x/a-arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/a/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4212, 3260, 214}

$$\frac{x}{a}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(-1),x]
```

```
[Out] x/a - (Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]
*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 4212

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x
] && NeQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{x}{a}-\frac{b\int \frac{1}{b+a\cosh^2(c+dx)}\,dx}{a}$$
$$=\frac{x}{a}-\frac{b\text{Subst}\left(\int \frac{1}{b-(a+b)x^2}\,dx,x,\coth(c+dx)\right)}{ad}$$
$$=\frac{x}{a}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}\,d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(46) = 92.
time = 0.17, size = 172, normalized size = 3.74

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^2(c+dx)\left(\sqrt{a+b}\,dx\sqrt{b(\cosh(c)-\sinh(c))^4}+b\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\,\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(-\cosh(2c)+\sinh(2c))\right)}{2a\sqrt{a+b}\,d\left(a+b\operatorname{sech}^2(c+dx)\right)\sqrt{b(\cosh(c)-\sinh(c))^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-1),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(Sqrt[a + b]*d*x*Sqrt[b*(C
osh[c] - Sinh[c])^4] + b*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2
*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c
])^4])]*(-Cosh[2*c] + Sinh[2*c])))/(2*a*Sqrt[a + b]*d*(a + b*Sech[c + d*x]^
2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(38) = 76.
time = 2.27, size = 142, normalized size = 3.09

method	result
risch	$\frac{x}{a}+\frac{\sqrt{b(a+b)}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{b(a+b)}+a+2b}{a}\right)}{2(a+b)da}-\frac{\sqrt{b(a+b)}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{b(a+b)}-a-2}{a}\right.}{2(a+b)da}$
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2b\left(-\frac{\ln\left(\sqrt{a+b}\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\ln\left(\right.\right.}{a}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2b\left(-\frac{\ln\left(\sqrt{a+b}\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\,\sqrt{a+b}}+\ln\left(\right.\right.}{a}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(tanh(1/2*d*x+1/2*c)+1)-1/a*ln(tanh(1/2*d*x+1/2*c)-1)+2/a*b*(-1/
4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1
/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*
d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(38) = 76$.
time = 0.49, size = 83, normalized size = 1.80

$$\frac{b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{2\,\sqrt{(a+b)b}\,ad}+\frac{dx+c}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x -
 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*a*d) + (d*x + c)/(a*
d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(38) = 76$.
time = 0.41, size = 436, normalized size = 9.48

$$\left[\frac{2\,dx+\sqrt{\frac{b}{a+b}}\log\left(\frac{a^2\cosh(dx+c)^4+4\,a^2\cosh(dx+c)\sinh(dx+c)^3+a^2\sinh(dx+c)^4+2\,(a^2+2\,ab)\cosh(dx+c)^2+2\,\left(3\,a^2\cosh(dx+c)^2+a^2+2\,ab\right)\sinh(dx+c)^2+a^2+8\,ab+8\,b^2+4\,\left(a^2\cosh(dx+c)^3+(a^2+2\,ab)\cosh(dx+c)\right)\sinh(dx+c)+4\,\left((a^2+ab)\cosh(dx+c)^2+2\,(a^2+ab)\cosh(dx+c)\sinh(dx+c)+(a^2+ab)\sinh(dx+c)^2+a^2+3\,ab+2\,b^2\right)\sqrt{\frac{b}{a+b}}}{a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2\,(a+2\,b)\cosh(dx+c)^2+2\,\left(3\,a\cosh(dx+c)^2+a+2\,b\right)\sinh(dx+c)^2+4\,\left(a\cosh(dx+c)^3+(a+2\,b)\cosh(dx+c)\right)\sinh(dx+c)+a}\right)}{2\,ad},\ \frac{dx-\sqrt{-\frac{b}{a+b}}\arctan\left(\frac{\left(a\cosh(dx+c)^2+2\,a\cosh(dx+c)\sinh(dx+c)+a\sinh(dx+c)^2+a+2\,b\right)\sqrt{-\frac{b}{a+b}}}{2\,b}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c
)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 +
 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*
b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) +
 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)
 + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*c
osh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*
(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)
^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)))/(
a*d), (d*x - sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x
```

```
+ c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b))/(a*d
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\operatorname{sech}^2(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(1/(a + b*sech(c + d*x)**2), x)
```

Giac [A]
time = 0.46, size = 64, normalized size = 1.39

$$-\frac{\frac{b \arctan\left(\frac{a e^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a} - \frac{dx+c}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(b*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b -
 b^2)*a) - (d*x + c)/a)/d
```

Mupad [B]
time = 2.16, size = 470, normalized size = 10.22

$$\frac{x}{a} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\left(a^5\sqrt{-a^3d^2-ba^2d^2}+a^4b\sqrt{-a^3d^2-ba^2d^2}\right)\left(e^{2c}e^{2dx}\left(\frac{2\left(a^2+8ab+8b^2\right)\left(8b^{5/2}\sqrt{-a^3d^2-ba^2d^2}+8ab^{3/2}\sqrt{-a^3d^2-ba^2d^2}+a^2\sqrt{b}\sqrt{-a^3d^2-ba^2d^2}\right)}{a^8d(a+b)^2\sqrt{-a^3d^2-ba^2d^2}}+\frac{4\sqrt{b}\,(2a+4b)\left(4da^3b+12d a^2b^2+8dab^3\right)}{a^7(a+b)\sqrt{-a^3d^2-ba^2d^2}\sqrt{-a^2d^2(a+b)}}\right)+\frac{2\left(2ab^{3/2}\sqrt{-a^3d^2-ba^2d^2}+a^2\sqrt{b}\sqrt{-a^3d^2-ba^2d^2}\right)\left(a^2+8ab+8b^2\right)}{a^8d(a+b)^2\sqrt{-a^3d^2-ba^2d^2}}+\frac{4\sqrt{b}\left(2da^3b+2da^2b^2\right)(2a+4b)}{a^7(a+b)\sqrt{-a^3d^2-ba^2d^2}\sqrt{-a^2d^2(a+b)}}\right)}{4b}\right)}{\sqrt{-a^3d^2-ba^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] x/a + (b^(1/2)*atan(((a^5*(- a^3*d^2 - a^2*b*d^2)^(1/2) + a^4*b*(- a^3*d^2
- a^2*b*d^2)^(1/2))*(exp(2*c)*exp(2*d*x)*((2*(8*a*b + a^2 + 8*b^2)*(8*b^(5/
2)*(- a^3*d^2 - a^2*b*d^2)^(1/2) + 8*a*b^(3/2)*(- a^3*d^2 - a^2*b*d^2)^(1/2
) + a^2*b^(1/2)*(- a^3*d^2 - a^2*b*d^2)^(1/2)))/(a^8*d*(a + b)^2*(- a^3*d^2
 - a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a + 4*b)*(12*a^2*b^2*d + 8*a*b^3*d + 4
*a^3*b*d))/(a^7*(a + b)*(- a^3*d^2 - a^2*b*d^2)^(1/2)*(-a^2*d^2*(a + b))^(1
/2))) + (2*(2*a*b^(3/2)*(- a^3*d^2 - a^2*b*d^2)^(1/2) + a^2*b^(1/2)*(- a^3*
d^2 - a^2*b*d^2)^(1/2))*(8*a*b + a^2 + 8*b^2))/(a^8*d*(a + b)^2*(- a^3*d^2
- a^2*b*d^2)^(1/2)) + (4*b^(1/2)*(2*a^2*b^2*d + 2*a^3*b*d)*(2*a + 4*b))/(a^
7*(a + b)*(- a^3*d^2 - a^2*b*d^2)^(1/2)*(-a^2*d^2*(a + b))^(1/2))))/(4*b)))
/(- a^3*d^2 - a^2*b*d^2)^(1/2)
```

3.144 $\int \frac{\coth(c+dx)}{a+b\,\text{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{b\log\left(b+a\cosh^2(c+dx)\right)}{2a(a+b)d}+\frac{\log(\sinh(c+dx))}{(a+b)d}$$

```
[Out] 1/2*b*ln(b+a*cosh(d*x+c)^2)/a/(a+b)/d+ln(sinh(d*x+c))/(a+b)/d
```

Rubi [A]
time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 457, 78}

$$\frac{\log(\sinh(c+dx))}{d(a+b)}+\frac{b\log\left(a\cosh^2(c+dx)+b\right)}{2ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] (b*Log[b + a*Cosh[c + d*x]^2])/(2*a*(a + b)*d) + Log[Sinh[c + d*x]]/((a + b
)*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^3}{(1-x^2)(b+ax^2)}\,dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{x}{(1-x)(b+ax)}\,dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{\operatorname{Subst}\left(\int \left(\frac{1}{(-a-b)(-1+x)} - \frac{b}{(a+b)(b+ax)}\right)\,dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{b\log\left(b+a\cosh^2(c+dx)\right)}{2a(a+b)d} + \frac{\log(\sinh(c+dx))}{(a+b)d}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 42, normalized size = 0.91

$$\frac{2a\log(\sinh(c+dx)) + b\log\left(a+b+a\sinh^2(c+dx)\right)}{2a^2d+2abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x]^2),x]
[Out] (2*a*Log[Sinh[c + d*x]] + b*Log[a + b + a*Sinh[c + d*x]^2])/(2*a^2*d + 2*a*
b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(44) = 88.
time = 2.20, size = 125, normalized size = 2.72

method	result
risch	$\frac{x}{a}-\frac{2x}{a+b}-\frac{2c}{d(a+b)}-\frac{2bx}{a(a+b)}-\frac{2bc}{ad(a+b)}+\frac{\ln\left(e^{2dx+2c}-1\right)}{d(a+b)}+\frac{b\ln\left(e^{4dx+4c}+\frac{2(2b+a)e^{2dx+2c}}{a}+1\right)}{2ad(a+b)}$
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a+b}+\frac{b\ln\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{2a(a+b)}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a+b}+\frac{b\ln\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{2a(a+b)}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/(a+b)*ln(tanh(1/2*d*x+1/2*c))+1/2/a*b
/(a+b)*ln(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+
1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)-1/a*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.
time = 0.28, size = 100, normalized size = 2.17

$$\frac{b \log \left(2 \, {\left(a + 2 \, b\right)} e^{\left(-2 \, d x - 2 \, c\right)} + a e^{\left(-4 \, d x - 4 \, c\right)} + a\right)}{2 \, {\left(a^{2} + a b\right)} d} + \frac{d x + c}{a d} + \frac{\log \left(e^{\left(-d x - c\right)} + 1\right)}{{\left(a + b\right)} d} + \frac{\log \left(e^{\left(-d x - c\right)} - 1\right)}{{\left(a + b\right)} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^2 + a*
b)*d) + (d*x + c)/(a*d) + log(e^(-d*x - c) + 1)/((a + b)*d) + log(e^(-d*x -
 c) - 1)/((a + b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(44) = 88$.
time = 0.42, size = 115, normalized size = 2.50

$$-\frac{2 \, {\left(a + b\right)} d x - b \log \left(\frac{2 \, {\left(a \cosh\left(d x + c\right)^{2} + a \sinh\left(d x + c\right)^{2} + a + 2 \, b\right)}}{\cosh\left(d x + c\right)^{2} - 2 \, \cosh\left(d x + c\right) \sinh\left(d x + c\right) + \sinh\left(d x + c\right)^{2}}\right) - 2 \, a \log \left(\frac{2 \, \sinh\left(d x + c\right)}{\cosh\left(d x + c\right) - \sinh\left(d x + c\right)}\right)}{2 \, {\left(a^{2} + a b\right)} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a + b)*d*x - b*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a +
2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) -
 2*a*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a^2 + a*b)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth{\left(c + d x \right)}}{a + b \operatorname{sech}^{2}{\left(c + d x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.82, size = 228, normalized size = 4.96

$$\frac{\ln\left(8\,a\,b^5-b^6-24\,a^2\,b^4+32\,a^3\,b^3-16\,a^4\,b^2+b^6\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-8\,a\,b^5\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+24\,a^2\,b^4\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-32\,a^3\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+16\,a^4\,b^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)}{a\,d+b\,d}-\frac{x}{a}+\frac{b\,\ln\left(2\,a^2-a\,b+4\,a^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+2\,a^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}-4\,b^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+6\,a\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-a\,b\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}\right)}{2\,d\,a^2+2\,b\,d\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] log(8*a*b^5 - b^6 - 24*a^2*b^4 + 32*a^3*b^3 - 16*a^4*b^2 + b^6*exp(2*c)*exp
(2*d*x) - 8*a*b^5*exp(2*c)*exp(2*d*x) + 24*a^2*b^4*exp(2*c)*exp(2*d*x) - 32
*a^3*b^3*exp(2*c)*exp(2*d*x) + 16*a^4*b^2*exp(2*c)*exp(2*d*x))/(a*d + b*d)
- x/a + (b*log(2*a^2 - a*b + 4*a^2*exp(2*c)*exp(2*d*x) + 2*a^2*exp(4*c)*exp
(4*d*x) - 4*b^2*exp(2*c)*exp(2*d*x) + 6*a*b*exp(2*c)*exp(2*d*x) - a*b*exp(4
*c)*exp(4*d*x)))/(2*a^2*d + 2*a*b*d)
```

3.145 $\int \frac{\coth^2(c+dx)}{a+b\mathrm{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{x}{a}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a(a+b)^{3/2}d}-\frac{\coth(c+dx)}{(a+b)d}$$

```
[Out] x/a-b^(3/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a/(a+b)^(3/2)/d-coth(d
*x+c)/(a+b)/d
```

Rubi [A]
time = 0.13, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4226, 2000, 491, 536, 212, 214}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ad(a+b)^{3/2}}-\frac{\coth(c+dx)}{d(a+b)}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] x/a - (b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*(a + b)^(3/
2)*d) - Coth[c + d*x]/((a + b)*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+b(1-x^2))}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{\coth(c+dx)}{(a+b)d} + \frac{\operatorname{Subst}\left(\int \frac{a+2b-bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{(a+b)d} \\
&= -\frac{\coth(c+dx)}{(a+b)d} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{ad} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \tanh(c+dx)\right)}{a(a+b)d} \\
&= \frac{x}{a} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a(a+b)^{3/2}d} - \frac{\coth(c+dx)}{(a+b)d}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(62) = 124.

time = 0.80, size = 193, normalized size = 3.11

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^2(c+dx)\left(b^2\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(-\cosh(2c)+\sinh(2c))+\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}\left((a+b)dx+a\operatorname{csch}(c)\operatorname{csch}(c+dx)\sinh(dx)\right)\right)}{2a(a+b)^{3/2}d\left(a+b\operatorname{sech}^2(c+dx)\right)\sqrt{b(\cosh(c)-\sinh(c))^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(b^2*ArcTanh[(Sech[d*x]*(C
osh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a
+ b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + Sqrt[a + b]
*Sqrt[b*(Cosh[c] - Sinh[c])^4]*((a + b)*d*x + a*Csch[c]*Csch[c + d*x]*Sinh[
d*x])))/(2*a*(a + b)^(3/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh
[c])^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(54) = 108$.
time = 2.56, size = 183, normalized size = 2.95

method	result
risch	$\frac{x}{a}-\frac{2}{d(a+b)\left(e^{2dx+2c}-1\right)}+\frac{\sqrt{b(a+b)}\,b\ln\left(e^{2dx+2c}+\frac{2\sqrt{b(a+b)}+a+2b}{a}\right)}{2(a+b)^2 da}-\frac{\sqrt{b(a+b)}\,b\ln\left(e^{2dx}\right.}{2(\ldots}$ …
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}+\frac{2b^2\left(-\frac{\ln\left(\sqrt{a+b}\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\left(\tanh^2\left(\frac{dx}{2}+\ldots\right.\right.}{4\sqrt{\ldots}}\right.}{a(a+b)}}{d}$ …
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}+\frac{2b^2\left(-\frac{\ln\left(\sqrt{a+b}\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}+\sqrt{a+b}\right)}{4\sqrt{b}\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\left(\tanh^2\left(\frac{dx}{2}+\ldots\right.\right.}{4\sqrt{\ldots}}\right.}{a(a+b)}}{d}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)*tanh(1/2*d*x+1/2*c)+2*b^2/a/(a+b)*(-1/4/b^(1/2)/(a+b)^(1/2)
*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(
1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1
/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)))-1/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/a*ln(ta
nh(1/2*d*x+1/2*c)+1)-1/2/(a+b)/tanh(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(54) = 108$.
time = 0.49, size = 429, normalized size = 6.92

$$\frac{b\log\left(ae^{(4\,dx+4\,c)}+2\,(a+2\,b)e^{(2\,dx+2\,c)}+a\right)}{4\,(a^2+ab)d}-\frac{b\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{4\,(a^2+ab)d}-\frac{(ab+2\,b^2)\log\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(2\,dx+2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,(a^2+ab)\sqrt{(a+b)b}\,d}+\frac{(ab+2\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{8\,(a^2+ab)\sqrt{(a+b)b}\,d}-\frac{b\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,\sqrt{(a+b)b}\,(a+b)d}+\frac{\log\left(e^{(2\,dx+2\,c)}-1\right)}{2\,(a+b)d}-\frac{\log\left(e^{(-2\,dx-2\,c)}-1\right)}{2\,(a+b)d}-\frac{1}{2\,((a+b)e^{(2\,dx+2\,c)}-a-b)d}+\frac{3}{2\,((a+b)e^{(-2\,dx-2\,c)}-a-b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*b*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/((a^2 + a*b)
*d) - 1/4*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^
2 + a*b)*d) - 1/8*(a*b + 2*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((
a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + a*b)*
sqrt((a + b)*b)*d) + 1/8*(a*b + 2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b -
2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^
2 + a*b)*sqrt((a + b)*b)*d) - 1/4*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*s
qrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((
a + b)*b)*(a + b)*d) + 1/2*log(e^(2*d*x + 2*c) - 1)/((a + b)*d) - 1/2*log(e
^(-2*d*x - 2*c) - 1)/((a + b)*d) - 1/2/(((a + b)*e^(2*d*x + 2*c) - a - b)*d
) + 3/2/(((a + b)*e^(-2*d*x - 2*c) - a - b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(54) = 108$.
time = 0.42, size = 749, normalized size = 12.08

$$\left[\frac{2(a+b)dx\cosh(dx+c)^2+4(a+b)dx\cosh(dx+c)\sinh(dx+c)+2(a+b)dx\sinh(dx+c)^2-2(a+b)dx+\left(b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2-b\right)\sqrt{\frac{b}{a+b}}\log\left(\frac{a^2\cosh(dx+c)^4+4a^2\cosh(dx+c)\sinh(dx+c)^3+a^2\sinh(dx+c)^4+2(a^2+2ab)\cosh(dx+c)^2+2\left(3a^2\cosh(dx+c)^2+a^2+2ab\right)\sinh(dx+c)^2+a^2+8ab+8b^2+4\left(a^2\cosh(dx+c)^3+(a^2+2ab)\cosh(dx+c)\right)\sinh(dx+c)+4\left((a^2+ab)\cosh(dx+c)^2+2(a^2+ab)\cosh(dx+c)\sinh(dx+c)+(a^2+ab)\sinh(dx+c)^2+a^2+3ab+2b^2\right)\sqrt{\frac{b}{a+b}}}{a\cosh(dx+c)^4+4a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2(a+2b)\cosh(dx+c)^2+2\left(3a\cosh(dx+c)^2+a+2b\right)\sinh(dx+c)^2+4\left(a\cosh(dx+c)^3+(a+2b)\cosh(dx+c)\right)\sinh(dx+c)+a}\right)-4a}{2\left((a^2+ab)d\cosh(dx+c)^2+2(a^2+ab)d\cosh(dx+c)\sinh(dx+c)+(a^2+ab)d\sinh(dx+c)^2-(a^2+ab)d\right)}, \frac{(a+b)dx\cosh(dx+c)^2+2(a+b)dx\cosh(dx+c)\sinh(dx+c)+(a+b)dx\sinh(dx+c)^2-(a+b)dx-\left(b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2-b\right)\sqrt{-\frac{b}{a+b}}\arctan\left(\frac{\left(a\cosh(dx+c)^2+2a\cosh(dx+c)\sinh(dx+c)+a\sinh(dx+c)^2+a+2b\right)\sqrt{-\frac{b}{a+b}}}{2b}\right)-2a}{(a^2+ab)d\cosh(dx+c)^2+2(a^2+ab)d\cosh(dx+c)\sinh(dx+c)+(a^2+ab)d\sinh(dx+c)^2-(a^2+ab)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a + b)*d*x*cosh(d*x + c)^2 + 4*(a + b)*d*x*cosh(d*x + c)*sinh(d*x
+ c) + 2*(a + b)*d*x*sinh(d*x + c)^2 - 2*(a + b)*d*x + (b*cosh(d*x + c)^2 +
 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(b/(a + b))*l
og((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*
x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2
 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 +
(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2
 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2
+ a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x +
 c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(
3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a
+ 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 4*a)/((a^2 + a*b)*d*cosh(d*x +
c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*sinh(d*x
 + c)^2 - (a^2 + a*b)*d), ((a + b)*d*x*cosh(d*x + c)^2 + 2*(a + b)*d*x*cosh
(d*x + c)*sinh(d*x + c) + (a + b)*d*x*sinh(d*x + c)^2 - (a + b)*d*x - (b*co
sh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sq
rt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x +
 c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) - 2*a)/((a^2 + a*b)*
d*cosh(d*x + c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*
b)*d*sinh(d*x + c)^2 - (a^2 + a*b)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*sech(c + d*x)**2), x)
```

Giac [A]
time = 1.03, size = 92, normalized size = 1.48

$$-\frac{\frac{b^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{(a^2+ab)\sqrt{-ab-b^2}}-\frac{dx+c}{a}+\frac{2}{(a+b)\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^2 + a*
b)*sqrt(-a*b - b^2)) - (d*x + c)/a + 2/((a + b)*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [B]
time = 3.39, size = 977, normalized size = 15.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] x/a - 2/((exp(2*c + 2*d*x) - 1)*(a*d + b*d)) + (atan(((exp(2*c)*exp(2*d*x)*
((8*(a + 2*b)*(20*a^2*b^4*d + 16*a^3*b^3*d + 4*a^4*b^2*d + 8*a*b^5*d))/(a^6
*(a + b)*(a*b^2 + 2*a^2*b + a^3)*(-a^2*d^2*(a + b)^3)^(1/2)*(- a^5*d^2 - 3*
a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)) + (2*(b^3)^(1/2)*(8*a*b + a
^2 + 8*b^2)*(a^2*(b^3)^(1/2)*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3
*b^2*d^2)^(1/2) + 8*b^2*(b^3)^(1/2)*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2
- 3*a^3*b^2*d^2)^(1/2) + 8*a*b*(b^3)^(1/2)*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b
^3*d^2 - 3*a^3*b^2*d^2)^(1/2)))/(a^7*b^2*d*(a + b)^3*(a*b^2 + 2*a^2*b + a^3
)*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2))) + (8*(a +
 2*b)*(2*a^2*b^4*d + 4*a^3*b^3*d + 2*a^4*b^2*d))/(a^6*(a + b)*(a*b^2 + 2*a^
2*b + a^3)*(-a^2*d^2*(a + b)^3)^(1/2)*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^
2 - 3*a^3*b^2*d^2)^(1/2)) + (2*(a^2*(b^3)^(1/2)*(- a^5*d^2 - 3*a^4*b*d^2 -
a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2) + 2*a*b*(b^3)^(1/2)*(- a^5*d^2 - 3*a^4*b
```

```
*d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2))*(b^3)^(1/2)*(8*a*b + a^2 + 8*b^2
))/(a^7*b^2*d*(a + b)^3*(a*b^2 + 2*a^2*b + a^3)*(- a^5*d^2 - 3*a^4*b*d^2 -
a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)))*(a^7*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^
3*d^2 - 3*a^3*b^2*d^2)^(1/2) + a^4*b^3*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d
^2 - 3*a^3*b^2*d^2)^(1/2) + 3*a^5*b^2*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^
2 - 3*a^3*b^2*d^2)^(1/2) + 3*a^6*b*(- a^5*d^2 - 3*a^4*b*d^2 - a^2*b^3*d^2 -
 3*a^3*b^2*d^2)^(1/2)))/(4*(b^3)^(1/2)))*(b^3)^(1/2))/(- a^5*d^2 - 3*a^4*b*
d^2 - a^2*b^3*d^2 - 3*a^3*b^2*d^2)^(1/2)
```

3.146 $\int \frac{\coth^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{\operatorname{csch}^2(c+dx)}{2(a+b)d}+\frac{b^2\log\left(b+a\cosh^2(c+dx)\right)}{2a(a+b)^2d}+\frac{(a+2b)\log(\sinh(c+dx))}{(a+b)^2d}$$

```
[Out] -1/2*csch(d*x+c)^2/(a+b)/d+1/2*b^2*ln(b+a*cosh(d*x+c)^2)/a/(a+b)^2/d+(a+2*b
)*ln(sinh(d*x+c))/(a+b)^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 90}

$$\frac{b^2\log\left(a\cosh^2(c+dx)+b\right)}{2ad(a+b)^2}-\frac{\operatorname{csch}^2(c+dx)}{2d(a+b)}+\frac{(a+2b)\log(\sinh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/2*Csch[c + d*x]^2/((a + b)*d) + (b^2*Log[b + a*Cosh[c + d*x]^2])/(2*a*(a
 + b)^2*d) + ((a + 2*b)*Log[Sinh[c + d*x]])/((a + b)^2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^5}{(1-x^2)^2(b+ax^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1-x)^2(b+ax)}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{(a+b)(-1+x)^2}+\frac{a+2b}{(a+b)^2(-1+x)}+\frac{b^2}{(a+b)^2(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= -\frac{\operatorname{csch}^2(c+dx)}{2(a+b)d}+\frac{b^2\log\left(b+a\cosh^2(c+dx)\right)}{2a(a+b)^2d}+\frac{(a+2b)\log(\sinh(c+dx))}{(a+b)^2d}$$

Mathematica [A]
time = 0.17, size = 100, normalized size = 1.37

$$-\frac{(a+2b+a\cosh(2(c+dx)))\left(a(a+b)\operatorname{csch}^2(c+dx)-2a(a+2b)\log(\sinh(c+dx))-b^2\log\left(a+b+a\sinh^2(c+dx)\right)\right)\operatorname{sech}^2(c+dx)}{4a(a+b)^2d\left(a+b\operatorname{sech}^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cosh[2*(c + d*x)])*(a*(a + b)*Csch[c + d*x]^2 - 2*a*(a +
 2*b)*Log[Sinh[c + d*x]] - b^2*Log[a + b + a*Sinh[c + d*x]^2])*Sech[c + d*x
]^2)/(a*(a + b)^2*d*(a + b*Sech[c + d*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(69) = 138.
time = 3.03, size = 171, normalized size = 2.34

method	result
derivativedivides	$\frac{-\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}+\frac{b^2\ln\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\cdots}{2a(a+b)^2}}{d}$
default	$\frac{-\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}+\frac{b^2\ln\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\cdots}{2a(a+b)^2}}{d}$
risch	$\frac{x}{a}-\frac{4bx}{a^2+2ab+b^2}-\frac{4bc}{d(a^2+2ab+b^2)}-\frac{2ax}{a^2+2ab+b^2}-\frac{2ac}{d(a^2+2ab+b^2)}-\frac{2b^2x}{a(a^2+2ab+b^2)}-\frac{2b^2c}{ad(a^2+2ab+b^2)}-\frac{\cdots}{d(a\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*tanh(1/2*d*x+1/2*c)^2/(a+b)-1/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/2*b^2
/a/(a+b)^2*ln(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*
d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)-1/8/(a+b)/tanh(1/2*d*x+1/2*c)^2
+1/4/(a+b)^2*(4*a+8*b)*ln(tanh(1/2*d*x+1/2*c))-1/a*ln(tanh(1/2*d*x+1/2*c)-1
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(69) = 138$.
time = 0.28, size = 187, normalized size = 2.56

$$\frac{b^2 \log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)} + ae^{(-4\,dx-4\,c)} + a\right)}{2\,(a^3 + 2\,a^2b + ab^2)d} + \frac{(a+2\,b)\log\left(e^{(-dx-c)} + 1\right)}{(a^2 + 2\,ab + b^2)d} + \frac{(a+2\,b)\log\left(e^{(-dx-c)} - 1\right)}{(a^2 + 2\,ab + b^2)d} + \frac{dx+c}{ad} + \frac{2\,e^{(-2\,dx-2\,c)}}{\left(2\,(a+b)e^{(-2\,dx-2\,c)} - (a+b)e^{(-4\,dx-4\,c)} - a - b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^3 +
2*a^2*b + a*b^2)*d) + (a + 2*b)*log(e^(-d*x - c) + 1)/((a^2 + 2*a*b + b^2)*
d) + (a + 2*b)*log(e^(-d*x - c) - 1)/((a^2 + 2*a*b + b^2)*d) + (d*x + c)/(a
*d) + 2*e^(-2*d*x - 2*c)/((2*(a + b)*e^(-2*d*x - 2*c) - (a + b)*e^(-4*d*x -
 4*c) - a - b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 862 vs. $2(69) = 138$.
time = 0.48, size = 862, normalized size = 11.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*d*x
*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^4
+ 2*(a^2 + 2*a*b + b^2)*d*x - 4*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(
d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*b +
b^2)*d*x + a^2 + a*b)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d
*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(
3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3
 - b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x
+ c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d
*x + c)^2)) - 2*((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 - 2*(a^2 + 2*a*b)*cosh(
d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 - a^2 - 2*a*b)*sinh(d*x + c
)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 - (a^2 + 2*a*b)*cosh(d
*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))
) + 8*((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - ((a^2 + 2*a*b + b^2)*d*x -
 a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d
```

```
*x + c)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^
3 + 2*a^2*b + a*b^2)*d*sinh(d*x + c)^4 - 2*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d
*x + c)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 - (a^3 + 2*a^2*b
 + a*b^2)*d)*sinh(d*x + c)^2 + (a^3 + 2*a^2*b + a*b^2)*d + 4*((a^3 + 2*a^2*
b + a*b^2)*d*cosh(d*x + c)^3 - (a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c))*sin
h(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*sech(c + d*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 2.07, size = 523, normalized size = 7.16

$$\frac{\ln\left(23\,a\,b^7+8\,a^7\,b-2\,b^8-72\,a^2\,b^6-10\,a^3\,b^5+184\,a^4\,b^4+180\,a^5\,b^3+64\,a^6\,b^2+2\,b^8\,e^{2\,c}\,e^{2\,d\,x}-23\,a\,b^7\,e^{2\,c}\,e^{2\,d\,x}-8\,a^7\,b\,e^{2\,c}\,e^{2\,d\,x}+72\,a^2\,b^6\,e^{2\,c}\,e^{2\,d\,x}+10\,a^3\,b^5\,e^{2\,c}\,e^{2\,d\,x}-184\,a^4\,b^4\,e^{2\,c}\,e^{2\,d\,x}-180\,a^5\,b^3\,e^{2\,c}\,e^{2\,d\,x}-64\,a^6\,b^2\,e^{2\,c}\,e^{2\,d\,x}\right)\,\left(a+2\,b\right)}{d\,a^2+2\,d\,a\,b+d\,b^2}-\frac{x}{a}-\frac{2}{\left(a\,d+b\,d\right)\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}+\frac{b^2\,\ln\left(a\,b^4+16\,a^4\,b+4\,a^5-8\,a^2\,b^3+12\,a^3\,b^2+8\,a^5\,e^{2\,c}\,e^{2\,d\,x}+4\,a^5\,e^{4\,c}\,e^{4\,d\,x}+4\,b^5\,e^{2\,c}\,e^{2\,d\,x}-30\,a\,b^4\,e^{2\,c}\,e^{2\,d\,x}+48\,a^4\,b\,e^{2\,c}\,e^{2\,d\,x}+a\,b^4\,e^{4\,c}\,e^{4\,d\,x}+16\,a^4\,b\,e^{4\,c}\,e^{4\,d\,x}+32\,a^2\,b^3\,e^{2\,c}\,e^{2\,d\,x}+88\,a^3\,b^2\,e^{2\,c}\,e^{2\,d\,x}-8\,a^2\,b^3\,e^{4\,c}\,e^{4\,d\,x}+12\,a^3\,b^2\,e^{4\,c}\,e^{4\,d\,x}\right)}{2\,d\,a^3+4\,d\,a^2\,b+2\,d\,a\,b^2}-\frac{2\,\left(a^2+b\,a\right)}{a\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)\,\left(a+b\right)\,\left(a\,d+b\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3/(a + b/cosh(c + d*x)^2),x)
```

```
[Out] (log(23*a*b^7 + 8*a^7*b - 2*b^8 - 72*a^2*b^6 - 10*a^3*b^5 + 184*a^4*b^4 + 1
80*a^5*b^3 + 64*a^6*b^2 + 2*b^8*exp(2*c)*exp(2*d*x) - 23*a*b^7*exp(2*c)*exp
(2*d*x) - 8*a^7*b*exp(2*c)*exp(2*d*x) + 72*a^2*b^6*exp(2*c)*exp(2*d*x) + 10
*a^3*b^5*exp(2*c)*exp(2*d*x) - 184*a^4*b^4*exp(2*c)*exp(2*d*x) - 180*a^5*b^
3*exp(2*c)*exp(2*d*x) - 64*a^6*b^2*exp(2*c)*exp(2*d*x))*(a + 2*b))/(a^2*d +
 b^2*d + 2*a*b*d) - x/a - 2/((a*d + b*d)*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*
d*x) + 1)) + (b^2*log(a*b^4 + 16*a^4*b + 4*a^5 - 8*a^2*b^3 + 12*a^3*b^2 + 8
```

```
*a^5*exp(2*c)*exp(2*d*x) + 4*a^5*exp(4*c)*exp(4*d*x) + 4*b^5*exp(2*c)*exp(2
*d*x) - 30*a*b^4*exp(2*c)*exp(2*d*x) + 48*a^4*b*exp(2*c)*exp(2*d*x) + a*b^4
*exp(4*c)*exp(4*d*x) + 16*a^4*b*exp(4*c)*exp(4*d*x) + 32*a^2*b^3*exp(2*c)*e
xp(2*d*x) + 88*a^3*b^2*exp(2*c)*exp(2*d*x) - 8*a^2*b^3*exp(4*c)*exp(4*d*x)
+ 12*a^3*b^2*exp(4*c)*exp(4*d*x)))/(2*a^3*d + 2*a*b^2*d + 4*a^2*b*d) - (2*(
a*b + a^2))/(a*(exp(2*c + 2*d*x) - 1)*(a + b)*(a*d + b*d))
```

3.147 $\int \frac{\coth^4(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$

Optimal. Leaf size=87

$$\frac{x}{a}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a(a+b)^{5/2}d}-\frac{(a+2b)\coth(c+dx)}{(a+b)^2d}-\frac{\coth^3(c+dx)}{3(a+b)d}$$

```
[Out] x/a-b^(5/2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a/(a+b)^(5/2)/d-(a+2*b
)*coth(d*x+c)/(a+b)^2/d-1/3*coth(d*x+c)^3/(a+b)/d
```

Rubi [A]
time = 0.20, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4226, 2000, 491, 597, 536, 212, 214}

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{ad(a+b)^{5/2}}-\frac{\coth^3(c+dx)}{3d(a+b)}-\frac{(a+2b)\coth(c+dx)}{d(a+b)^2}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] x/a - (b^(5/2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a*(a + b)^(5/
2)*d) - ((a + 2*b)*Coth[c + d*x])/((a + b)^2*d) - Coth[c + d*x]^3/(3*(a + b
)*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
```

```
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^4(c+dx)}{a+b\operatorname{sech}^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+b(1-x^2))}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{\coth^3(c+dx)}{3(a+b)d} + \frac{\text{Subst}\left(\int \frac{3(a+2b)-3bx^2}{x^2(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+dx)\right)}{3(a+b)d} \\
&= -\frac{(a+2b)\coth(c+dx)}{(a+b)^2 d} - \frac{\coth^3(c+dx)}{3(a+b)d} - \frac{\text{Subst}\left(\int \frac{-3(a^2+3ab+3b^2)+3b(a+2b)x^2}{(1-x^2)(a+b-bx^2)}\,dx,\right.}{3(a+b)^2 d} \\
&= -\frac{(a+2b)\coth(c+dx)}{(a+b)^2 d} - \frac{\coth^3(c+dx)}{3(a+b)d} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{ad} - \\
&= \frac{x}{a} - \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{a(a+b)^{5/2}d} - \frac{(a+2b)\coth(c+dx)}{(a+b)^2 d} - \frac{\coth^3(c+dx)}{3(a+b)d}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 380 vs. 2(87) = 174.
time = 2.22, size = 380, normalized size = 4.37

$$
\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^2(c+dx)\left(3b^3\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(-\cosh(2c)+\sinh(2c))+\frac{1}{8}\sqrt{a+b}\operatorname{csch}(c)\operatorname{csch}^3(c+dx)\sqrt{b(\cosh(c)-\sinh(c))^4}\,(9(a+b)^2dx\cosh(dx)-9(a+b)^2dx\cosh(2c+dx)-3a^2dx\cosh(2c+3dx)-6abdx\cosh(2c+3dx)-3b^2dx\cosh(2c+3dx)+3a^2dx\cosh(4c+3dx)+6abdx\cosh(4c+3dx)+3b^2dx\cosh(4c+3dx)-12a^2\sinh(dx)-24ab\sinh(dx)-12a^2\sinh(2c+dx)-18ab\sinh(2c+dx)+8a^2\sinh(2c+3dx)+14ab\sinh(2c+3dx))\right)}{6a(a+b)^{5/2}d\left(a+b\operatorname{sech}^2(c+dx)\right)\sqrt{b(\cosh(c)-\sinh(c))^4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*(3*b^3*ArcTanh[(Sech[d*x]*
(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(-Cosh[2*c] + Sinh[2*c]) + (Sqrt[a +
 b]*Csch[c]*Csch[c + d*x]^3*Sqrt[b*(Cosh[c] - Sinh[c])^4]*(9*(a + b)^2*d*x*
Cosh[d*x] - 9*(a + b)^2*d*x*Cosh[2*c + d*x] - 3*a^2*d*x*Cosh[2*c + 3*d*x] -
 6*a*b*d*x*Cosh[2*c + 3*d*x] - 3*b^2*d*x*Cosh[2*c + 3*d*x] + 3*a^2*d*x*Cosh
[4*c + 3*d*x] + 6*a*b*d*x*Cosh[4*c + 3*d*x] + 3*b^2*d*x*Cosh[4*c + 3*d*x] -
 12*a^2*Sinh[d*x] - 24*a*b*Sinh[d*x] - 12*a^2*Sinh[2*c + d*x] - 18*a*b*Sinh
[2*c + d*x] + 8*a^2*Sinh[2*c + 3*d*x] + 14*a*b*Sinh[2*c + 3*d*x]))/8))/(6*a
*(a + b)^(5/2)*d*(a + b*Sech[c + d*x]^2)*Sqrt[b*(Cosh[c] - Sinh[c])^4])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(77) = 154.
time = 3.10, size = 252, normalized size = 2.90

method	result
risch	$\frac{x}{a}-\frac{2(6a\,e^{4dx+4c}+9b\,e^{4dx+4c}-6a\,e^{2dx+2c}-12b\,e^{2dx+2c}+4a+7b)}{3d(a+b)^2(e^{2dx+2c}-1)^3}+\frac{\sqrt{b(a+b)}\,b^2\ln\left(e^{2dx+2c}+\frac{2\sqrt{b(a+b}}{a}\right.}{2(a+b)^3da}$
derivativedivides	$\frac{-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+9b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)^2}+\frac{2b^3\left(-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\right.}{4\sqrt{b}\,\sqrt{a+}}\right.}{}}{}$
default	$\frac{-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+9b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a+b)^2}+\frac{2b^3\left(-\frac{\ln\left(\sqrt{a+b}\,\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tanh\right.}{4\sqrt{b}\,\sqrt{a+}}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/(a+b)^2*(1/3*a*tanh(1/2*d*x+1/2*c)^3+1/3*b*tanh(1/2*d*x+1/2*c)^3+
5*a*tanh(1/2*d*x+1/2*c)+9*b*tanh(1/2*d*x+1/2*c))+2*b^3/a/(a+b)^2*(-1/4/b^(1
/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*
b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/
2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)))+1/a*ln(tanh(1/2*d*x+1/2*
c)+1)-1/a*ln(tanh(1/2*d*x+1/2*c)-1)-1/24/(a+b)/tanh(1/2*d*x+1/2*c)^3-1/8*(5
*a+9*b)/(a+b)^2/tanh(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1435 vs. $2(77) = 154$.
time = 0.59, size = 1435, normalized size = 16.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 3/16*a*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*
x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*
b)*d) + 1/8*(a*b + 2*b^2)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*
c) + a)/((a^3 + 2*a^2*b + a*b^2)*d) - 1/4*b*log(a*e^(4*d*x + 4*c) + 2*(a +
2*b)*e^(2*d*x + 2*c) + a)/((a^2 + 2*a*b + b^2)*d) - 1/8*(a*b + 2*b^2)*log(2
*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^3 + 2*a^2*b + a*b
^2)*d) + 1/4*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(
(a^2 + 2*a*b + b^2)*d) + 1/4*(2*a + 3*b)*log(e^(2*d*x + 2*c) - 1)/((a^2 + 2
*a*b + b^2)*d) + 1/2*b*log(e^(2*d*x + 2*c) - 1)/((a^2 + 2*a*b + b^2)*d) - 1
/4*(2*a + 3*b)*log(e^(-2*d*x - 2*c) - 1)/((a^2 + 2*a*b + b^2)*d) - 1/2*b*lo
```

```
g(e^(-2*d*x - 2*c) - 1)/((a^2 + 2*a*b + b^2)*d) - 1/32*(a^2*b + 8*a*b^2 + 8
*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2
*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 2*a^2*b + a*b^2)*sqrt((a + b)*b
)*d) + 1/8*(a*b + 2*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*
b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)
*sqrt((a + b)*b)*d) + 1/32*(a^2*b + 8*a*b^2 + 8*b^3)*log((a*e^(-2*d*x - 2*c
) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a
+ b)*b)))/((a^3 + 2*a^2*b + a*b^2)*sqrt((a + b)*b)*d) - 1/8*(a*b + 2*b^2)*l
og((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) +
 a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b)*d) + 1/
24*(3*(12*a + 13*b)*e^(4*d*x + 4*c) - 6*(9*a + 10*b)*e^(2*d*x + 2*c) + 22*a
 + 25*b)/((a^2 + 2*a*b + b^2 - (a^2 + 2*a*b + b^2)*e^(6*d*x + 6*c) + 3*(a^2
 + 2*a*b + b^2)*e^(4*d*x + 4*c) - 3*(a^2 + 2*a*b + b^2)*e^(2*d*x + 2*c))*d)
 + 1/6*(3*(4*a + 5*b)*e^(4*d*x + 4*c) - 6*(2*a + 3*b)*e^(2*d*x + 2*c) + 4*a
 + 7*b)/((a^2 + 2*a*b + b^2 - (a^2 + 2*a*b + b^2)*e^(6*d*x + 6*c) + 3*(a^2
+ 2*a*b + b^2)*e^(4*d*x + 4*c) - 3*(a^2 + 2*a*b + b^2)*e^(2*d*x + 2*c))*d)
+ 1/24*(6*(9*a + 10*b)*e^(-2*d*x - 2*c) - 3*(12*a + 13*b)*e^(-4*d*x - 4*c)
- 22*a - 25*b)/((a^2 + 2*a*b + b^2 - 3*(a^2 + 2*a*b + b^2)*e^(-2*d*x - 2*c)
 + 3*(a^2 + 2*a*b + b^2)*e^(-4*d*x - 4*c) - (a^2 + 2*a*b + b^2)*e^(-6*d*x -
 6*c))*d) + 1/6*(6*(2*a + 3*b)*e^(-2*d*x - 2*c) - 3*(4*a + 5*b)*e^(-4*d*x -
 4*c) - 4*a - 7*b)/((a^2 + 2*a*b + b^2 - 3*(a^2 + 2*a*b + b^2)*e^(-2*d*x -
2*c) + 3*(a^2 + 2*a*b + b^2)*e^(-4*d*x - 4*c) - (a^2 + 2*a*b + b^2)*e^(-6*d
*x - 6*c))*d) - 1/4*(6*a*e^(-2*d*x - 2*c) + 3*b*e^(-4*d*x - 4*c) - 2*a + b)
/((a^2 + 2*a*b + b^2 - 3*(a^2 + 2*a*b + b^2)*e^(-2*d*x - 2*c) + 3*(a^2 + 2*
a*b + b^2)*e^(-4*d*x - 4*c) - (a^2 + 2*a*b + b^2)*e^(-6*d*x - 6*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1214 vs. $2(77) = 154$.
time = 0.45, size = 2705, normalized size = 31.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 36*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^5 + 6*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^6
 - 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^4 + 6*(15*(a
^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - 3*(a^2 + 2*a*b + b^2)*d*x - 4*a^2 -
 6*a*b)*sinh(d*x + c)^4 + 24*(5*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - (
3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 -
 6*(a^2 + 2*a*b + b^2)*d*x + 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 8*a*b)*
cosh(d*x + c)^2 + 6*(15*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 3*(a^2 +
2*a*b + b^2)*d*x - 6*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x +
 c)^2 + 4*a^2 + 8*a*b)*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cos
```

h(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 +
3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4
*(5*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*c
osh(d*x + c)^4 - 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^
2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c
))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*c
osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^
2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 +
a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a
*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c
)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*c
osh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*c
osh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) - 16*a^2 - 28
*a*b + 12*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 - 2*(3*(a^2 + 2*a*b +
b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x + 4*
a^2 + 8*a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 2*a^2*b + a*b^2)*d*cosh(
d*x + c)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a
^3 + 2*a^2*b + a*b^2)*d*sinh(d*x + c)^6 - 3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(
d*x + c)^4 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 - (a^3 + 2*a^2*
b + a*b^2)*d)*sinh(d*x + c)^4 + 3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2
 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^3 - 3*(a^3 + 2*a^2*b + a*b^
2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d
x + c)^4 - 6(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 + (a^3 + 2*a^2*b +
a*b^2)*d)*sinh(d*x + c)^2 - (a^3 + 2*a^2*b + a*b^2)*d + 6*((a^3 + 2*a^2*b +
 a*b^2)*d*cosh(d*x + c)^5 - 2*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^3 + (
a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/3*(3*(a^2 + 2*a*b
 + b^2)*d*x*cosh(d*x + c)^6 + 18*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)*sinh
(d*x + c)^5 + 3*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^6 - 3*(3*(a^2 + 2*a*b
 + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^4 + 3*(15*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)^2 - 3*(a^2 + 2*a*b + b^2)*d*x - 4*a^2 - 6*a*b)*sinh(d*x + c
)^4 + 12*(5*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - (3*(a^2 + 2*a*b + b^2
)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^
2)*d*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 8*a*b)*cosh(d*x + c)^2 + 3*
(15*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x - 6
(3(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^2 + 4*a^2 + 8*a*
b)*sinh(d*x + c)^2 - 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x
+ c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x +
c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c
)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 - 6*b
^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c)^5 -
2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*
arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*
x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) - 8*a^2 - 14*a*b + 6*(3*(a^2 + 2*a*
b + b^2)*d*x*cosh(d*x + c)^5 - 2*(3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b

```
)*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 8*a*b)*cosh(d*x +
c))*sinh(d*x + c))/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^6 + 6*(a^3 + 2*
a^2*b + a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + 2*a^2*b + a*b^2)*d*
sinh(d*x + c)^6 - 3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^4 + 3*(5*(a^3 +
 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 - (a^3 + 2*a^2*b + a*b^2)*d)*sinh(d*x +
 c)^4 + 3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 + 4*(5*(a^3 + 2*a^2*b +
 a*b^2)*d*cosh(d*x + c)^3 - 3*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c))*sinh
(d*x + c)^3 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^4 - 6*(a^3 + 2*a
^2*b + a*b^2)*d*cosh(d*x + c)^2 + (a^3 + 2*a^2*b + a*b^2)*d)*sinh(d*x + c)^
2 - (a^3 + 2*a^2*b + a*b^2)*d + 6*((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^
5 - 2*(a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c)^3 + (a^3 + 2*a^2*b + a*b^2)*d
*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(c+dx)}{a+b\,\mathrm{sech}^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4/(a+b*sech(d*x+c)**2),x)
```

```
[Out] Integral(coth(c + d*x)**4/(a + b*sech(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(77) = 154.
time = 1.60, size = 164, normalized size = 1.89

$$-\frac{\frac{3\,b^3\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{(a^3+2\,a^2b+ab^2)\sqrt{-ab-b^2}}-\frac{3\,(dx+c)}{a}+\frac{2\,\left(6\,ae^{(4\,dx+4\,c)}+9\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}-12\,be^{(2\,dx+2\,c)}+4\,a+7\,b\right)}{(a^2+2\,ab+b^2)\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^3*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^
3 + 2*a^2*b + a*b^2)*sqrt(-a*b - b^2)) - 3*(d*x + c)/a + 2*(6*a*e^(4*d*x +
4*c) + 9*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 12*b*e^(2*d*x + 2*c) + 4
*a + 7*b)/((a^2 + 2*a*b + b^2)*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [B]
time = 4.15, size = 779, normalized size = 8.95

$$\frac{x}{a}-\frac{8}{3\,(a\,d+b\,d)\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}-\frac{\mathrm{atan}\left(\left(e^{2\,c}\,e^{2\,d\,x}\left(\frac{4\,b^3}{a^2\,d\,(a+b)^2\,\sqrt{b^5}\,(a^3+2\,a^2\,b+a\,b^2)}+\frac{(a+2\,b)\left(a^4\,d\,\sqrt{b^5}+2\,a\,b^3\,d\,\sqrt{b^5}+4\,a^3\,b\,d\,\sqrt{b^5}+5\,a^2\,b^2\,d\,\sqrt{b^5}\right)}{a^2\,b^3\,(a^3+2\,a^2\,b+a\,b^2)\,\sqrt{-a^2\,d^2\,(a+b)^5}\,\sqrt{-a^7\,d^2-5\,a^6\,b\,d^2-10\,a^5\,b^2\,d^2-10\,a^4\,b^3\,d^2-5\,a^3\,b^4\,d^2-a^2\,b^5\,d^2}}\right)+\frac{(a+2\,b)\left(a^4\,d\,\sqrt{b^5}+2\,a^3\,b\,d\,\sqrt{b^5}+a^2\,b^2\,d\,\sqrt{b^5}\right)}{a^2\,b^3\,(a^3+2\,a^2\,b+a\,b^2)\,\sqrt{-a^2\,d^2\,(a+b)^5}\,\sqrt{-a^7\,d^2-5\,a^6\,b\,d^2-10\,a^5\,b^2\,d^2-10\,a^4\,b^3\,d^2-5\,a^3\,b^4\,d^2-a^2\,b^5\,d^2}}\right)\left(\frac{a^4\,\sqrt{-a^7\,d^2-5\,a^6\,b\,d^2-10\,a^5\,b^2\,d^2-10\,a^4\,b^3\,d^2-5\,a^3\,b^4\,d^2-a^2\,b^5\,d^2}}{2}+\frac{a^2\,b^2\,\sqrt{-a^7\,d^2-5\,a^6\,b\,d^2-10\,a^5\,b^2\,d^2-10\,a^4\,b^3\,d^2-5\,a^3\,b^4\,d^2-a^2\,b^5\,d^2}}{2}+a^3\,b\,\sqrt{-a^7\,d^2-5\,a^6\,b\,d^2-10\,a^5\,b^2\,d^2-10\,a^4\,b^3\,d^2-5\,a^3\,b^4\,d^2-a^2\,b^5\,d^2}\right)\right)\sqrt{b^5}}{\sqrt{-a^7\,d^2-5\,a^6\,b\,d^2-10\,a^5\,b^2\,d^2-10\,a^4\,b^3\,d^2-5\,a^3\,b^4\,d^2-a^2\,b^5\,d^2}}-\frac{4\,(a^2+b\,a)}{a\,(a+b)\,(a\,d+b\,d)\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{2\,(2\,a^2+3\,b\,a)}{a\,\left(e^{2\,c+2\,d\,x}-1\right)\,(a+b)\,(a\,d+b\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4/(a + b/cosh(c + d*x)^2),x)

[Out] x/a - 8/(3*(a*d + b*d)*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c +
 6*d*x) - 1)) - (atan((exp(2*c)*exp(2*d*x)*((4*b^3)/(a^3*d*(a + b)^2*(b^5)^
(1/2)*(a*b^2 + 2*a^2*b + a^3)) + ((a + 2*b)*(a^4*d*(b^5)^(1/2) + 2*a*b^3*d*
(b^5)^(1/2) + 4*a^3*b*d*(b^5)^(1/2) + 5*a^2*b^2*d*(b^5)^(1/2)))/(a^2*b^3*(a
*b^2 + 2*a^2*b + a^3)*(-a^2*d^2*(a + b)^5)^(1/2)*(- a^7*d^2 - 5*a^6*b*d^2 -
 a^2*b^5*d^2 - 5*a^3*b^4*d^2 - 10*a^4*b^3*d^2 - 10*a^5*b^2*d^2)^(1/2))) + (
(a + 2*b)*(a^4*d*(b^5)^(1/2) + 2*a^3*b*d*(b^5)^(1/2) + a^2*b^2*d*(b^5)^(1/2
)))/(a^2*b^3*(a*b^2 + 2*a^2*b + a^3)*(-a^2*d^2*(a + b)^5)^(1/2)*(- a^7*d^2
- 5*a^6*b*d^2 - a^2*b^5*d^2 - 5*a^3*b^4*d^2 - 10*a^4*b^3*d^2 - 10*a^5*b^2*d
^2)^(1/2)))*((a^4*(- a^7*d^2 - 5*a^6*b*d^2 - a^2*b^5*d^2 - 5*a^3*b^4*d^2 -
10*a^4*b^3*d^2 - 10*a^5*b^2*d^2)^(1/2))/2 + (a^2*b^2*(- a^7*d^2 - 5*a^6*b*d
^2 - a^2*b^5*d^2 - 5*a^3*b^4*d^2 - 10*a^4*b^3*d^2 - 10*a^5*b^2*d^2)^(1/2))/
2 + a^3*b*(- a^7*d^2 - 5*a^6*b*d^2 - a^2*b^5*d^2 - 5*a^3*b^4*d^2 - 10*a^4*b
^3*d^2 - 10*a^5*b^2*d^2)^(1/2)))*(b^5)^(1/2))/(- a^7*d^2 - 5*a^6*b*d^2 - a^
2*b^5*d^2 - 5*a^3*b^4*d^2 - 10*a^4*b^3*d^2 - 10*a^5*b^2*d^2)^(1/2) - (4*(a*
b + a^2))/(a*(a + b)*(a*d + b*d)*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1
)) - (2*(3*a*b + 2*a^2))/(a*(exp(2*c + 2*d*x) - 1)*(a + b)*(a*d + b*d))
```

3.148 $\int \frac{\tanh^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=76

$$\frac{(a+b)^2}{2a^2bd\left(b+a\cosh^2(c+dx)\right)}+\frac{\log(\cosh(c+dx))}{b^2d}+\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cosh^2(c+dx)\right)}{2d}$$

```
[Out] 1/2*(a+b)^2/a^2/b/d/(b+a*cosh(d*x+c)^2)+ln(cosh(d*x+c))/b^2/d+1/2*(1/a^2-1/
b^2)*ln(b+a*cosh(d*x+c)^2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 457, 90}

$$\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cosh^2(c+dx)+b\right)}{2d}+\frac{(a+b)^2}{2a^2bd\left(a\cosh^2(c+dx)+b\right)}+\frac{\log(\cosh(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (a + b)^2/(2*a^2*b*d*(b + a*Cosh[c + d*x]^2)) + Log[Cosh[c + d*x]]/(b^2*d)
+ ((a^(-2) - b^(-2))*Log[b + a*Cosh[c + d*x]^2])/(2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^5(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x(b+ax)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2x}-\frac{(a+b)^2}{ab(b+ax)^2}+\frac{-a^2+b^2}{ab^2(b+ax)}\right)\,dx, x, \cosh^2(c+dx)\right)}{2d}\\ &= \frac{(a+b)^2}{2a^2bd\left(b+a\cosh^2(c+dx)\right)}+\frac{\log(\cosh(c+dx))}{b^2d}+\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cosh\right.}{2d}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 109, normalized size = 1.43

$$\frac{(a+2b+a\cosh(2c+2dx))^2\left(\frac{(a+b)^2}{a^2b(b+a\cosh^2(c+dx))}+\frac{2\log(\cosh(c+dx))}{b^2}+\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cosh^2(c+dx)\right)\right)\text{sech}^4(c+dx)}{8d\left(a+b\text{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*c + 2*d*x])^2*((a + b)^2/(a^2*b*(b + a*Cosh[c + d*x]^2
)) + (2*Log[Cosh[c + d*x]])/b^2 + (a^(-2) - b^(-2))*Log[b + a*Cosh[c + d*x]
^2])*Sech[c + d*x]^4)/(8*d*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(72) = 144.
time = 2.36, size = 209, normalized size = 2.75

method	result
risch	$-\frac{x}{a^2}-\frac{2c}{a^2d}+\frac{2(a^2+2ab+b^2)e^{2dx+2c}}{a^2bd(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a)}+\frac{\ln(1+e^{2dx+2c})}{b^2d}-\frac{\ln\left(e^{4dx+4c}+\frac{2(2b+a)e^{2dx+2c}}{a}+1\right.}{2b^2d}$
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{(a+b)\left(\frac{2ab\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(a-b)}{}\right.}{a^2b^2}}{}$

default	$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{(a+b)\left(\frac{2ab\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(a-b)\ln\left(\ldots\right.}{\ldots}\right.}{a^2b^2}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/a^2/b^2*(a+b)*(2*a*b*tanh(1/2*d*x+1
/2*c)^2/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1
/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)+1/2*(a-b)*ln(a*tanh(1/2*d*x+1/2*c)^4
+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^
2+a+b))-1/a^2*ln(tanh(1/2*d*x+1/2*c)+1)+1/b^2*ln(tanh(1/2*d*x+1/2*c)^2+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(72) = 144$.
time = 0.48, size = 154, normalized size = 2.03

$$\frac{2\left(a^2+2ab+b^2\right)e^{(-2dx-2c)}}{\left(a^3be^{(-4dx-4c)}+a^3b+2\left(a^3b+2a^2b^2\right)e^{(-2dx-2c)}\right)d}+\frac{dx+c}{a^2d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{b^2d}-\frac{\left(a^2-b^2\right)\log\left(2(a+2b)e^{(-2dx-2c)}+ae^{(-4dx-4c)}+a\right)}{2a^2b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*(a^2 + 2*a*b + b^2)*e^(-2*d*x - 2*c)/((a^3*b*e^(-4*d*x - 4*c) + a^3*b + 2
*(a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2*c))*d) + (d*x + c)/(a^2*d) + log(e^(-2*d
*x - 2*c) + 1)/(b^2*d) - 1/2*(a^2 - b^2)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) +
 a*e^(-4*d*x - 4*c) + a)/(a^2*b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 853 vs. $2(72) = 144$.
time = 0.46, size = 853, normalized size = 11.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b^2*d*x*cosh(d*x + c)^4 + 8*a*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)
^3 + 2*a*b^2*d*x*sinh(d*x + c)^4 + 2*a*b^2*d*x - 4*(a^2*b + 2*a*b^2 + b^3 -
 (a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*a*b^2*d*x*cosh(d*x + c)^2 - a^
2*b - 2*a*b^2 - b^3 + (a*b^2 + 2*b^3)*d*x)*sinh(d*x + c)^2 + ((a^3 - a*b^2)
*cosh(d*x + c)^4 + 4*(a^3 - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 - a
*b^2)*sinh(d*x + c)^4 + a^3 - a*b^2 + 2*(a^3 + 2*a^2*b - a*b^2 - 2*b^3)*cos
h(d*x + c)^2 + 2*(a^3 + 2*a^2*b - a*b^2 - 2*b^3 + 3*(a^3 - a*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 4*((a^3 - a*b^2)*cosh(d*x + c)^3 + (a^3 + 2*a^2*b
 - a*b^2 - 2*b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 +
```

```
a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2)) - 2*(a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*x + c)*sinh(
d*x + c)^3 + a^3*sinh(d*x + c)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(d*x + c)^2
+ 2*(3*a^3*cosh(d*x + c)^2 + a^3 + 2*a^2*b)*sinh(d*x + c)^2 + 4*(a^3*cosh(d
*x + c)^3 + (a^3 + 2*a^2*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x +
c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(a*b^2*d*x*cosh(d*x + c)^3 - (a^2*b
 + 2*a*b^2 + b^3 - (a*b^2 + 2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(a^3*
b^2*d*cosh(d*x + c)^4 + 4*a^3*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b^2
*d*sinh(d*x + c)^4 + a^3*b^2*d + 2*(a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2
+ 2*(3*a^3*b^2*d*cosh(d*x + c)^2 + (a^3*b^2 + 2*a^2*b^3)*d)*sinh(d*x + c)^2
 + 4*(a^3*b^2*d*cosh(d*x + c)^3 + (a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*si
nh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(tanh(c + d*x)**5/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^4\,\tanh(c+d\,x)^5}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^4*tanh(c + d*x)^5)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.149 $\int \frac{\tanh^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2} \, dx$

Optimal. Leaf size=91

$$\frac{x}{a^2}+\frac{(a-2b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}d}-\frac{(a+b)\tanh(c+dx)}{2abd\left(a+b-b\tanh^2(c+dx)\right)}$$

```
[Out] x/a^2+1/2*(a-2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*(a+b)^(1/2)/a^2/
b^(3/2)/d-1/2*(a+b)*tanh(d*x+c)/a/b/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.14, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4226, 2000, 481, 536, 212, 214}

$$\frac{(a-2b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}d}+\frac{x}{a^2}-\frac{(a+b)\tanh(c+dx)}{2abd\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] x/a^2 + ((a - 2*b)*Sqrt[a + b]*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]
)/(2*a^2*b^(3/2)*d) - ((a + b)*Tanh[c + d*x])/(2*a*b*d*(a + b - b*Tanh[c +
d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
```

```
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+b(1-x^2))^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{(a+b)\tanh(c+dx)}{2abd\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a+b+(-a+b)x^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+
\end{aligned}$$

$$\begin{aligned}
&= -\frac{(a+b)\tanh(c+dx)}{2abd\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{a^2d} + \frac{((
\end{aligned}$$

$$\begin{aligned}
&= \frac{x}{a^2} + \frac{(a-2b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}d} - \frac{(a+b)\tanh(c+dx)}{2abd\left(a+b-b\tanh^2(c+
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 228 vs. 2(91) = 182.
time = 1.46, size = 228, normalized size = 2.51

$$\frac{(a+2b+a\cosh(2(c+dx)))\mathrm{sech}^4(c+dx)\left(2x(a+2b+a\cosh(2(c+dx)))+\frac{(a^2-ab-2b^2)\tanh^{-1}\left(\frac{\mathrm{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(a+2b+a\cosh(2(c+dx)))(\cosh(2c)-\sinh(2c))}{b\sqrt{a+b}\,d\sqrt{b(\cosh(c)-\sinh(c))^4}}+\frac{(a+b)\mathrm{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{bd}\right)}{8a^2\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(2*x*(a + 2*b + a*Cosh[2*(
c + d*x)]) + ((a^2 - a*b - 2*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c]
)*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c]
 - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))/(
b*Sqrt[a + b]*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + ((a + b)*Sech[2*c]*((a + 2
*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(b*d)))/(8*a^2*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(79) = 158.
time = 2.78, size = 259, normalized size = 2.85

method	result
derivativedivides	$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{2\left(-\frac{a(a+b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b}-\frac{a(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2b}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}$
default	$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{2\left(-\frac{a(a+b)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b}-\frac{a(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2b}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}$
risch	$\frac{x}{a^2}+\frac{a^2 e^{2dx+2c}+3ab\,e^{2dx+2c}+2b^2 e^{2dx+2c}+a^2+ab}{a^2 bd\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)}-\frac{\sqrt{b(a+b)}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{b(a+b)}-a-2b}{a}\right)}{2bd\,a^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+2/a^2*
((-1/2*a*(a+b)/b*tanh(1/2*d*x+1/2*c)^3-1/2*a*(a+b)/b*tanh(1/2*d*x+1/2*c))/(
a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2
*b*tanh(1/2*d*x+1/2*c)^2+a+b)+1/2*(a^2-a*b-2*b^2)/b*(-1/4/b^(1/2)/(a+b)^(1/
2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)
```

```
^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh
(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(82) = 164$.
time = 0.66, size = 1053, normalized size = 11.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(a^3 - 6*a^2*b - 24*a*b^2 - 16*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b -
 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^
3*b + a^2*b^2)*sqrt((a + b)*b)*d) + 1/16*a*log((a*e^(2*d*x + 2*c) + a + 2*b
 - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(s
qrt((a + b)*b)*(a*b + b^2)*d) - 1/64*(a^3 - 6*a^2*b - 24*a*b^2 - 16*b^3)*lo
g((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) +
a + 2*b + 2*sqrt((a + b)*b)))/((a^3*b + a^2*b^2)*sqrt((a + b)*b)*d) - 3/32*
(a + 2*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d
*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a*b + b^2)*d) -
 1/16*a*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x
 - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/(sqrt((a + b)*b)*(a*b + b^2)*d) + 1
/16*(a^3 + 8*a^2*b + 8*a*b^2 + (a^3 + 18*a^2*b + 48*a*b^2 + 32*b^3)*e^(2*d*
x + 2*c))/((a^4*b + a^3*b^2 + (a^4*b + a^3*b^2)*e^(4*d*x + 4*c) + 2*(a^4*b
+ 3*a^3*b^2 + 2*a^2*b^3)*e^(2*d*x + 2*c))*d) - 1/16*(a^3 + 8*a^2*b + 8*a*b^
2 + (a^3 + 18*a^2*b + 48*a*b^2 + 32*b^3)*e^(-2*d*x - 2*c))/((a^4*b + a^3*b^
2 + 2*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*e^(-2*d*x - 2*c) + (a^4*b + a^3*b^2)*
e^(-4*d*x - 4*c))*d) + 1/4*(a^2 + 2*a*b + (a^2 + 8*a*b + 8*b^2)*e^(2*d*x +
2*c))/((a^3*b + a^2*b^2 + (a^3*b + a^2*b^2)*e^(4*d*x + 4*c) + 2*(a^3*b + 3*
a^2*b^2 + 2*a*b^3)*e^(2*d*x + 2*c))*d) - 1/4*(a^2 + 2*a*b + (a^2 + 8*a*b +
8*b^2)*e^(-2*d*x - 2*c))/((a^3*b + a^2*b^2 + 2*(a^3*b + 3*a^2*b^2 + 2*a*b^3
)*e^(-2*d*x - 2*c) + (a^3*b + a^2*b^2)*e^(-4*d*x - 4*c))*d) - 3/8*((a + 2*b
)*e^(-2*d*x - 2*c) + a)/((a^2*b + a*b^2 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*e^(-2
*d*x - 2*c) + (a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) + 1/4*log(a*e^(4*d*x + 4
*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^2*d) - 1/4*log(2*(a + 2*b)*e^(-2*
d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(82) = 164$.
time = 0.46, size = 1479, normalized size = 16.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*b*d*x*cosh(d*x + c)^4 + 16*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3
+ 4*a*b*d*x*sinh(d*x + c)^4 + 4*a*b*d*x + 4*(2*(a*b + 2*b^2)*d*x + a^2 + 3*
a*b + 2*b^2)*cosh(d*x + c)^2 + 4*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(a*b + 2*b^
2)*d*x + a^2 + 3*a*b + 2*b^2)*sinh(d*x + c)^2 - ((a^2 - 2*a*b)*cosh(d*x + c
)^4 + 4*(a^2 - 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - 2*a*b)*sinh(d*
x + c)^4 + 2*(a^2 - 4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - 2*a*b)*cosh(d*x +
c)^2 + a^2 - 4*b^2)*sinh(d*x + c)^2 + a^2 - 2*a*b + 4*((a^2 - 2*a*b)*cosh(d
*x + c)^3 + (a^2 - 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a + b)/b)*log
((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x
+ c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 +
 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a
^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*c
osh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b + 2*b^2)*sqrt((a + b
)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x +
 c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sin
h(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c
) + a)) + 4*a^2 + 4*a*b + 8*(2*a*b*d*x*cosh(d*x + c)^3 + (2*(a*b + 2*b^2)*d
*x + a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a^3*b*d*cosh(d*x +
 c)^4 + 4*a^3*b*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b*d*sinh(d*x + c)^4 +
 a^3*b*d + 2*(a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*a^3*b*d*cosh(d*x
+ c)^2 + (a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + 4*(a^3*b*d*cosh(d*x + c)^
3 + (a^3*b + 2*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*d*x*cos
h(d*x + c)^4 + 8*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a*b*d*x*sinh(d*x
 + c)^4 + 2*a*b*d*x + 2*(2*(a*b + 2*b^2)*d*x + a^2 + 3*a*b + 2*b^2)*cosh(d*
x + c)^2 + 2*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(a*b + 2*b^2)*d*x + a^2 + 3*a*b
 + 2*b^2)*sinh(d*x + c)^2 + ((a^2 - 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 - 2*a*b
)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 -
4*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - 2*a*b)*cosh(d*x + c)^2 + a^2 - 4*b^2)*
sinh(d*x + c)^2 + a^2 - 2*a*b + 4*((a^2 - 2*a*b)*cosh(d*x + c)^3 + (a^2 - 4
*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a + b)/b)*arctan(1/2*(a*cosh(d*x
 + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sq
rt(-(a + b)/b)/(a + b)) + 2*a^2 + 2*a*b + 4*(2*a*b*d*x*cosh(d*x + c)^3 + (2
*(a*b + 2*b^2)*d*x + a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a^
3*b*d*cosh(d*x + c)^4 + 4*a^3*b*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b*d*s
inh(d*x + c)^4 + a^3*b*d + 2*(a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*a
^3*b*d*cosh(d*x + c)^2 + (a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + 4*(a^3*b*
d*cosh(d*x + c)^3 + (a^3*b + 2*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(82) = 164.
time = 1.81, size = 173, normalized size = 1.90

$$\frac{\frac{2\,(dx+c)}{a^2}+\frac{(a^2-ab-2\,b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^2b}+\frac{2\,(a^2e^{(2\,dx+2\,c)}+3\,abe^{(2\,dx+2\,c)}+2\,b^2e^{(2\,dx+2\,c)}+a^2+ab)}{(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a)a^2b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/a^2 + (a^2 - a*b - 2*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) +
a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a^2*b) + 2*(a^2*e^(2*d*x + 2*c
) + 3*a*b*e^(2*d*x + 2*c) + 2*b^2*e^(2*d*x + 2*c) + a^2 + a*b)/((a*e^(4*d*x
 + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)*a^2*b))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\cosh(c+d\,x)^2-1\right)^2}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^2 - 1)^2/(b + a*cosh(c + d*x)^2)^2, x)
```

3.150 $\int \frac{\tanh^3(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=51

$$\frac{a+b}{2a^2d\left(b+a\cosh^2(c+dx)\right)}+\frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^2d}$$

```
[Out] 1/2*(a+b)/a^2/d/(b+a*cosh(d*x+c)^2)+1/2*ln(b+a*cosh(d*x+c)^2)/a^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 455, 45}

$$\frac{a+b}{2a^2d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (a + b)/(2*a^2*d*(b + a*Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a
^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x(1-x^2)}{(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{1-x}{(b+ax)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(\frac{a+b}{a(b+ax)^2} - \frac{1}{a(b+ax)}\right)\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{a+b}{2a^2d\left(b+a\cosh^2(c+dx)\right)} + \frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^2d}$$

Mathematica [A]
time = 0.52, size = 81, normalized size = 1.59

$$\frac{2(a+b)+(a+2b)\log(a+2b+a\cosh(2(c+dx)))+a\cosh(2(c+dx))\log(a+2b+a\cosh(2(c+dx)))}{2a^2d(a+2b+a\cosh(2(c+dx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a*Cosh[2*(c + d
*x)]*Log[a + 2*b + a*Cosh[2*(c + d*x)]])/(2*a^2*d*(a + 2*b + a*Cosh[2*(c +
d*x)]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(47) = 94$.
time = 2.32, size = 178, normalized size = 3.49

method	result
risch	$-\frac{x}{a^2}-\frac{2c}{a^2d}+\frac{2(a+b)e^{2dx+2c}}{a^2d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)}+\frac{\ln\left(e^{4dx+4c}+\frac{2(2b+a)e^{2dx+2c}}{a}+1\right)}{2a^2d}$
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}+\frac{-\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{\ln\left(a\left(\tanh^4\left(\cdots\right.\right.\right.}{\cdots}}{a^2}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}+\frac{-\frac{2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{\ln\left(a\left(\tanh^4\left(\cdots\right.\right.\right.}{\cdots}}{a^2}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2*ln(tanh(1/2*d*x+1/2*c)+1)+1/a^2*(-2*a*tanh(1/2*d*x+1/2*c)^2/(a*
tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b
*tanh(1/2*d*x+1/2*c)^2+a+b)+1/2*ln(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1
/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b))-1/a^2*ln(
tanh(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(47) = 94$.
time = 0.29, size = 108, normalized size = 2.12

$$\frac{2\,(a+b)e^{(-2\,dx-2\,c)}}{\left(a^3e^{(-4\,dx-4\,c)}+a^3+2\,(a^3+2\,a^2b)e^{(-2\,dx-2\,c)}\right)d}+\frac{dx+c}{a^2d}+\frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*(a + b)*e^(-2*d*x - 2*c)/((a^3*e^(-4*d*x - 4*c) + a^3 + 2*(a^3 + 2*a^2*b)
*e^(-2*d*x - 2*c))*d) + (d*x + c)/(a^2*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x -
 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(47) = 94$.
time = 0.39, size = 485, normalized size = 9.51

$$-\frac{2\,adx\cosh(dx+c)^4+8\,adx\cosh(dx+c)\sinh(dx+c)^3+2\,adx\sinh(dx+c)^4+2\,adx+4\left((a+2\,b)dx-a-b\right)\cosh(dx+c)^2+4\left(3\,adx\cosh(dx+c)^2+(a+2\,b)dx-a-b\right)\sinh(dx+c)^2-\left(a\cosh(dx+c)^4+4\,a\cosh(dx+c)\sinh(dx+c)^3+a\sinh(dx+c)^4+2\,(a+2\,b)\cosh(dx+c)^2+2\left(3\,a\cosh(dx+c)^2+a+2\,b\right)\sinh(dx+c)^2+4\left(a\cosh(dx+c)^3+(a+2\,b)\cosh(dx+c)\right)\sinh(dx+c)+a\right)\log\left(\frac{2\left(a\cosh(dx+c)^2+a\sinh(dx+c)^2+a+2\,b\right)}{\cosh(dx+c)^2-2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)+8\left(adx\cosh(dx+c)^3+\left((a+2\,b)dx-a-b\right)\cosh(dx+c)\right)\sinh(dx+c)}{2\left(a^3d\cosh(dx+c)^4+4\,a^3d\cosh(dx+c)\sinh(dx+c)^3+a^3d\sinh(dx+c)^4+a^3d+2\,(a^3+2\,a^2b)d\cosh(dx+c)^2+2\left(3\,a^3d\cosh(dx+c)^2+(a^3+2\,a^2b)d\right)\sinh(dx+c)^2+4\left(a^3d\cosh(dx+c)^3+(a^3+2\,a^2b)d\cosh(dx+c)\right)\sinh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*x*cosh(d*x + c)^4 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a
*d*x*sinh(d*x + c)^4 + 2*a*d*x + 4*((a + 2*b)*d*x - a - b)*cosh(d*x + c)^2
+ 4*(3*a*d*x*cosh(d*x + c)^2 + (a + 2*b)*d*x - a - b)*sinh(d*x + c)^2 - (a*
cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2
*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c
)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*lo
g(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*
cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*(a*d*x*cosh(d*x + c)^3
+ ((a + 2*b)*d*x - a - b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c
)^4 + 4*a^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4 + a^3*d
 + 2*(a^3 + 2*a^2*b)*d*cosh(d*x + c)^2 + 2*(3*a^3*d*cosh(d*x + c)^2 + (a^3
+ 2*a^2*b)*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 + (a^3 + 2*a^2*b)*
d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(c+d\,x)^4\tanh(c+d\,x)^3}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^4*tanh(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.151 $\int \frac{\tanh^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=85

$$\frac{x}{a^2}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\sqrt{a+b}\,d}-\frac{\tanh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)}$$

```
[Out] x/a^2-1/2*(a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^2/d/b^(1/2)/(a
+b)^(1/2)-1/2*tanh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]

time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4226, 2000, 482, 536, 212, 214}

$$-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\,d\sqrt{a+b}}+\frac{x}{a^2}-\frac{\tanh(c+dx)}{2ad\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] x/a^2 - ((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(2*a^2*Sqr
t[b]*Sqrt[a + b]*d) - Tanh[c + d*x]/(2*a*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b(1-x^2))^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\tanh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1+x^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \tanh(c+d\ldots\right.}{2ad}$$

$$= -\frac{\tanh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{a^2d} - \frac{(a - \ldots}{}$$

$$= \frac{x}{a^2} - \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\,\sqrt{a+b}\,d} - \frac{\tanh(c+dx)}{2ad\left(a+b-b\tanh^2(c+dx)\right)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 326 vs. 2(85) = 170.
time = 3.07, size = 326, normalized size = 3.84

$$\frac{(a+2b+a\cosh(2(c+dx)))^2\operatorname{sech}^4(c+dx)\left(\frac{16x}{a^2}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}(a+b)^{3/2}d}+\frac{(a^3-6a^2b-24ab^2-16b^3)\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(\cosh(2c)-\sinh(2c))}{a^2b(a+b)^{3/2}d\sqrt{b(\cosh(c)-\sinh(c))^4}}+\frac{(a^2+8ab+8b^2)\operatorname{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{a^2b(a+b)d(a+2b+a\cosh(2(c+dx)))}+\frac{a\sinh(2(c+dx))}{b(a+b)d(a+2b+a\cosh(2(c+dx)))}\right)}{64\left(a+b\operatorname{sech}^2(c+dx)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*((16*x)/a^2 - ((a + 2*b)
*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(b^(3/2)*(a + b)^(3/2)*d) +
((a^3 - 6*a^2*b - 24*a*b^2 - 16*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2
*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh
[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(a^2*b*(a + b)^(3/2)*d*Sqrt[b*
(Cosh[c] - Sinh[c])^4]) + ((a^2 + 8*a*b + 8*b^2)*Sech[2*c]*((a + 2*b)*Sinh[
2*c] - a*Sinh[2*d*x]))/(a^2*b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)])) +
(a*Sinh[2*(c + d*x)])/(b*(a + b)*d*(a + 2*b + a*Cosh[2*(c + d*x)]))))/(64*(
a + b*Sech[c + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(73) = 146.
time = 2.30, size = 236, normalized size = 2.78

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{2\left(-\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{2\left(-\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a}$
risch	$\frac{x}{a^2}+\frac{a\,e^{2dx+2c}+2b\,e^{2dx+2c}+a}{a^2d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)}+\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{ab+b^2}+2b\sqrt{ab+b^2}+2ab+2b^2}{a\sqrt{ab+b^2}}\right)b}{2\sqrt{ab+b^2}\,d\,a^2}+\frac{\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+2/a^2*
((-1/2*a*tanh(1/2*d*x+1/2*c)^3-1/2*a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1
/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+
1/2*c)^2+a+b)+1/2*(2*b+a)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2
```

```
*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^
(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a
+b)^(1/2)))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(76) = 152$.
time = 0.54, size = 597, normalized size = 7.02

$$-\frac{(a^2+6ab+4b^2)\log\left(\frac{ae^{(2dx+2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(2dx+2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{16(a^3+a^2b)\sqrt{(a+b)b}\,d}+\frac{(a^2+6ab+4b^2)\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{16(a^3+a^2b)\sqrt{(a+b)b}\,d}+\frac{a^2+2ab+(a^2+8ab+8b^2)e^{(2dx+2c)}}{4(a^4+a^3b+(a^4+a^3b)e^{(4dx+4c)}+2(a^4+3a^3b+2a^2b^2)e^{(2dx+2c)})d}-\frac{a^2+2ab+(a^2+8ab+8b^2)e^{(-2dx-2c)}}{4(a^4+a^3b+2(a^4+3a^3b+2a^2b^2)e^{(-2dx-2c)}+(a^4+a^3b)e^{(-4dx-4c)})d}-\frac{(a+2b)e^{(-2dx-2c)}+a}{2(a^3+a^2b+2(a^3+3a^2b+2ab^2)e^{(-2dx-2c)}+(a^3+a^2b)e^{(-4dx-4c)})d}+\frac{\log\left(\frac{ae^{(-2dx-2c)}+a+2b-2\sqrt{(a+b)b}}{ae^{(-2dx-2c)}+a+2b+2\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}\,(a+b)d}+\frac{\log\left(ae^{(4dx+4c)}+2(a+2b)e^{(2dx+2c)}+a\right)}{4a^2d}-\frac{\log\left(2(a+2b)e^{(-2dx-2c)}+ae^{(-4dx-4c)}+a\right)}{4a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(a^2 + 6*a*b + 4*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a +
b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*sq
rt((a + b)*b)*d) + 1/16*(a^2 + 6*a*b + 4*b^2)*log((a*e^(-2*d*x - 2*c) + a +
 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)
))/((a^3 + a^2*b)*sqrt((a + b)*b)*d) + 1/4*(a^2 + 2*a*b + (a^2 + 8*a*b + 8*
b^2)*e^(2*d*x + 2*c))/((a^4 + a^3*b + (a^4 + a^3*b)*e^(4*d*x + 4*c) + 2*(a^
4 + 3*a^3*b + 2*a^2*b^2)*e^(2*d*x + 2*c))*d) - 1/4*(a^2 + 2*a*b + (a^2 + 8*
a*b + 8*b^2)*e^(-2*d*x - 2*c))/((a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2
)*e^(-2*d*x - 2*c) + (a^4 + a^3*b)*e^(-4*d*x - 4*c))*d) - 1/2*((a + 2*b)*e^
(-2*d*x - 2*c) + a)/((a^3 + a^2*b + 2*(a^3 + 3*a^2*b + 2*a*b^2)*e^(-2*d*x -
 2*c) + (a^3 + a^2*b)*e^(-4*d*x - 4*c))*d) + 1/8*log((a*e^(-2*d*x - 2*c) +
a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)
*b)))/(sqrt((a + b)*b)*(a + b)*d) + 1/4*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)
*e^(2*d*x + 2*c) + a)/(a^2*d) - 1/4*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^
(-4*d*x - 4*c) + a)/(a^2*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(76) = 152$.
time = 0.55, size = 1846, normalized size = 21.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 16*(a^2*b + a*b^2)*d*x*cosh(d
*x + c)*sinh(d*x + c)^3 + 4*(a^2*b + a*b^2)*d*x*sinh(d*x + c)^4 + 4*a^2*b +
 4*a*b^2 + 4*(a^2*b + a*b^2)*d*x + 4*(a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b +
3*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(6*(a^2*b + a*b^2)*d*x*cosh(d*x +
 c)^2 + a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*sinh(d*x
 + c)^2 + ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*a*b + 4*b^2)*cos
h(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*si
```

```
nh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2 + 4*a
*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x
 + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^
2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d
*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*co
sh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(
d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4
 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh
(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh
(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(2*(a^2*b +
a*b^2)*d*x*cosh(d*x + c)^3 + (a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2
+ 2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + a^3*b^2)*d*cosh(d*x +
 c)^4 + 4*(a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + a^3*
b^2)*d*sinh(d*x + c)^4 + 2*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^
2 + 2*(3*(a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^4*b + 3*a^3*b^2 + 2*a^2*b
^3)*d)*sinh(d*x + c)^2 + (a^4*b + a^3*b^2)*d + 4*((a^4*b + a^3*b^2)*d*cosh(
d*x + c)^3 + (a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)
), 1/2*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2*b + a*b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + 2*(a^2*b + a*b^2)*d*x*sinh(d*x + c)^4 + 2*a^2*b
+ 2*a*b^2 + 2*(a^2*b + a*b^2)*d*x + 2*(a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b +
 3*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 2*(6*(a^2*b + a*b^2)*d*x*cosh(d*x
+ c)^2 + a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*sinh(d*
x + c)^2 - ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*s
inh(d*x + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(a^2 + 4*a*b + 4*b^2)*co
sh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 + a^2 + 4*a*b + 4*b^2)*s
inh(d*x + c)^2 + a^2 + 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 + (a^2 + 4*
a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b - b^2)*arctan(1/2*(a*c
osh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a +
2*b)*sqrt(-a*b - b^2)/(a*b + b^2)) + 4*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)
^3 + (a^2*b + 3*a*b^2 + 2*b^3 + 2*(a^2*b + 3*a*b^2 + 2*b^3)*d*x)*cosh(d*x +
 c))*sinh(d*x + c))/((a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^4*b + a^3*b
^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + a^3*b^2)*d*sinh(d*x + c)^4 +
 2*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + a^3*b^
2)*d*cosh(d*x + c)^2 + (a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*d)*sinh(d*x + c)^2 +
 (a^4*b + a^3*b^2)*d + 4*((a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 + (a^4*b + 3*
a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [A]
time = 1.21, size = 137, normalized size = 1.61

$$-\frac{\frac{(a+2\,b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^2}-\frac{2\,(dx+c)}{a^2}-\frac{2\,\left(ae^{(2\,dx+2\,c)}+2\,be^{(2\,dx+2\,c)}+a\right)}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((a + 2*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/
(sqrt(-a*b - b^2)*a^2) - 2*(d*x + c)/a^2 - 2*(a*e^(2*d*x + 2*c) + 2*b*e^(2*
d*x + 2*c) + a)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x +
2*c) + a)*a^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^2\,\left(\cosh(c+d\,x)^2-1\right)}{{\left(a\cosh\left(c+d\,x\right)^2+b\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^2*(cosh(c + d*x)^2 - 1))/(b + a*cosh(c + d*x)^2)^2, x)
```

3.152 $\int \frac{\tanh(c+dx)}{\left(a+b\,\mathbf{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=49

$$\frac{b}{2a^2d\left(b+a\cosh^2(c+dx)\right)}+\frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^2d}$$

```
[Out] 1/2*b/a^2/d/(b+a*cosh(d*x+c)^2)+1/2*ln(b+a*cosh(d*x+c)^2)/a^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4223, 272, 45}

$$\frac{b}{2a^2d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] b/(2*a^2*d*(b + a*Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(b+ax)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{b}{a(b+ax)^2}+\frac{1}{a(b+ax)}\right)\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{b}{2a^2d\left(b+a\cosh^2(c+dx)\right)} + \frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^2d}$$

Mathematica [**A**]
time = 0.41, size = 79, normalized size = 1.61

$$\frac{2b+(a+2b)\log(a+2b+a\cosh(2(c+dx)))+a\cosh(2(c+dx))\log(a+2b+a\cosh(2(c+dx)))}{2a^2d(a+2b+a\cosh(2(c+dx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] (2*b + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a*Cosh[2*(c + d*x)]*L
og[a + 2*b + a*Cosh[2*(c + d*x)]])/(2*a^2*d*(a + 2*b + a*Cosh[2*(c + d*x)])
)
```

Maple [**A**]
time = 0.95, size = 62, normalized size = 1.27

method	result	size
derivativedivides	$-\frac{\frac{\ln(\text{sech}(dx+c))}{a^2}-\frac{b\left(-\frac{a}{b\left(a+b\,\text{sech}(dx+c)^2\right)}+\frac{\ln\left(a+b\,\text{sech}(dx+c)^2\right)}{b}\right)}{2a^2}}{d}$	62
default	$-\frac{\frac{\ln(\text{sech}(dx+c))}{a^2}-\frac{b\left(-\frac{a}{b\left(a+b\,\text{sech}(dx+c)^2\right)}+\frac{\ln\left(a+b\,\text{sech}(dx+c)^2\right)}{b}\right)}{2a^2}}{d}$	62
risch	$-\frac{x}{a^2}-\frac{2c}{a^2d}+\frac{2b\,e^{2dx+2c}}{a^2d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)}+\frac{\ln\left(e^{4dx+4c}+\frac{2(2b+a)e^{2dx+2c}}{a}+1\right)}{2a^2d}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(1/a^2*ln(sech(d*x+c))-1/2*b/a^2*(-a/b/(a+b*sech(d*x+c)^2)+1/b*ln(a+b*
sech(d*x+c)^2)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(45) = 90$.
time = 0.27, size = 106, normalized size = 2.16

$$\frac{2\,be^{(-2\,dx-2\,c)}}{\left(a^3e^{(-4\,dx-4\,c)}+a^3+2\,(a^3+2\,a^2b)e^{(-2\,dx-2\,c)}\right)d}+\frac{dx+c}{a^2d}+\frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*b*e^(-2*d*x - 2*c)/((a^3*e^(-4*d*x - 4*c) + a^3 + 2*(a^3 + 2*a^2*b)*e^(-2
*d*x - 2*c))*d) + (d*x + c)/(a^2*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x - 2*c)
+ a*e^(-4*d*x - 4*c) + a)/(a^2*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(45) = 90$.
time = 0.44, size = 476, normalized size = 9.71

$$-\frac{2\,adx\cosh\left(dx+c\right)^4+8\,adx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+2\,adx\sinh\left(dx+c\right)^4+2\,adx+4\,\left((a+2\,b)dx-b\right)\cosh\left(dx+c\right)^2+4\,\left(3\,adx\cosh\left(dx+c\right)^2+(a+2\,b)dx-b\right)\sinh\left(dx+c\right)^2-\left(a\cosh\left(dx+c\right)^4+4\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+a\sinh\left(dx+c\right)^4+2\,(a+2\,b)\cosh\left(dx+c\right)^2+2\,\left(3\,a\cosh\left(dx+c\right)^2+a+2\,b\right)\sinh\left(dx+c\right)^2+4\,\left(a\cosh\left(dx+c\right)^3+(a+2\,b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+a\right)\log\left(\frac{2\,\left(a\cosh(dx+c)^2+a\sinh(dx+c)^2+a+2\,b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)+8\,\left(adx\cosh\left(dx+c\right)^3+\left((a+2\,b)dx-b\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{2\,\left(a^3d\cosh\left(dx+c\right)^4+4\,a^3d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+a^3d\sinh\left(dx+c\right)^4+a^3d+2\,(a^3+2\,a^2b)d\cosh\left(dx+c\right)^2+2\,\left(3\,a^3d\cosh\left(dx+c\right)^2+(a^3+2\,a^2b)d\right)\sinh\left(dx+c\right)^2+4\,\left(a^3d\cosh\left(dx+c\right)^3+(a^3+2\,a^2b)d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*x*cosh(d*x + c)^4 + 8*a*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a
*d*x*sinh(d*x + c)^4 + 2*a*d*x + 4*((a + 2*b)*d*x - b)*cosh(d*x + c)^2 + 4*
(3*a*d*x*cosh(d*x + c)^2 + (a + 2*b)*d*x - b)*sinh(d*x + c)^2 - (a*cosh(d*x
 + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*
b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*
(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)*log(2*(a*c
osh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x
 + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*(a*d*x*cosh(d*x + c)^3 + ((a +
2*b)*d*x - b)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*
d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4 + a^3*d + 2*(a^3 +
2*a^2*b)*d*cosh(d*x + c)^2 + 2*(3*a^3*d*cosh(d*x + c)^2 + (a^3 + 2*a^2*b)*d
)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 + (a^3 + 2*a^2*b)*d*cosh(d*x +
 c))*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.73, size = 53, normalized size = 1.08

$$\frac{\ln\left(\cosh(c+d\,x)^2\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)\right)}{2\,a^2\,d}-\frac{1}{2\,a\,d\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] log(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2))/(2*a^2*d) - 1/(2*a*d*(a + b/co
sh(c + d*x)^2))
```

3.153 $\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2} dx$

Optimal. Leaf size=93

$$\frac{x}{a^2}-\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d}-\frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

```
[Out] x/a^2-1/2*(3*a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^(1/2)/a^2/(a
+b)^(3/2)/d-1/2*b*tanh(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4213, 425, 536, 212, 214}

$$-\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tanh(c+dx)}{2ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(-2),x]
```

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/
(2*a^2*(a + b)^(3/2)*d) - (b*Tanh[c + d*x])/(2*a*(a + b)*d*(a + b - b*Tanh[
c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2a-b-bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \text{tanh}\right.}{2a(a+b)d}$$

$$= -\frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{a^2d}$$

$$= \frac{x}{a^2} - \frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d} - \frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(\right.}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 221 vs. 2(93) = 186.
time = 1.41, size = 221, normalized size = 2.38

$$\frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^4(c+dx)\left(2x(a+2b+a\cosh(2(c+dx))) - \frac{b(3a+2b)\tanh^{-1}\left(\frac{\text{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(a+2b+a\cosh(2(c+dx)))(\cosh(2c)-\sinh(2c))}{(a+b)^{3/2}d\sqrt{b(\cosh(c)-\sinh(c))^4}} + \frac{b\text{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{(a+b)d}\right)}{8a^2\left(a+b\text{sech}^2(c+dx)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*(2*x*(a + 2*b + a*Cosh[2*(
c + d*x)]) - (b*(3*a + 2*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a
+ 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sin
h[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))/((a + b
)^(3/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (b*Sech[2*c]*((a + 2*b)*Sinh[2*c
] - a*Sinh[2*d*x]))/((a + b)*d)))/(8*a^2*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(81) = 162$.
time = 2.42, size = 254, normalized size = 2.73

method	result
derivativedivides	$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2} + \frac{2b\left(\frac{-\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)}-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(3a+2b)\left(-\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2} + \frac{2b\left(\frac{-\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)}-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(3a+2b)\left(-\cdots\right.}{\cdots}\right.}{\cdots}$
risch	$\frac{x}{a^2} + \frac{b\left(a\,e^{2dx+2c}+2b\,e^{2dx+2c}+a\right)}{a^2(a+b)d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)} + \frac{3\sqrt{b(a+b)}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{b(a+b)}+a+2b}{a}\right)}{4(a+b)^2da} - \cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+2*b/a^2*((-1/2/(a+b)*a*tanh(1/2*d*x+1
/2*c)^3-1/2/(a+b)*a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/
2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)+1/2
*(3*a+2*b)/(a+b)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*
c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln(
(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)
)))+1/a^2*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(84) = 168$.
time = 0.50, size = 187, normalized size = 2.01

$$\frac{(3\,ab+2\,b^2)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{4\,(a^3+a^2b)\sqrt{(a+b)b}\,d} - \frac{ab+(ab+2\,b^2)e^{(-2\,dx-2\,c)}}{(a^4+a^3b+2\,(a^4+3\,a^3b+2\,a^2b^2)e^{(-2\,dx-2\,c)}+(a^4+a^3b)e^{(-4\,dx-4\,c)})d} + \frac{dx+c}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(3*a*b + 2*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/
(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((a
+ b)*b)*d) - (a*b + (a*b + 2*b^2)*e^(-2*d*x - 2*c))/((a^4 + a^3*b + 2*(a^4
+ 3*a^3*b + 2*a^2*b^2)*e^(-2*d*x - 2*c) + (a^4 + a^3*b)*e^(-4*d*x - 4*c))*d
) + (d*x + c)/(a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(84) = 168$.
time = 0.50, size = 1690, normalized size = 18.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 16*(a^2 + a*b)*d*x*cosh(d*x + c)*
sinh(d*x + c)^3 + 4*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 4*(a^2 + a*b)*d*x + 4
*(2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^2 + 4*(6*(a^2 +
a*b)*d*x*cosh(d*x + c)^2 + 2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*sinh(
d*x + c)^2 + ((3*a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(3*a^2 + 2*a*b)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (3*a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*(3*a^2 + 8*a*b +
 4*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*
a*b + 4*b^2)*sinh(d*x + c)^2 + 3*a^2 + 2*a*b + 4*((3*a^2 + 2*a*b)*cosh(d*x
+ c)^3 + (3*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a +
b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*si
nh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2
+ a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)
^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x +
 c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x +
c)^2 + a^2 + 3*a*b + 2*b^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(
d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2
+ 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3
+ (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 4*a*b + 8*(2*(a^2 + a*b)*d
*x*cosh(d*x + c)^3 + (2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*cosh(d*x +
 c))*sinh(d*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^4 + 4*(a^4 + a^3*b)*d*co
sh(d*x + c)*sinh(d*x + c)^3 + (a^4 + a^3*b)*d*sinh(d*x + c)^4 + 2*(a^4 + 3*
a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + a^3*b)*d*cosh(d*x + c)^2
 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + (a^4 + a^3*b)*d + 4*((a
^4 + a^3*b)*d*cosh(d*x + c)^3 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)
)*sinh(d*x + c)), 1/2*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 8*(a^2 + a*b)*d*
x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 2*(a^
2 + a*b)*d*x + 2*(2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b^2)*cosh(d*x + c)^
```

```
2 + 2*(6*(a^2 + a*b)*d*x*cosh(d*x + c)^2 + 2*(a^2 + 3*a*b + 2*b^2)*d*x + a*
b + 2*b^2)*sinh(d*x + c)^2 - ((3*a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(3*a^2 +
2*a*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 2*a*b)*sinh(d*x + c)^4 + 2*
(3*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 2*a*b)*cosh(d*x + c
)^2 + 3*a^2 + 8*a*b + 4*b^2)*sinh(d*x + c)^2 + 3*a^2 + 2*a*b + 4*((3*a^2 +
2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 4*b^2)*cosh(d*x + c))*sinh(d*x +
c))*sqrt(-b/(a + b))*arctan(1/2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh
(d*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(-b/(a + b))/b) + 2*a*b + 4*(2
*(a^2 + a*b)*d*x*cosh(d*x + c)^3 + (2*(a^2 + 3*a*b + 2*b^2)*d*x + a*b + 2*b
^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + a^3*b)*d*cosh(d*x + c)^4 + 4*(a^4
 + a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + a^3*b)*d*sinh(d*x + c)^4
 + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + a^3*b)*d*c
osh(d*x + c)^2 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d)*sinh(d*x + c)^2 + (a^4 + a^
3*b)*d + 4*((a^4 + a^3*b)*d*cosh(d*x + c)^3 + (a^4 + 3*a^3*b + 2*a^2*b^2)*d
*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**(-2), x)
```

Giac [A]
time = 0.51, size = 163, normalized size = 1.75

$$-\frac{\frac{\left(3\,ab+2\,b^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\left(a^3+a^2b\right)\sqrt{-ab-b^2}}-\frac{2\left(abe^{(2\,dx+2\,c)}+2\,b^2e^{(2\,dx+2\,c)}+ab\right)}{\left(a^3+a^2b\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}-\frac{2\,(dx+c)}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a*b + 2*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b -
b^2))/((a^3 + a^2*b)*sqrt(-a*b - b^2)) - 2*(a*b*e^(2*d*x + 2*c) + 2*b^2*e^(
2*d*x + 2*c) + a*b)/((a^3 + a^2*b)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c)
 + 4*b*e^(2*d*x + 2*c) + a)) - 2*(d*x + c)/a^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{b}{{\mathrm{cosh}\left(c+d\,x\right)}^2}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x)^2)^2,x)
[Out] int(1/(a + b/cosh(c + d*x)^2)^2, x)
```

3.154 $\int \frac{\coth(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{b^2}{2a^2(a+b)d\left(b+a\cosh^2(c+dx)\right)}+\frac{b(2a+b)\log\left(b+a\cosh^2(c+dx)\right)}{2a^2(a+b)^2d}+\frac{\log(\sinh(c+dx))}{(a+b)^2d}$$

```
[Out] 1/2*b^2/a^2/(a+b)/d/(b+a*cosh(d*x+c)^2)+1/2*b*(2*a+b)*ln(b+a*cosh(d*x+c)^2)
/a^2/(a+b)^2/d+ln(sinh(d*x+c))/(a+b)^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4223, 457, 90}

$$\frac{b^2}{2a^2d(a+b)\left(a\cosh^2(c+dx)+b\right)}+\frac{b(2a+b)\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d(a+b)^2}+\frac{\log(\sinh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] b^2/(2*a^2*(a + b)*d*(b + a*Cosh[c + d*x]^2)) + (b*(2*a + b)*Log[b + a*Cosh
[c + d*x]^2])/(2*a^2*(a + b)^2*d) + Log[Sinh[c + d*x]]/((a + b)^2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(b+ax)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)}+\frac{b^2}{a(a+b)(b+ax)^2}-\frac{b(2a+b)}{a(a+b)^2(b+ax)}\right)\,dx, x, \cosh^2(c+dx\right.}{2d}$$
$$= \frac{b^2}{2a^2(a+b)d\left(b+a\cosh^2(c+dx)\right)}+\frac{b(2a+b)\log\left(b+a\cosh^2(c+dx)\right)}{2a^2(a+b)^2d}+\frac{\text{lo}}{}$$

Mathematica [A]
time = 0.21, size = 115, normalized size = 1.39

$$\frac{(a+b)\left(2a^2\log(\sinh(c+dx))+b\left(b+(2a+b)\log\left(a+b+a\sinh^2(c+dx)\right)\right)\right)+a\left(2a^2\log(\sinh(c+dx))+b(2a+b)\log\left(a+b+a\sinh^2(c+dx)\right)\right)\sinh^2(c+dx)}{a^2(a+b)^2d(a+2b+a\cosh(2(c+dx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*(2*a^2*Log[Sinh[c + d*x]] + b*(b + (2*a + b)*Log[a + b + a*Sinh[c
+ d*x]^2])) + a*(2*a^2*Log[Sinh[c + d*x]] + b*(2*a + b)*Log[a + b + a*Sinh[
c + d*x]^2])*Sinh[c + d*x]^2)/(a^2*(a + b)^2*d*(a + 2*b + a*Cosh[2*(c + d*x
)]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(79) = 158.
time = 2.68, size = 206, normalized size = 2.48

method	result
derivativedivides	$\frac{b\left(-\frac{2ab\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(2a+b)\ln\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\text{ta}\right.\right.}{}\right.}{a^2(a+b)^2}$
default	$\frac{b\left(-\frac{2ab\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(2a+b)\ln\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\text{ta}\right.\right.}{}\right.}{a^2(a+b)^2}$

risch	$\frac{x}{a^2} - \frac{2x}{a^2+2ab+b^2} - \frac{2c}{d(a^2+2ab+b^2)} - \frac{4bx}{a(a^2+2ab+b^2)} - \frac{4bc}{ad(a^2+2ab+b^2)} - \frac{2b^2x}{a^2(a^2+2ab+b^2)} - \frac{2b^2c}{a^2d(a^2+2ab+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b/a^2/(a+b)^2*(-2*a*b*tanh(1/2*d*x+1/2*c)^2/(a*tanh(1/2*d*x+1/2*c)^4+b
*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+
a+b)+1/2*(2*a+b)*ln(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tan
h(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b))+1/(a+b)^2*ln(tanh(1/2*d*
x+1/2*c))-1/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(79) = 158$.
time = 0.28, size = 209, normalized size = 2.52

$$\frac{2\,b^2e^{(-2\,dx-2\,c)}}{(a^4+a^3b+2\,(a^4+3\,a^3b+2\,a^2b^2)e^{(-2\,dx-2\,c)}+(a^4+a^3b)e^{(-4\,dx-4\,c)})d} + \frac{(2\,ab+b^2)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,(a^4+2\,a^3b+a^2b^2)d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{(a^2+2\,ab+b^2)d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{(a^2+2\,ab+b^2)d} + \frac{dx+c}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*b^2*e^(-2*d*x - 2*c)/((a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*e^(-2*
d*x - 2*c) + (a^4 + a^3*b)*e^(-4*d*x - 4*c))*d) + 1/2*(2*a*b + b^2)*log(2*(
a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^4 + 2*a^3*b + a^2*b
^2)*d) + log(e^(-d*x - c) + 1)/((a^2 + 2*a*b + b^2)*d) + log(e^(-d*x - c) -
 1)/((a^2 + 2*a*b + b^2)*d) + (d*x + c)/(a^2*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(79) = 158$.
time = 0.48, size = 1031, normalized size = 12.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^3 + 2*a^2*b + a*
b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^3 + 2*a^2*b + a*b^2)*d*x*sinh
(d*x + c)^4 + 2*(a^3 + 2*a^2*b + a*b^2)*d*x - 4*(a*b^2 + b^3 - (a^3 + 4*a^2
*b + 5*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*(a^3 + 2*a^2*b + a*b^2)*d
*x*cosh(d*x + c)^2 - a*b^2 - b^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*d*x)*s
inh(d*x + c)^2 - ((2*a^2*b + a*b^2)*cosh(d*x + c)^4 + 4*(2*a^2*b + a*b^2)*c
osh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b + a*b^2)*sinh(d*x + c)^4 + 2*a^2*b
+ a*b^2 + 2*(2*a^2*b + 5*a*b^2 + 2*b^3)*cosh(d*x + c)^2 + 2*(2*a^2*b + 5*a*
```

```
b^2 + 2*b^3 + 3*(2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((2*
a^2*b + a*b^2)*cosh(d*x + c)^3 + (2*a^2*b + 5*a*b^2 + 2*b^3)*cosh(d*x + c))
*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(co
sh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(a^3*
cosh(d*x + c)^4 + 4*a^3*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)^4
 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(d*x + c)^2 + 2*(3*a^3*cosh(d*x + c)^2 + a^3
 + 2*a^2*b)*sinh(d*x + c)^2 + 4*(a^3*cosh(d*x + c)^3 + (a^3 + 2*a^2*b)*cosh
(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c
))) + 8*((a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 - (a*b^2 + b^3 - (a^3
+ 4*a^2*b + 5*a*b^2 + 2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 2*a
^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a^5 + 2*a^4*b + a^3*b^2)*d*sinh(d*x + c)^4 + 2*(a^5
 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 2*a^4*b
 + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d)*
sinh(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b^2)*d + 4*((a^5 + 2*a^4*b + a^3*b^2
)*d*cosh(d*x + c)^3 + (a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*d*cosh(d*x +
c))*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth (c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^4\,\mathrm{coth}\left(c+d\,x\right)}{{\left(a\,{\mathrm{cosh}\left(c+d\,x\right)}^2+b\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(a + b/cosh(c + d*x)^2)^2,x)
[Out] int((cosh(c + d*x)^4*coth(c + d*x))/(b + a*cosh(c + d*x)^2)^2, x)
```

3.155 $\int \frac{\coth^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=121

$$\frac{x}{a^2}-\frac{b^{3/2}(5a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{5/2}d}-\frac{(2a-b)\coth(c+dx)}{2a(a+b)^2d}-\frac{b\coth(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)}$$

```
[Out] x/a^2-1/2*b^(3/2)*(5*a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^2/(a
+b)^(5/2)/d-1/2*(2*a-b)*coth(d*x+c)/a/(a+b)^2/d-1/2*b*coth(d*x+c)/a/(a+b)/d
/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.20, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4226, 2000, 483, 597, 536, 212, 214}

$$-\frac{b^{3/2}(5a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{5/2}}+\frac{x}{a^2}-\frac{(2a-b)\coth(c+dx)}{2ad(a+b)^2}-\frac{b\coth(c+dx)}{2ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] x/a^2 - (b^(3/2)*(5*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/
(2*a^2*(a + b)^(5/2)*d) - ((2*a - b)*Coth[c + d*x])/(2*a*(a + b)^2*d) - (b*
Coth[c + d*x])/(2*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
```

```
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+b(1-x^2))^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{b\coth(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2a+b-3bx^2}{x^2(1-x^2)(a+b-bx^2)}\,dx, x, \tan\right.}{2a(a+b)d} \\
&= -\frac{(2a-b)\coth(c+dx)}{2a(a+b)^2d} - \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int\right.}{} \\
&= -\frac{(2a-b)\coth(c+dx)}{2a(a+b)^2d} - \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int\right.}{} \\
&= \frac{x}{a^2} - \frac{b^{3/2}(5a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{5/2}d} - \frac{(2a-b)\coth(c+dx)}{2a(a+b)^2d} - \frac{}{2a(a}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 268 vs. 2(121) = 242.
time = 1.91, size = 268, normalized size = 2.21

$$\frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^4(c+dx)\left(\frac{2x(a+2b+a\cosh(2(c+dx)))}{a^2} - \frac{b^2(5a+2b)\tanh^{-1}\left(\frac{\text{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(a+2b+a\cosh(2(c+dx)))(\cosh(2c)-\sinh(2c))}{a^2(a+b)^{5/2}d\sqrt{b(\cosh(c)-\sinh(c))^4}} + \frac{2(a+2b+a\cosh(2(c+dx)))\text{csch}(c)\text{csch}(c+dx)\sinh(dx)}{(a+b)^2d} + \frac{b^2\text{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{a^2(a+b)^2d}\right)}{8\left(a+b\text{sech}^2(c+dx)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((2*x*(a + 2*b + a*Cosh[2*
(c + d*x)]))/a^2 - (b^2*(5*a + 2*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*
c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[
c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c]))
/(a^2*(a + b)^(5/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (2*(a + 2*b + a*Cosh
[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x])/((a + b)^2*d) + (b^2*Sech[2
*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(a^2*(a + b)^2*d)))/(8*(a + b*Se
ch[c + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(107) = 214.
time = 2.71, size = 288, normalized size = 2.38

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2+2ab+b^2\right)}+\frac{2b^2\left(\frac{-\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(5a+2b)\left(-\frac{\ln\left(\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2+2ab+b^2\right)}+\frac{2b^2\left(\frac{-\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{(5a+2b)\left(-\frac{\ln\left(\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}$
risch	$\frac{x}{a^2}-\frac{2a^3e^{4dx+4c}-ab^2e^{4dx+4c}-2b^3e^{4dx+4c}+4a^3e^{2dx+2c}+8a^2be^{2dx+2c}+2b^3e^{2dx+2c}+2a^3+ab^2}{d(a+b)^2\left(e^{2dx+2c}-1\right)a^2\left(ae^{4dx+4c}+2ae^{2dx+2c}+4be^{2dx+2c}+a\right)}+\frac{5\sqrt{b(a+b)}\,b\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+2*b^2/(a+b)^2/a^2*((-1/2*a*ta
nh(1/2*d*x+1/2*c)^3-1/2*a*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*t
anh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+
b)+1/2*(5*a+2*b)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*
c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln(
(a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2)
)))-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/2/(a+
b)^2/tanh(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1070 vs. $2(110) = 220$.
time = 0.55, size = 1070, normalized size = 8.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a*b + b^2)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/
((a^4 + 2*a^3*b + a^2*b^2)*d) - 1/4*(2*a*b + b^2)*log(2*(a + 2*b)*e^(-2*d*x
 - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^4 + 2*a^3*b + a^2*b^2)*d) - 1/16*(3*a
^2*b + 10*a*b^2 + 4*b^3)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*
```

```
b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + 2*a^3*b + a^
2*b^2)*sqrt((a + b)*b)*d) + 1/16*(3*a^2*b + 10*a*b^2 + 4*b^3)*log((a*e^(-2*
d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2
*sqrt((a + b)*b)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt((a + b)*b)*d) - 3/8*b*lo
g((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) +
a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b)*d) + 1/4
*(2*a^3 + a^2*b + 2*a*b^2 + (2*a^3 - a^2*b - 8*a*b^2 - 8*b^3)*e^(4*d*x + 4*
c) + 2*(2*a^3 + 4*a^2*b + 3*a*b^2 + 4*b^3)*e^(2*d*x + 2*c))/((a^5 + 2*a^4*b
 + a^3*b^2 - (a^5 + 2*a^4*b + a^3*b^2)*e^(6*d*x + 6*c) - (a^5 + 6*a^4*b + 9
*a^3*b^2 + 4*a^2*b^3)*e^(4*d*x + 4*c) + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*
b^3)*e^(2*d*x + 2*c))*d) - 1/4*(2*a^3 + a^2*b + 2*a*b^2 + 2*(2*a^3 + 4*a^2*
b + 3*a*b^2 + 4*b^3)*e^(-2*d*x - 2*c) + (2*a^3 - a^2*b - 8*a*b^2 - 8*b^3)*e
^(-4*d*x - 4*c))/((a^5 + 2*a^4*b + a^3*b^2 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4
*a^2*b^3)*e^(-2*d*x - 2*c) - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*e^(-4*
d*x - 4*c) - (a^5 + 2*a^4*b + a^3*b^2)*e^(-6*d*x - 6*c))*d) - 1/2*(2*a^2 -
a*b + 2*(2*a^2 + 4*a*b - b^2)*e^(-2*d*x - 2*c) + (2*a^2 + a*b + 2*b^2)*e^(-
4*d*x - 4*c))/((a^4 + 2*a^3*b + a^2*b^2 + (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*
b^3)*e^(-2*d*x - 2*c) - (a^4 + 6*a^3*b + 9*a^2*b^2 + 4*a*b^3)*e^(-4*d*x - 4
*c) - (a^4 + 2*a^3*b + a^2*b^2)*e^(-6*d*x - 6*c))*d) + 1/2*log(e^(2*d*x + 2
*c) - 1)/((a^2 + 2*a*b + b^2)*d) - 1/2*log(e^(-2*d*x - 2*c) - 1)/((a^2 + 2*
a*b + b^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1674 vs. $2(110) = 220$.
time = 0.45, size = 3624, normalized size = 29.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^6 + 24*(a^3 + 2*a^2*b + a
*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(a^3 + 2*a^2*b + a*b^2)*d*x*sin
h(d*x + c)^6 - 4*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)
*d*x)*cosh(d*x + c)^4 + 4*(15*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^2 -
 2*a^3 + a*b^2 + 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*sinh(d*x +
c)^4 + 16*(5*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 - (2*a^3 - a*b^2 -
 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c
)^3 - 8*a^3 - 4*a*b^2 - 4*(a^3 + 2*a^2*b + a*b^2)*d*x - 4*(4*a^3 + 8*a^2*b
+ 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^2 + 4*(15*(a
^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 - 4*a^3 - 8*a^2*b - 2*b^3 - (a^3
+ 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x - 6*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*
b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((5*a^2*b + 2*
a*b^2)*cosh(d*x + c)^6 + 6*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^
5 + (5*a^2*b + 2*a*b^2)*sinh(d*x + c)^6 + (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh
```

```
(d*x + c)^4 + (5*a^2*b + 22*a*b^2 + 8*b^3 + 15*(5*a^2*b + 2*a*b^2)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + (5*a^
2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*a^2*b - 2*a*b^2
- (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^2 + (15*(5*a^2*b + 2*a*b^2)*co
sh(d*x + c)^4 - 5*a^2*b - 22*a*b^2 - 8*b^3 + 6*(5*a^2*b + 22*a*b^2 + 8*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^
5 + 2*(5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^3 - (5*a^2*b + 22*a*b^2 +
8*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x + c)
^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2
*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x +
c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*
x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh
(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b^2
)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 +
 a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 +
 a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c))
*sinh(d*x + c) + a)) + 8*(3*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^5 - 2
*(2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x +
 c)^3 - (4*a^3 + 8*a^2*b + 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*c
osh(d*x + c))*sinh(d*x + c))/((a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^6 +
 6*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^5 + 2*a^4
*b + a^3*b^2)*d*sinh(d*x + c)^6 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d
*cosh(d*x + c)^4 + (15*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 +
 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d)*sinh(d*x + c)^4 - (a^5 + 6*a^4*b + 9*a
^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + 2*a^4*b + a^3*b^2)*d*co
sh(d*x + c)^3 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + (15*(a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 + 6*(a^5 +
6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*cosh(d*x + c)^2 - (a^5 + 6*a^4*b + 9*a^3
*b^2 + 4*a^2*b^3)*d)*sinh(d*x + c)^2 - (a^5 + 2*a^4*b + a^3*b^2)*d + 2*(3*(
a^5 + 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^5 + 2*(a^5 + 6*a^4*b + 9*a^3*b^2 +
 4*a^2*b^3)*d*cosh(d*x + c)^3 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*d*c
osh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x +
 c)^6 + 12*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a
^3 + 2*a^2*b + a*b^2)*d*x*sinh(d*x + c)^6 - 2*(2*a^3 - a*b^2 - 2*b^3 - (a^3
 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^4 + 2*(15*(a^3 + 2*a^2*b +
 a*b^2)*d*x*cosh(d*x + c)^2 - 2*a^3 + a*b^2 + 2*b^3 + (a^3 + 6*a^2*b + 9*a*
b^2 + 4*b^3)*d*x)*sinh(d*x + c)^4 + 8*(5*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d
*x + c)^3 - (2*a^3 - a*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)
*cosh(d*x + c))*sinh(d*x + c)^3 - 4*a^3 - 2*a*b^2 - 2*(a^3 + 2*a^2*b + a*b^
2)*d*x - 2*(4*a^3 + 8*a^2*b + 2*b^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x
)*cosh(d*x + c)^2 + 2*(15*(a^3 + 2*a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 - 4*a
^3 - 8*a^2*b - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x - 6*(2*a^3 - a
*b^2 - 2*b^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*d*x)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 - ((5*a^2*b + 2*a*b^2)*cosh(d*x + c)^6 + 6*(5*a^2*b + 2*a*b^2)*
cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^2*b + 2*a*b^2)*sinh(d*x + c)^6 + (5*a^
```

```
2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^4 + (5*a^2*b + 22*a*b^2 + 8*b^3 + 15*
(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 2*a*
b^2)*cosh(d*x + c)^3 + (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^3 - 5*a^2*b - 2*a*b^2 - (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^2
+ (15*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^4 - 5*a^2*b - 22*a*b^2 - 8*b^3 + 6*
(5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2
*b + 2*a*b^2)*cosh(d*x + c)^5 + 2*(5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c
)^3 - (5*a^2*b + 22*a*b^2 + 8*b^3)*cosh(d*x + c...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(110) = 220.
time = 1.19, size = 273, normalized size = 2.26

$$-\frac{\frac{(5\,ab^2+2\,b^3)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{(a^4+2\,a^3b+a^2b^2)\sqrt{-ab-b^2}}+\frac{2\,(2\,a^3e^{(4\,dx+4\,c)}-ab^2e^{(4\,dx+4\,c)}-2\,b^3e^{(4\,dx+4\,c)}+4\,a^3e^{(2\,dx+2\,c)}+8\,a^2be^{(2\,dx+2\,c)}+2\,b^3e^{(2\,dx+2\,c)}+2\,a^3+ab^2)}{(a^4+2\,a^3b+a^2b^2)(ae^{(6\,dx+6\,c)}+ae^{(4\,dx+4\,c)}+4\,be^{(4\,dx+4\,c)}-ae^{(2\,dx+2\,c)}-4\,be^{(2\,dx+2\,c)}-a)}-\frac{2\,(dx+c)}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((5*a*b^2 + 2*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*b
- b^2))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(-a*b - b^2)) + 2*(2*a^3*e^(4*d*x +
4*c) - a*b^2*e^(4*d*x + 4*c) - 2*b^3*e^(4*d*x + 4*c) + 4*a^3*e^(2*d*x + 2*c
) + 8*a^2*b*e^(2*d*x + 2*c) + 2*b^3*e^(2*d*x + 2*c) + 2*a^3 + a*b^2)/((a^4
+ 2*a^3*b + a^2*b^2)*(a*e^(6*d*x + 6*c) + a*e^(4*d*x + 4*c) + 4*b*e^(4*d*x
+ 4*c) - a*e^(2*d*x + 2*c) - 4*b*e^(2*d*x + 2*c) - a)) - 2*(d*x + c)/a^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^4\,\coth(c+d\,x)^2}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(a + b/cosh(c + d*x)^2)^2,x)
```

```
[Out] int((cosh(c + d*x)^4*coth(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.156 $\int \frac{\coth^3(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=110

$$\frac{b^3}{2a^2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)}-\frac{\mathrm{csch}^2(c+dx)}{2(a+b)^2d}+\frac{b^2(3a+b)\log\left(b+a\cosh^2(c+dx)\right)}{2a^2(a+b)^3d}+\frac{(a+3b)\log(\sinh(c+dx))}{(a+b)^3d}$$

```
[Out] 1/2*b^3/a^2/(a+b)^2/d/(b+a*cosh(d*x+c)^2)-1/2*csch(d*x+c)^2/(a+b)^2/d+1/2*b
^2*(3*a+b)*ln(b+a*cosh(d*x+c)^2)/a^2/(a+b)^3/d+(a+3*b)*ln(sinh(d*x+c))/(a+b
)^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 457, 90}

$$\frac{b^3}{2a^2d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}+\frac{b^2(3a+b)\log\left(a\cosh^2(c+dx)+b\right)}{2a^2d(a+b)^3}-\frac{\mathrm{csch}^2(c+dx)}{2d(a+b)^2}+\frac{(a+3b)\log(\sinh(c+dx))}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] b^3/(2*a^2*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)) - Csch[c + d*x]^2/(2*(a + b
)^2*d) + (b^2*(3*a + b)*Log[b + a*Cosh[c + d*x]^2])/(2*a^2*(a + b)^3*d) + (
(a + 3*b)*Log[Sinh[c + d*x]])/((a + b)^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
```

```
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^7}{(1-x^2)^2(b+ax^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(1-x)^2(b+ax)^2}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^2(-1+x)^2} + \frac{a+3b}{(a+b)^3(-1+x)} - \frac{b^3}{a(a+b)^2(b+ax)^2} + \frac{b^2(3a+b)}{a(a+b)^3(b+ax)}\right) dx, x, \cdots}{2d}$$
$$= \frac{b^3}{2a^2(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} - \frac{\text{csch}^2(c+dx)}{2(a+b)^2d} + \frac{b^2(3a+b)\log\left(b+a\cdots\right.}{2a^2(a+b\cdots}$$

Mathematica [A]
time = 0.88, size = 130, normalized size = 1.18

$$\frac{(a+2b+a\cosh(2(c+dx)))^2\text{sech}^4(c+dx)\left(-((a+b)\text{csch}^2(c+dx))+2(a+3b)\log(\sinh(c+dx))+\frac{b^2(3a+b)\log\left(a+b+a\sinh^2(c+dx)\right)}{a^2}+\frac{b^3(a+b)}{a^2\left(a+b+a\sinh^2(c+dx)\right)}\right)}{8(a+b)^3d\left(a+b\text{sech}^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^2*Sech[c + d*x]^4*(-((a + b)*Csch[c + d*x]
^2) + 2*(a + 3*b)*Log[Sinh[c + d*x]] + (b^2*(3*a + b)*Log[a + b + a*Sinh[c
+ d*x]^2])/a^2 + (b^3*(a + b))/(a^2*(a + b + a*Sinh[c + d*x]^2))))/(8*(a +
b)^3*d*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(104) = 208$.
time = 3.27, size = 260, normalized size = 2.36

method	result
derivativedivides	$-\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^2+2ab+b^2\right)} - \frac{1}{8(a+b)^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2} + \frac{(4a+12b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^3} - \frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2} + \frac{b^2\left(-\frac{\cdots}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\cdots\right.}\cdots}{\cdots}$
default	$-\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^2+2ab+b^2\right)} - \frac{1}{8(a+b)^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2} + \frac{(4a+12b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4(a+b)^3} - \frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2} + \frac{b^2\left(-\frac{\cdots}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\cdots\right.}\cdots}{\cdots}$

risch	$\frac{x}{a^2} - \frac{2ax}{a^3+3a^2b+3a\,b^2+b^3} - \frac{2ac}{d(a^3+3a^2b+3a\,b^2+b^3)} - \frac{6bx}{a^3+3a^2b+3a\,b^2+b^3} - \frac{6bc}{d(a^3+3a^2b+3a\,b^2+b^3)} - \frac{6}{a(a^3+3a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*tanh(1/2*d*x+1/2*c)^2/(a^2+2*a*b+b^2)-1/8/(a+b)^2/tanh(1/2*d*x+1/
2*c)^2+1/4/(a+b)^3*(4*a+12*b)*ln(tanh(1/2*d*x+1/2*c))-1/a^2*ln(tanh(1/2*d*x
+1/2*c)+1)+b^2/a^2/(a+b)^3*(-2*a*b*tanh(1/2*d*x+1/2*c)^2/(a*tanh(1/2*d*x+1/
2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1
/2*c)^2+a+b)+1/2*(3*a+b)*ln(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4
+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b))-1/a^2*ln(tanh(1/
2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(104) = 208$.
time = 0.30, size = 384, normalized size = 3.49

$$\frac{(3\,ab^2+b^3)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)d}+\frac{(a+3\,b)\log\left(e^{(-dx-c)}+1\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)d}+\frac{(a+3\,b)\log\left(e^{(-dx-c)}-1\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)d}-\frac{2\left((a^3-b^3)e^{(-2\,dx-2\,c)}+2\,(a^3+2\,a^2b+b^3)e^{(-4\,dx-4\,c)}+(a^3-b^3)e^{(-6\,dx-6\,c)}\right)}{(a^5+2\,a^4b+a^3b^2+4\,(a^4b+2\,a^3b^2+a^2b^3)e^{(-2\,dx-2\,c)}-2\,(a^5+6\,a^4b+9\,a^3b^2+4\,a^2b^3)e^{(-4\,dx-4\,c)}+4\,(a^4b+2\,a^3b^2+a^2b^3)e^{(-6\,dx-6\,c)}+(a^5+2\,a^4b+a^3b^2)e^{(-8\,dx-8\,c)})d}+\frac{dx+c}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*b^2 + b^3)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) +
 a)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d) + (a + 3*b)*log(e^(-d*x - c)
+ 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + (a + 3*b)*log(e^(-d*x - c) - 1)/
((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 2*((a^3 - b^3)*e^(-2*d*x - 2*c) + 2*(
a^3 + 2*a^2*b + b^3)*e^(-4*d*x - 4*c) + (a^3 - b^3)*e^(-6*d*x - 6*c))/((a^5
 + 2*a^4*b + a^3*b^2 + 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*e^(-2*d*x - 2*c) - 2
*(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*e^(-4*d*x - 4*c) + 4*(a^4*b + 2*a^
3*b^2 + a^2*b^3)*e^(-6*d*x - 6*c) + (a^5 + 2*a^4*b + a^3*b^2)*e^(-8*d*x - 8
*c))*d) + (d*x + c)/(a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3624 vs. $2(104) = 208$.
time = 0.59, size = 3624, normalized size = 32.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^8 + 16*(a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^4 +
```

3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*sinh(d*x + c)^8 + 4*(a^4 + a^3*b - a*b^3 -
b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c)^6 + 4*(14*(
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^2 + a^4 + a^3*b - a*b^
3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*sinh(d*x + c)^6 + 8*(1
4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^3 + 3*(a^4 + a^3*b
- a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c))*s
inh(d*x + c)^5 + 4*(2*a^4 + 6*a^3*b + 4*a^2*b^2 + 2*a*b^3 + 2*b^4 - (a^4 +
7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*cosh(d*x + c)^4 + 4*(35*(a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^4 + 2*a^4 + 6*a^3*b + 4*a^
2*b^2 + 2*a*b^3 + 2*b^4 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d
x + 15(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*
d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*d*x*cosh(d*x + c)^5 + 5*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^
2*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c)^3 + (2*a^4 + 6*a^3*b + 4*a^2*b^2
+ 2*a*b^3 + 2*b^4 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^3 + 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x +
4*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x)*c
osh(d*x + c)^2 + 4*(14*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c
)^6 + 15*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)
*d*x)*cosh(d*x + c)^4 + a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 +
3*a*b^3 + b^4)*d*x + 6*(2*a^4 + 6*a^3*b + 4*a^2*b^2 + 2*a*b^3 + 2*b^4 - (a^
4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 - ((3*a^2*b^2 + a*b^3)*cosh(d*x + c)^8 + 8*(3*a^2*b^2 + a*b^3)*cosh
(d*x + c)*sinh(d*x + c)^7 + (3*a^2*b^2 + a*b^3)*sinh(d*x + c)^8 + 4*(3*a*b^
3 + b^4)*cosh(d*x + c)^6 + 4*(3*a*b^3 + b^4 + 7*(3*a^2*b^2 + a*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b^2 + a*b^3)*cosh(d*x + c)^3 + 3*(3
*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(3*a^2*b^2 + 13*a*b^3 + 4*
b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^2*b^2 + a*b^3)*cosh(d*x + c)^4 - 3*a^2*b^
2 - 13*a*b^3 - 4*b^4 + 30*(3*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4
+ 3*a^2*b^2 + a*b^3 + 8*(7*(3*a^2*b^2 + a*b^3)*cosh(d*x + c)^5 + 10*(3*a*b^
3 + b^4)*cosh(d*x + c)^3 - (3*a^2*b^2 + 13*a*b^3 + 4*b^4)*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(3*a*b^3 + b^4)*cosh(d*x + c)^2 + 4*(7*(3*a^2*b^2 + a*b^3
)*cosh(d*x + c)^6 + 15*(3*a*b^3 + b^4)*cosh(d*x + c)^4 + 3*a*b^3 + b^4 - 3*
(3*a^2*b^2 + 13*a*b^3 + 4*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^2
*b^2 + a*b^3)*cosh(d*x + c)^7 + 3*(3*a*b^3 + b^4)*cosh(d*x + c)^5 - (3*a^2*
b^2 + 13*a*b^3 + 4*b^4)*cosh(d*x + c)^3 + (3*a*b^3 + b^4)*cosh(d*x + c))*si
nh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(
d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^4 +
3*a^3*b)*cosh(d*x + c)^8 + 8*(a^4 + 3*a^3*b)*cosh(d*x + c)*sinh(d*x + c)^7
+ (a^4 + 3*a^3*b)*sinh(d*x + c)^8 + 4*(a^3*b + 3*a^2*b^2)*cosh(d*x + c)^6 +
4*(a^3*b + 3*a^2*b^2 + 7*(a^4 + 3*a^3*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6
+ 8*(7*(a^4 + 3*a^3*b)*cosh(d*x + c)^3 + 3*(a^3*b + 3*a^2*b^2)*cosh(d*x + c
))*sinh(d*x + c)^5 - 2*(a^4 + 7*a^3*b + 12*a^2*b^2)*cosh(d*x + c)^4 + 2*(35
*(a^4 + 3*a^3*b)*cosh(d*x + c)^4 - a^4 - 7*a^3*b - 12*a^2*b^2 + 30*(a^3*b +
3*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 3*a^3*b + 8*(7*(a^4 +

```
3*a^3*b)*cosh(d*x + c)^5 + 10*(a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 - (a^4 +
7*a^3*b + 12*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^3*b + 3*a^2*b^2
)*cosh(d*x + c)^2 + 4*(7*(a^4 + 3*a^3*b)*cosh(d*x + c)^6 + 15*(a^3*b + 3*a^
2*b^2)*cosh(d*x + c)^4 + a^3*b + 3*a^2*b^2 - 3*(a^4 + 7*a^3*b + 12*a^2*b^2)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 3*a^3*b)*cosh(d*x + c)^7 + 3*
(a^3*b + 3*a^2*b^2)*cosh(d*x + c)^5 - (a^4 + 7*a^3*b + 12*a^2*b^2)*cosh(d*x
 + c)^3 + (a^3*b + 3*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x
+ c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a
*b^3)*d*x*cosh(d*x + c)^7 + 3*(a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2
*b^2 + 3*a*b^3 + b^4)*d*x)*cosh(d*x + c)^5 + 2*(2*a^4 + 6*a^3*b + 4*a^2*b^2
 + 2*a*b^3 + 2*b^4 - (a^4 + 7*a^3*b + 15*a^2*b^2 + 13*a*b^3 + 4*b^4)*d*x)*c
osh(d*x + c)^3 + (a^4 + a^3*b - a*b^3 - b^4 + 2*(a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a
^3*b^3)*d*cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(
d*x + c)*sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*sinh(d*x
 + c)^8 + 4*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^6 + 4
*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 + (a^5*b + 3*a^
4*b^2 + 3*a^3*b^3 + a^2*b^4)*d)*sinh(d*x + c)^6 - 2*(a^6 + 7*a^5*b + 15*a^4
*b^2 + 13*a^3*b^3 + 4*a^2*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*
a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^3 + 3*(a^5*b...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^4\,\coth(c+d\,x)^3}{{\left(a\cosh\left(c+d\,x\right)^2+b\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3/(a + b/cosh(c + d*x)^2)^2,x)
[Out] int((cosh(c + d*x)^4*coth(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.157 $\int \frac{\coth^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=161

$$\frac{x}{a^2}-\frac{b^{5/2}(7a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{7/2}d}-\frac{(2a^2+6ab-b^2)\coth(c+dx)}{2a(a+b)^3d}-\frac{(2a-3b)\coth^3(c+dx)}{6a(a+b)^2d}-\frac{}{2a(a+}$$

```
[Out] x/a^2-1/2*b^(5/2)*(7*a+2*b)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^2/(a
+b)^(7/2)/d-1/2*(2*a^2+6*a*b-b^2)*coth(d*x+c)/a/(a+b)^3/d-1/6*(2*a-3*b)*cot
h(d*x+c)^3/a/(a+b)^2/d-1/2*b*coth(d*x+c)^3/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.29, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4226, 2000, 483, 597, 536, 212, 214}

$$-\frac{b^{5/2}(7a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{7/2}}-\frac{(2a^2+6ab-b^2)\coth(c+dx)}{2ad(a+b)^3}+\frac{x}{a^2}-\frac{(2a-3b)\coth^3(c+dx)}{6ad(a+b)^2}-\frac{b\coth^3(c+dx)}{2ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] x/a^2 - (b^(5/2)*(7*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/
(2*a^2*(a + b)^(7/2)*d) - ((2*a^2 + 6*a*b - b^2)*Coth[c + d*x])/(2*a*(a + b
)^3*d) - ((2*a - 3*b)*Coth[c + d*x]^3)/(6*a*(a + b)^2*d) - (b*Coth[c + d*x]
^3)/(2*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
```

```
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\coth^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+b(1-x^2))^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2a+3b-5bx^2}{x^4(1-x^2)(a+b-bx^2)}\,dx, x, \text{tanl}\right.}{2a(a+b)d}$$

$$= -\frac{(2a-3b)\coth^3(c+dx)}{6a(a+b)^2d} - \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(2a^2+6ab-b^2)\coth(c+dx)}{2a(a+b)^3d} - \frac{(2a-3b)\coth^3(c+dx)}{6a(a+b)^2d} - \frac{b\,\text{cot}}{2a(a+b)d\,(a+}$$

$$= -\frac{(2a^2+6ab-b^2)\coth(c+dx)}{2a(a+b)^3d} - \frac{(2a-3b)\coth^3(c+dx)}{6a(a+b)^2d} - \frac{b\,\text{cot}}{2a(a+b)d\,(a+}$$

$$= \frac{x}{a^2} - \frac{b^{5/2}(7a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{7/2}d} - \frac{(2a^2+6ab-b^2)\coth(c+dx)}{2a(a+b)^3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 350 vs. 2(161) = 322.

time = 3.37, size = 350, normalized size = 2.17

$$\frac{(a+2b+a\cosh(2(c+dx)))\text{sech}^4(c+dx)\left(\frac{6x(a+2b+a\cosh(2(c+dx)))}{a^2} - \frac{2(a+2b+a\cosh(2(c+dx)))\coth(c)\text{csch}^2(c+dx)}{(a+b)^2d} - \frac{3b^3(7a+2b)\tanh^{-1}\left(\frac{\text{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(a+2b+a\cosh(2(c+dx)))(\cosh(2c)-\sinh(2c))}{a^2(a+b)^{7/2}d\sqrt{b(\cosh(c)-\sinh(c))^4}} + \frac{4(2a+5b)(a+2b+a\cosh(2(c+dx)))\text{csch}(c)\text{csch}(c+dx)\sinh(dx)}{(a+b)^3d} + \frac{2(a+2b+a\cosh(2(c+dx)))\text{csch}(c)\text{csch}^3(c+dx)\sinh(dx)}{(a+b)^2d} + \frac{3b^3\text{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{a^2(a+b)^3d}\right)}{24\left(a+b\text{sech}^2(c+dx)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^4*((6*x*(a + 2*b + a*Cosh[2*
(c + d*x)]))/a^2 - (2*(a + 2*b + a*Cosh[2*(c + d*x)])*Coth[c]*Csch[c + d*x]
^2)/((a + b)^2*d) - (3*b^3*(7*a + 2*b)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh
[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Co
sh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])*(Cosh[2*c] - Sinh[2*c
]))/(a^2*(a + b)^(7/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (4*(2*a + 5*b)*(a
 + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]*Sinh[d*x])/((a + b)^3*d
) + (2*(a + 2*b + a*Cosh[2*(c + d*x)])*Csch[c]*Csch[c + d*x]^3*Sinh[d*x])/(
(a + b)^2*d) + (3*b^3*Sech[2*c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/(a^2
*(a + b)^3*d)))/(24*(a + b*Sech[c + d*x]^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(145) = 290$.
time = 3.15, size = 365, normalized size = 2.27

method	result
derivativedivides	$-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+13b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^2+2ab+b^2\right)(a+b)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}+\frac{2b^3\left(\frac{\cdots}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}\cdots\right.}{\cdots}$
default	$-\frac{\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+13b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^2+2ab+b^2\right)(a+b)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}+\frac{2b^3\left(\frac{\cdots}{a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}\cdots\right.}{\cdots}$
risch	$\frac{x}{a^2}-\frac{12a^4e^{8dx+8c}+24a^3b\,e^{8dx+8c}-3a\,b^3e^{8dx+8c}-6b^4e^{8dx+8c}+12a^4e^{6dx+6c}+60a^3b\,e^{6dx+6c}+96a^2b^2e^{6dx+6c}+6a\,b^3e^{\cdots}}{3\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/(a^2+2*a*b+b^2)/(a+b)*(1/3*a*tanh(1/2*d*x+1/2*c)^3+1/3*b*tanh(1/2
*d*x+1/2*c)^3+5*a*tanh(1/2*d*x+1/2*c)+13*b*tanh(1/2*d*x+1/2*c))+1/a^2*ln(ta
nh(1/2*d*x+1/2*c)+1)+2*b^3/a^2/(a+b)^3*((-1/2*a*tanh(1/2*d*x+1/2*c)^3-1/2*a
*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*
tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)+1/2*(7*a+2*b)*(-1/4/b^
(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c
)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d*x
+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))))-1/a^2*ln(tanh(1/2*d*
x+1/2*c)-1)-1/24/(a+b)^2/tanh(1/2*d*x+1/2*c)^3-1/8*(5*a+13*b)/(a+b)^3/tanh(
1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2961 vs. $2(148) = 296$.
time = 0.77, size = 2961, normalized size = 18.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/4*(a^2*b + 3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x +
2*c) + a)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d) - 1/2*b*log(a*e^(4*d*x
+ 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
d) - 1/4*(a^2*b + 3*a*b^2 + b^3)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4
*d*x - 4*c) + a)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d) + 1/2*b*log(2*(a
 + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d) + 1/2*(a + 2*b)*log(e^(2*d*x + 2*c) - 1)/((a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*d) + b*log(e^(2*d*x + 2*c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*d) - 1/2*(a + 2*b)*log(e^(-2*d*x - 2*c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*d) - b*log(e^(-2*d*x - 2*c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) -
1/64*(3*a^3*b + 38*a^2*b^2 + 56*a*b^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a
+ 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)
))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt((a + b)*b)*d) + 1/16*(3*a*b
+ 8*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x
+ 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqr
t((a + b)*b)*d) + 1/64*(3*a^3*b + 38*a^2*b^2 + 56*a*b^3 + 16*b^4)*log((a*e^
(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b
 + 2*sqrt((a + b)*b)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt((a + b)*
b)*d) - 1/16*(3*a*b + 8*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a
+ b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*sqrt((a + b)*b)*d) + 3/32*(3*a*b - 2*b^2)*log((a*e^(-2*d
*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*
sqrt((a + b)*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt((a + b)*b)*d) + 1/4
8*(44*a^4 + 59*a^3*b + 24*a^2*b^2 + 24*a*b^3 + 3*(24*a^4 + 27*a^3*b - 18*a^
2*b^2 - 48*a*b^3 - 32*b^4)*e^(8*d*x + 8*c) + 6*(6*a^4 + 55*a^3*b + 79*a^2*b
^2 + 68*a*b^3 + 48*b^4)*e^(6*d*x + 6*c) - 2*(50*a^4 + 278*a^3*b + 309*a^2*b
^2 + 180*a*b^3 + 144*b^4)*e^(4*d*x + 4*c) - 2*(10*a^4 - 75*a^3*b - 103*a^2*
b^2 - 36*a*b^3 - 48*b^4)*e^(2*d*x + 2*c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3
*b^3 - (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*e^(10*d*x + 10*c) + (a^6 - a^5
*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*e^(8*d*x + 8*c) + 2*(a^6 + 9*a^5*b
 + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*e^(6*d*x + 6*c) - 2*(a^6 + 9*a^5*b
+ 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*e^(4*d*x + 4*c) - (a^6 - a^5*b - 9*a
^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*e^(2*d*x + 2*c))*d) - 1/48*(44*a^4 + 59*a^
3*b + 24*a^2*b^2 + 24*a*b^3 - 2*(10*a^4 - 75*a^3*b - 103*a^2*b^2 - 36*a*b^3
 - 48*b^4)*e^(-2*d*x - 2*c) - 2*(50*a^4 + 278*a^3*b + 309*a^2*b^2 + 180*a*b
^3 + 144*b^4)*e^(-4*d*x - 4*c) + 6*(6*a^4 + 55*a^3*b + 79*a^2*b^2 + 68*a*b^
3 + 48*b^4)*e^(-6*d*x - 6*c) + 3*(24*a^4 + 27*a^3*b - 18*a^2*b^2 - 48*a*b^3
 - 32*b^4)*e^(-8*d*x - 8*c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 -
 a^5*b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*e^(-2*d*x - 2*c) - 2*(a^6 + 9*
a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*e^(-4*d*x - 4*c) + 2*(a^6 + 9*
a^5*b + 21*a^4*b^2 + 19*a^3*b^3 + 6*a^2*b^4)*e^(-6*d*x - 6*c) + (a^6 - a^5*
b - 9*a^4*b^2 - 11*a^3*b^3 - 4*a^2*b^4)*e^(-8*d*x - 8*c) - (a^6 + 3*a^5*b +
 3*a^4*b^2 + a^3*b^3)*e^(-10*d*x - 10*c))*d) + 1/12*(8*a^3 + 17*a^2*b - 6*a
```

```
*b^2 + 3*(8*a^3 + 13*a^2*b + 8*a*b^2 + 8*b^3)*e^(8*d*x + 8*c) + 6*(4*a^3 +
19*a^2*b + 13*a*b^2 - 12*b^3)*e^(6*d*x + 6*c) - 2*(8*a^3 + 68*a^2*b + 69*a*
b^2 - 36*b^3)*e^(4*d*x + 4*c) - 2*(4*a^3 - 15*a^2*b - 37*a*b^2 + 12*b^3)*e^
(2*d*x + 2*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 - (a^5 + 3*a^4*b + 3*a
^3*b^2 + a^2*b^3)*e^(10*d*x + 10*c) + (a^5 - a^4*b - 9*a^3*b^2 - 11*a^2*b^3
 - 4*a*b^4)*e^(8*d*x + 8*c) + 2*(a^5 + 9*a^4*b + 21*a^3*b^2 + 19*a^2*b^3 +
6*a*b^4)*e^(6*d*x + 6*c) - 2*(a^5 + 9*a^4*b + 21*a^3*b^2 + 19*a^2*b^3 + 6*a
*b^4)*e^(4*d*x + 4*c) - (a^5 - a^4*b - 9*a^3*b^2 - 11*a^2*b^3 - 4*a*b^4)*e^
(2*d*x + 2*c))*d) - 1/12*(8*a^3 + 17*a^2*b - 6*a*b^2 - 2*(4*a^3 - 15*a^2*b
- 37*a*b^2 + 12*b^3)*e^(-2*d*x - 2*c) - 2*(8*a^3 + 68*a^2*b + 69*a*b^2 - 36
*b^3)*e^(-4*d*x - 4*c) + 6*(4*a^3 + 19*a^2*b + 13*a*b^2 - 12*b^3)*e^(-6*d*x
 - 6*c) + 3*(8*a^3 + 13*a^2*b + 8*a*b^2 + 8*b^3)*e^(-8*d*x - 8*c))/((a^5 +
3*a^4*b + 3*a^3*b^2 + a^2*b^3 - (a^5 - a^4*b - 9*a^3*b^2 - 11*a^2*b^3 - 4*a
*b^4)*e^(-2*d*x - 2*c) - 2*(a^5 + 9*a^4*b + 21*a^3*b^2 + 19*a^2*b^3 + 6*a*b
^4)*e^(-4*d*x - 4*c) + 2*(a^5 + 9*a^4*b + 21*a^3*b^2 + 19*a^2*b^3 + 6*a*b^4
)*e^(-6*d*x - 6*c) + (a^5 - a^4*b - 9*a^3*b^2 - 11*a^2*b^3 - 4*a*b^4)*e^(-8
*d*x - 8*c) - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(-10*d*x - 10*c))*d)
+ 1/8*(4*a^2 - 11*a*b - 2*(2*a^2 - 9*a*b + 19*b^2)*e^(-2*d*x - 2*c) - 2*(10
*a^2 + 22*a*b - 33*b^2)*e^(-4*d*x - 4*c) - 6*(2*a^2 + 3*a*b + 11*b^2)*e^(-6
*d*x - 6*c) - 3*(3*a*b - 2*b^2)*e^(-8*d*x - 8*c))/((a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*e^(-2*d*x - 2*c)
- 2*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*e^(-4*d*x - 4*c) + 2*(a
^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*e^(-6*d*x - 6*c) + (a^4 - a^3
*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*e^(-8*d*x - ...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4786 vs. $2(148) = 296$.
time = 0.49, size = 9849, normalized size = 61.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^10 + 120*(a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 12*(a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*sinh(d*x + c)^10 - 12*(4*a^4 + 8*a^3*
b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(
d*x + c)^8 + 12*(45*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^2
 - 4*a^4 - 8*a^3*b + a*b^3 + 2*b^4 - (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 -
4*b^4)*d*x)*sinh(d*x + c)^8 + 96*(15*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*
x*cosh(d*x + c)^3 - (4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2
*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 24*(2*a^4 +
10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*
b^3 + 6*b^4)*d*x)*cosh(d*x + c)^6 + 24*(105*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*
```

b^3)*d*x*cosh(d*x + c)^4 - 2*a^4 - 10*a^3*b - 16*a^2*b^2 - a*b^3 - 3*b^4 -
(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x - 14*(4*a^4 + 8*a^3*b -
a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x
+ c)^2)*sinh(d*x + c)^6 + 48*(63*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*c
osh(d*x + c)^5 - 14*(4*a^4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2
*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^3 - 3*(2*a^4 + 10*a^3*b + 16*a^
2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x
)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4
+ 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^4 +
8*(315*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^6 - 105*(4*a^
4 + 8*a^3*b - a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*
d*x)*cosh(d*x + c)^4 + 2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a
^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x - 45*(2*a^4 + 10*a^3*b + 16*a^2*b
^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*c
osh(d*x + c)^2)*sinh(d*x + c)^4 - 32*a^4 - 80*a^3*b - 12*a*b^3 + 32*(45*(a^
4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^7 - 21*(4*a^4 + 8*a^3*b
- a*b^3 - 2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*
x + c)^5 - 15*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3
*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^3 + (2*a^4 + 38*a^3*
b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^2 + 19*a*b^3 + 6*b^4)*
d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 12*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3
)*d*x - 4*(4*a^4 + 36*a^3*b + 80*a^2*b^2 - 6*a*b^3 + 6*b^4 - 3*(a^4 - a^3*b
- 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^2 + 4*(135*(a^4 + 3*a^3
*b + 3*a^2*b^2 + a*b^3)*d*x*cosh(d*x + c)^8 - 84*(4*a^4 + 8*a^3*b - a*b^3 -
2*b^4 + (a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x)*cosh(d*x + c)^6
- 90*(2*a^4 + 10*a^3*b + 16*a^2*b^2 + a*b^3 + 3*b^4 + (a^4 + 9*a^3*b + 21*a
^2*b^2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^4 - 4*a^4 - 36*a^3*b - 80*a^2
*b^2 + 6*a*b^3 - 6*b^4 + 3*(a^4 - a^3*b - 9*a^2*b^2 - 11*a*b^3 - 4*b^4)*d*x
+ 12*(2*a^4 + 38*a^3*b + 72*a^2*b^2 + 9*b^4 + 3*(a^4 + 9*a^3*b + 21*a^2*b^
2 + 19*a*b^3 + 6*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((7*a^2*b^2
+ 2*a*b^3)*cosh(d*x + c)^10 + 10*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)*sinh(
d*x + c)^9 + (7*a^2*b^2 + 2*a*b^3)*sinh(d*x + c)^10 - (7*a^2*b^2 - 26*a*b^3
- 8*b^4)*cosh(d*x + c)^8 - (7*a^2*b^2 - 26*a*b^3 - 8*b^4 - 45*(7*a^2*b^2 +
2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(7*a^2*b^2 + 2*a*b^3)*co
sh(d*x + c)^3 - (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)
^7 - 2*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^6 + 2*(105*(7*a^2*b^2
+ 2*a*b^3)*cosh(d*x + c)^4 - 7*a^2*b^2 - 44*a*b^3 - 12*b^4 - 14*(7*a^2*b^2
- 26*a*b^3 - 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(7*a^2*b^2 + 2
*a*b^3)*cosh(d*x + c)^5 - 14*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^3
- 3*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(7*
a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^4 + 2*(105*(7*a^2*b^2 + 2*a*b^3)
*cosh(d*x + c)^6 - 35*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^4 + 7*a^
2*b^2 + 44*a*b^3 + 12*b^4 - 15*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c
)^2)*sinh(d*x + c)^4 - 7*a^2*b^2 - 2*a*b^3 + 8*(15*(7*a^2*b^2 + 2*a*b^3)*co
sh(d*x + c)^7 - 7*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^5 - 5*(7*a^2

```
*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x + c)^3 + (7*a^2*b^2 + 44*a*b^3 + 12*b^4)
*cosh(d*x + c))*sinh(d*x + c)^3 + (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x +
 c)^2 + (45*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^8 - 28*(7*a^2*b^2 - 26*a*b^
3 - 8*b^4)*cosh(d*x + c)^6 - 30*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x +
c)^4 + 7*a^2*b^2 - 26*a*b^3 - 8*b^4 + 12*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^9
 - 4*(7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c)^7 - 6*(7*a^2*b^2 + 44*a*b
^3 + 12*b^4)*cosh(d*x + c)^5 + 4*(7*a^2*b^2 + 44*a*b^3 + 12*b^4)*cosh(d*x +
 c)^3 + (7*a^2*b^2 - 26*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b
/(a + b))*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 +
a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4/(a+b*sech(d*x+c)**2)**2,x)
```

```
[Out] Integral(coth(c + d*x)**4/(a + b*sech(c + d*x)**2)**2, x)
```

Giac [A]
time = 1.88, size = 293, normalized size = 1.82

$$-\frac{\frac{3\left(7\,ab^3+2\,b^4\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)\sqrt{-ab-b^2}}-\frac{6\left(ab^3e^{(2\,dx+2\,c)}+2\,b^4e^{(2\,dx+2\,c)}+ab^3\right)}{\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)}-\frac{6\,(dx+c)}{a^2}+\frac{8\left(3\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}-3\,ae^{(2\,dx+2\,c)}-9\,be^{(2\,dx+2\,c)}+2\,a+5\,b\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\left(e^{(2\,dx+2\,c)}-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*(7*a*b^3 + 2*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqrt(-a*
b - b^2))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(-a*b - b^2)) - 6*(a*b
^3*e^(2*d*x + 2*c) + 2*b^4*e^(2*d*x + 2*c) + a*b^3)/((a^5 + 3*a^4*b + 3*a^3
*b^2 + a^2*b^3)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2
*c) + a)) - 6*(d*x + c)/a^2 + 8*(3*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c)
- 3*a*e^(2*d*x + 2*c) - 9*b*e^(2*d*x + 2*c) + 2*a + 5*b)/((a^3 + 3*a^2*b +
3*a*b^2 + b^3)*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^4\,{\mathrm{coth}\left(c+d\,x\right)}^4}{{\left(a\,{\mathrm{cosh}\left(c+d\,x\right)}^2+b\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4/(a + b/cosh(c + d*x)^2)^2,x)
[Out] int((cosh(c + d*x)^4*coth(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^2, x)
```

3.158 $\int \frac{\tanh^6(c+dx)}{\left(a+b\mathbf{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=148

$$\frac{x}{a^3}-\frac{\sqrt{a+b}\,(3a^2-4ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}d}-\frac{(a+b)\tanh^3(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2}+\frac{(3a-4b)(a+}{8a^2b^2d\left(a+b-\right.}$$

```
[Out] x/a^3-1/8*(3*a^2-4*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*(a+b
)^(1/2)/a^3/b^(5/2)/d-1/4*(a+b)*tanh(d*x+c)^3/a/b/d/(a+b-b*tanh(d*x+c)^2)^2
+1/8*(3*a-4*b)*(a+b)*tanh(d*x+c)/a^2/b^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.22, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4226, 2000, 481, 592, 536, 212, 214}

$$\frac{x}{a^3}+\frac{(3a-4b)(a+b)\tanh(c+dx)}{8a^2b^2d\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\sqrt{a+b}\,(3a^2-4ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}d}-\frac{(a+b)\tanh^3(c+dx)}{4abd\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] x/a^3 - (Sqrt[a + b]*(3*a^2 - 4*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x]
)/Sqrt[a + b]])/(8*a^3*b^(5/2)*d) - ((a + b)*Tanh[c + d*x]^3)/(4*a*b*d*(a +
 b - b*Tanh[c + d*x]^2)^2) + ((3*a - 4*b)*(a + b)*Tanh[c + d*x])/(8*a^2*b^2
*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
```

```
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)(a+b(1-x^2))^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\tanh^3(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(-3a+b)x^2)}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh\right.}{4abd}$$

$$= -\frac{(a+b)\tanh^3(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(3a-4b)(a+b)\tanh(c+dx)}{8a^2b^2d\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Su}}{}$$

$$= -\frac{(a+b)\tanh^3(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(3a-4b)(a+b)\tanh(c+dx)}{8a^2b^2d\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Su}}{}$$

$$= \frac{x}{a^3} - \frac{\sqrt{a+b}\,(3a^2-4ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}d} - \frac{(a+b)\tan}{4abd\,(a+b-b\,t}$$

<u>Mathematica</u> [**B**] Leaf count is larger than twice the leaf count of optimal. 515 vs. 2(148) = 296.
time = 3.82, size = 515, normalized size = 3.48

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[c + d*x]^6/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*((-2*(3*a^3 - a^2*b + 4*a*
b^2 + 8*b^3)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x
] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a +
 2*b + a*Cosh[2*(c + d*x)])^2*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*
(Cosh[c] - Sinh[c])^4]) + Sech[2*c]*(8*b^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*Cosh
[2*c] + 16*a*b^2*(a + 2*b)*d*x*Cosh[2*d*x] + 4*a^2*b^2*d*x*Cosh[2*(c + 2*d*
x)] + 16*a^2*b^2*d*x*Cosh[4*c + 2*d*x] + 32*a*b^3*d*x*Cosh[4*c + 2*d*x] + 4
*a^2*b^2*d*x*Cosh[6*c + 4*d*x] - 9*a^4*Sinh[2*c] - 15*a^3*b*Sinh[2*c] + 18*
a^2*b^2*Sinh[2*c] + 72*a*b^3*Sinh[2*c] + 48*b^4*Sinh[2*c] + 9*a^4*Sinh[2*d*
x] + 13*a^3*b*Sinh[2*d*x] - 28*a^2*b^2*Sinh[2*d*x] - 32*a*b^3*Sinh[2*d*x] +
 3*a^4*Sinh[2*(c + 2*d*x)] - 3*a^3*b*Sinh[2*(c + 2*d*x)] - 6*a^2*b^2*Sinh[2
*(c + 2*d*x)] - 3*a^4*Sinh[4*c + 2*d*x] + a^3*b*Sinh[4*c + 2*d*x] + 20*a^2*
b^2*Sinh[4*c + 2*d*x] + 16*a*b^3*Sinh[4*c + 2*d*x])))/(128*a^3*b^2*d*(a + b
*Sech[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(134) = 268$.
time = 3.08, size = 389, normalized size = 2.63

method	result
derivativedivides	$\frac{2\left(\frac{a(3a^3+2a^2b-5a\,b^2-4b^3)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8b^2}+\frac{(9a^3-14a^2b-19a\,b^2+4b^3)a\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8b^2}+\frac{(9a^3-14a^2b-19a\,b^2+4b^3)a\left(\tanh^3\right.}{8b^2}\right.}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.}$
default	$\frac{2\left(\frac{a(3a^3+2a^2b-5a\,b^2-4b^3)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8b^2}+\frac{(9a^3-14a^2b-19a\,b^2+4b^3)a\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8b^2}+\frac{(9a^3-14a^2b-19a\,b^2+4b^3)a\left(\tanh^3\right.}{8b^2}\right.}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.}$
risch	$\frac{x}{a^3}-\frac{3a^4e^{6dx+6c}-a^3b\,e^{6dx+6c}-20a^2b^2e^{6dx+6c}-16a\,b^3e^{6dx+6c}+9a^4e^{4dx+4c}+15a^3b\,e^{4dx+4c}-18a^2b^2e^{4dx+4c}-72a\,b^3e^{4}}{4a^3b^2d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^3*((1/8*a*(3*a^3+2*a^2*b-5*a*b^2-4*b^3)/b^2*tanh(1/2*d*x+1/2*c)^7+
1/8*(9*a^3-14*a^2*b-19*a*b^2+4*b^3)*a/b^2*tanh(1/2*d*x+1/2*c)^5+1/8*(9*a^3-
14*a^2*b-19*a*b^2+4*b^3)*a/b^2*tanh(1/2*d*x+1/2*c)^3+1/8*a*(3*a^3+2*a^2*b-5
*a*b^2-4*b^3)/b^2*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*
d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)^2+1/8
*(3*a^3-a^2*b+4*a*b^2+8*b^3)/b^2*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*t
anh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)
/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(
1/2)+(a+b)^(1/2))))+1/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-1/a^3*ln(tanh(1/2*d*x+1
/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3239 vs. $2(140) = 280$.
time = 1.13, size = 3239, normalized size = 21.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -45/1024*(a + 2*b)*a*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/
(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 + 2*a*b^3 + b^
4)*sqrt((a + b)*b)*d) - 9/512*a^2*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt
```

((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 +
 2*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 45/1024*(a + 2*b)*a*log((a*e^(-2*d*x -
 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt
((a + b)*b)))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 9/512*a^2*log
((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a
 + 2*b + 2*sqrt((a + b)*b)))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)*d)
- 1/1024*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^5
)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c)
+ a + 2*b + 2*sqrt((a + b)*b)))/((a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*sqrt((a +
b)*b)*d) + 1/1024*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4
+ 256*b^5)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*
d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^5*b^2 + 2*a^4*b^3 + a^3*b^4)
*sqrt((a + b)*b)*d) + 5/256*(3*a^2 + 8*a*b + 8*b^2)*log((a*e^(-2*d*x - 2*c)
 + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a +
 b)*b)))/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)*d) - 1/256*(3*a^6 - 12*
a^5*b - 204*a^4*b^2 - 384*a^3*b^3 - 192*a^2*b^4 + (3*a^6 - 10*a^5*b - 560*a
^4*b^2 - 2080*a^3*b^3 - 2560*a^2*b^4 - 1024*a*b^5)*e^(6*d*x + 6*c) + (9*a^6
 - 12*a^5*b - 1100*a^4*b^2 - 5248*a^3*b^3 - 10304*a^2*b^4 - 9216*a*b^5 - 30
72*b^6)*e^(4*d*x + 4*c) + (9*a^6 - 14*a^5*b - 864*a^4*b^2 - 3136*a^3*b^3 -
3840*a^2*b^4 - 1536*a*b^5)*e^(2*d*x + 2*c))/((a^7*b^2 + 2*a^6*b^3 + a^5*b^4
 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*e^(8*d*x + 8*c) + 4*(a^7*b^2 + 4*a^6*b^3
 + 5*a^5*b^4 + 2*a^4*b^5)*e^(6*d*x + 6*c) + 2*(3*a^7*b^2 + 14*a^6*b^3 + 27*
a^5*b^4 + 24*a^4*b^5 + 8*a^3*b^6)*e^(4*d*x + 4*c) + 4*(a^7*b^2 + 4*a^6*b^3
+ 5*a^5*b^4 + 2*a^4*b^5)*e^(2*d*x + 2*c))*d) + 1/256*(3*a^6 - 12*a^5*b - 20
4*a^4*b^2 - 384*a^3*b^3 - 192*a^2*b^4 + (9*a^6 - 14*a^5*b - 864*a^4*b^2 - 3
136*a^3*b^3 - 3840*a^2*b^4 - 1536*a*b^5)*e^(-2*d*x - 2*c) + (9*a^6 - 12*a^5
*b - 1100*a^4*b^2 - 5248*a^3*b^3 - 10304*a^2*b^4 - 9216*a*b^5 - 3072*b^6)*e
^(-4*d*x - 4*c) + (3*a^6 - 10*a^5*b - 560*a^4*b^2 - 2080*a^3*b^3 - 2560*a^2
*b^4 - 1024*a*b^5)*e^(-6*d*x - 6*c))/((a^7*b^2 + 2*a^6*b^3 + a^5*b^4 + 4*(a
^7*b^2 + 4*a^6*b^3 + 5*a^5*b^4 + 2*a^4*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^7*b^2
 + 14*a^6*b^3 + 27*a^5*b^4 + 24*a^4*b^5 + 8*a^3*b^6)*e^(-4*d*x - 4*c) + 4*(
a^7*b^2 + 4*a^6*b^3 + 5*a^5*b^4 + 2*a^4*b^5)*e^(-6*d*x - 6*c) + (a^7*b^2 +
2*a^6*b^3 + a^5*b^4)*e^(-8*d*x - 8*c))*d) - 3/128*(3*a^5 - 2*a^4*b - 24*a^3
*b^2 - 16*a^2*b^3 + (3*a^5 - 128*a^3*b^2 - 256*a^2*b^3 - 128*a*b^4)*e^(6*d*
x + 6*c) + (9*a^5 + 18*a^4*b - 128*a^3*b^2 - 512*a^2*b^3 - 640*a*b^4 - 256*
b^5)*e^(4*d*x + 4*c) + (9*a^5 + 16*a^4*b - 112*a^3*b^2 - 256*a^2*b^3 - 128*
a*b^4)*e^(2*d*x + 2*c))/((a^6*b^2 + 2*a^5*b^3 + a^4*b^4 + (a^6*b^2 + 2*a^5*
b^3 + a^4*b^4)*e^(8*d*x + 8*c) + 4*(a^6*b^2 + 4*a^5*b^3 + 5*a^4*b^4 + 2*a^3
*b^5)*e^(6*d*x + 6*c) + 2*(3*a^6*b^2 + 14*a^5*b^3 + 27*a^4*b^4 + 24*a^3*b^5
 + 8*a^2*b^6)*e^(4*d*x + 4*c) + 4*(a^6*b^2 + 4*a^5*b^3 + 5*a^4*b^4 + 2*a^3*
b^5)*e^(2*d*x + 2*c))*d) + 3/128*(3*a^5 - 2*a^4*b - 24*a^3*b^2 - 16*a^2*b^3
 + (9*a^5 + 16*a^4*b - 112*a^3*b^2 - 256*a^2*b^3 - 128*a*b^4)*e^(-2*d*x - 2
*c) + (9*a^5 + 18*a^4*b - 128*a^3*b^2 - 512*a^2*b^3 - 640*a*b^4 - 256*b^5)*
e^(-4*d*x - 4*c) + (3*a^5 - 128*a^3*b^2 - 256*a^2*b^3 - 128*a*b^4)*e^(-6*d*
x - 6*c))/((a^6*b^2 + 2*a^5*b^3 + a^4*b^4 + 4*(a^6*b^2 + 4*a^5*b^3 + 5*a^4*

```
b^4 + 2*a^3*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^6*b^2 + 14*a^5*b^3 + 27*a^4*b^4
+ 24*a^3*b^5 + 8*a^2*b^6)*e^(-4*d*x - 4*c) + 4*(a^6*b^2 + 4*a^5*b^3 + 5*a^4
*b^4 + 2*a^3*b^5)*e^(-6*d*x - 6*c) + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*e^(-8*
d*x - 8*c))*d) - 15/256*(3*a^4 + 4*a^3*b + 4*a^2*b^2 + 3*(a^4 + 2*a^3*b)*e^
(6*d*x + 6*c) + (9*a^4 + 36*a^3*b + 100*a^2*b^2 + 128*a*b^3 + 64*b^4)*e^(4*
d*x + 4*c) + (9*a^4 + 34*a^3*b + 48*a^2*b^2 + 32*a*b^3)*e^(2*d*x + 2*c))/((
a^5*b^2 + 2*a^4*b^3 + a^3*b^4 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*e^(8*d*x +
8*c) + 4*(a^5*b^2 + 4*a^4*b^3 + 5*a^3*b^4 + 2*a^2*b^5)*e^(6*d*x + 6*c) + 2*
(3*a^5*b^2 + 14*a^4*b^3 + 27*a^3*b^4 + 24*a^2*b^5 + 8*a*b^6)*e^(4*d*x + 4*c
) + 4*(a^5*b^2 + 4*a^4*b^3 + 5*a^3*b^4 + 2*a^2*b^5)*e^(2*d*x + 2*c))*d) + 1
5/256*(3*a^4 + 4*a^3*b + 4*a^2*b^2 + (9*a^4 + 34*a^3*b + 48*a^2*b^2 + 32*a*
b^3)*e^(-2*d*x - 2*c) + (9*a^4 + 36*a^3*b + 100*a^2*b^2 + 128*a*b^3 + 64*b^
4)*e^(-4*d*x - 4*c) + 3*(a^4 + 2*a^3*b)*e^(-6*d*x - 6*c))/((a^5*b^2 + 2*a^4
*b^3 + a^3*b^4 + 4*(a^5*b^2 + 4*a^4*b^3 + 5*a^3*b^4 + 2*a^2*b^5)*e^(-2*d*x
- 2*c) + 2*(3*a^5*b^2 + 14*a^4*b^3 + 27*a^3*b^4 + 24*a^2*b^5 + 8*a*b^6)*e^(
-4*d*x - 4*c) + 4*(a^5*b^2 + 4*a^4*b^3 + 5*a^3*b^4 + 2*a^2*b^5)*e^(-6*d*x -
 6*c) + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*e^(-8*d*x - 8*c))*d) + 5/64*(3*a^3
+ 6*a^2*b + (9*a^3 + 40*a^2*b + 40*a*b^2)*e^(-2*d*x - 2*c) + 3*(3*a^3 + 14*
a^2*b + 24*a*b^2 + 16*b^3)*e^(-4*d*x - 4*c) + (...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2604 vs. $2(140) = 280$.
time = 0.51, size = 5463, normalized size = 36.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*a^2*b^2*d*x*cosh(d*x + c)^8 + 128*a^2*b^2*d*x*cosh(d*x + c)*sinh(
d*x + c)^7 + 16*a^2*b^2*d*x*sinh(d*x + c)^8 - 4*(3*a^4 - a^3*b - 20*a^2*b^2
 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^6 + 4*(112*a^2*b^2*
d*x*cosh(d*x + c)^2 - 3*a^4 + a^3*b + 20*a^2*b^2 + 16*a*b^3 + 16*(a^2*b^2 +
 2*a*b^3)*d*x)*sinh(d*x + c)^6 + 16*a^2*b^2*d*x + 8*(112*a^2*b^2*d*x*cosh(d
*x + c)^3 - 3*(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^
3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(9*a^4 + 15*a^3*b - 18*a^2*b^2 -
 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^4 +
 4*(280*a^2*b^2*d*x*cosh(d*x + c)^4 - 9*a^4 - 15*a^3*b + 18*a^2*b^2 + 72*a*
b^3 + 48*b^4 + 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x - 15*(3*a^4 - a^3*b - 20
*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 - 12*a^4 + 12*a^3*b + 24*a^2*b^2 + 16*(56*a^2*b^2*d*x*cosh(d*x + c)
^5 - 5*(3*a^4 - a^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)
*cosh(d*x + c)^3 - (9*a^4 + 15*a^3*b - 18*a^2*b^2 - 72*a*b^3 - 48*b^4 - 8*(
3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(9*a^4
 + 13*a^3*b - 28*a^2*b^2 - 32*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x
```

```
+ c)^2 + 4*(112*a^2*b^2*d*x*cosh(d*x + c)^6 - 15*(3*a^4 - a^3*b - 20*a^2*b^
2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^4 - 9*a^4 - 13*a^3
*b + 28*a^2*b^2 + 32*a*b^3 + 16*(a^2*b^2 + 2*a*b^3)*d*x - 6*(9*a^4 + 15*a^3
*b - 18*a^2*b^2 - 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*d*x)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x
+ c)^8 + 8*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3
*a^4 - 4*a^3*b + 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(3*a^4 + 2*a^3*b + 16*a*b^3
)*cosh(d*x + c)^6 + 4*(3*a^4 + 2*a^3*b + 16*a*b^3 + 7*(3*a^4 - 4*a^3*b + 8*
a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 - 4*a^3*b + 8*a^2*b
^2)*cosh(d*x + c)^3 + 3*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c))*sinh(d*
x + c)^5 + 2*(9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cosh(d*x +
 c)^4 + 2*(35*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 9*a^4 + 12*a^
3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4 + 30*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 - 4*a^3*b + 8*a^2*b^2 + 8*(7*(3*a^4 -
4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh
(d*x + c)^3 + (9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 4*(3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c)^2 + 4*
(7*(3*a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(3*a^4 + 2*a^3*b + 16
*a*b^3)*cosh(d*x + c)^4 + 3*a^4 + 2*a^3*b + 16*a*b^3 + 3*(9*a^4 + 12*a^3*b
+ 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*
a^4 - 4*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(3*a^4 + 2*a^3*b + 16*a*b^3)
*cosh(d*x + c)^5 + (9*a^4 + 12*a^3*b + 16*a^2*b^2 + 32*a*b^3 + 64*b^4)*cosh
(d*x + c)^3 + (3*a^4 + 2*a^3*b + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sq
rt((a + b)/b)*log((a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^
3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d
*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cos
h(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x
 + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b + 2
*b^2)*sqrt((a + b)/b))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)
^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)
^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x +
 c))*sinh(d*x + c) + a)) + 8*(16*a^2*b^2*d*x*cosh(d*x + c)^7 - 3*(3*a^4 - a
^3*b - 20*a^2*b^2 - 16*a*b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^5
- 2*(9*a^4 + 15*a^3*b - 18*a^2*b^2 - 72*a*b^3 - 48*b^4 - 8*(3*a^2*b^2 + 8*a
*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^3 - (9*a^4 + 13*a^3*b - 28*a^2*b^2 - 32*a*
b^3 - 16*(a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(a^5*b^2*d*
cosh(d*x + c)^8 + 8*a^5*b^2*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*b^2*d*sin
h(d*x + c)^8 + a^5*b^2*d + 4*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^6 + 4*(7
*a^5*b^2*d*cosh(d*x + c)^2 + (a^5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^6 + 2*(
3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^4 + 8*(7*a^5*b^2*d*cosh(
d*x + c)^3 + 3*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(
35*a^5*b^2*d*cosh(d*x + c)^4 + 30*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^2 +
 (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(a^5*b^2 + 2*a^
4*b^3)*d*cosh(d*x + c)^2 + 8*(7*a^5*b^2*d*cosh(d*x + c)^5 + 10*(a^5*b^2 + 2
*a^4*b^3)*d*cosh(d*x + c)^3 + (3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*
```

```
x + c))*sinh(d*x + c)^3 + 4*(7*a^5*b^2*d*cosh(d*x + c)^6 + 15*(a^5*b^2 + 2*
a^4*b^3)*d*cosh(d*x + c)^4 + 3*(3*a^5*b^2 + 8*a^4*b^3 + 8*a^3*b^4)*d*cosh(d
*x + c)^2 + (a^5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^2 + 8*(a^5*b^2*d*cosh(d*
x + c)^7 + 3*(a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c)^5 + (3*a^5*b^2 + 8*a^4*b
^3 + 8*a^3*b^4)*d*cosh(d*x + c)^3 + (a^5*b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*
sinh(d*x + c)), 1/8*(8*a^2*b^2*d*x*cosh(d*x + c...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**6/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(tanh(c + d*x)**6/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(140) = 280.
time = 3.26, size = 354, normalized size = 2.39

$$\frac{\frac{8\,(dx+c)}{a^3}-\frac{(3\,a^3-a^2b+4\,ab^2+8\,b^3)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^3b^2}-\frac{2\,(3\,a^4e^{(6\,dx+6\,c)}-a^3be^{(6\,dx+6\,c)}-20\,a^2b^2e^{(6\,dx+6\,c)}-16\,ab^3e^{(6\,dx+6\,c)}+9\,a^4e^{(4\,dx+4\,c)}+15\,a^3be^{(4\,dx+4\,c)}-18\,a^2b^2e^{(4\,dx+4\,c)}-72\,ab^3e^{(4\,dx+4\,c)}-48\,b^4e^{(4\,dx+4\,c)}+9\,a^4e^{(2\,dx+2\,c)}+13\,a^3be^{(2\,dx+2\,c)}-28\,a^2b^2e^{(2\,dx+2\,c)}-32\,ab^3e^{(2\,dx+2\,c)}+3\,a^4-3\,a^3b-6\,a^2b^2)}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)^2a^3b^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^6/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*(d*x + c)/a^3 - (3*a^3 - a^2*b + 4*a*b^2 + 8*b^3)*arctan(1/2*(a*e^(2
*d*x + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a^3*b^2) - 2*(3*
a^4*e^(6*d*x + 6*c) - a^3*b*e^(6*d*x + 6*c) - 20*a^2*b^2*e^(6*d*x + 6*c) -
16*a*b^3*e^(6*d*x + 6*c) + 9*a^4*e^(4*d*x + 4*c) + 15*a^3*b*e^(4*d*x + 4*c)
 - 18*a^2*b^2*e^(4*d*x + 4*c) - 72*a*b^3*e^(4*d*x + 4*c) - 48*b^4*e^(4*d*x
+ 4*c) + 9*a^4*e^(2*d*x + 2*c) + 13*a^3*b*e^(2*d*x + 2*c) - 28*a^2*b^2*e^(2
*d*x + 2*c) - 32*a*b^3*e^(2*d*x + 2*c) + 3*a^4 - 3*a^3*b - 6*a^2*b^2)/((a*e
^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2*a^3*b^2))
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\cosh(c+d\,x)^2-1\right)^3}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^6/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^2 - 1)^3/(b + a*cosh(c + d*x)^2)^3, x)
```

3.159 $\int \frac{\tanh^5(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3} \, dx$

Optimal. Leaf size=77

$$-\frac{(a+b)^2}{4a^3d\left(b+a\cosh^2(c+dx)\right)^2}+\frac{a+b}{a^3d\left(b+a\cosh^2(c+dx)\right)}+\frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^3d}$$

```
[Out] -1/4*(a+b)^2/a^3/d/(b+a*cosh(d*x+c)^2)^2+(a+b)/a^3/d/(b+a*cosh(d*x+c)^2)+1/
2*ln(b+a*cosh(d*x+c)^2)/a^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 455, 45}

$$-\frac{(a+b)^2}{4a^3d\left(a\cosh^2(c+dx)+b\right)^2}+\frac{a+b}{a^3d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/4*(a + b)^2/(a^3*d*(b + a*Cosh[c + d*x]^2)^2) + (a + b)/(a^3*d*(b + a*Co
sh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^5(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x(1-x^2)^2}{(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{(1-x)^2}{(b+ax)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{(a+b)^2}{a^2(b+ax)^3}-\frac{2(a+b)}{a^2(b+ax)^2}+\frac{1}{a^2(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{(a+b)^2}{4a^3d\left(b+a\cosh^2(c+dx)\right)^2}+\frac{a+b}{a^3d\left(b+a\cosh^2(c+dx)\right)}+\frac{\log\left(b+a\cosh^2(\right.}{2a^3d}
\end{aligned}
$$

Mathematica [A]
time = 1.41, size = 136, normalized size = 1.77

$$
\frac{2(a^2+4ab+3b^2)+(a+2b)^2\log(a+2b+a\cosh(2(c+dx)))+a^2\cosh^2(2(c+dx))\log(a+2b+a\cosh(2(c+dx)))+2a\cosh(2(c+dx))(2(a+b)+(a+2b)\log(a+2b+a\cosh(2(c+dx))))}{2a^3d(a+2b+a\cosh(2(c+dx)))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (2*(a^2 + 4*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] +
 a^2*Cosh[2*(c + d*x)]^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + 2*a*Cosh[2*(c
 + d*x)]*(2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]]))/(2*a^3
*d*(a + 2*b + a*Cosh[2*(c + d*x)])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(73) = 146$.
time = 2.52, size = 226, normalized size = 2.94

method	result
risch	$-\frac{x}{a^3}-\frac{2c}{a^3d}+\frac{4\,e^{2dx+2c}\left(a^2e^{4dx+4c}+ab\,e^{4dx+4c}+a^2e^{2dx+2c}+4ab\,e^{2dx+2c}+3b^2e^{2dx+2c}+a^2+ab\right)}{a^3d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)^2}+\frac{\ln\left(e^{4dx+4c}+\frac{2(2\right.}{2a}$
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3}+\frac{\frac{\left(-2a^2-2ab\right)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4(-b+2a)a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)(a+b)}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{\ln\left(a\left(\tanh^4\left(\right.\right.\right.}{}}{a^3}}{d}$

default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3}+\frac{\frac{\left(-2a^2-2ab\right)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4(-b+2a)a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)(a+b)}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{\ln\left(a\left(\tanh\right.\right.}{}}{a^3}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^3*ln(tanh(1/2*d*x+1/2*c)+1)+1/a^3*(((-2*a^2-2*a*b)*tanh(1/2*d*x+1
/2*c)^6-4*(-b+2*a)*a*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*c)^2*(a+b))
/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2
-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)^2+1/2*ln(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2
*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b))-1/a
^3*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(73) = 146$.
time = 0.28, size = 206, normalized size = 2.68

$$\frac{4\left((a^2+ab)e^{(-2\,dx-2\,c)}+(a^2+4\,ab+3\,b^2)e^{(-4\,dx-4\,c)}+(a^2+ab)e^{(-6\,dx-6\,c)}\right)}{\left(a^5e^{(-8\,dx-8\,c)}+a^5+4\,(a^5+2\,a^4b)e^{(-2\,dx-2\,c)}+2\,(3\,a^5+8\,a^4b+8\,a^3b^2)e^{(-4\,dx-4\,c)}+4\,(a^5+2\,a^4b)e^{(-6\,dx-6\,c)}\right)d}+\frac{dx+c}{a^3d}+\frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 4*((a^2 + a*b)*e^(-2*d*x - 2*c) + (a^2 + 4*a*b + 3*b^2)*e^(-4*d*x - 4*c) +
(a^2 + a*b)*e^(-6*d*x - 6*c))/((a^5*e^(-8*d*x - 8*c) + a^5 + 4*(a^5 + 2*a^4
*b)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*e^(-4*d*x - 4*c) + 4
*(a^5 + 2*a^4*b)*e^(-6*d*x - 6*c))*d) + (d*x + c)/(a^3*d) + 1/2*log(2*(a +
2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1741 vs. $2(73) = 146$.
time = 0.42, size = 1741, normalized size = 22.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^8 + 16*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7
+ 2*a^2*d*x*sinh(d*x + c)^8 + 8*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x +
c)^6 + 8*(7*a^2*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b)*d*x - a^2 - a*b)*sinh(d
*x + c)^6 + 16*(7*a^2*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b)*d*x - a^2 - a*
b)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2
- 8*a*b - 6*b^2)*cosh(d*x + c)^4 + 4*(35*a^2*d*x*cosh(d*x + c)^4 + (3*a^2 +
 8*a*b + 8*b^2)*d*x + 30*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^2 -
```

```
2*a^2 - 8*a*b - 6*b^2)*sinh(d*x + c)^4 + 2*a^2*d*x + 16*(7*a^2*d*x*cosh(d*x
 + c)^5 + 10*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^3 + ((3*a^2 + 8*
a*b + 8*b^2)*d*x - 2*a^2 - 8*a*b - 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
8*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 8*(7*a^2*d*x*cosh(d*x +
 c)^6 + 15*((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)^4 + (a^2 + 2*a*b)*
d*x + 3*((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 8*a*b - 6*b^2)*cosh(d*x + c)
^2 - a^2 - a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c
)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^6 +
 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*
x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 + 8*a*
b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b)*c
osh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x
 + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d
*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(
d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*c
osh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^
7 + 3*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)
^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 +
 a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x +
 c) + sinh(d*x + c)^2)) + 16*(a^2*d*x*cosh(d*x + c)^7 + 3*((a^2 + 2*a*b)*d*
x - a^2 - a*b)*cosh(d*x + c)^5 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 8*a
*b - 6*b^2)*cosh(d*x + c)^3 + ((a^2 + 2*a*b)*d*x - a^2 - a*b)*cosh(d*x + c)
)*sinh(d*x + c))/(a^5*d*cosh(d*x + c)^8 + 8*a^5*d*cosh(d*x + c)*sinh(d*x +
c)^7 + a^5*d*sinh(d*x + c)^8 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + 4*(7*a
^5*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^6 + a^5*d + 2*(3*a^
5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^4 + 8*(7*a^5*d*cosh(d*x + c)^3 + 3
*(a^5 + 2*a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^5*d*cosh(d*x +
c)^4 + 30*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)
*d)*sinh(d*x + c)^4 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + 8*(7*a^5*d*cosh
(d*x + c)^5 + 10*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a
^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^5*d*cosh(d*x + c)^6 + 15*
(a^5 + 2*a^4*b)*d*cosh(d*x + c)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(
d*x + c)^2 + (a^5 + 2*a^4*b)*d)*sinh(d*x + c)^2 + 8*(a^5*d*cosh(d*x + c)^7
+ 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cos
h(d*x + c)^3 + (a^5 + 2*a^4*b)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^6\,\tanh(c+d\,x)^5}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*tanh(c + d*x)^5)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.160 $\int \frac{\tanh^4(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=139

$$\frac{x}{a^3}+\frac{(a^2-4ab-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}\sqrt{a+b}\,d}-\frac{(a+b)\tanh(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2}+\frac{(a-4b)\tanh(c+dx)}{8a^2bd\left(a+b-b\tanh^2(c+\right.}$$

```
[Out] x/a^3+1/8*(a^2-4*a*b-8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^3/b^
(3/2)/d/(a+b)^(1/2)-1/4*(a+b)*tanh(d*x+c)/a/b/d/(a+b-b*tanh(d*x+c)^2)^2+1/8
*(a-4*b)*tanh(d*x+c)/a^2/b/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]

time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4226, 2000, 481, 541, 536, 212, 214}

$$\frac{x}{a^3}+\frac{(a-4b)\tanh(c+dx)}{8a^2bd\left(a-b\tanh^2(c+dx)+b\right)}+\frac{(a^2-4ab-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}d\sqrt{a+b}}-\frac{(a+b)\tanh(c+dx)}{4abd\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] x/a^3 + ((a^2 - 4*a*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]]
)/(8*a^3*b^(3/2)*Sqrt[a + b]*d) - ((a + b)*Tanh[c + d*x])/(4*a*b*d*(a + b -
 b*Tanh[c + d*x]^2)^2) + ((a - 4*b)*Tanh[c + d*x])/(8*a^2*b*d*(a + b - b*Ta
nh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
```

```
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+b(1-x^2))^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\tanh(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+b+(-a+3b)x^2}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+\right.}{4abd}$$

$$= -\frac{(a+b)\tanh(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(a-4b)\tanh(c+dx)}{8a^2bd\left(a+b-b\tanh^2(c+dx)\right)} - \frac{\text{Subs}}{}$$

$$= -\frac{(a+b)\tanh(c+dx)}{4abd\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{(a-4b)\tanh(c+dx)}{8a^2bd\left(a+b-b\tanh^2(c+dx)\right)} + \frac{\text{Subs}}{}$$

$$= \frac{x}{a^3} + \frac{(a^2-4ab-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}\sqrt{a+b}\,d} - \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b-b\tanh^2(c+\right.}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1317 vs. $2(139) = 278$.

time = 9.75, size = 1317, normalized size = 9.47

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((6*a*(a + 2*b)*ArcTanh[
(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) + (4*(3*a^2 + 8*a*b + 8
*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (4*a*Sq
rt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*(c
 + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) - (2*Sqrt[b]*(3*a^3
 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cosh[2*(c + d*x
)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) + (Sqr
t[b]*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^
5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh
[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - S
inh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (Sech[2*c]*(256*b^
2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*Cosh[2*c] + 512*a*b^2*(a + b)^2*(a
+ 2*b)*d*x*Cosh[2*d*x] + 128*a^4*b^2*d*x*Cosh[2*(c + 2*d*x)] + 256*a^3*b^3*
d*x*Cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*x*Cosh[2*(c + 2*d*x)] + 512*a^4*b^2
```

```
*d*x*Cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*x*Cosh[4*c + 2*d*x] + 2560*a^2*b^4*
d*x*Cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*Cosh[4*c + 2*d*x] + 128*a^4*b^2*d*x*
Cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*Cosh[6*c + 4*d*x] + 128*a^2*b^4*d*x*Cos
h[6*c + 4*d*x] - 9*a^6*Sinh[2*c] + 12*a^5*b*Sinh[2*c] + 684*a^4*b^2*Sinh[2*
c] + 2880*a^3*b^3*Sinh[2*c] + 5280*a^2*b^4*Sinh[2*c] + 4608*a*b^5*Sinh[2*c]
 + 1536*b^6*Sinh[2*c] + 9*a^6*Sinh[2*d*x] - 14*a^5*b*Sinh[2*d*x] - 608*a^4*
b^2*Sinh[2*d*x] - 2112*a^3*b^3*Sinh[2*d*x] - 2560*a^2*b^4*Sinh[2*d*x] - 102
4*a*b^5*Sinh[2*d*x] + 3*a^6*Sinh[2*(c + 2*d*x)] - 12*a^5*b*Sinh[2*(c + 2*d*
x)] - 204*a^4*b^2*Sinh[2*(c + 2*d*x)] - 384*a^3*b^3*Sinh[2*(c + 2*d*x)] - 1
92*a^2*b^4*Sinh[2*(c + 2*d*x)] - 3*a^6*Sinh[4*c + 2*d*x] + 10*a^5*b*Sinh[4*
c + 2*d*x] + 304*a^4*b^2*Sinh[4*c + 2*d*x] + 1056*a^3*b^3*Sinh[4*c + 2*d*x]
 + 1280*a^2*b^4*Sinh[4*c + 2*d*x] + 512*a*b^5*Sinh[4*c + 2*d*x]))/(a + 2*b
+ a*Cosh[2*(c + d*x)])^2))/(a^3*(a + b)^2) - (2*Sqrt[b]*((6*a^2*ArcTanh[(Se
ch[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/
(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sq
rt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (a*Sech[2*c]*((-9*a^4 - 16*a^3*b
 + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sinh[2*d*x] + a*(-3*a^3 + 2*a^2*b + 24*
a*b^2 + 16*b^3)*Sinh[2*(c + 2*d*x)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*
b^4)*Sinh[4*c + 2*d*x]) + (9*a^5 + 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 32
0*a*b^4 - 128*b^5)*Tanh[2*c])/(a^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(a
+ b)^2))/(4096*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(125) = 250$.

time = 2.71, size = 343, normalized size = 2.47

method	result
derivativedivides	$\frac{2\left(-\frac{a\left(a^2+5ab+4b^2\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8b}-\frac{\left(3a^2+19ab-4b^2\right)a\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8b}-\frac{\left(3a^2+19ab-4b^2\right)a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8b}-\frac{a\left(a^2+5ab+\right.}{}\right.}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}$
default	$\frac{2\left(-\frac{a\left(a^2+5ab+4b^2\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8b}-\frac{\left(3a^2+19ab-4b^2\right)a\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8b}-\frac{\left(3a^2+19ab-4b^2\right)a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8b}-\frac{a\left(a^2+5ab+\right.}{}\right.}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}$
risch	$\frac{x}{a^3}+\frac{a^3e^{6dx+6c}+12a^2be^{6dx+6c}+16ab^2e^{6dx+6c}+3a^3e^{4dx+4c}+26a^2be^{4dx+4c}+56ab^2e^{4dx+4c}+48b^3e^{4dx+4c}+3a^3e^{2dx-}}{4a^3d\left(ae^{4dx+4c}+2ae^{2dx+2c}+4be^{2dx+2c}+a\right)^2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^3*((-1/8*a*(a^2+5*a*b+4*b^2)/b*tanh(1/2*d*x+1/2*c)^7-1/8*(3*a^2+19
*a*b-4*b^2)*a/b*tanh(1/2*d*x+1/2*c)^5-1/8*(3*a^2+19*a*b-4*b^2)*a/b*tanh(1/2
```

```
*d*x+1/2*c)^3-1/8*a*(a^2+5*a*b+4*b^2)/b*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*
x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d
*x+1/2*c)^2+a+b)^2+1/8*(a^2-4*a*b-8*b^2)/b*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+
b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1
/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+
1/2*c)*b^(1/2)+(a+b)^(1/2))))-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/a^3*ln(tanh
(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2201 vs. $2(131) = 262$.
time = 0.82, size = 2201, normalized size = 15.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/256*(a^4 - 20*a^3*b - 120*a^2*b^2 - 160*a*b^3 - 64*b^4)*log((a*e^(2*d*x +
 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt(
(a + b)*b)))/((a^5*b + 2*a^4*b^2 + a^3*b^3)*sqrt((a + b)*b)*d) + 1/64*(a -
2*b)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*
c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b + 2*a*b^2 + b^3)*sqrt((a + b)*b)
*d) - 1/256*(a^4 - 20*a^3*b - 120*a^2*b^2 - 160*a*b^3 - 64*b^4)*log((a*e^(-
2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b +
 2*sqrt((a + b)*b)))/((a^5*b + 2*a^4*b^2 + a^3*b^3)*sqrt((a + b)*b)*d) - 3/
128*(a + 4*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(
-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b + 2*a*b^2 + b^3)*sqrt
((a + b)*b)*d) - 1/64*(a - 2*b)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt(
(a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2*b + 2
*a*b^2 + b^3)*sqrt((a + b)*b)*d) + 1/64*(a^5 + 38*a^4*b + 88*a^3*b^2 + 48*a
^2*b^3 + (a^5 + 76*a^4*b + 392*a^3*b^2 + 576*a^2*b^3 + 256*a*b^4)*e^(6*d*x
+ 6*c) + (3*a^5 + 186*a^4*b + 1024*a^3*b^2 + 2240*a^2*b^3 + 2176*a*b^4 + 76
8*b^5)*e^(4*d*x + 4*c) + (3*a^5 + 148*a^4*b + 648*a^3*b^2 + 896*a^2*b^3 + 3
84*a*b^4)*e^(2*d*x + 2*c))/((a^7*b + 2*a^6*b^2 + a^5*b^3 + (a^7*b + 2*a^6*b
^2 + a^5*b^3)*e^(8*d*x + 8*c) + 4*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^
4)*e^(6*d*x + 6*c) + 2*(3*a^7*b + 14*a^6*b^2 + 27*a^5*b^3 + 24*a^4*b^4 + 8*
a^3*b^5)*e^(4*d*x + 4*c) + 4*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*e^
(2*d*x + 2*c))*d) - 1/64*(a^5 + 38*a^4*b + 88*a^3*b^2 + 48*a^2*b^3 + (3*a^5
 + 148*a^4*b + 648*a^3*b^2 + 896*a^2*b^3 + 384*a*b^4)*e^(-2*d*x - 2*c) + (3
*a^5 + 186*a^4*b + 1024*a^3*b^2 + 2240*a^2*b^3 + 2176*a*b^4 + 768*b^5)*e^(-
4*d*x - 4*c) + (a^5 + 76*a^4*b + 392*a^3*b^2 + 576*a^2*b^3 + 256*a*b^4)*e^(
-6*d*x - 6*c))/((a^7*b + 2*a^6*b^2 + a^5*b^3 + 4*(a^7*b + 4*a^6*b^2 + 5*a^5
*b^3 + 2*a^4*b^4)*e^(-2*d*x - 2*c) + 2*(3*a^7*b + 14*a^6*b^2 + 27*a^5*b^3 +
 24*a^4*b^4 + 8*a^3*b^5)*e^(-4*d*x - 4*c) + 4*(a^7*b + 4*a^6*b^2 + 5*a^5*b^
3 + 2*a^4*b^4)*e^(-6*d*x - 6*c) + (a^7*b + 2*a^6*b^2 + a^5*b^3)*e^(-8*d*x -
```

```
 8*c))*d) + 1/16*(a^4 + 8*a^3*b + 4*a^2*b^2 + (a^4 + 30*a^3*b + 64*a^2*b^2
+ 32*a*b^3)*e^(6*d*x + 6*c) + (3*a^4 + 64*a^3*b + 180*a^2*b^2 + 192*a*b^3 +
 64*b^4)*e^(4*d*x + 4*c) + (3*a^4 + 42*a^3*b + 80*a^2*b^2 + 32*a*b^3)*e^(2*
d*x + 2*c))/((a^6*b + 2*a^5*b^2 + a^4*b^3 + (a^6*b + 2*a^5*b^2 + a^4*b^3)*e
^(8*d*x + 8*c) + 4*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 + 2*a^3*b^4)*e^(6*d*x + 6
*c) + 2*(3*a^6*b + 14*a^5*b^2 + 27*a^4*b^3 + 24*a^3*b^4 + 8*a^2*b^5)*e^(4*d
*x + 4*c) + 4*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 + 2*a^3*b^4)*e^(2*d*x + 2*c))*
d) - 1/16*(a^4 + 8*a^3*b + 4*a^2*b^2 + (3*a^4 + 42*a^3*b + 80*a^2*b^2 + 32*
a*b^3)*e^(-2*d*x - 2*c) + (3*a^4 + 64*a^3*b + 180*a^2*b^2 + 192*a*b^3 + 64*
b^4)*e^(-4*d*x - 4*c) + (a^4 + 30*a^3*b + 64*a^2*b^2 + 32*a*b^3)*e^(-6*d*x
- 6*c))/((a^6*b + 2*a^5*b^2 + a^4*b^3 + 4*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 +
2*a^3*b^4)*e^(-2*d*x - 2*c) + 2*(3*a^6*b + 14*a^5*b^2 + 27*a^4*b^3 + 24*a^3
*b^4 + 8*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 + 2*a
^3*b^4)*e^(-6*d*x - 6*c) + (a^6*b + 2*a^5*b^2 + a^4*b^3)*e^(-8*d*x - 8*c))*
d) - 3/32*(a^3 - 2*a^2*b + (3*a^3 - 4*a^2*b - 16*a*b^2)*e^(-2*d*x - 2*c) +
(3*a^3 + 2*a^2*b - 8*a*b^2 - 16*b^3)*e^(-4*d*x - 4*c) + (a^3 + 4*a^2*b)*e^(
-6*d*x - 6*c))/((a^5*b + 2*a^4*b^2 + a^3*b^3 + 4*(a^5*b + 4*a^4*b^2 + 5*a^3
*b^3 + 2*a^2*b^4)*e^(-2*d*x - 2*c) + 2*(3*a^5*b + 14*a^4*b^2 + 27*a^3*b^3 +
 24*a^2*b^4 + 8*a*b^5)*e^(-4*d*x - 4*c) + 4*(a^5*b + 4*a^4*b^2 + 5*a^3*b^3
+ 2*a^2*b^4)*e^(-6*d*x - 6*c) + (a^5*b + 2*a^4*b^2 + a^3*b^3)*e^(-8*d*x - 8
*c))*d) + 1/4*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^3
*d) - 1/4*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3111 vs. $2(131) = 262$.
time = 0.46, size = 6464, normalized size = 46.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 128*(a^3*b^2 + a^2*b^3)
*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^3*b^2 + a^2*b^3)*d*x*sinh(d*x +
c)^8 + 4*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*
b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^6 + 4*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 +
16*a*b^4 + 112*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 + 16*(a^3*b^2 + 3*a^
2*b^3 + 2*a*b^4)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^3*b^2 + a^2*b^3)*d*x*cosh
(d*x + c)^3 + 3*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 +
 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*a^4*b + 28*a^
3*b^2 + 24*a^2*b^3 + 4*(3*a^4*b + 29*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*
b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 +
4*(280*(a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 + 3*a^4*b + 29*a^3*b^2 + 82*
a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 + 8*b^5
```

)*d*x + 15*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^
2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(56*(a^3*b^2 +
a^2*b^3)*d*x*cosh(d*x + c)^5 + 5*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^
4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^3 + (3*a^4*b + 29
*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16
*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^3*b^2 + a^2*b^3
)*d*x + 4*(3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^4 + 16*(a^3*b^2 + 3*a
^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^3*b^2 + a^2*b^3)*d*x*cos
h(d*x + c)^6 + 3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^4 + 15*(a^4*b + 1
3*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)
*cosh(d*x + c)^4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x + 6*(3*a^4*b + 29
*a^3*b^2 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16
*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((a^4 - 4*a^3*b - 8
*a^2*b^2)*cosh(d*x + c)^8 + 8*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)*sin
h(d*x + c)^7 + (a^4 - 4*a^3*b - 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(a^4 - 2*a^3
*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^6 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2
 - 16*a*b^3 + 7*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^
6 + 8*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(a^4 - 2*a^3*b - 1
6*a^2*b^2 - 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 - 4*a^3*b -
 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*cosh(d*x + c)^4 + 2*(35*(a^4 - 4*a^3*b - 8
*a^2*b^2)*cosh(d*x + c)^4 + 3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^
4 + 30*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + a^4 - 4*a^3*b - 8*a^2*b^2 + 8*(7*(a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*
x + c)^5 + 10*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^3 + (3*
a^4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(a^4
 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(a^4 - 2*a^3*b - 16*a^2*b^2 -
16*a*b^3)*cosh(d*x + c)^4 + a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3 + 3*(3*a^
4 - 4*a^3*b - 48*a^2*b^2 - 96*a*b^3 - 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 8*((a^4 - 4*a^3*b - 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(a^4 - 2*a^3*b - 1
6*a^2*b^2 - 16*a*b^3)*cosh(d*x + c)^5 + (3*a^4 - 4*a^3*b - 48*a^2*b^2 - 96*
a*b^3 - 64*b^4)*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - 16*a^2*b^2 - 16*a*b^3)*c
osh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x + c)^4 + 4*a
^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b)*co
sh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a
^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*
sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*
sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4 + 4*a*cosh(d
*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 +
 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 +
 (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(16*(a^3*b^2 + a^2*b^3)*d
*x*cosh(d*x + c)^7 + 3*(a^4*b + 13*a^3*b^2 + 28*a^2*b^3 + 16*a*b^4 + 16*(a^
3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^5 + 2*(3*a^4*b + 29*a^3*b^2
 + 82*a^2*b^3 + 104*a*b^4 + 48*b^5 + 8*(3*a^3*b^2 + 11*a^2*b^3 + 16*a*b^4 +
 8*b^5)*d*x)*cosh(d*x + c)^3 + (3*a^4*b + 23*a^3*b^2 + 52*a^2*b^3 + 32*a*b^

```
4 + 16*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(
(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 + a^5*b^3)*d*cosh(d*x +
c)*sinh(d*x + c)^7 + (a^6*b^2 + a^5*b^3)*d*sinh(d*x + c)^8 + 4*(a^6*b^2 + 3
*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + a^5*b^3)*d*cosh(d
*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d)*sinh(d*x + c)^6 + 2*(3*a^6
*b^2 + 11*a^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b
^2 + a^5*b^3)*d*cosh(d*x + c)^3 + 3*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^4 +
 30*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^2 + (3*a^6*b^2 + 11*a
^5*b^3 + 16*a^4*b^4 + 8*a^3*b^5)*d)*sinh(d*x + ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(131) = 262.
time = 2.30, size = 281, normalized size = 2.02

$$\frac{\frac{8\,(dx+c)}{a^3}+\frac{(a^2-4\,ab-8\,b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^3b}+\frac{2\,(a^3e^{(6\,dx+6\,c)}+12\,a^2be^{(6\,dx+6\,c)}+16\,ab^2e^{(6\,dx+6\,c)}+3\,a^3e^{(4\,dx+4\,c)}+26\,a^2be^{(4\,dx+4\,c)}+56\,ab^2e^{(4\,dx+4\,c)}+48\,b^3e^{(4\,dx+4\,c)}+3\,a^3e^{(2\,dx+2\,c)}+20\,a^2be^{(2\,dx+2\,c)}+32\,ab^2e^{(2\,dx+2\,c)}+a^3+6\,a^2b)}{\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)^2a^3b}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*(d*x + c)/a^3 + (a^2 - 4*a*b - 8*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c)
+ a + 2*b)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a^3*b) + 2*(a^3*e^(6*d*x + 6
*c) + 12*a^2*b*e^(6*d*x + 6*c) + 16*a*b^2*e^(6*d*x + 6*c) + 3*a^3*e^(4*d*x
+ 4*c) + 26*a^2*b*e^(4*d*x + 4*c) + 56*a*b^2*e^(4*d*x + 4*c) + 48*b^3*e^(4*
d*x + 4*c) + 3*a^3*e^(2*d*x + 2*c) + 20*a^2*b*e^(2*d*x + 2*c) + 32*a*b^2*e^
(2*d*x + 2*c) + a^3 + 6*a^2*b)/((a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) +
4*b*e^(2*d*x + 2*c) + a)^2*a^3*b))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^6\,\tanh(c+d\,x)^4}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*tanh(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.161 $\int \frac{\tanh^3(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=81

$$-\frac{b(a+b)}{4a^3d\left(b+a\cosh^2(c+dx)\right)^2}+\frac{a+2b}{2a^3d\left(b+a\cosh^2(c+dx)\right)}+\frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^3d}$$

```
[Out] -1/4*b*(a+b)/a^3/d/(b+a*cosh(d*x+c)^2)^2+1/2*(a+2*b)/a^3/d/(b+a*cosh(d*x+c)
^2)+1/2*ln(b+a*cosh(d*x+c)^2)/a^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4223, 457, 78}

$$-\frac{b(a+b)}{4a^3d\left(a\cosh^2(c+dx)+b\right)^2}+\frac{a+2b}{2a^3d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/4*(b*(a + b))/(a^3*d*(b + a*Cosh[c + d*x]^2)^2) + (a + 2*b)/(2*a^3*d*(b
+ a*Cosh[c + d*x]^2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a^3*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
```

```
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^3(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^3(1-x^2)}{(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \frac{(1-x)x}{(b+ax)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{\text{Subst}\left(\int \left(-\frac{b(a+b)}{a^2(b+ax)^3}+\frac{a+2b}{a^2(b+ax)^2}-\frac{1}{a^2(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d} \\
&= -\frac{b(a+b)}{4a^3d\left(b+a\cosh^2(c+dx)\right)^2}+\frac{a+2b}{2a^3d\left(b+a\cosh^2(c+dx)\right)}+\frac{\log\left(b+a\cosh\right.}{2a^3d}
\end{aligned}
$$

Mathematica [A]
time = 1.01, size = 131, normalized size = 1.62

$$
\frac{2(a^2+3ab+3b^2)+(a+2b)^2\log(a+2b+a\cosh(2(c+dx)))+a^2\cosh^2(2(c+dx))\log(a+2b+a\cosh(2(c+dx)))+2a(a+2b)\cosh(2(c+dx))(1+\log(a+2b+a\cosh(2(c+dx))))}{2a^3d(a+2b+a\cosh(2(c+dx)))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]

[Out] (2*(a^2 + 3*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] +
 a^2*Cosh[2*(c + d*x)]^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + 2*a*(a + 2*b)
*Cosh[2*(c + d*x)]*(1 + Log[a + 2*b + a*Cosh[2*(c + d*x)]]))/(2*a^3*d*(a +
2*b + a*Cosh[2*(c + d*x)])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(75) = 150$.
time = 2.44, size = 236, normalized size = 2.91

method	result
risch	$-\frac{x}{a^3}-\frac{2c}{a^3d}+\frac{2\,e^{2dx+2c}\left(a^2e^{4dx+4c}+2ab\,e^{4dx+4c}+2a^2e^{2dx+2c}+6ab\,e^{2dx+2c}+6b^2e^{2dx+2c}+a^2+2ab\right)}{a^3d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)^2}+\frac{\ln\left(e^{4dx+4c}+\right.}{}$
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{\left(-2a^2-2ab\right)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4a\left(a^2+ab-b^2\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a+b}-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)(a+b)}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2 a^3}+\frac{\ln\left(a\left(\tanh^4\right.\right.}{}}{d}$

default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3}+\frac{\frac{\left(-2a^2-2ab\right)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4a\left(a^2+ab-b^2\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a+b}-2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)(a+b)}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{\ln\left(a\left(\tan\right.\right.}{}}{a^3}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/a^3*(((-2*a^2-2*a*b)*tanh(1/2*d*x+1
/2*c)^6-4*a*(a^2+a*b-b^2)/(a+b)*tanh(1/2*d*x+1/2*c)^4-2*a*tanh(1/2*d*x+1/2*
c)^2*(a+b))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d
*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)^2+1/2*ln(a*tanh(1/2*d*x+1/2*c)^4
+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^
2+a+b))-1/a^3*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(75) = 150$.
time = 0.29, size = 209, normalized size = 2.58

$$\frac{2\left((a^2+2ab)e^{(-2dx-2c)}+2(a^2+3ab+3b^2)e^{(-4dx-4c)}+(a^2+2ab)e^{(-6dx-6c)}\right)}{\left(a^5e^{(-8dx-8c)}+a^5+4(a^5+2a^4b)e^{(-2dx-2c)}+2(3a^5+8a^4b+8a^3b^2)e^{(-4dx-4c)}+4(a^5+2a^4b)e^{(-6dx-6c)}\right)d}+\frac{dx+c}{a^3d}+\frac{\log\left(2(a+2b)e^{(-2dx-2c)}+ae^{(-4dx-4c)}+a\right)}{2a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 2*((a^2 + 2*a*b)*e^(-2*d*x - 2*c) + 2*(a^2 + 3*a*b + 3*b^2)*e^(-4*d*x - 4*c
) + (a^2 + 2*a*b)*e^(-6*d*x - 6*c))/((a^5*e^(-8*d*x - 8*c) + a^5 + 4*(a^5 +
 2*a^4*b)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*e^(-4*d*x - 4*
c) + 4*(a^5 + 2*a^4*b)*e^(-6*d*x - 6*c))*d) + (d*x + c)/(a^3*d) + 1/2*log(2
*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1753 vs. $2(75) = 150$.
time = 0.41, size = 1753, normalized size = 21.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^8 + 16*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7
+ 2*a^2*d*x*sinh(d*x + c)^8 + 4*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*
x + c)^6 + 4*(14*a^2*d*x*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*
b)*sinh(d*x + c)^6 + 8*(14*a^2*d*x*cosh(d*x + c)^3 + 3*(2*(a^2 + 2*a*b)*d*x
 - a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 + 8*a*b + 8*b^2)
*d*x - 2*a^2 - 6*a*b - 6*b^2)*cosh(d*x + c)^4 + 4*(35*a^2*d*x*cosh(d*x + c)
```

```
^4 + (3*a^2 + 8*a*b + 8*b^2)*d*x + 15*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*c
osh(d*x + c)^2 - 2*a^2 - 6*a*b - 6*b^2)*sinh(d*x + c)^4 + 2*a^2*d*x + 16*(7
*a^2*d*x*cosh(d*x + c)^5 + 5*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x +
 c)^3 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 6*a*b - 6*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^3 + 4*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^2 +
 4*(14*a^2*d*x*cosh(d*x + c)^6 + 15*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cos
h(d*x + c)^4 + 2*(a^2 + 2*a*b)*d*x + 6*((3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2
 - 6*a*b - 6*b^2)*cosh(d*x + c)^2 - a^2 - 2*a*b)*sinh(d*x + c)^2 - (a^2*cos
h(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*sinh(d*x + c)^8 +
4*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*s
inh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*
sinh(d*x + c)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cos
h(d*x + c)^4 + 30*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*si
nh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3
 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)
*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x + c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c
)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c
)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 + 3*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a
^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x
 + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x +
c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*(2*a^2*d*x*cos
h(d*x + c)^7 + 3*(2*(a^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c)^5 + 2*((
3*a^2 + 8*a*b + 8*b^2)*d*x - 2*a^2 - 6*a*b - 6*b^2)*cosh(d*x + c)^3 + (2*(a
^2 + 2*a*b)*d*x - a^2 - 2*a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^5*d*cosh(d*
x + c)^8 + 8*a^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*d*sinh(d*x + c)^8 +
4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + 4*(7*a^5*d*cosh(d*x + c)^2 + (a^5 + 2
*a^4*b)*d)*sinh(d*x + c)^6 + a^5*d + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh
(d*x + c)^4 + 8*(7*a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x + c
))*sinh(d*x + c)^5 + 2*(35*a^5*d*cosh(d*x + c)^4 + 30*(a^5 + 2*a^4*b)*d*cos
h(d*x + c)^2 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d)*sinh(d*x + c)^4 + 4*(a^5 +
2*a^4*b)*d*cosh(d*x + c)^2 + 8*(7*a^5*d*cosh(d*x + c)^5 + 10*(a^5 + 2*a^4*b
)*d*cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c))*sinh(d
*x + c)^3 + 4*(7*a^5*d*cosh(d*x + c)^6 + 15*(a^5 + 2*a^4*b)*d*cosh(d*x + c)
^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 2*a^4*b)*d)
*sinh(d*x + c)^2 + 8*(a^5*d*cosh(d*x + c)^7 + 3*(a^5 + 2*a^4*b)*d*cosh(d*x
+ c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 2*a^4*b)*
d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

[Out] Timed out

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^6\,\tanh(c+d\,x)^3}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*tanh(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.162 $\int \frac{\tanh^2(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=139

$$\frac{x}{a^3}-\frac{(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}\,(a+b)^{3/2}d}-\frac{\tanh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2}-\frac{(3a+4b)\tanh(c-}{8a^2(a+b)d\left(a+b-b\tan\right.}$$

```
[Out] x/a^3-1/8*(3*a^2+12*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))/a^3
/(a+b)^(3/2)/d/b^(1/2)-1/4*tanh(d*x+c)/a/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*(3*a
+4*b)*tanh(d*x+c)/a^2/(a+b)/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.18, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {4226, 2000, 482, 541, 536, 212, 214}

$$\frac{x}{a^3}-\frac{(3a+4b)\tanh(c+dx)}{8a^2d(a+b)\left(a-b\tanh^2(c+dx)+b\right)}-\frac{(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}\,d(a+b)^{3/2}}-\frac{\tanh(c+dx)}{4ad\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] x/a^3 - ((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a +
b]])/(8*a^3*Sqrt[b]*(a + b)^(3/2)*d) - Tanh[c + d*x]/(4*a*d*(a + b - b*Tanh
[c + d*x]^2)^2) - ((3*a + 4*b)*Tanh[c + d*x])/(8*a^2*(a + b)*d*(a + b - b*T
anh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
```

```
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b(1-x^2))^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\tanh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{1+3x^2}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tanh(c+d$$

$$= -\frac{\tanh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{(3a+4b)\tanh(c+dx)}{8a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} -$$

$$= -\frac{\tanh(c+dx)}{4ad\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{(3a+4b)\tanh(c+dx)}{8a^2(a+b)d\left(a+b-b\tanh^2(c+dx)\right)} +$$

$$= \frac{x}{a^3} - \frac{(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}\,(a+b)^{3/2}d} - \frac{\tanh(c+dx)}{4ad\left(a+b-b\tanh^2(c+\right.}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1317 vs. $2(139) = 278$.
time = 9.55, size = 1317, normalized size = 9.47

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*((6*a*(a + 2*b)*ArcTanh[
(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) - (4*(3*a^2 + 8*a*b + 8
*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a + b]])/(a + b)^(5/2) + (4*a*Sq
rt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cosh[2*(c + d*x)])*Sinh[2*(c
 + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) - (2*Sqrt[b]*(3*a^3
 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cosh[2*(c + d*x
)])*Sinh[2*(c + d*x)])/((a + b)^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2) + (Sqr
t[b]*((-2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^4 + 256*b^
5)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh
[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - S
inh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (Sech[2*c]*(256*b^
2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*d*x*Cosh[2*c] + 512*a*b^2*(a + b)^2*(a
+ 2*b)*d*x*Cosh[2*d*x] + 128*a^4*b^2*d*x*Cosh[2*(c + 2*d*x)] + 256*a^3*b^3*
d*x*Cosh[2*(c + 2*d*x)] + 128*a^2*b^4*d*x*Cosh[2*(c + 2*d*x)] + 512*a^4*b^2
```

```
*d*x*Cosh[4*c + 2*d*x] + 2048*a^3*b^3*d*x*Cosh[4*c + 2*d*x] + 2560*a^2*b^4*
d*x*Cosh[4*c + 2*d*x] + 1024*a*b^5*d*x*Cosh[4*c + 2*d*x] + 128*a^4*b^2*d*x*
Cosh[6*c + 4*d*x] + 256*a^3*b^3*d*x*Cosh[6*c + 4*d*x] + 128*a^2*b^4*d*x*Cos
h[6*c + 4*d*x] - 9*a^6*Sinh[2*c] + 12*a^5*b*Sinh[2*c] + 684*a^4*b^2*Sinh[2*
c] + 2880*a^3*b^3*Sinh[2*c] + 5280*a^2*b^4*Sinh[2*c] + 4608*a*b^5*Sinh[2*c]
 + 1536*b^6*Sinh[2*c] + 9*a^6*Sinh[2*d*x] - 14*a^5*b*Sinh[2*d*x] - 608*a^4*
b^2*Sinh[2*d*x] - 2112*a^3*b^3*Sinh[2*d*x] - 2560*a^2*b^4*Sinh[2*d*x] - 102
4*a*b^5*Sinh[2*d*x] + 3*a^6*Sinh[2*(c + 2*d*x)] - 12*a^5*b*Sinh[2*(c + 2*d*
x)] - 204*a^4*b^2*Sinh[2*(c + 2*d*x)] - 384*a^3*b^3*Sinh[2*(c + 2*d*x)] - 1
92*a^2*b^4*Sinh[2*(c + 2*d*x)] - 3*a^6*Sinh[4*c + 2*d*x] + 10*a^5*b*Sinh[4*
c + 2*d*x] + 304*a^4*b^2*Sinh[4*c + 2*d*x] + 1056*a^3*b^3*Sinh[4*c + 2*d*x]
 + 1280*a^2*b^4*Sinh[4*c + 2*d*x] + 512*a*b^5*Sinh[4*c + 2*d*x]))/(a + 2*b
+ a*Cosh[2*(c + d*x)])^2))/(a^3*(a + b)^2) + (2*Sqrt[b]*((6*a^2*ArcTanh[(Se
ch[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/
(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(Cosh[2*c] - Sinh[2*c]))/(Sq
rt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4]) + (a*Sech[2*c]*((-9*a^4 - 16*a^3*b
 + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sinh[2*d*x] + a*(-3*a^3 + 2*a^2*b + 24*
a*b^2 + 16*b^3)*Sinh[2*(c + 2*d*x)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*
b^4)*Sinh[4*c + 2*d*x]) + (9*a^5 + 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 32
0*a*b^4 - 128*b^5)*Tanh[2*c])/(a^2*(a + 2*b + a*Cosh[2*(c + d*x)])^2)))/(a
+ b)^2))/(4096*b^(5/2)*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(125) = 250$.

time = 2.61, size = 335, normalized size = 2.41

method	result
derivativedivides	$\frac{\frac{2\left(\left(-\frac{5}{8}a^2-\frac{1}{2}ab\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a\left(15a^2+15ab-4b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}-\frac{a\left(15a^2+15ab-4b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}+\left(-\frac{5}{8}a^2-\frac{1}{2}ab\right.}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}}{}$
default	$\frac{\frac{2\left(\left(-\frac{5}{8}a^2-\frac{1}{2}ab\right)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a\left(15a^2+15ab-4b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}-\frac{a\left(15a^2+15ab-4b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}+\left(-\frac{5}{8}a^2-\frac{1}{2}ab\right.}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}}{}$
risch	$\frac{x}{a^3}+\frac{5a^3e^{6dx+6c}+20a^2b\,e^{6dx+6c}+16a\,b^2e^{6dx+6c}+15a^3e^{4dx+4c}+58a^2b\,e^{4dx+4c}+88a\,b^2e^{4dx+4c}+48b^3e^{4dx+4c}+15a^3e}{4a^3d\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)^2(a+b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^3*(((-5/8*a^2-1/2*a*b)*tanh(1/2*d*x+1/2*c)^7-1/8*a*(15*a^2+15*a*b-
4*b^2)/(a+b)*tanh(1/2*d*x+1/2*c)^5-1/8*a*(15*a^2+15*a*b-4*b^2)/(a+b)*tanh(1
```

```
/2*d*x+1/2*c)^3+(-5/8*a^2-1/2*a*b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2
*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/
2*c)^2+a+b)^2+1/8*(3*a^2+12*a*b+8*b^2)/(a+b)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((
a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))
+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*
x+1/2*c)*b^(1/2)+(a+b)^(1/2))))-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/a^3*ln(ta
nh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1255 vs. $2(131) = 262$.
time = 0.62, size = 1255, normalized size = 9.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(3*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*log((a*e^(2*d*x + 2*c) + a + 2
*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/
((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d) + 1/64*(3*a^3 + 30*a^2*b + 40
*a*b^2 + 16*b^3)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*
e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^5 + 2*a^4*b + a^3*b^2)
*sqrt((a + b)*b)*d) + 1/16*(5*a^4 + 20*a^3*b + 12*a^2*b^2 + (5*a^4 + 66*a^3
*b + 128*a^2*b^2 + 64*a*b^3)*e^(6*d*x + 6*c) + (15*a^4 + 164*a^3*b + 460*a^
2*b^2 + 512*a*b^3 + 192*b^4)*e^(4*d*x + 4*c) + (15*a^4 + 118*a^3*b + 208*a^
2*b^2 + 96*a*b^3)*e^(2*d*x + 2*c))/((a^7 + 2*a^6*b + a^5*b^2 + (a^7 + 2*a^6
*b + a^5*b^2)*e^(8*d*x + 8*c) + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e
^(6*d*x + 6*c) + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)
*e^(4*d*x + 4*c) + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(2*d*x + 2*c
))*d) - 1/16*(5*a^4 + 20*a^3*b + 12*a^2*b^2 + (15*a^4 + 118*a^3*b + 208*a^2
*b^2 + 96*a*b^3)*e^(-2*d*x - 2*c) + (15*a^4 + 164*a^3*b + 460*a^2*b^2 + 512
*a*b^3 + 192*b^4)*e^(-4*d*x - 4*c) + (5*a^4 + 66*a^3*b + 128*a^2*b^2 + 64*a
*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^
5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24
*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a
^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b + a^5*b^2)*e^(-8*d*x - 8*c))*d) -
 1/8*(5*a^3 + 2*a^2*b + (15*a^3 + 32*a^2*b + 8*a*b^2)*e^(-2*d*x - 2*c) + (1
5*a^3 + 46*a^2*b + 56*a*b^2 + 16*b^3)*e^(-4*d*x - 4*c) + (5*a^3 + 16*a^2*b
+ 8*a*b^2)*e^(-6*d*x - 6*c))/((a^6 + 2*a^5*b + a^4*b^2 + 4*(a^6 + 4*a^5*b +
 5*a^4*b^2 + 2*a^3*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^6 + 14*a^5*b + 27*a^4*b^2
 + 24*a^3*b^3 + 8*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(a^6 + 4*a^5*b + 5*a^4*b^2
+ 2*a^3*b^3)*e^(-6*d*x - 6*c) + (a^6 + 2*a^5*b + a^4*b^2)*e^(-8*d*x - 8*c))
*d) + 3/32*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*
d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^2 + 2*a*b + b^2)*sqrt((a + b
)*b)*d) + 1/4*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/(a^3
```

```
*d) - 1/4*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/(a^3*d
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3459 vs. $2(131) = 262$.
time = 0.45, size = 7158, normalized size = 51.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 128*(a^4*b +
2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^4*b + 2*a^3*
b^2 + a^2*b^3)*d*x*sinh(d*x + c)^8 + 4*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 +
 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)
^6 + 4*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 112*(a^4*b + 2*a^3*b
^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a
*b^4)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(
d*x + c)^3 + 3*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b +
4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 20*a
^4*b + 44*a^3*b^2 + 24*a^2*b^3 + 4*(15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 1
36*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5
)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x
+ c)^4 + 15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^
4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x + 15*(5*a^4*b + 25*a^
3*b^2 + 36*a^2*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(56*(a^4*b + 2*a^3*b^2 + a^2*b
^3)*d*x*cosh(d*x + c)^5 + 5*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^3 + 16*a*b^4 +
 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^3 + (15*a^
4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b
^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 1
6*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x + 4*(15*a^4*b + 59*a^3*b^2 + 76*a^2*b^3
 + 32*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x +
c)^2 + 4*(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^6 + 15*a^4*b
+ 59*a^3*b^2 + 76*a^2*b^3 + 32*a*b^4 + 15*(5*a^4*b + 25*a^3*b^2 + 36*a^2*b^
3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x +
 c)^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x + 6*(15*a^4*b + 73
*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*(3*a^4*b + 14*a^3*b^2 + 27*
a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4
 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^8 + 8*(3*a^4 + 12*a^3*b + 8*a^2*b^2)
*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 12*a^3*b + 8*a^2*b^2)*sinh(d*x +
c)^8 + 4*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^6 + 4*(3*
a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3 + 7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*c
osh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(
```

```
d*x + c)^3 + 3*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(9*a^4 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cos
h(d*x + c)^4 + 2*(35*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 9*a^4
 + 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4 + 30*(3*a^4 + 18*a^3*b + 32*
a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 + 12*a^3*b + 8
*a^2*b^2 + 8*(7*(3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(3*a^4
+ 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (9*a^4 + 60*a^3*b + 1
44*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4
+ 18*a^3*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(3*a^4 + 12*a^3*
b + 8*a^2*b^2)*cosh(d*x + c)^6 + 15*(3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b
^3)*cosh(d*x + c)^4 + 3*a^4 + 18*a^3*b + 32*a^2*b^2 + 16*a*b^3 + 3*(9*a^4 +
 60*a^3*b + 144*a^2*b^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 8*((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(3*a^4 + 18*a^3
*b + 32*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (9*a^4 + 60*a^3*b + 144*a^2*b
^2 + 160*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (3*a^4 + 18*a^3*b + 32*a^2*b^2 +
 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b + b^2)*log((a^2*cosh(d*x
+ c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2
 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*
x + c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cos
h(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d
*x + c) + a*sinh(d*x + c)^2 + a + 2*b)*sqrt(a*b + b^2))/(a*cosh(d*x + c)^4
+ 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(
d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(
d*x + c)^3 + (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a)) + 8*(16*(a^4*b +
2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^7 + 3*(5*a^4*b + 25*a^3*b^2 + 36*a^2
*b^3 + 16*a*b^4 + 16*(a^4*b + 4*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*
x + c)^5 + 2*(15*a^4*b + 73*a^3*b^2 + 146*a^2*b^3 + 136*a*b^4 + 48*b^5 + 8*
(3*a^4*b + 14*a^3*b^2 + 27*a^2*b^3 + 24*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^3
 + (15*a^4*b + 59*a^3*b^2 + 76*a^2*b^3 + 32*a*b^4 + 16*(a^4*b + 4*a^3*b^2 +
 5*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7*b + 2*a^6*b^
2 + a^5*b^3)*d*cosh(d*x + c)^8 + 8*(a^7*b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x
 + c)*sinh(d*x + c)^7 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*d*sinh(d*x + c)^8 + 4
*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^7*
b + 2*a^6*b^2 + a^5*b^3)*d*cosh(d*x + c)^2 + (a...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(131) = 262.
time = 1.84, size = 296, normalized size = 2.13

$$-\frac{\frac{(3a^2+12ab+8b^2)\arctan\left(\frac{ae^{(2dx+2c)}+a+2b}{2\sqrt{-ab-b^2}}\right)}{(a^4+a^3b)\sqrt{-ab-b^2}}-\frac{2\left(5a^3e^{(6dx+6c)}+20a^2be^{(6dx+6c)}+16ab^2e^{(6dx+6c)}+15a^3e^{(4dx+4c)}+58a^2be^{(4dx+4c)}+88ab^2e^{(4dx+4c)}+48b^3e^{(4dx+4c)}+15a^3e^{(2dx+2c)}+44a^2be^{(2dx+2c)}+32ab^2e^{(2dx+2c)}+5a^3+6a^2b\right)}{(a^4+a^3b)\left(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a\right)^2}-\frac{8(dx+c)}{a^3}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((3*a^2 + 12*a*b + 8*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b)/sqr
t(-a*b - b^2))/((a^4 + a^3*b)*sqrt(-a*b - b^2)) - 2*(5*a^3*e^(6*d*x + 6*c)
+ 20*a^2*b*e^(6*d*x + 6*c) + 16*a*b^2*e^(6*d*x + 6*c) + 15*a^3*e^(4*d*x + 4
*c) + 58*a^2*b*e^(4*d*x + 4*c) + 88*a*b^2*e^(4*d*x + 4*c) + 48*b^3*e^(4*d*x
 + 4*c) + 15*a^3*e^(2*d*x + 2*c) + 44*a^2*b*e^(2*d*x + 2*c) + 32*a*b^2*e^(2
*d*x + 2*c) + 5*a^3 + 6*a^2*b)/((a^4 + a^3*b)*(a*e^(4*d*x + 4*c) + 2*a*e^(2
*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2) - 8*(d*x + c)/a^3)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^4\,\left(\cosh(c+d\,x)^2-1\right)}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^4*(cosh(c + d*x)^2 - 1))/(b + a*cosh(c + d*x)^2)^3, x)
```

3.163 $\int \frac{\tanh(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=73

$$-\frac{b^2}{4a^3d\left(b+a\cosh^2(c+dx)\right)^2}+\frac{b}{a^3d\left(b+a\cosh^2(c+dx)\right)}+\frac{\log\left(b+a\cosh^2(c+dx)\right)}{2a^3d}$$

```
[Out] -1/4*b^2/a^3/d/(b+a*cosh(d*x+c)^2)^2+b/a^3/d/(b+a*cosh(d*x+c)^2)+1/2*ln(b+a
*cosh(d*x+c)^2)/a^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 272, 45}

$$-\frac{b^2}{4a^3d\left(a\cosh^2(c+dx)+b\right)^2}+\frac{b}{a^3d\left(a\cosh^2(c+dx)+b\right)}+\frac{\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/4*b^2/(a^3*d*(b + a*Cosh[c + d*x]^2)^2) + b/(a^3*d*(b + a*Cosh[c + d*x]^
2)) + Log[b + a*Cosh[c + d*x]^2]/(2*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^5}{(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(b+ax)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{b^2}{a^2(b+ax)^3}-\frac{2b}{a^2(b+ax)^2}+\frac{1}{a^2(b+ax)}\right)dx, x, \cosh^2(c+dx)\right)}{2d}\\ &= -\frac{b^2}{4a^3d\left(b+a\cosh^2(c+dx)\right)^2}+\frac{b}{a^3d\left(b+a\cosh^2(c+dx)\right)}+\frac{\log\left(b+a\cosh\right.}{2a^3d}\end{aligned}$$

Mathematica [A]
time = 1.33, size = 129, normalized size = 1.77

$$\frac{2b(2a+3b)+(a+2b)^2\log(a+2b+a\cosh(2(c+dx)))+a^2\cosh^2(2(c+dx))\log(a+2b+a\cosh(2(c+dx)))+2a\cosh(2(c+dx))(2b+(a+2b)\log(a+2b+a\cosh(2(c+dx))))}{2a^3d(a+2b+a\cosh(2(c+dx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] (2*b*(2*a + 3*b) + (a + 2*b)^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + a^2*Cos
h[2*(c + d*x)]^2*Log[a + 2*b + a*Cosh[2*(c + d*x)]] + 2*a*Cosh[2*(c + d*x)]
*(2*b + (a + 2*b)*Log[a + 2*b + a*Cosh[2*(c + d*x)]]))/(2*a^3*d*(a + 2*b +
a*Cosh[2*(c + d*x)])^2)
```

Maple [A]
time = 1.12, size = 84, normalized size = 1.15

method	result	size
derivativedivides	$-\frac{-\frac{b\left(-\frac{a^2}{2b\left(a+b\operatorname{sech}(dx+c)^2\right)^2}+\frac{\ln\left(a+b\operatorname{sech}(dx+c)^2\right)}{b}-\frac{a}{b\left(a+b\operatorname{sech}(dx+c)^2\right)}\right)}{2a^3}+\frac{\ln(\operatorname{sech}(dx+c))}{a^3}}{d}$	84
default	$-\frac{-\frac{b\left(-\frac{a^2}{2b\left(a+b\operatorname{sech}(dx+c)^2\right)^2}+\frac{\ln\left(a+b\operatorname{sech}(dx+c)^2\right)}{b}-\frac{a}{b\left(a+b\operatorname{sech}(dx+c)^2\right)}\right)}{2a^3}+\frac{\ln(\operatorname{sech}(dx+c))}{a^3}}{d}$	84
risch	$-\frac{x}{a^3}-\frac{2c}{a^3d}+\frac{4\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+3b\,e^{2dx+2c}+a\right)e^{2dx+2c}b}{a^3\left(a\,e^{4dx+4c}+2a\,e^{2dx+2c}+4b\,e^{2dx+2c}+a\right)^2d}+\frac{\ln\left(e^{4dx+4c}+\frac{2(2b+a)e^{2dx+2c}}{a}+1\right)}{2a^3d}$	150

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-1/2/a^3*b*(-1/2*a^2/b/(a+b*sech(d*x+c)^2)^2+1/b*ln(a+b*sech(d*x+c)^2
)-a/b/(a+b*sech(d*x+c)^2))+1/a^3*ln(sech(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(69) = 138$.
time = 0.28, size = 193, normalized size = 2.64

$$\frac{4\left(abe^{(-2\,dx-2\,c)}+abe^{(-6\,dx-6\,c)}+(2\,ab+3\,b^2)e^{(-4\,dx-4\,c)}\right)}{\left(a^5e^{(-8\,dx-8\,c)}+a^5+4\,(a^5+2\,a^4b)e^{(-2\,dx-2\,c)}+2\,(3\,a^5+8\,a^4b+8\,a^3b^2)e^{(-4\,dx-4\,c)}+4\,(a^5+2\,a^4b)e^{(-6\,dx-6\,c)}\right)d}+\frac{dx+c}{a^3d}+\frac{\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 4*(a*b*e^(-2*d*x - 2*c) + a*b*e^(-6*d*x - 6*c) + (2*a*b + 3*b^2)*e^(-4*d*x
- 4*c))/((a^5*e^(-8*d*x - 8*c) + a^5 + 4*(a^5 + 2*a^4*b)*e^(-2*d*x - 2*c) +
 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*e^(-4*d*x - 4*c) + 4*(a^5 + 2*a^4*b)*e^(-6
*d*x - 6*c))*d) + (d*x + c)/(a^3*d) + 1/2*log(2*(a + 2*b)*e^(-2*d*x - 2*c)
+ a*e^(-4*d*x - 4*c) + a)/(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1666 vs. $2(69) = 138$.
time = 0.40, size = 1666, normalized size = 22.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^8 + 16*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7
+ 2*a^2*d*x*sinh(d*x + c)^8 + 8*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^6 +
 8*(7*a^2*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b)*d*x - a*b)*sinh(d*x + c)^6 +
16*(7*a^2*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c))*
sinh(d*x + c)^5 + 4*((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*x
+ c)^4 + 4*(35*a^2*d*x*cosh(d*x + c)^4 + (3*a^2 + 8*a*b + 8*b^2)*d*x + 30*(
(a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^2 - 4*a*b - 6*b^2)*sinh(d*x + c)^4 +
 2*a^2*d*x + 16*(7*a^2*d*x*cosh(d*x + c)^5 + 10*((a^2 + 2*a*b)*d*x - a*b)*c
osh(d*x + c)^3 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*x + c
))*sinh(d*x + c)^3 + 8*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^2 + 8*(7*a^2
*d*x*cosh(d*x + c)^6 + 15*((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c)^4 + (a^2
+ 2*a*b)*d*x + 3*((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*x + c
)^2 - a*b)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sin
h(d*x + c)^7 + a^2*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^6 + 4*(7
*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^6 + 8*(7*a^2*cosh(d*x + c
)^3 + 3*(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 + 8*a*b + 8
*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b)*cosh(d
*x + c)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)
^5 + 10*(a^2 + 2*a*b)*cosh(d*x + c)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x +
```

```
c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b)*cosh(d*x + c)^2 + 4*(7*a^2*cosh(d*x +
 c)^6 + 15*(a^2 + 2*a*b)*cosh(d*x + c)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(d
*x + c)^2 + a^2 + 2*a*b)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 + 3
*(a^2 + 2*a*b)*cosh(d*x + c)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(d*x + c)^3 +
(a^2 + 2*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c)^2 + a*si
nh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)) + 16*(a^2*d*x*cosh(d*x + c)^7 + 3*((a^2 + 2*a*b)*d*x - a
*b)*cosh(d*x + c)^5 + ((3*a^2 + 8*a*b + 8*b^2)*d*x - 4*a*b - 6*b^2)*cosh(d*
x + c)^3 + ((a^2 + 2*a*b)*d*x - a*b)*cosh(d*x + c))*sinh(d*x + c))/(a^5*d*c
osh(d*x + c)^8 + 8*a^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*d*sinh(d*x + c
)^8 + 4*(a^5 + 2*a^4*b)*d*cosh(d*x + c)^6 + 4*(7*a^5*d*cosh(d*x + c)^2 + (a
^5 + 2*a^4*b)*d)*sinh(d*x + c)^6 + a^5*d + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*
d*cosh(d*x + c)^4 + 8*(7*a^5*d*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*d*cosh(d
*x + c))*sinh(d*x + c)^5 + 2*(35*a^5*d*cosh(d*x + c)^4 + 30*(a^5 + 2*a^4*b)
*d*cosh(d*x + c)^2 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d)*sinh(d*x + c)^4 + 4*(
a^5 + 2*a^4*b)*d*cosh(d*x + c)^2 + 8*(7*a^5*d*cosh(d*x + c)^5 + 10*(a^5 + 2
*a^4*b)*d*cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c))*
sinh(d*x + c)^3 + 4*(7*a^5*d*cosh(d*x + c)^6 + 15*(a^5 + 2*a^4*b)*d*cosh(d*
x + c)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 + 2*a^4
*b)*d)*sinh(d*x + c)^2 + 8*(a^5*d*cosh(d*x + c)^7 + 3*(a^5 + 2*a^4*b)*d*cos
h(d*x + c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*d*cosh(d*x + c)^3 + (a^5 + 2*a
^4*b)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 1.60, size = 94, normalized size = 1.29

$$\frac{\ln\left(\cosh(c+d\,x)^2\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)\right)}{2\,a^3\,d}-\frac{b^2}{4\,a^3\,d\cosh\left(c+d\,x\right)^4\,{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)}^2}+\frac{b}{a^3\,d\cosh\left(c+d\,x\right)^2\,\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] log(cosh(c + d*x)^2*(a + b/cosh(c + d*x)^2))/(2*a^3*d) - b^2/(4*a^3*d*cosh(
c + d*x)^4*(a + b/cosh(c + d*x)^2)^2) + b/(a^3*d*cosh(c + d*x)^2*(a + b/cos
h(c + d*x)^2))
```

3.164 $\int \frac{1}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=146

$$\frac{x}{a^3}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}d}-\frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}-\frac{b(7a}{8a^2(a+b)^2}$$

```
[Out] x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(1/2))*b^
(1/2)/a^3/(a+b)^(5/2)/d-1/4*b*tanh(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2
-1/8*b*(7*a+4*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]

time = 0.14, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4213, 425, 541, 536, 212, 214}

$$\frac{x}{a^3}-\frac{b(7a+4b)\tanh(c+dx)}{8a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{5/2}}-\frac{b\tanh(c+dx)}{4ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(-3),x]
```

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a + b]])/(8*a^3*(a + b)^(5/2)*d) - (b*Tanh[c + d*x])/(4*a*(a + b)*d*(a
 + b - b*Tanh[c + d*x]^2)^2) - (b*(7*a + 4*b)*Tanh[c + d*x])/(8*a^2*(a + b)
^2*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-4a-b-3bx^2}{(1-x^2)(a+b-bx^2)^2}\,dx, x, \tan\right.}{4a(a+b)d}$$

$$= -\frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(7a+4b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b-b\tanh^2(c+\right.}$$

$$= -\frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(7a+4b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b-b\tanh^2(c+\right.}$$

$$= \frac{x}{a^3} - \frac{\sqrt{b}\,(15a^2+20ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}d} - \frac{b\tanh(}{4a(a+b)d\,(a+b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 301 vs. 2(146) = 292.

time = 4.08, size = 301, normalized size = 2.06

$$\frac{(a+2b+a\cosh(2(c+dx)))\operatorname{sech}^6(c+dx)\left(8x(a+2b+a\cosh(2(c+dx)))^2-\frac{b(15a^2+20ab+8b^2)\tanh^{-1}\left(\frac{\operatorname{sech}(dx)(\cosh(2c)-\sinh(2c))((a+2b)\sinh(dx)-a\sinh(2c+dx))}{2\sqrt{a+b}\sqrt{b(\cosh(c)-\sinh(c))^4}}\right)(a+2b+a\cosh(2(c+dx)))^2(\cosh(2c)-\sinh(2c))}{(a+b)^{5/2}d\sqrt{b(\cosh(c)-\sinh(c))^4}}-\frac{4b^2\operatorname{sech}(2c)((a+2b)\sinh(2c)-a\sinh(2dx))}{(a+b)d}+\frac{b(a+2b+a\cosh(2(c+dx)))\operatorname{sech}(2c)((9a^2+28ab+16b^2)\sinh(2c)-3a(3a+2b)\sinh(2dx))}{(a+b)^2d}\right)}{64a^3\left(a+b\operatorname{sech}^2(c+dx)\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-3),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*(8*x*(a + 2*b + a*Cosh[2*(
c + d*x)])^2 - (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c] -
 Sinh[2*c])*((a + 2*b)*Sinh[d*x] - a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[
b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b + a*Cosh[2*(c + d*x)])^2*(Cosh[2*c] - S
inh[2*c]))/((a + b)^(5/2)*d*Sqrt[b*(Cosh[c] - Sinh[c])^4]) - (4*b^2*Sech[2*
c]*((a + 2*b)*Sinh[2*c] - a*Sinh[2*d*x]))/((a + b)*d) + (b*(a + 2*b + a*Cos
h[2*(c + d*x)])*Sech[2*c]*((9*a^2 + 28*a*b + 16*b^2)*Sinh[2*c] - 3*a*(3*a +
 2*b)*Sinh[2*d*x]))/((a + b)^2*d)))/(64*a^3*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(132) = 264.

time = 2.52, size = 355, normalized size = 2.43

method	result

derivativedivides	$\frac{2b\left(\frac{-\frac{a(9a+4b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}-\frac{a\left(27a^2+11ab-4b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2}-\frac{a\left(27a^2+11ab-4b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2}-\frac{a(9a+4b)\tanh\left(\frac{dx}{2}\cdots\right)}{8(a+b)}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}\cdots\right.}{\cdots}$
default	$\frac{2b\left(\frac{-\frac{a(9a+4b)\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)}-\frac{a\left(27a^2+11ab-4b^2\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2}-\frac{a\left(27a^2+11ab-4b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8(a+b)^2}-\frac{a(9a+4b)\tanh\left(\frac{dx}{2}\cdots\right)}{8(a+b)}}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}\cdots\right.}{\cdots}$
risch	$\frac{x}{a^3}+\frac{b\left(9a^3\mathrm{e}^{6dx+6c}+28a^2b\,\mathrm{e}^{6dx+6c}+16a\,b^2\mathrm{e}^{6dx+6c}+27a^3\mathrm{e}^{4dx+4c}+90a^2b\,\mathrm{e}^{4dx+4c}+120a\,b^2\mathrm{e}^{4dx+4c}+48b^3\mathrm{e}^{4dx+4c}+27a^3\cdots\right.}{4a^3(a+b)^2d\left(a\,\mathrm{e}^{4dx+4c}+2a\,\mathrm{e}^{2dx+2c}+4b\,\mathrm{e}^{2dx+2c}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^3*b*((-1/8*a*(9*a+4*b)/(a+b)*tanh(1/2*d*x+1/2*c)^7-1/8*a*(27*a^2+1
1*a*b-4*b^2)/(a+b)^2*tanh(1/2*d*x+1/2*c)^5-1/8*a*(27*a^2+11*a*b-4*b^2)/(a+b
)^2*tanh(1/2*d*x+1/2*c)^3-1/8*a*(9*a+4*b)/(a+b)*tanh(1/2*d*x+1/2*c))/(a*tan
h(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*ta
nh(1/2*d*x+1/2*c)^2+a+b)^2+1/8*(15*a^2+20*a*b+8*b^2)/(a^2+2*a*b+b^2)*(-1/4/
b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2
*c)*b^(1/2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*d
*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)-(a+b)^(1/2))))-1/a^3*ln(tanh(1/2*
d*x+1/2*c)-1)+1/a^3*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(138) = 276$.
time = 0.53, size = 402, normalized size = 2.75

$$\frac{(15\,a^2b+20\,ab^2+8\,b^3)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{16\,(a^5+2\,a^4b+a^3b^2)\sqrt{(a+b)b}\,d}-\frac{9\,a^3b+6\,a^2b^2+(27\,a^3b+68\,a^2b^2+32\,ab^3)e^{(-2\,dx-2\,c)}+3\,(9\,a^3b+30\,a^2b^2+40\,ab^3+16\,b^4)e^{(-4\,dx-4\,c)}+(9\,a^3b+28\,a^2b^2+16\,ab^3)e^{(-6\,dx-6\,c)}}{4\,(a^7+2\,a^6b+a^5b^2+4\,(a^7+4\,a^6b+5\,a^5b^2+2\,a^4b^3)e^{(-2\,dx-2\,c)}+2\,(3\,a^7+14\,a^6b+27\,a^5b^2+24\,a^4b^3+8\,a^3b^4)e^{(-4\,dx-4\,c)}+4\,(a^7+4\,a^6b+5\,a^5b^2+2\,a^4b^3)e^{(-6\,dx-6\,c)}+(a^7+2\,a^6b+a^5b^2)e^{(-8\,dx-8\,c)})d}+\frac{dx+c}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(15*a^2*b + 20*a*b^2 + 8*b^3)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sq
rt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^5 +
2*a^4*b + a^3*b^2)*sqrt((a + b)*b)*d) - 1/4*(9*a^3*b + 6*a^2*b^2 + (27*a^3*
b + 68*a^2*b^2 + 32*a*b^3)*e^(-2*d*x - 2*c) + 3*(9*a^3*b + 30*a^2*b^2 + 40*
a*b^3 + 16*b^4)*e^(-4*d*x - 4*c) + (9*a^3*b + 28*a^2*b^2 + 16*a*b^3)*e^(-6*
```

```
d*x - 6*c))/((a^7 + 2*a^6*b + a^5*b^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^
4*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8
*a^3*b^4)*e^(-4*d*x - 4*c) + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-
6*d*x - 6*c) + (a^7 + 2*a^6*b + a^5*b^2)*e^(-8*d*x - 8*c))*d) + (d*x + c)/(
a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3131 vs. $2(138) = 276$.
time = 0.48, size = 6538, normalized size = 44.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^8 + 128*(a^4 + 2*a^3*
b + a^2*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^4 + 2*a^3*b + a^2*b^
2)*d*x*sinh(d*x + c)^8 + 4*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a
^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^6 + 4*(112*(a^4 + 2*a^3*b +
a^2*b^2)*d*x*cosh(d*x + c)^2 + 9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 +
4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^4 + 2*a^3*b
 + a^2*b^2)*d*x*cosh(d*x + c)^3 + 3*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(
a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 +
4*(27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^
2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^4 + 2*a^3*b + a^
2*b^2)*d*x*cosh(d*x + c)^4 + 27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4 + 8
*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x + 15*(9*a^3*b + 28*
a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 36*a^3*b + 24*a^2*b^2 + 16*(56*(a^4 + 2*a^3*b +
a^2*b^2)*d*x*cosh(d*x + c)^5 + 5*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4
 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^3 + (27*a^3*b + 90*a^2
*b^2 + 120*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8
*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^4 + 2*a^3*b + a^2*b^2)*d*
x + 4*(27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2
*a*b^3)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*
x + c)^6 + 15*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*
b^2 + 2*a*b^3)*d*x)*cosh(d*x + c)^4 + 27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16
*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*d*x + 6*(27*a^3*b + 90*a^2*b^2 + 120
*a*b^3 + 48*b^4 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d
*x + c)^8 + 8*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^7
 + (15*a^4 + 20*a^3*b + 8*a^2*b^2)*sinh(d*x + c)^8 + 4*(15*a^4 + 50*a^3*b +
 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^6 + 4*(15*a^4 + 50*a^3*b + 48*a^2*b^2
 + 16*a*b^3 + 7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x +
 c)^6 + 8*(7*(15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^3 + 3*(15*a^4 +
```

```
50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a^
4 + 180*a^3*b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^4 + 2*(35*(
15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^4 + 45*a^4 + 180*a^3*b + 304*a
^2*b^2 + 224*a*b^3 + 64*b^4 + 30*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 20*a^3*b + 8*a^2*b^2 + 8*(7*(
15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^5 + 10*(15*a^4 + 50*a^3*b + 48
*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^3 + (45*a^4 + 180*a^3*b + 304*a^2*b^2 +
224*a*b^3 + 64*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 + 50*a^3*b +
 48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^2 + 4*(7*(15*a^4 + 20*a^3*b + 8*a^2*b
^2)*cosh(d*x + c)^6 + 15*(15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3)*cosh(d
*x + c)^4 + 15*a^4 + 50*a^3*b + 48*a^2*b^2 + 16*a*b^3 + 3*(45*a^4 + 180*a^3
*b + 304*a^2*b^2 + 224*a*b^3 + 64*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8
*((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cosh(d*x + c)^7 + 3*(15*a^4 + 50*a^3*b +
48*a^2*b^2 + 16*a*b^3)*cosh(d*x + c)^5 + (45*a^4 + 180*a^3*b + 304*a^2*b^2
+ 224*a*b^3 + 64*b^4)*cosh(d*x + c)^3 + (15*a^4 + 50*a^3*b + 48*a^2*b^2 + 1
6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log((a^2*cosh(d*x +
c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*(a^2 +
 2*a*b)*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2 + 2*a*b)*sinh(d*x
+ c)^2 + a^2 + 8*a*b + 8*b^2 + 4*(a^2*cosh(d*x + c)^3 + (a^2 + 2*a*b)*cosh(
d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*co
sh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 + 3*a*b + 2*b
^2)*sqrt(b/(a + b)))/(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
 + a*sinh(d*x + c)^4 + 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2
 + a + 2*b)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + (a + 2*b)*cosh(d*x + c
))*sinh(d*x + c) + a)) + 8*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^
7 + 3*(9*a^3*b + 28*a^2*b^2 + 16*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*
a*b^3)*d*x)*cosh(d*x + c)^5 + 2*(27*a^3*b + 90*a^2*b^2 + 120*a*b^3 + 48*b^4
 + 8*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*d*x)*cosh(d*x + c)^
3 + (27*a^3*b + 68*a^2*b^2 + 32*a*b^3 + 16*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a
*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 2*a^6*b + a^5*b^2)*d*cosh(
d*x + c)^8 + 8*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 +
(a^7 + 2*a^6*b + a^5*b^2)*d*sinh(d*x + c)^8 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2
+ 2*a^4*b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x
+ c)^2 + (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d)*sinh(d*x + c)^6 + 2*(3*
a^7 + 14*a^6*b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*d*cosh(d*x + c)^4 + 8
*(7*(a^7 + 2*a^6*b + a^5*b^2)*d*cosh(d*x + c)^3 + 3*(a^7 + 4*a^6*b + 5*a^5*
b^2 + 2*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\operatorname{sech}^2\left(c + dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**(-3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(138) = 276.
time = 0.67, size = 327, normalized size = 2.24

$$-\frac{\frac{(15\,a^2b+20\,ab^2+8\,b^3)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{-ab-b^2}}-\frac{2\,(9\,a^3be^{(6\,dx+6\,c)}+28\,a^2b^2e^{(6\,dx+6\,c)}+16\,ab^3e^{(6\,dx+6\,c)}+27\,a^3be^{(4\,dx+4\,c)}+90\,a^2b^2e^{(4\,dx+4\,c)}+120\,ab^3e^{(4\,dx+4\,c)}+48\,b^4e^{(4\,dx+4\,c)}+27\,a^3be^{(2\,dx+2\,c)}+68\,a^2b^2e^{(2\,dx+2\,c)}+32\,ab^3e^{(2\,dx+2\,c)}+9\,a^3b+6\,a^2b^2)}{(a^5+2\,a^4b+a^3b^2)(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a)^2}-\frac{8\,(dx+c)}{a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2*b
)/sqrt(-a*b - b^2))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(-a*b - b^2)) - 2*(9*a^3
*b*e^(6*d*x + 6*c) + 28*a^2*b^2*e^(6*d*x + 6*c) + 16*a*b^3*e^(6*d*x + 6*c)
+ 27*a^3*b*e^(4*d*x + 4*c) + 90*a^2*b^2*e^(4*d*x + 4*c) + 120*a*b^3*e^(4*d*
x + 4*c) + 48*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 68*a^2*b^2*e
^(2*d*x + 2*c) + 32*a*b^3*e^(2*d*x + 2*c) + 9*a^3*b + 6*a^2*b^2)/((a^5 + 2*
a^4*b + a^3*b^2)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x +
2*c) + a)^2) - 8*(d*x + c)/a^3)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{b}{{\mathrm{cosh}\left(c+d\,x\right)}^2}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int(1/(a + b/cosh(c + d*x)^2)^3, x)
```

3.165 $\int \frac{\coth(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=130

$$-\frac{b^3}{4a^3(a+b)d\left(b+a\cosh^2(c+dx)\right)^2}+\frac{b^2(3a+2b)}{2a^3(a+b)^2d\left(b+a\cosh^2(c+dx)\right)}+\frac{b(3a^2+3ab+b^2)\log\left(b+a\cosh^2\right.}{2a^3(a+b)^3d}$$

```
[Out] -1/4*b^3/a^3/(a+b)/d/(b+a*cosh(d*x+c)^2)^2+1/2*b^2*(3*a+2*b)/a^3/(a+b)^2/d/
(b+a*cosh(d*x+c)^2)+1/2*b*(3*a^2+3*a*b+b^2)*ln(b+a*cosh(d*x+c)^2)/a^3/(a+b)
^3/d+ln(sinh(d*x+c))/(a+b)^3/d
```

Rubi [A]
time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4223, 457, 90}

$$-\frac{b^3}{4a^3d(a+b)\left(a\cosh^2(c+dx)+b\right)^2}+\frac{b^2(3a+2b)}{2a^3d(a+b)^2\left(a\cosh^2(c+dx)+b\right)}+\frac{b(3a^2+3ab+b^2)\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d(a+b)^3}+\frac{\log(\sinh(c+dx))}{d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/4*b^3/(a^3*(a + b)*d*(b + a*Cosh[c + d*x]^2)^2) + (b^2*(3*a + 2*b))/(2*a
^3*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)) + (b*(3*a^2 + 3*a*b + b^2)*Log[b +
a*Cosh[c + d*x]^2])/(2*a^3*(a + b)^3*d) + Log[Sinh[c + d*x]]/((a + b)^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
```

```
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^7}{(1-x^2)(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x)(b+ax)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} - \frac{b^3}{a^2(a+b)(b+ax)^3} + \frac{b^2(3a+2b)}{a^2(a+b)^2(b+ax)^2} - \frac{b(3a^2+3ab+b^2)}{a^2(a+b)^3(b+ax)}\right) \ldots}{2d}$$

$$= -\frac{b^3}{4a^3(a+b)d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{b^2(3a+2b)}{2a^3(a+b)^2d\left(b+a\cosh^2(c+dx)\right)} + \ldots$$

Mathematica [A]
time = 0.69, size = 155, normalized size = 1.19

$$\frac{(a+2b+a\cosh(2(c+dx)))^3\text{sech}^6(c+dx)\left(4\log(\sinh(c+dx)) + \frac{2b(3a^2+3ab+b^2)\log(a+b+a\sinh^2(c+dx))}{a^3} - \frac{b^3(a+b)^2}{a^3(a+b+a\sinh^2(c+dx))^2} + \frac{2b^2(a+b)(3a+2b)}{a^3(a+b+a\sinh^2(c+dx))}\right)}{32(a+b)^3d\left(a+b\text{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x]^2)^3,x]
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*(4*Log[Sinh[c + d*x]] +
(2*b*(3*a^2 + 3*a*b + b^2)*Log[a + b + a*Sinh[c + d*x]^2])/a^3 - (b^3*(a +
b)^2)/(a^3*(a + b + a*Sinh[c + d*x]^2)^2) + (2*b^2*(a + b)*(3*a + 2*b))/(a^
3*(a + b + a*Sinh[c + d*x]^2))))/(32*(a + b)^3*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(124) = 248$.
time = 2.88, size = 292, normalized size = 2.25

method	result
derivativedivides	$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3} + \frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a+b)^3} - \frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3} + \frac{b\left(\frac{\left(-6a^3b-8a^2b^2-2a\,b^3\right)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\left(3a^2-ab-b^2 \ldots\right.}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a \ldots\right.}}{\ldots}$
default	$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3} + \frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a+b)^3} - \frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3} + \frac{b\left(\frac{\left(-6a^3b-8a^2b^2-2a\,b^3\right)\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\left(3a^2-ab-b^2 \ldots\right.}{\left(a\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a \ldots\right.}}{\ldots}$

risch	$\frac{x}{a^3} - \frac{2x}{a^3+3a^2b+3a\,b^2+b^3} - \frac{2c}{d(a^3+3a^2b+3a\,b^2+b^3)} - \frac{6bx}{a(a^3+3a^2b+3a\,b^2+b^3)} - \frac{6bc}{ad(a^3+3a^2b+3a\,b^2+b^3)} - \frac{}{a^2(a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/(a+b)^3*ln(tanh(1/2*d*x+1/2*c))-1/a
^3*ln(tanh(1/2*d*x+1/2*c)+1)+b/a^3/(a+b)^3*(((-6*a^3*b-8*a^2*b^2-2*a*b^3)*t
anh(1/2*d*x+1/2*c)^6-4*(3*a^2-a*b-b^2)*a*b*tanh(1/2*d*x+1/2*c)^4-2*(3*a^2+4
*a*b+b^2)*a*b*tanh(1/2*d*x+1/2*c)^2)/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*
x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)^2+1/2*(
3*a^2+3*a*b+b^2)*ln(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tan
h(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(124) = 248$.
time = 0.30, size = 419, normalized size = 3.22

$$\frac{(3\,a^2b+3\,ab^2+b^3)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)d}+\frac{2\left((3\,a^2b^2+2\,ab^3)e^{(-2\,dx-2\,c)}+2\,(3\,a^2b^2+7\,ab^3+3\,b^4)e^{(-4\,dx-4\,c)}+(3\,a^2b^2+2\,ab^3)e^{(-6\,dx-6\,c)}\right)}{(a^7+2\,a^6b+a^5b^2+4\,(a^7+4\,a^6b+5\,a^5b^2+2\,a^4b^3)e^{(-2\,dx-2\,c)}+2\,(3\,a^7+14\,a^6b+27\,a^5b^2+24\,a^4b^3+8\,a^3b^4)e^{(-4\,dx-4\,c)}+4\,(a^7+4\,a^6b+5\,a^5b^2+2\,a^4b^3)e^{(-6\,dx-6\,c)}+(a^7+2\,a^6b+a^5b^2)e^{(-8\,dx-8\,c)})d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)d}+\frac{dx+c}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a^2*b + 3*a*b^2 + b^3)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*
x - 4*c) + a)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + 2*((3*a^2*b^2 + 2
*a*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^2*b^2 + 7*a*b^3 + 3*b^4)*e^(-4*d*x - 4*c)
 + (3*a^2*b^2 + 2*a*b^3)*e^(-6*d*x - 6*c))/((a^7 + 2*a^6*b + a^5*b^2 + 4*(a
^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 14*a^6*
b + 27*a^5*b^2 + 24*a^4*b^3 + 8*a^3*b^4)*e^(-4*d*x - 4*c) + 4*(a^7 + 4*a^6*
b + 5*a^5*b^2 + 2*a^4*b^3)*e^(-6*d*x - 6*c) + (a^7 + 2*a^6*b + a^5*b^2)*e^(
-8*d*x - 8*c))*d) + log(e^(-d*x - c) + 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
d) + log(e^(-d*x - c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + (d*x + c)/
(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4132 vs. $2(124) = 248$.
time = 0.63, size = 4132, normalized size = 31.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 16*(a^5
 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^
```

5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*sinh(d*x + c)^8 - 4*(3*a^3*b^2 + 5*a
^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)
*cosh(d*x + c)^6 - 4*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 14*(a^5 + 3*a^4*b +
3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 +
7*a^2*b^3 + 2*a*b^4)*d*x)*sinh(d*x + c)^6 + 8*(14*(a^5 + 3*a^4*b + 3*a^3*b^
2 + a^2*b^3)*d*x*cosh(d*x + c)^3 - 3*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(
a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c))*sinh(d
x + c)^5 - 4(6*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 6*b^5 - (3*a^5 + 17*a^4*
b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 + 4*(3
5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 - 6*a^3*b^2 - 2
0*a^2*b^3 - 20*a*b^4 - 6*b^5 + (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3
+ 32*a*b^4 + 8*b^5)*d*x - 15*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*
a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c
)^4 + 16*(7*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 - 5*(
3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3
+ 2*a*b^4)*d*x)*cosh(d*x + c)^3 - (6*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 6*b^
5 - (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^3 + 2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x
- 4*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2
*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^2 + 4*(14*(a^5 + 3*a^4*b + 3*a^3*b^2 + a
^2*b^3)*d*x*cosh(d*x + c)^6 - 3*a^3*b^2 - 5*a^2*b^3 - 2*a*b^4 - 15*(3*a^3*b
^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b
^4)*d*x)*cosh(d*x + c)^4 + 2*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b
^4)*d*x - 6*(6*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 6*b^5 - (3*a^5 + 17*a^4*b
+ 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 - ((3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^8 + 8*(3*a^4*b +
3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4*b + 3*a^3*b^2 +
a^2*b^3)*sinh(d*x + c)^8 + 4*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*c
osh(d*x + c)^6 + 4*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 7*(3*a^4*b
+ 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4*b + 3
*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^3 + 3*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 +
2*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + 2
*(9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*cosh(d*x + c)^4 + 2
*(9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5 + 35*(3*a^4*b + 3*a^
3*b^2 + a^2*b^3)*cosh(d*x + c)^4 + 30*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*
a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(3*a^4*b + 3*a^3*b^2 + a^2*b
^3)*cosh(d*x + c)^5 + 10*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh(d
*x + c)^3 + (9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 4*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh
(d*x + c)^2 + 4*(7*(3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^6 + 3*a^4*
b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 15*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 +
2*a*b^4)*cosh(d*x + c)^4 + 3*(9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4
+ 8*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4*b + 3*a^3*b^2 + a^2*
b^3)*cosh(d*x + c)^7 + 3*(3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh(d
*x + c)^5 + (9*a^4*b + 33*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*cosh(d*x

```
 + c)^3 + (3*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh(d*x + c))*sinh(d
*x + c))*log(2*(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x
+ c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(a^5*cosh(d*
x + c)^8 + 8*a^5*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*sinh(d*x + c)^8 + 4*(a
^5 + 2*a^4*b)*cosh(d*x + c)^6 + 4*(7*a^5*cosh(d*x + c)^2 + a^5 + 2*a^4*b)*s
inh(d*x + c)^6 + 8*(7*a^5*cosh(d*x + c)^3 + 3*(a^5 + 2*a^4*b)*cosh(d*x + c)
)*sinh(d*x + c)^5 + a^5 + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(d*x + c)^4 +
 2*(35*a^5*cosh(d*x + c)^4 + 3*a^5 + 8*a^4*b + 8*a^3*b^2 + 30*(a^5 + 2*a^4*
b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a^5*cosh(d*x + c)^5 + 10*(a^5 +
2*a^4*b)*cosh(d*x + c)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(a^5 + 2*a^4*b)*cosh(d*x + c)^2 + 4*(7*a^5*cosh(d*x + c)^6
 + a^5 + 2*a^4*b + 15*(a^5 + 2*a^4*b)*cosh(d*x + c)^4 + 3*(3*a^5 + 8*a^4*b
+ 8*a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a^5*cosh(d*x + c)^7 + 3*
(a^5 + 2*a^4*b)*cosh(d*x + c)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(d*x +
c)^3 + (a^5 + 2*a^4*b)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(c
osh(d*x + c) - sinh(d*x + c))) + 8*(2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)
*d*x*cosh(d*x + c)^7 - 3*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4 - 2*(a^5 + 5*a^4*
b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*d*x)*cosh(d*x + c)^5 - 2*(6*a^3*b^2 +
20*a^2*b^3 + 20*a*b^4 + 6*b^5 - (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3
 + 32*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^3 - (3*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^6\,\coth(c+d\,x)}{{\left(a\,{\cosh\left(c+d\,x\right)}^2+b\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(a + b/cosh(c + d*x)^2)^3,x)
[Out] int((cosh(c + d*x)^6*coth(c + d*x))/(b + a*cosh(c + d*x)^2)^3, x)
```

3.166 $\int \frac{\coth^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=182

$$\frac{x}{a^3}-\frac{b^{3/2}(35a^2+28ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{7/2}d}-\frac{(8a^2-11ab-4b^2)\coth(c+dx)}{8a^2(a+b)^3d}-\frac{b\coth(\cdots}{4a(a+b)d\,(a+b-\cdots}$$

```
[Out] x/a^3-1/8*b^(3/2)*(35*a^2+28*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(
1/2))/a^3/(a+b)^(7/2)/d-1/8*(8*a^2-11*a*b-4*b^2)*coth(d*x+c)/a^2/(a+b)^3/d-
1/4*b*coth(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*b*(9*a+4*b)*coth(d*
x+c)/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.28, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4226, 2000, 483, 593, 597, 536, 212, 214}

$$\frac{x}{a^3}-\frac{(8a^2-11ab-4b^2)\coth(c+dx)}{8a^2d(a+b)^3}-\frac{b(9a+4b)\coth(c+dx)}{8a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{b^{3/2}(35a^2+28ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{7/2}}-\frac{b\coth(c+dx)}{4ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] x/a^3 - (b^(3/2)*(35*a^2 + 28*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a + b]])/(8*a^3*(a + b)^(7/2)*d) - ((8*a^2 - 11*a*b - 4*b^2)*Coth[c +
d*x])/(8*a^2*(a + b)^3*d) - (b*Coth[c + d*x])/(4*a*(a + b)*d*(a + b - b*Tan
h[c + d*x]^2)^2) - (b*(9*a + 4*b)*Coth[c + d*x])/(8*a^2*(a + b)^2*d*(a + b
- b*Tanh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
```

```
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
```

```
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+b(1-x^2))^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\coth(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{-4a+b-5bx^2}{x^2(1-x^2)(a+b-bx^2)^2}\,dx, x, \text{ta}\right.}{4a(a+b)d}$$

$$= -\frac{b\coth(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(9a+4b)\coth(c+dx)}{8a^2(a+b)^2d\left(a+b-b\tanh^2(c+\right.}$$

$$= -\frac{(8a^2-11ab-4b^2)\coth(c+dx)}{8a^2(a+b)^3d} - \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} -$$

$$= -\frac{(8a^2-11ab-4b^2)\coth(c+dx)}{8a^2(a+b)^3d} - \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} -$$

$$= \frac{x}{a^3} - \frac{b^{3/2}(35a^2+28ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{7/2}d} - \frac{(8a^2-11ab-4b^2)\,\text{c}}{8a^2(a+b)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1769 vs. 2(182) = 364.
time = 6.07, size = 1769, normalized size = 9.72

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cosh[2*(c + d*x)])*Sech[c + d*x]^6*((-8*b^2*(35*a^2 + 28*a*b
+ 8*b^2)*ArcTanh[(Sech[d*x]*(Cosh[2*c] - Sinh[2*c])*((a + 2*b)*Sinh[d*x] -
a*Sinh[2*c + d*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cosh[c] - Sinh[c])^4])]*(a + 2*b
 + a*Cosh[2*(c + d*x)])^2*(Cosh[2*c] - Sinh[2*c]))/(Sqrt[a + b]*Sqrt[b*(Cos
h[c] - Sinh[c])^4]) + Csch[c]*Csch[c + d*x]*Sech[2*c]*(8*(a + b)^3*(a^2 + 4
*a*b + 8*b^2)*d*x*Cosh[d*x] - 4*a*(a + b)^3*(3*a + 8*b)*d*x*Cosh[3*d*x] - 8
*a^5*d*x*Cosh[2*c - d*x] - 56*a^4*b*d*x*Cosh[2*c - d*x] - 184*a^3*b^2*d*x*C
```

```
osh[2*c - d*x] - 296*a^2*b^3*d*x*Cosh[2*c - d*x] - 224*a*b^4*d*x*Cosh[2*c -
 d*x] - 64*b^5*d*x*Cosh[2*c - d*x] - 8*a^5*d*x*Cosh[2*c + d*x] - 56*a^4*b*d
*x*Cosh[2*c + d*x] - 184*a^3*b^2*d*x*Cosh[2*c + d*x] - 296*a^2*b^3*d*x*Cosh
[2*c + d*x] - 224*a*b^4*d*x*Cosh[2*c + d*x] - 64*b^5*d*x*Cosh[2*c + d*x] +
8*a^5*d*x*Cosh[4*c + d*x] + 56*a^4*b*d*x*Cosh[4*c + d*x] + 184*a^3*b^2*d*x*
Cosh[4*c + d*x] + 296*a^2*b^3*d*x*Cosh[4*c + d*x] + 224*a*b^4*d*x*Cosh[4*c
+ d*x] + 64*b^5*d*x*Cosh[4*c + d*x] + 12*a^5*d*x*Cosh[2*c + 3*d*x] + 68*a^4
*b*d*x*Cosh[2*c + 3*d*x] + 132*a^3*b^2*d*x*Cosh[2*c + 3*d*x] + 108*a^2*b^3*
d*x*Cosh[2*c + 3*d*x] + 32*a*b^4*d*x*Cosh[2*c + 3*d*x] - 12*a^5*d*x*Cosh[4*
c + 3*d*x] - 68*a^4*b*d*x*Cosh[4*c + 3*d*x] - 132*a^3*b^2*d*x*Cosh[4*c + 3*
d*x] - 108*a^2*b^3*d*x*Cosh[4*c + 3*d*x] - 32*a*b^4*d*x*Cosh[4*c + 3*d*x] +
 12*a^5*d*x*Cosh[6*c + 3*d*x] + 68*a^4*b*d*x*Cosh[6*c + 3*d*x] + 132*a^3*b^
2*d*x*Cosh[6*c + 3*d*x] + 108*a^2*b^3*d*x*Cosh[6*c + 3*d*x] + 32*a*b^4*d*x*
Cosh[6*c + 3*d*x] - 4*a^5*d*x*Cosh[2*c + 5*d*x] - 12*a^4*b*d*x*Cosh[2*c + 5
*d*x] - 12*a^3*b^2*d*x*Cosh[2*c + 5*d*x] - 4*a^2*b^3*d*x*Cosh[2*c + 5*d*x]
+ 4*a^5*d*x*Cosh[4*c + 5*d*x] + 12*a^4*b*d*x*Cosh[4*c + 5*d*x] + 12*a^3*b^2
*d*x*Cosh[4*c + 5*d*x] + 4*a^2*b^3*d*x*Cosh[4*c + 5*d*x] - 4*a^5*d*x*Cosh[6
*c + 5*d*x] - 12*a^4*b*d*x*Cosh[6*c + 5*d*x] - 12*a^3*b^2*d*x*Cosh[6*c + 5*
d*x] - 4*a^2*b^3*d*x*Cosh[6*c + 5*d*x] + 4*a^5*d*x*Cosh[8*c + 5*d*x] + 12*a
^4*b*d*x*Cosh[8*c + 5*d*x] + 12*a^3*b^2*d*x*Cosh[8*c + 5*d*x] + 4*a^2*b^3*d
*x*Cosh[8*c + 5*d*x] - 32*a^5*Sinh[d*x] - 64*a^4*b*Sinh[d*x] - 30*a^2*b^3*S
inh[d*x] - 120*a*b^4*Sinh[d*x] - 48*b^5*Sinh[d*x] + 32*a^5*Sinh[3*d*x] + 64
*a^4*b*Sinh[3*d*x] + 26*a^3*b^2*Sinh[3*d*x] + 86*a^2*b^3*Sinh[3*d*x] + 32*a
*b^4*Sinh[3*d*x] - 48*a^5*Sinh[2*c - d*x] - 128*a^4*b*Sinh[2*c - d*x] - 128
*a^3*b^2*Sinh[2*c - d*x] - 30*a^2*b^3*Sinh[2*c - d*x] - 120*a*b^4*Sinh[2*c
- d*x] - 48*b^5*Sinh[2*c - d*x] + 48*a^5*Sinh[2*c + d*x] + 128*a^4*b*Sinh[2
*c + d*x] + 102*a^3*b^2*Sinh[2*c + d*x] - 86*a^2*b^3*Sinh[2*c + d*x] - 136*
a*b^4*Sinh[2*c + d*x] - 48*b^5*Sinh[2*c + d*x] - 32*a^5*Sinh[4*c + d*x] - 6
4*a^4*b*Sinh[4*c + d*x] + 26*a^3*b^2*Sinh[4*c + d*x] + 86*a^2*b^3*Sinh[4*c
+ d*x] + 136*a*b^4*Sinh[4*c + d*x] + 48*b^5*Sinh[4*c + d*x] - 8*a^5*Sinh[2*
c + 3*d*x] - 26*a^3*b^2*Sinh[2*c + 3*d*x] - 86*a^2*b^3*Sinh[2*c + 3*d*x] -
32*a*b^4*Sinh[2*c + 3*d*x] + 32*a^5*Sinh[4*c + 3*d*x] + 64*a^4*b*Sinh[4*c +
 3*d*x] - 13*a^3*b^2*Sinh[4*c + 3*d*x] - 36*a^2*b^3*Sinh[4*c + 3*d*x] - 16*
a*b^4*Sinh[4*c + 3*d*x] - 8*a^5*Sinh[6*c + 3*d*x] + 13*a^3*b^2*Sinh[6*c + 3
*d*x] + 36*a^2*b^3*Sinh[6*c + 3*d*x] + 16*a*b^4*Sinh[6*c + 3*d*x] + 8*a^5*S
inh[2*c + 5*d*x] + 13*a^3*b^2*Sinh[2*c + 5*d*x] + 6*a^2*b^3*Sinh[2*c + 5*d*
x] - 13*a^3*b^2*Sinh[4*c + 5*d*x] - 6*a^2*b^3*Sinh[4*c + 5*d*x] + 8*a^5*Sin
h[6*c + 5*d*x])))/(512*a^3*(a + b)^3*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(166) = 332$.

time = 3.10, size = 394, normalized size = 2.16 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)+2*b^2/(a+b)^3/a^3*(
((-13/8*a^3-17/8*a^2*b-1/2*a*b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(39*a^2+7*a*b-4
*b^2)*a*tanh(1/2*d*x+1/2*c)^5-1/8*(39*a^2+7*a*b-4*b^2)*a*tanh(1/2*d*x+1/2*c
)^3+(-13/8*a^3-17/8*a^2*b-1/2*a*b^2)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1
/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+
1/2*c)^2+a+b)^2+1/8*(35*a^2+28*a*b+8*b^2)*(-1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b
)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))+1/
4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1
/2*c)*b^(1/2)+(a+b)^(1/2))))-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)-1/2/(a+b)^3/ta
nh(1/2*d*x+1/2*c)+1/a^3*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1971 vs. $2(172) = 344$.
time = 0.68, size = 1971, normalized size = 10.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*a^2*b + 3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x
+ 2*c) + a)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) - 1/4*(3*a^2*b + 3*a*
b^2 + b^3)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^6
 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) - 1/64*(15*a^3*b + 70*a^2*b^2 + 56*a*b
^3 + 16*b^4)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*
d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^
3*b^3)*sqrt((a + b)*b)*d) + 1/64*(15*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4
)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c
) + a + 2*b + 2*sqrt((a + b)*b)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sq
rt((a + b)*b)*d) - 15/32*b*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a +
b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*sqrt((a + b)*b)*d) + 1/16*(8*a^5 + 9*a^4*b + 28*a^3*b^2 +
12*a^2*b^3 + (8*a^5 - 9*a^4*b - 98*a^3*b^2 - 160*a^2*b^3 - 64*a*b^4)*e^(8*d
*x + 8*c) + 2*(16*a^5 + 23*a^4*b - 77*a^3*b^2 - 246*a^2*b^3 - 288*a*b^4 - 9
6*b^5)*e^(6*d*x + 6*c) + 2*(24*a^5 + 64*a^4*b + 99*a^3*b^2 + 190*a^2*b^3 +
272*a*b^4 + 96*b^5)*e^(4*d*x + 4*c) + 2*(16*a^5 + 41*a^4*b + 77*a^3*b^2 + 1
30*a^2*b^3 + 48*a*b^4)*e^(2*d*x + 2*c))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b
^3 - (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*e^(10*d*x + 10*c) - (3*a^8 + 17*
a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*e^(8*d*x + 8*c) - 2*(a^8 + 7*a
^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^(6*d*x + 6*c) +
2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^(4*d
*x + 4*c) + (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*e^(2*d
*x + 2*c))*d) - 1/16*(8*a^5 + 9*a^4*b + 28*a^3*b^2 + 12*a^2*b^3 + 2*(16*a^5
 + 41*a^4*b + 77*a^3*b^2 + 130*a^2*b^3 + 48*a*b^4)*e^(-2*d*x - 2*c) + 2*(24
*a^5 + 64*a^4*b + 99*a^3*b^2 + 190*a^2*b^3 + 272*a*b^4 + 96*b^5)*e^(-4*d*x
```

```
- 4*c) + 2*(16*a^5 + 23*a^4*b - 77*a^3*b^2 - 246*a^2*b^3 - 288*a*b^4 - 96*b
^5)*e^(-6*d*x - 6*c) + (8*a^5 - 9*a^4*b - 98*a^3*b^2 - 160*a^2*b^3 - 64*a*b
^4)*e^(-8*d*x - 8*c))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + (3*a^8 + 17*a
^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*e^(-2*d*x - 2*c) + 2*(a^8 + 7*a
^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^(-4*d*x - 4*c) -
 2*(a^8 + 7*a^7*b + 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^(-6
*d*x - 6*c) - (3*a^8 + 17*a^7*b + 33*a^6*b^2 + 27*a^5*b^3 + 8*a^4*b^4)*e^(-
8*d*x - 8*c) - (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*e^(-10*d*x - 10*c))*d)
 - 1/8*(8*a^4 - 9*a^3*b - 2*a^2*b^2 + 2*(16*a^4 + 23*a^3*b - 27*a^2*b^2 - 4
*a*b^3)*e^(-2*d*x - 2*c) + 2*(24*a^4 + 64*a^3*b + 53*a^2*b^2 - 40*a*b^3 - 8
*b^4)*e^(-4*d*x - 4*c) + 2*(16*a^4 + 41*a^3*b + 27*a^2*b^2 + 40*a*b^3 + 8*b
^4)*e^(-6*d*x - 6*c) + (8*a^4 + 9*a^3*b + 24*a^2*b^2 + 8*a*b^3)*e^(-8*d*x -
 8*c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3 + (3*a^7 + 17*a^6*b + 33*a^5*b
^2 + 27*a^4*b^3 + 8*a^3*b^4)*e^(-2*d*x - 2*c) + 2*(a^7 + 7*a^6*b + 23*a^5*b
^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*e^(-4*d*x - 4*c) - 2*(a^7 + 7*a^6
*b + 23*a^5*b^2 + 37*a^4*b^3 + 28*a^3*b^4 + 8*a^2*b^5)*e^(-6*d*x - 6*c) - (
3*a^7 + 17*a^6*b + 33*a^5*b^2 + 27*a^4*b^3 + 8*a^3*b^4)*e^(-8*d*x - 8*c) -
(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*e^(-10*d*x - 10*c))*d) + 1/2*log(e^(2
*d*x + 2*c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/2*log(e^(-2*d*x -
2*c) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5665 vs. $2(172) = 344$.
time = 0.52, size = 11606, normalized size = 63.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^10 + 160*
(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 1
6*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*sinh(d*x + c)^10 - 4*(8*a^5 - 1
3*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a
^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^8 - 4*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3
 - 16*a*b^4 - 180*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2
 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*sinh(d*x +
 c)^8 + 32*(60*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 -
(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*
b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 8*(16*a^5
 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*
b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^6 + 8*(4
20*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 - 16*a^5 - 32*
a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 68*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23
*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x - 14*(8*a^5 - 13*a^3*b^2 - 36
```

```
*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b
^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(252*(a^5 + 3*a^4*b + 3*a^3*
b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 - 14*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 -
16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*co
sh(d*x + c)^3 - 3*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 68*a*b^4 -
 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*
x)*cosh(d*x + c))*sinh(d*x + c)^5 - 32*a^5 - 52*a^3*b^2 - 24*a^2*b^3 - 8*(2
4*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7
*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 +
 8*(420*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^6 - 24*a^5
- 64*a^4*b - 64*a^3*b^2 - 15*a^2*b^3 - 60*a*b^4 - 24*b^5 - 35*(8*a^5 - 13*a
^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*
b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^4 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^
2*b^3 + 28*a*b^4 + 8*b^5)*d*x - 15*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2
*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*
a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(60*(a^5 + 3*a^4*
b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^7 - 7*(8*a^5 - 13*a^3*b^2 - 36*a
^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4
)*d*x)*cosh(d*x + c)^5 - 5*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 43*a^2*b^3 - 6
8*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 +
8*b^5)*d*x)*cosh(d*x + c)^3 - (24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b^3
+ 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4
 + 8*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 16*(a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*d*x - 8*(16*a^5 + 32*a^4*b + 13*a^3*b^2 + 43*a^2*b^3 + 16*a*b
^4 + 2*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x
 + c)^2 + 8*(90*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 -
 14*(8*a^5 - 13*a^3*b^2 - 36*a^2*b^3 - 16*a*b^4 - 4*(3*a^5 + 17*a^4*b + 33*
a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*d*x)*cosh(d*x + c)^6 - 16*a^5 - 32*a^4*b -
13*a^3*b^2 - 43*a^2*b^3 - 16*a*b^4 - 15*(16*a^5 + 32*a^4*b - 13*a^3*b^2 - 4
3*a^2*b^3 - 68*a*b^4 - 24*b^5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3
+ 28*a*b^4 + 8*b^5)*d*x)*cosh(d*x + c)^4 - 2*(3*a^5 + 17*a^4*b + 33*a^3*b^2
 + 27*a^2*b^3 + 8*a*b^4)*d*x - 6*(24*a^5 + 64*a^4*b + 64*a^3*b^2 + 15*a^2*b
^3 + 60*a*b^4 + 24*b^5 + 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*
b^4 + 8*b^5)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((35*a^4*b + 28*a^3*b^
2 + 8*a^2*b^3)*cosh(d*x + c)^10 + 10*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*co
sh(d*x + c)*sinh(d*x + c)^9 + (35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*sinh(d*x
+ c)^10 + (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^
8 + (105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^4 + 45*(35*a^4*b + 28*a
^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 28
*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^3 + (105*a^4*b + 364*a^3*b^2 + 248*a^2*
b^3 + 64*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(35*a^4*b + 168*a^3*b^2
+ 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x + c)^6 + 2*(35*a^4*b + 168*a^3
*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5 + 105*(35*a^4*b + 28*a^3*b^2 + 8*a^
2*b^3)*cosh(d*x + c)^4 + 14*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b
^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(35*a^4*b + 28*a^3*b^2 + 8*a^2
```

```
*b^3)*cosh(d*x + c)^5 + 14*(105*a^4*b + 364*a^3*b^2 + 248*a^2*b^3 + 64*a*b^
4)*cosh(d*x + c)^3 + 3*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 +
64*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 35*a^4*b - 28*a^3*b^2 - 8*a^2*b^3
- 2*(35*a^4*b + 168*a^3*b^2 + 400*a^2*b^3 + 256*a*b^4 + 64*b^5)*cosh(d*x +
c)^4 + 2*(105*(35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cosh(d*x + c)^6 - 35*a^4*
b - 168*a^3*b^2 - 400*a^2*b^3 - 256*a*b^4 - 64*b^5 + 35*(105*a^4*b + 364*a^
3*b^2 + 248*a^2*b^3 + 64*a*b^4)*cosh(d*x + c)^4...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(172) = 344.
time = 1.76, size = 389, normalized size = 2.14

$$-\frac{\frac{(35\,a^2b^2+28\,ab^3+8\,b^4)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\sqrt{-ab-b^2}}-\frac{2\,(13\,a^3b^2e^{(6\,dx+6\,c)}+36\,a^2b^3e^{(6\,dx+6\,c)}+16\,ab^4e^{(6\,dx+6\,c)}+39\,a^3b^2e^{(4\,dx+4\,c)}+122\,a^2b^3e^{(4\,dx+4\,c)}+152\,ab^4e^{(4\,dx+4\,c)}+48\,b^5e^{(4\,dx+4\,c)}+39\,a^3b^2e^{(2\,dx+2\,c)}+92\,a^2b^3e^{(2\,dx+2\,c)}+32\,ab^4e^{(2\,dx+2\,c)}+13\,a^3b^2+6\,a^2b^3)}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\left(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a\right)^2}-\frac{8\,(dx+c)}{a^3}+\frac{16}{(a^3+3\,a^2b+3\,ab^2+b^3)\left(e^{(2\,dx+2\,c)}-1\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((35*a^2*b^2 + 28*a*b^3 + 8*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + a + 2
*b)/sqrt(-a*b - b^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt(-a*b - b^
2)) - 2*(13*a^3*b^2*e^(6*d*x + 6*c) + 36*a^2*b^3*e^(6*d*x + 6*c) + 16*a*b^4
*e^(6*d*x + 6*c) + 39*a^3*b^2*e^(4*d*x + 4*c) + 122*a^2*b^3*e^(4*d*x + 4*c)
 + 152*a*b^4*e^(4*d*x + 4*c) + 48*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d*x
 + 2*c) + 92*a^2*b^3*e^(2*d*x + 2*c) + 32*a*b^4*e^(2*d*x + 2*c) + 13*a^3*b^
2 + 6*a^2*b^3)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*(a*e^(4*d*x + 4*c) +
2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^2) - 8*(d*x + c)/a^3 + 16/((
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^6\,\coth(c+d\,x)^2}{\left(a\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(a + b/cosh(c + d*x)^2)^3,x)
```

```
[Out] int((cosh(c + d*x)^6*coth(c + d*x)^2)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.167 $\int \frac{\coth^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{b^4}{4a^3(a+b)^2d\left(b+a\cosh^2(c+dx)\right)^2}+\frac{b^3(2a+b)}{a^3(a+b)^3d\left(b+a\cosh^2(c+dx)\right)}-\frac{\operatorname{csch}^2(c+dx)}{2(a+b)^3d}+\frac{b^2(6a^2+4ab+b^2}{2a}$$

```
[Out] -1/4*b^4/a^3/(a+b)^2/d/(b+a*cosh(d*x+c)^2)^2+b^3*(2*a+b)/a^3/(a+b)^3/d/(b+a
*cosh(d*x+c)^2)-1/2*csch(d*x+c)^2/(a+b)^3/d+1/2*b^2*(6*a^2+4*a*b+b^2)*ln(b+
a*cosh(d*x+c)^2)/a^3/(a+b)^4/d+(a+4*b)*ln(sinh(d*x+c))/(a+b)^4/d
```

Rubi [A]
time = 0.17, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4223, 457, 90}

$$-\frac{b^4}{4a^3d(a+b)^2\left(a\cosh^2(c+dx)+b\right)^2}+\frac{b^3(2a+b)}{a^3d(a+b)^3\left(a\cosh^2(c+dx)+b\right)}+\frac{b^2(6a^2+4ab+b^2)\log\left(a\cosh^2(c+dx)+b\right)}{2a^3d(a+b)^4}-\frac{\operatorname{csch}^2(c+dx)}{2d(a+b)^3}+\frac{(a+4b)\log(\sinh(c+dx))}{d(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/4*b^4/(a^3*(a + b)^2*d*(b + a*Cosh[c + d*x]^2)^2) + (b^3*(2*a + b))/(a^3
*(a + b)^3*d*(b + a*Cosh[c + d*x]^2)) - Csch[c + d*x]^2/(2*(a + b)^3*d) + (
b^2*(6*a^2 + 4*a*b + b^2)*Log[b + a*Cosh[c + d*x]^2])/(2*a^3*(a + b)^4*d) +
 ((a + 4*b)*Log[Sinh[c + d*x]])/((a + b)^4*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4223

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-(f
```

```
*ff^(m + n*p - 1))^(-1), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x
)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^9}{(1-x^2)^2(b+ax^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1-x)^2(b+ax)^3}\,dx, x, \cosh^2(c+dx)\right)}{2d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{(a+b)^3(-1+x)^2} + \frac{a+4b}{(a+b)^4(-1+x)} + \frac{b^4}{a^2(a+b)^2(b+ax)^3} - \frac{2b^3(2a+b)}{a^2(a+b)^3(b+ax)^2} + \frac{b^2(6}{a^2(}\right.\right.}{2d}$$

$$= -\frac{b^4}{4a^3(a+b)^2d\left(b+a\cosh^2(c+dx)\right)^2} + \frac{b^3(2a+b)}{a^3(a+b)^3d\left(b+a\cosh^2(c+dx)\right)} -$$

Mathematica [A]
time = 1.24, size = 172, normalized size = 1.13

$$-\frac{(a+2b+a\cosh(2(c+dx)))^3\operatorname{sech}^6(c+dx)\left(2(a+b)\operatorname{csch}^2(c+dx)-4(a+4b)\log(\sinh(c+dx))-\frac{2b^2(6a^2+4ab+b^2)\log\left(a+b+a\sinh^2(c+dx)\right)}{a^3}+\frac{b^4(a+b)^2}{a^3\left(a+b+a\sinh^2(c+dx)\right)^2}-\frac{4b^3(a+b)(2a+b)}{a^3\left(a+b+a\sinh^2(c+dx)\right)}\right)}{32(a+b)^4d\left(a+b\operatorname{sech}^2(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] -1/32*((a + 2*b + a*Cosh[2*(c + d*x)])^3*Sech[c + d*x]^6*(2*(a + b)*Csch[c
+ d*x]^2 - 4*(a + 4*b)*Log[Sinh[c + d*x]] - (2*b^2*(6*a^2 + 4*a*b + b^2)*Lo
g[a + b + a*Sinh[c + d*x]^2])/a^3 + (b^4*(a + b)^2)/(a^3*(a + b + a*Sinh[c
+ d*x]^2)^2) - (4*b^3*(a + b)*(2*a + b))/(a^3*(a + b + a*Sinh[c + d*x]^2)))
)/((a + b)^4*d*(a + b*Sech[c + d*x]^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(146) = 292$.
time = 3.37, size = 354, normalized size = 2.33 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*tanh(1/2*d*x+1/2*c)^2/(a^3+3*a^2*b+3*a*b^2+b^3)+b^2/(a+b)^4/a^3*(
((-8*a^3*b-10*a^2*b^2-2*a*b^3)*tanh(1/2*d*x+1/2*c)^6-4*(4*a^2-2*a*b-b^2)*a*
b*tanh(1/2*d*x+1/2*c)^4-2*(4*a^2+5*a*b+b^2)*a*b*tanh(1/2*d*x+1/2*c)^2)/(a*t
```

```
anh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*
tanh(1/2*d*x+1/2*c)^2+a+b)^2+1/2*(6*a^2+4*a*b+b^2)*ln(a*tanh(1/2*d*x+1/2*c)
^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c
)^2+a+b))-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)-1/8/(a+b)^3/tanh(1/2*d*x+1/2*c)^2
+1/4/(a+b)^4*(4*a+16*b)*ln(tanh(1/2*d*x+1/2*c))-1/a^3*ln(tanh(1/2*d*x+1/2*c
)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(146) = 292$.
time = 0.31, size = 692, normalized size = 4.55

$$\frac{(6\,a^2b^2+4\,ab^3+b^4)\log\left(2\,(a+2\,b)e^{(-2\,dx-2\,c)}+ae^{(-4\,dx-4\,c)}+a\right)}{2\,(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4)d}+\frac{(a+4\,b)\log\left(e^{(-dx-c)}+1\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)d}+\frac{(a+4\,b)\log\left(e^{(-dx-c)}-1\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)d}-\frac{2\left((a^5-4\,a^2b^3-2\,ab^4)e^{(-2\,dx-2\,c)}+2\,(2\,a^5+4\,a^4b-7\,ab^4-3\,b^5)e^{(-4\,dx-4\,c)}+2\,(3\,a^5+8\,a^4b+8\,a^3b^2+4\,a^2b^3+16\,ab^4+6\,b^5)e^{(-6\,dx-6\,c)}+2\,(2\,a^5+4\,a^4b-7\,ab^4-3\,b^5)e^{(-8\,dx-8\,c)}+(a^5-4\,a^2b^3-2\,ab^4)e^{(-10\,dx-10\,c)}\right)}{\left(a^8+3\,a^7b+3\,a^6b^2+a^5b^3+2\,(a^8+7\,a^7b+15\,a^6b^2+13\,a^5b^3+4\,a^4b^4)e^{(-2\,dx-2\,c)}-(a^8+3\,a^7b-13\,a^6b^2-47\,a^5b^3-48\,a^4b^4-16\,a^3b^5)e^{(-4\,dx-4\,c)}-4\,(a^8+7\,a^7b+23\,a^6b^2+37\,a^5b^3+28\,a^4b^4+8\,a^3b^5)e^{(-6\,dx-6\,c)}-(a^8+3\,a^7b-13\,a^6b^2-47\,a^5b^3-48\,a^4b^4-16\,a^3b^5)e^{(-8\,dx-8\,c)}+2\,(a^8+7\,a^7b+15\,a^6b^2+13\,a^5b^3+4\,a^4b^4)e^{(-10\,dx-10\,c)}+(a^8+3\,a^7b+3\,a^6b^2+a^5b^3)e^{(-12\,dx-12\,c)}\right)d}+\frac{dx+c}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*a^2*b^2 + 4*a*b^3 + b^4)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*
d*x - 4*c) + a)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d) + (a
+ 4*b)*log(e^(-d*x - c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d
) + (a + 4*b)*log(e^(-d*x - c) - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
 b^4)*d) - 2*((a^5 - 4*a^2*b^3 - 2*a*b^4)*e^(-2*d*x - 2*c) + 2*(2*a^5 + 4*a
^4*b - 7*a*b^4 - 3*b^5)*e^(-4*d*x - 4*c) + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2 +
 4*a^2*b^3 + 16*a*b^4 + 6*b^5)*e^(-6*d*x - 6*c) + 2*(2*a^5 + 4*a^4*b - 7*a*
b^4 - 3*b^5)*e^(-8*d*x - 8*c) + (a^5 - 4*a^2*b^3 - 2*a*b^4)*e^(-10*d*x - 10
*c))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^8 + 7*a^7*b + 15*a^6*b^2
+ 13*a^5*b^3 + 4*a^4*b^4)*e^(-2*d*x - 2*c) - (a^8 + 3*a^7*b - 13*a^6*b^2 -
47*a^5*b^3 - 48*a^4*b^4 - 16*a^3*b^5)*e^(-4*d*x - 4*c) - 4*(a^8 + 7*a^7*b +
 23*a^6*b^2 + 37*a^5*b^3 + 28*a^4*b^4 + 8*a^3*b^5)*e^(-6*d*x - 6*c) - (a^8
+ 3*a^7*b - 13*a^6*b^2 - 47*a^5*b^3 - 48*a^4*b^4 - 16*a^3*b^5)*e^(-8*d*x -
8*c) + 2*(a^8 + 7*a^7*b + 15*a^6*b^2 + 13*a^5*b^3 + 4*a^4*b^4)*e^(-10*d*x -
 10*c) + (a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*e^(-12*d*x - 12*c))*d) + (d*
x + c)/(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 10255 vs. $2(146) = 292$.
time = 0.92, size = 10255, normalized size = 67.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)
^12 + 24*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c
)*sinh(d*x + c)^11 + 2*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*
x*sinh(d*x + c)^12 + 4*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^
```

6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x
 + c)^10 + 4*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + 33*(a^6 + 4*a
^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^2 + (a^6 + 8*a^5*
b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*sinh(d*x + c)^10 +
 40*(11*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)
^3 + (a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a
^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c))*sinh(d*x +
c)^9 + 2*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 -
(a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*
d*x)*cosh(d*x + c)^8 + 2*(495*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*
b^4)*d*x*cosh(d*x + c)^4 + 8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*
a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64
*a*b^5 - 16*b^6)*d*x + 90*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 +
(a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(
d*x + c)^2)*sinh(d*x + c)^8 + 16*(99*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3
 + a^2*b^4)*d*x*cosh(d*x + c)^5 + 30*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 -
 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)
*d*x)*cosh(d*x + c)^3 + (8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*
b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a
*b^5 - 16*b^6)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 + 8*(3*a^6 + 11*a^5*b +
16*a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b +
30*a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*cosh(d*x + c)
^6 + 8*(231*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x
+ c)^6 + 3*a^6 + 11*a^5*b + 16*a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5
 + 6*b^6 + 105*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^
5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^4
- (a^6 + 8*a^5*b + 30*a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)
*d*x + 7*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 -
(a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*
d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(99*(a^6 + 4*a^5*b + 6*a^4*b^2 +
 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^7 + 63*(a^6 + a^5*b - 4*a^3*b^3 - 6
*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4
+ 4*a*b^5)*d*x)*cosh(d*x + c)^5 + 7*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2
*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^
2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^3 + 3*(3*a^6 + 11*a^5*b + 16*
a^4*b^2 + 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30*
a^4*b^2 + 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*cosh(d*x + c))*s
inh(d*x + c)^5 + 2*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 -
 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5
- 16*b^6)*d*x)*cosh(d*x + c)^4 + 2*(495*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*
b^3 + a^2*b^4)*d*x*cosh(d*x + c)^8 + 420*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b
^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*
b^5)*d*x)*cosh(d*x + c)^6 + 8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40
*a*b^5 - 12*b^6 + 70*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5
 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^

```
5 - 16*b^6)*d*x)*cosh(d*x + c)^4 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3
 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x + 60*(3*a^6 + 11*a^5*b + 16*a^4*b^2
+ 12*a^3*b^3 + 20*a^2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30*a^4*b^2
+ 60*a^3*b^3 + 65*a^2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 8*(55*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh
(d*x + c)^9 + 60*(a^6 + a^5*b - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*
a^5*b + 22*a^4*b^2 + 28*a^3*b^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^
7 + 14*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a
^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*
x)*cosh(d*x + c)^5 + 20*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 12*a^3*b^3 + 20*a^
2*b^4 + 22*a*b^5 + 6*b^6 - (a^6 + 8*a^5*b + 30*a^4*b^2 + 60*a^3*b^3 + 65*a^
2*b^4 + 36*a*b^5 + 8*b^6)*d*x)*cosh(d*x + c)^3 + (8*a^6 + 24*a^5*b + 16*a^4
*b^2 - 28*a^2*b^4 - 40*a*b^5 - 12*b^6 - (a^6 + 4*a^5*b - 10*a^4*b^2 - 60*a^
3*b^3 - 95*a^2*b^4 - 64*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3
 + 2*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x + 4*(a^6 + a^5*b
 - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5 + (a^6 + 8*a^5*b + 22*a^4*b^2 + 28*a^3*b
^3 + 17*a^2*b^4 + 4*a*b^5)*d*x)*cosh(d*x + c)^2 + 4*(33*(a^6 + 4*a^5*b + 6*
a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^6\,\coth(c+d\,x)^3}{\left(a\,\cosh\left(c+d\,x\right)^2+b\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3/(a + b/cosh(c + d*x)^2)^3,x)
[Out] int((cosh(c + d*x)^6*coth(c + d*x)^3)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.168 $\int \frac{\coth^4(c+dx)}{\left(a+b\operatorname{sech}^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=232

$$\frac{x}{a^3}-\frac{b^{5/2}(63a^2+36ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{9/2}d}-\frac{(8a^3+32a^2b-15ab^2-4b^3)\coth(c+dx)}{8a^2(a+b)^4d}-\frac{(8a^2-39$$

```
[Out] x/a^3-1/8*b^(5/2)*(63*a^2+36*a*b+8*b^2)*arctanh(b^(1/2)*tanh(d*x+c)/(a+b)^(
1/2))/a^3/(a+b)^(9/2)/d-1/8*(8*a^3+32*a^2*b-15*a*b^2-4*b^3)*coth(d*x+c)/a^2
/(a+b)^4/d-1/24*(8*a^2-39*a*b-12*b^2)*coth(d*x+c)^3/a^2/(a+b)^3/d-1/4*b*cot
h(d*x+c)^3/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^2-1/8*b*(11*a+4*b)*coth(d*x+c)^3
/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)
```

Rubi [A]
time = 0.35, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {4226, 2000, 483, 593, 597, 536, 212, 214}

$$\frac{x}{a^3}-\frac{(8a^2-39ab-12b^2)\coth^3(c+dx)}{24a^2d(a+b)^3}-\frac{b(11a+4b)\coth^3(c+dx)}{8a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)}-\frac{b^{5/2}(63a^2+36ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3d(a+b)^{9/2}}-\frac{(8a^3+32a^2b-15ab^2-4b^3)\coth(c+dx)}{8a^2d(a+b)^4}-\frac{b\coth^3(c+dx)}{4ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
```

```
[Out] x/a^3 - (b^(5/2)*(63*a^2 + 36*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a + b]])/(8*a^3*(a + b)^(9/2)*d) - ((8*a^3 + 32*a^2*b - 15*a*b^2 - 4*b
^3)*Coth[c + d*x])/(8*a^2*(a + b)^4*d) - ((8*a^2 - 39*a*b - 12*b^2)*Coth[c
+ d*x]^3)/(24*a^2*(a + b)^3*d) - (b*Coth[c + d*x]^3)/(4*a*(a + b)*d*(a + b
- b*Tanh[c + d*x]^2)^2) - (b*(11*a + 4*b)*Coth[c + d*x]^3)/(8*a^2*(a + b)^2
*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\coth^4(c+dx)}{\left(a+b\text{sech}^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+b(1-x^2))^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-4a+3b-7bx^2}{x^4(1-x^2)(a+b-bx^2)^2}\,dx, x, \text{ta}\right.}{4a(a+b)d}$$

$$= -\frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2} - \frac{b(11a+4b)\coth^3(c+dx)}{8a^2(a+b)^2d\left(a+b-b\tanh^2(c+\right.}$$

$$= -\frac{(8a^2-39ab-12b^2)\coth^3(c+dx)}{24a^2(a+b)^3d} - \frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^2}$$

$$= -\frac{(8a^3+32a^2b-15ab^2-4b^3)\coth(c+dx)}{8a^2(a+b)^4d} - \frac{(8a^2-39ab-12b^2)\coth^3(c+}{24a^2(a+b)^3d}$$

$$= -\frac{(8a^3+32a^2b-15ab^2-4b^3)\coth(c+dx)}{8a^2(a+b)^4d} - \frac{(8a^2-39ab-12b^2)\coth^3(c+}{24a^2(a+b)^3d}$$

$$= \frac{x}{a^3} - \frac{b^{5/2}(63a^2+36ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{9/2}d} - \frac{(8a^3+32a^2b-15ab}{8a^2(a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.16, size = 3334, normalized size = 14.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Sech[c + d*x]^2)^3,x]
[Out] ((63*a^2 + 36*a*b + 8*b^2)*(a + 2*b + a*Cosh[2*c + 2*d*x])^3*Sech[c + d*x]^
6*(((I/64)*b^3*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*C
```

osh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c]
- b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Cosh
[2*c])/(a^3*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) - ((I/64)*b^3*Ar
cTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh
[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]))*
(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c])/(a^3*Sqrt[
a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/((a + b)^4*(a + b*Sech[c + d*x]
^2)^3) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Csch[c]*Csch[c + d*x]^3*Sech[2*c]
*Sech[c + d*x]^6*(-36*a^6*d*x*Cosh[d*x] - 336*a^5*b*d*x*Cosh[d*x] - 1560*a^
4*b^2*d*x*Cosh[d*x] - 3600*a^3*b^3*d*x*Cosh[d*x] - 4260*a^2*b^4*d*x*Cosh[d*
x] - 2496*a*b^5*d*x*Cosh[d*x] - 576*b^6*d*x*Cosh[d*x] + 36*a^6*d*x*Cosh[3*d
*x] + 240*a^5*b*d*x*Cosh[3*d*x] + 408*a^4*b^2*d*x*Cosh[3*d*x] - 48*a^3*b^3*
d*x*Cosh[3*d*x] - 732*a^2*b^4*d*x*Cosh[3*d*x] - 672*a*b^5*d*x*Cosh[3*d*x] -
192*b^6*d*x*Cosh[3*d*x] + 36*a^6*d*x*Cosh[2*c - d*x] + 336*a^5*b*d*x*Cosh[
2*c - d*x] + 1560*a^4*b^2*d*x*Cosh[2*c - d*x] + 3600*a^3*b^3*d*x*Cosh[2*c -
d*x] + 4260*a^2*b^4*d*x*Cosh[2*c - d*x] + 2496*a*b^5*d*x*Cosh[2*c - d*x] +
576*b^6*d*x*Cosh[2*c - d*x] + 36*a^6*d*x*Cosh[2*c + d*x] + 336*a^5*b*d*x*C
osh[2*c + d*x] + 1560*a^4*b^2*d*x*Cosh[2*c + d*x] + 3600*a^3*b^3*d*x*Cosh[2
*c + d*x] + 4260*a^2*b^4*d*x*Cosh[2*c + d*x] + 2496*a*b^5*d*x*Cosh[2*c + d*
x] + 576*b^6*d*x*Cosh[2*c + d*x] - 36*a^6*d*x*Cosh[4*c + d*x] - 336*a^5*b*d
*x*Cosh[4*c + d*x] - 1560*a^4*b^2*d*x*Cosh[4*c + d*x] - 3600*a^3*b^3*d*x*Co
sh[4*c + d*x] - 4260*a^2*b^4*d*x*Cosh[4*c + d*x] - 2496*a*b^5*d*x*Cosh[4*c
+ d*x] - 576*b^6*d*x*Cosh[4*c + d*x] - 36*a^6*d*x*Cosh[2*c + 3*d*x] - 240*a
^5*b*d*x*Cosh[2*c + 3*d*x] - 408*a^4*b^2*d*x*Cosh[2*c + 3*d*x] + 48*a^3*b^3
*d*x*Cosh[2*c + 3*d*x] + 732*a^2*b^4*d*x*Cosh[2*c + 3*d*x] + 672*a*b^5*d*x*
Cosh[2*c + 3*d*x] + 192*b^6*d*x*Cosh[2*c + 3*d*x] + 36*a^6*d*x*Cosh[4*c + 3
*d*x] + 240*a^5*b*d*x*Cosh[4*c + 3*d*x] + 408*a^4*b^2*d*x*Cosh[4*c + 3*d*x]
- 48*a^3*b^3*d*x*Cosh[4*c + 3*d*x] - 732*a^2*b^4*d*x*Cosh[4*c + 3*d*x] - 6
72*a*b^5*d*x*Cosh[4*c + 3*d*x] - 192*b^6*d*x*Cosh[4*c + 3*d*x] - 36*a^6*d*x
*Cosh[6*c + 3*d*x] - 240*a^5*b*d*x*Cosh[6*c + 3*d*x] - 408*a^4*b^2*d*x*Cosh
[6*c + 3*d*x] + 48*a^3*b^3*d*x*Cosh[6*c + 3*d*x] + 732*a^2*b^4*d*x*Cosh[6*c
+ 3*d*x] + 672*a*b^5*d*x*Cosh[6*c + 3*d*x] + 192*b^6*d*x*Cosh[6*c + 3*d*x]
- 12*a^6*d*x*Cosh[2*c + 5*d*x] - 144*a^5*b*d*x*Cosh[2*c + 5*d*x] - 456*a^4
*b^2*d*x*Cosh[2*c + 5*d*x] - 624*a^3*b^3*d*x*Cosh[2*c + 5*d*x] - 396*a^2*b^
4*d*x*Cosh[2*c + 5*d*x] - 96*a*b^5*d*x*Cosh[2*c + 5*d*x] + 12*a^6*d*x*Cosh[
4*c + 5*d*x] + 144*a^5*b*d*x*Cosh[4*c + 5*d*x] + 456*a^4*b^2*d*x*Cosh[4*c +
5*d*x] + 624*a^3*b^3*d*x*Cosh[4*c + 5*d*x] + 396*a^2*b^4*d*x*Cosh[4*c + 5*
d*x] + 96*a*b^5*d*x*Cosh[4*c + 5*d*x] - 12*a^6*d*x*Cosh[6*c + 5*d*x] - 144*
a^5*b*d*x*Cosh[6*c + 5*d*x] - 456*a^4*b^2*d*x*Cosh[6*c + 5*d*x] - 624*a^3*b
^3*d*x*Cosh[6*c + 5*d*x] - 396*a^2*b^4*d*x*Cosh[6*c + 5*d*x] - 96*a*b^5*d*x
*Cosh[6*c + 5*d*x] + 12*a^6*d*x*Cosh[8*c + 5*d*x] + 144*a^5*b*d*x*Cosh[8*c
+ 5*d*x] + 456*a^4*b^2*d*x*Cosh[8*c + 5*d*x] + 624*a^3*b^3*d*x*Cosh[8*c + 5
*d*x] + 396*a^2*b^4*d*x*Cosh[8*c + 5*d*x] + 96*a*b^5*d*x*Cosh[8*c + 5*d*x]
- 12*a^6*d*x*Cosh[4*c + 7*d*x] - 48*a^5*b*d*x*Cosh[4*c + 7*d*x] - 72*a^4*b^
2*d*x*Cosh[4*c + 7*d*x] - 48*a^3*b^3*d*x*Cosh[4*c + 7*d*x] - 12*a^2*b^4*d*x

```
*Cosh[4*c + 7*d*x] + 12*a^6*d*x*Cosh[6*c + 7*d*x] + 48*a^5*b*d*x*Cosh[6*c +
 7*d*x] + 72*a^4*b^2*d*x*Cosh[6*c + 7*d*x] + 48*a^3*b^3*d*x*Cosh[6*c + 7*d*
x] + 12*a^2*b^4*d*x*Cosh[6*c + 7*d*x] - 12*a^6*d*x*Cosh[8*c + 7*d*x] - 48*a
^5*b*d*x*Cosh[8*c + 7*d*x] - 72*a^4*b^2*d*x*Cosh[8*c + 7*d*x] - 48*a^3*b^3*
d*x*Cosh[8*c + 7*d*x] - 12*a^2*b^4*d*x*Cosh[8*c + 7*d*x] + 12*a^6*d*x*Cosh[
10*c + 7*d*x] + 48*a^5*b*d*x*Cosh[10*c + 7*d*x] + 72*a^4*b^2*d*x*Cosh[10*c
+ 7*d*x] + 48*a^3*b^3*d*x*Cosh[10*c + 7*d*x] + 12*a^2*b^4*d*x*Cosh[10*c + 7
*d*x] - 128*a^6*Sinh[d*x] - 440*a^5*b*Sinh[d*x] - 1152*a^4*b^2*Sinh[d*x] -
1920*a^3*b^3*Sinh[d*x] + 228*a^2*b^4*Sinh[d*x] + 1320*a*b^5*Sinh[d*x] + 432
*b^6*Sinh[d*x] + 48*a^6*Sinh[3*d*x] + 104*a^5*b*Sinh[3*d*x] + 640*a^4*b^2*S
inh[3*d*x] + 1511*a^3*b^3*Sinh[3*d*x] - 528*a^2*b^4*Sinh[3*d*x] + 264*a*b^5
*Sinh[3*d*x] + 144*b^6*Sinh[3*d*x] - 32*a^6*Sinh[2*c - d*x] + 384*a^5*b*Sin
h[2*c - d*x] + 2048*a^4*b^2*Sinh[2*c - d*x] + 3072*a^3*b^3*Sinh[2*c - d*x]
+ 228*a^2*b^4*Sinh[2*c - d*x] + 1320*a*b^5*Sinh[2*c - d*x] + 432*b^6*Sinh[2
*c - d*x] + 32*a^6*Sinh[2*c + d*x] - 384*a^5*b*Sinh[2*c + d*x] - 2048*a^4*b
^2*Sinh[2*c + d*x] - 2919*a^3*b^3*Sinh[2*c + d*x] + 642*a^2*b^4*Sinh[2*c +
d*x] + 1416*a*b^5*Sinh[2*c + d*x] + 432*b^6*Sinh[2*c + d*x] - 128*a^6*Sinh[
4*c + d*x] - 440*a^5*b*Sinh[4*c + d*x] - 1152*a^4*b^2*Sinh[4*c + d*x] - 207
3*a^3*b^3*Sinh[4*c + d*x] - 642*a^2*b^4*Sinh[4*c + d*x] - 1416*a*b^5*Sinh[4
*c + d*x] - 432*b^6*Sinh[4*c + d*x] - 144*a^6*S...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(214) = 428$.
time = 3.47, size = 468, normalized size = 2.02 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8/(a^3+3*a^2*b+3*a*b^2+b^3)/(a+b)*(1/3*a*tanh(1/2*d*x+1/2*c)^3+1/3*
b*tanh(1/2*d*x+1/2*c)^3+5*a*tanh(1/2*d*x+1/2*c)+17*b*tanh(1/2*d*x+1/2*c))+2
*b^3/(a+b)^4/a^3*(((-17/8*a^3-21/8*a^2*b-1/2*a*b^2)*tanh(1/2*d*x+1/2*c)^7-1
/8*(51*a^2+3*a*b-4*b^2)*a*tanh(1/2*d*x+1/2*c)^5-1/8*(51*a^2+3*a*b-4*b^2)*a*
tanh(1/2*d*x+1/2*c)^3+(-17/8*a^3-21/8*a^2*b-1/2*a*b^2)*tanh(1/2*d*x+1/2*c))
/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2
-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)^2+1/8*(63*a^2+36*a*b+8*b^2)*(-1/4/b^(1/2)/(
a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/
2)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^
2-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))-1/24/(a+b)^3/tanh(1/2*d*x+1/
2*c)^3-1/8*(5*a+17*b)/(a+b)^4/tanh(1/2*d*x+1/2*c)+1/a^3*ln(tanh(1/2*d*x+1/2
*c)+1)-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4920 vs. $2(220) = 440$.
time = 1.11, size = 4920, normalized size = 21.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/8*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3 + 2*b^4)*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d) - 3/4*b*log(a*e^(4*d*x + 4*c) + 2*(a + 2*b)*e^(2*d*x + 2*c) + a)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 1/8*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3 + 2*b^4)*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d) + 3/4*b*log(2*(a + 2*b)*e^(-2*d*x - 2*c) + a*e^(-4*d*x - 4*c) + a)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) + 1/4*(2*a + 5*b)*log(e^(2*d*x + 2*c) - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) + 3/2*b*log(e^(2*d*x + 2*c) - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 1/4*(2*a + 5*b)*log(e^(-2*d*x - 2*c) - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 3/2*b*log(e^(-2*d*x - 2*c) - 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 1/256*(15*a^4*b + 260*a^3*b^2 + 504*a^2*b^3 + 288*a*b^4 + 64*b^5)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*sqrt((a + b)*b)*d) + 5/64*(3*a*b + 10*b^2)*log((a*e^(2*d*x + 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(2*d*x + 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 1/256*(15*a^4*b + 260*a^3*b^2 + 504*a^2*b^3 + 288*a*b^4 + 64*b^5)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*sqrt((a + b)*b)*d) - 5/64*(3*a*b + 10*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 15/128*(3*a*b - 4*b^2)*log((a*e^(-2*d*x - 2*c) + a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)*b)))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)*d) + 1/192*(176*a^6 + 275*a^5*b + 306*a^4*b^2 + 456*a^3*b^3 + 144*a^2*b^4 + 3*(96*a^6 + 111*a^5*b - 220*a^4*b^2 - 776*a^3*b^3 - 832*a^2*b^4 - 256*a*b^5)*e^(12*d*x + 12*c) + 6*(120*a^6 + 528*a^5*b + 525*a^4*b^2 - 52*a^3*b^3 - 896*a^2*b^4 - 1216*a*b^5 - 384*b^6)*e^(10*d*x + 10*c) + (176*a^6 + 1337*a^5*b + 7554*a^4*b^2 + 16416*a^3*b^3 + 26880*a^2*b^4 + 25344*a*b^5 + 6912*b^6)*e^(8*d*x + 8*c) - 4*(184*a^6 + 1056*a^5*b + 2993*a^4*b^2 + 4122*a^3*b^3 + 5892*a^2*b^4 + 6144*a*b^5 + 1728*b^6)*e^(6*d*x + 6*c) - (384*a^6 + 1177*a^5*b - 736*a^4*b^2 + 112*a^3*b^3 - 624*a^2*b^4 - 6144*a*b^5 - 2304*b^6)*e^(4*d*x + 4*c) + 2*(136*a^6 + 912*a^5*b + 1211*a^4*b^2 + 1440*a^3*b^3 + 1896*a^2*b^4 + 576*a*b^5)*e^(2*d*x + 2*c))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4 - (a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*e^(14*d*x + 14*c) - (a^9 + 12*a^8*b + 38*a^7*b^2 + 52*a^6*b^3 + 33*a^5*b^4 + 8*a^4*b^5)*e^(12*d*x + 12*c) + (3*a^9 + 20*a^8*b + 34*a^7*b^2 - 4*a^6*b^3 - 61*a^5*b^4 - 56*a^4*b^5 - 16*a^3*b^6)*e^(10*d*x + 10*c) + (3*a^9 + 28*a^8*b + 130*a^7*b^2 + 300*a^6*b^3 + 355*a^5*b^4 + 208*a^4*b^5 + 48*a^3*b^6)*e^(8

```
*d*x + 8*c) - (3*a^9 + 28*a^8*b + 130*a^7*b^2 + 300*a^6*b^3 + 355*a^5*b^4 +
 208*a^4*b^5 + 48*a^3*b^6)*e^(6*d*x + 6*c) - (3*a^9 + 20*a^8*b + 34*a^7*b^2
 - 4*a^6*b^3 - 61*a^5*b^4 - 56*a^4*b^5 - 16*a^3*b^6)*e^(4*d*x + 4*c) + (a^9
 + 12*a^8*b + 38*a^7*b^2 + 52*a^6*b^3 + 33*a^5*b^4 + 8*a^4*b^5)*e^(2*d*x +
2*c))*d) - 1/192*(176*a^6 + 275*a^5*b + 306*a^4*b^2 + 456*a^3*b^3 + 144*a^2
*b^4 + 2*(136*a^6 + 912*a^5*b + 1211*a^4*b^2 + 1440*a^3*b^3 + 1896*a^2*b^4
+ 576*a*b^5)*e^(-2*d*x - 2*c) - (384*a^6 + 1177*a^5*b - 736*a^4*b^2 + 112*a
^3*b^3 - 624*a^2*b^4 - 6144*a*b^5 - 2304*b^6)*e^(-4*d*x - 4*c) - 4*(184*a^6
 + 1056*a^5*b + 2993*a^4*b^2 + 4122*a^3*b^3 + 5892*a^2*b^4 + 6144*a*b^5 + 1
728*b^6)*e^(-6*d*x - 6*c) + (176*a^6 + 1337*a^5*b + 7554*a^4*b^2 + 16416*a^
3*b^3 + 26880*a^2*b^4 + 25344*a*b^5 + 6912*b^6)*e^(-8*d*x - 8*c) + 6*(120*a
^6 + 528*a^5*b + 525*a^4*b^2 - 52*a^3*b^3 - 896*a^2*b^4 - 1216*a*b^5 - 384*
b^6)*e^(-10*d*x - 10*c) + 3*(96*a^6 + 111*a^5*b - 220*a^4*b^2 - 776*a^3*b^3
 - 832*a^2*b^4 - 256*a*b^5)*e^(-12*d*x - 12*c))/((a^9 + 4*a^8*b + 6*a^7*b^2
 + 4*a^6*b^3 + a^5*b^4 + (a^9 + 12*a^8*b + 38*a^7*b^2 + 52*a^6*b^3 + 33*a^5
*b^4 + 8*a^4*b^5)*e^(-2*d*x - 2*c) - (3*a^9 + 20*a^8*b + 34*a^7*b^2 - 4*a^6
*b^3 - 61*a^5*b^4 - 56*a^4*b^5 - 16*a^3*b^6)*e^(-4*d*x - 4*c) - (3*a^9 + 28
*a^8*b + 130*a^7*b^2 + 300*a^6*b^3 + 355*a^5*b^4 + 208*a^4*b^5 + 48*a^3*b^6
)*e^(-6*d*x - 6*c) + (3*a^9 + 28*a^8*b + 130*a^7*b^2 + 300*a^6*b^3 + 355*a^
5*b^4 + 208*a^4*b^5 + 48*a^3*b^6)*e^(-8*d*x - 8*c) + (3*a^9 + 20*a^8*b + 34
*a^7*b^2 - 4*a^6*b^3 - 61*a^5*b^4 - 56*a^4*b^5 - 16*a^3*b^6)*e^(-10*d*x - 1
0*c) - (a^9 + 12*a^8*b + 38*a^7*b^2 + 52*a^6*b^3 + 33*a^5*b^4 + 8*a^4*b^5)*
e^(-12*d*x - 12*c) - (a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*e^(-
14*d*x - 14*c))*d) + 1/48*(32*a^5 + 77*a^4*b - 72*a^3*b^2 - 12*a^2*b^3 + 3*
(32*a^5 + 65*a^4*b + 94*a^3*b^2 + 128*a^2*b^3 + 32*a*b^4)*e^(12*d*x + 12*c)
 + 6*(48*a^5 + 200*a^4*b + 203*a^3*b^2 + 90*a^2...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11993 vs. $2(220) = 440$.
time = 0.64, size = 24263, normalized size = 104.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(48*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x +
c)^14 + 672*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x
+ c)*sinh(d*x + c)^13 + 48*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4
)*d*x*sinh(d*x + c)^14 - 12*(16*a^6 + 40*a^5*b - 17*a^3*b^3 - 44*a^2*b^4 -
16*a*b^5 - 4*(a^6 + 12*a^5*b + 38*a^4*b^2 + 52*a^3*b^3 + 33*a^2*b^4 + 8*a*b
^5)*d*x)*cosh(d*x + c)^12 - 12*(16*a^6 + 40*a^5*b - 17*a^3*b^3 - 44*a^2*b^4
 - 16*a*b^5 - 364*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cos
h(d*x + c)^2 - 4*(a^6 + 12*a^5*b + 38*a^4*b^2 + 52*a^3*b^3 + 33*a^2*b^4 + 8
*a*b^5)*d*x)*sinh(d*x + c)^12 + 48*(364*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*
```

b^3 + a^2*b^4)*d*x*cosh(d*x + c)^3 - 3*(16*a^6 + 40*a^5*b - 17*a^3*b^3 - 44
*a^2*b^4 - 16*a*b^5 - 4*(a^6 + 12*a^5*b + 38*a^4*b^2 + 52*a^3*b^3 + 33*a^2*
b^4 + 8*a*b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^11 - 24*(24*a^6 + 112*a^5*
b + 160*a^4*b^2 - 11*a^2*b^4 - 68*a*b^5 - 24*b^6 + 2*(3*a^6 + 20*a^5*b + 34
*a^4*b^2 - 4*a^3*b^3 - 61*a^2*b^4 - 56*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^1
0 + 24*(2002*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x
 + c)^4 - 24*a^6 - 112*a^5*b - 160*a^4*b^2 + 11*a^2*b^4 + 68*a*b^5 + 24*b^6
 - 2*(3*a^6 + 20*a^5*b + 34*a^4*b^2 - 4*a^3*b^3 - 61*a^2*b^4 - 56*a*b^5 - 1
6*b^6)*d*x - 33*(16*a^6 + 40*a^5*b - 17*a^3*b^3 - 44*a^2*b^4 - 16*a*b^5 - 4
*(a^6 + 12*a^5*b + 38*a^4*b^2 + 52*a^3*b^3 + 33*a^2*b^4 + 8*a*b^5)*d*x)*cos
h(d*x + c)^2)*sinh(d*x + c)^10 + 48*(2002*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^
3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^5 - 55*(16*a^6 + 40*a^5*b - 17*a^3*b^3 -
 44*a^2*b^4 - 16*a*b^5 - 4*(a^6 + 12*a^5*b + 38*a^4*b^2 + 52*a^3*b^3 + 33*a
^2*b^4 + 8*a*b^5)*d*x)*cosh(d*x + c)^3 - 5*(24*a^6 + 112*a^5*b + 160*a^4*b^
2 - 11*a^2*b^4 - 68*a*b^5 - 24*b^6 + 2*(3*a^6 + 20*a^5*b + 34*a^4*b^2 - 4*a
^3*b^3 - 61*a^2*b^4 - 56*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c))*sinh(d*x + c)^
9 - 4*(128*a^6 + 440*a^5*b + 1152*a^4*b^2 + 2073*a^3*b^3 + 642*a^2*b^4 + 14
16*a*b^5 + 432*b^6 + 12*(3*a^6 + 28*a^5*b + 130*a^4*b^2 + 300*a^3*b^3 + 355
*a^2*b^4 + 208*a*b^5 + 48*b^6)*d*x)*cosh(d*x + c)^8 + 4*(36036*(a^6 + 4*a^5
*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^6 - 128*a^6 - 440*a
^5*b - 1152*a^4*b^2 - 2073*a^3*b^3 - 642*a^2*b^4 - 1416*a*b^5 - 432*b^6 - 1
485*(16*a^6 + 40*a^5*b - 17*a^3*b^3 - 44*a^2*b^4 - 16*a*b^5 - 4*(a^6 + 12*a
^5*b + 38*a^4*b^2 + 52*a^3*b^3 + 33*a^2*b^4 + 8*a*b^5)*d*x)*cosh(d*x + c)^4
 - 12*(3*a^6 + 28*a^5*b + 130*a^4*b^2 + 300*a^3*b^3 + 355*a^2*b^4 + 208*a*b
^5 + 48*b^6)*d*x - 270*(24*a^6 + 112*a^5*b + 160*a^4*b^2 - 11*a^2*b^4 - 68*
a*b^5 - 24*b^6 + 2*(3*a^6 + 20*a^5*b + 34*a^4*b^2 - 4*a^3*b^3 - 61*a^2*b^4
 - 56*a*b^5 - 16*b^6)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 32*(5148*(a^6
 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^7 - 297*(16*
a^6 + 40*a^5*b - 17*a^3*b^3 - 44*a^2*b^4 - 16*a*b^5 - 4*(a^6 + 12*a^5*b + 3
8*a^4*b^2 + 52*a^3*b^3 + 33*a^2*b^4 + 8*a*b^5)*d*x)*cosh(d*x + c)^5 - 90*(2
4*a^6 + 112*a^5*b + 160*a^4*b^2 - 11*a^2*b^4 - 68*a*b^5 - 24*b^6 + 2*(3*a^6
 + 20*a^5*b + 34*a^4*b^2 - 4*a^3*b^3 - 61*a^2*b^4 - 56*a*b^5 - 16*b^6)*d*x)
*cosh(d*x + c)^3 - (128*a^6 + 440*a^5*b + 1152*a^4*b^2 + 2073*a^3*b^3 + 642
*a^2*b^4 + 1416*a*b^5 + 432*b^6 + 12*(3*a^6 + 28*a^5*b + 130*a^4*b^2 + 300*
a^3*b^3 + 355*a^2*b^4 + 208*a*b^5 + 48*b^6)*d*x)*cosh(d*x + c))*sinh(d*x +
c)^7 - 16*(8*a^6 - 96*a^5*b - 512*a^4*b^2 - 768*a^3*b^3 - 57*a^2*b^4 - 330*
a*b^5 - 108*b^6 - 3*(3*a^6 + 28*a^5*b + 130*a^4*b^2 + 300*a^3*b^3 + 355*a^2
*b^4 + 208*a*b^5 + 48*b^6)*d*x)*cosh(d*x + c)^6 + 16*(9009*(a^6 + 4*a^5*b +
 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^8 - 693*(16*a^6 + 40*a^
5*b - 17*a^3*b^3 - 44*a^2*b^4 - 16*a*b^5 - 4*(a^6 + 12*a^5*b + 38*a^4*b^2 +
 52*a^3*b^3 + 33*a^2*b^4 + 8*a*b^5)*d*x)*cosh(d*x + c)^6 - 8*a^6 + 96*a^5*b
 + 512*a^4*b^2 + 768*a^3*b^3 + 57*a^2*b^4 + 330*a*b^5 + 108*b^6 - 315*(24*a
^6 + 112*a^5*b + 160*a^4*b^2 - 11*a^2*b^4 - 68*a*b^5 - 24*b^6 + 2*(3*a^6 +
20*a^5*b + 34*a^4*b^2 - 4*a^3*b^3 - 61*a^2*b^4 - 56*a*b^5 - 16*b^6)*d*x)*co
sh(d*x + c)^4 + 3*(3*a^6 + 28*a^5*b + 130*a^4*b^2 + 300*a^3*b^3 + 355*a^2*b

```
^4 + 208*a*b^5 + 48*b^6)*d*x - 7*(128*a^6 + 440*a^5*b + 1152*a^4*b^2 + 2073
*a^3*b^3 + 642*a^2*b^4 + 1416*a*b^5 + 432*b^6 + 12*(3*a^6 + 28*a^5*b + 130*
a^4*b^2 + 300*a^3*b^3 + 355*a^2*b^4 + 208*a*b^5 + 48*b^6)*d*x)*cosh(d*x + c
)^2)*sinh(d*x + c)^6 - 128*a^6 - 416*a^5*b - 204*a^3*b^3 - 72*a^2*b^4 + 32*
(3003*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*x*cosh(d*x + c)^9
 - 297*(16*a^6 + 40*a^5*b - 17*a^3*b^3 - 44*a^2*b^4 - 16*a*b^5 - 4*(a^6 + 1
2*a^5*b + 38*a^4*b^2 + 52*a^3*b^3 + 33*a^2*b^4 + 8*a*b^5)*d*x)*cosh(d*x + c
)^7 - 189*(24*a^6 + 112*a^5*b + 160*a^4*b^2 - 11*a^2*b^4 - 68*a*b^5 - 24*b^
6 + 2*(3*a^6 + 20*a^5*b + 34*a^4*b^2 - 4*a^3*b^3 - 61*a^2*b^4 - 56*a*b^5 -
16*b^6)*d*x)*cosh(d*x + c)^5 - 7*(128*a^6 + 440*a^5*b + 1152*a^4*b^2 + 2073
*a^3*b^3 + 642*a^2*b^4 + 1416*a*b^5 + 432*b^6 + 12*(3*a^6 + 28*a^5*b + 130*
a^4*b^2 + 300*a^3*b^3 + 355*a^2*b^4 + 208*a*b^5 + 48*b^6)*d*x)*cosh(d*x + c
)^3 - 3*(8*a^6 - 96*a^5*b - 512*a^4*b^2 - 768*a...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(c+dx)}{\left(a+b\,\text{sech}^2(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4/(a+b*sech(d*x+c)**2)**3,x)
```

```
[Out] Integral(coth(c + d*x)**4/(a + b*sech(c + d*x)**2)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(220) = 440.
time = 2.51, size = 469, normalized size = 2.02

$$-\frac{\frac{3\left(63a^2b^3+36ab^4+8b^5\right)\arctan\left(\frac{ae^{(2dx+2c)}+a+2b}{2\sqrt{-ab-b^2}}\right)}{\left(a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4\right)\sqrt{-ab-b^2}}-\frac{6\left(17a^3b^3e^{(6dx+6c)}+44a^2b^4e^{(6dx+6c)}+16ab^5e^{(6dx+6c)}+51a^3b^3e^{(4dx+4c)}+154a^2b^4e^{(4dx+4c)}+184ab^5e^{(4dx+4c)}+48b^6e^{(4dx+4c)}+51a^3b^3e^{(2dx+2c)}+116a^2b^4e^{(2dx+2c)}+32ab^5e^{(2dx+2c)}+17a^3b^3+6a^2b^4\right)}{\left(a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4\right)\left(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a\right)^2}-\frac{24(dx+c)}{a^3}+\frac{16\left(6ae^{(4dx+4c)}+15be^{(4dx+4c)}-6ae^{(2dx+2c)}-24be^{(2dx+2c)}+4a+13b\right)}{\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\left(e^{(2dx+2c)}-1\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(63*a^2*b^3 + 36*a*b^4 + 8*b^5)*arctan(1/2*(a*e^(2*d*x + 2*c) + a
+ 2*b)/sqrt(-a*b - b^2))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)
*sqrt(-a*b - b^2)) - 6*(17*a^3*b^3*e^(6*d*x + 6*c) + 44*a^2*b^4*e^(6*d*x +
6*c) + 16*a*b^5*e^(6*d*x + 6*c) + 51*a^3*b^3*e^(4*d*x + 4*c) + 154*a^2*b^4*
e^(4*d*x + 4*c) + 184*a*b^5*e^(4*d*x + 4*c) + 48*b^6*e^(4*d*x + 4*c) + 51*a
^3*b^3*e^(2*d*x + 2*c) + 116*a^2*b^4*e^(2*d*x + 2*c) + 32*a*b^5*e^(2*d*x +
2*c) + 17*a^3*b^3 + 6*a^2*b^4)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^
3*b^4)*(a*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^
2) - 24*(d*x + c)/a^3 + 16*(6*a*e^(4*d*x + 4*c) + 15*b*e^(4*d*x + 4*c) - 6*
a*e^(2*d*x + 2*c) - 24*b*e^(2*d*x + 2*c) + 4*a + 13*b)/((a^4 + 4*a^3*b + 6*
a^2*b^2 + 4*a*b^3 + b^4)*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^6\,\coth(c+d\,x)^4}{{\left(a\,{\cosh\left(c+d\,x\right)}^2+b\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4/(a + b/cosh(c + d*x)^2)^3,x)
[Out] int((cosh(c + d*x)^6*coth(c + d*x)^4)/(b + a*cosh(c + d*x)^2)^3, x)
```

3.169 $\int \frac{1}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^4}\,dx$

Optimal. Leaf size=207

$$\frac{x}{a^4}-\frac{\sqrt{b}\,(35a^3+70a^2b+56ab^2+16b^3)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{16a^4(a+b)^{7/2}d}-\frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^3}-\ 24$$

```
[Out] x/a^4-1/16*(35*a^3+70*a^2*b+56*a*b^2+16*b^3)*arctanh(b^(1/2)*tanh(d*x+c)/(a
+b)^(1/2))*b^(1/2)/a^4/(a+b)^(7/2)/d-1/6*b*tanh(d*x+c)/a/(a+b)/d/(a+b-b*tan
h(d*x+c)^2)^3-1/24*b*(11*a+6*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c
)^2)^2-1/16*b*(19*a^2+22*a*b+8*b^2)*tanh(d*x+c)/a^3/(a+b)^3/d/(a+b-b*tanh(d
*x+c)^2)
```

Rubi [A]
time = 0.24, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4213, 425, 541, 536, 212, 214}

$$\frac{x}{a^4}-\frac{b(11a+6b)\tanh(c+dx)}{24a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)^2}-\frac{b(19a^2+22ab+8b^2)\tanh(c+dx)}{16a^3d(a+b)^3\left(a-b\tanh^2(c+dx)+b\right)}-\frac{\sqrt{b}\,(35a^3+70a^2b+56ab^2+16b^3)\tanh^{-1}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b}}\right)}{16a^4d(a+b)^{7/2}}-\frac{b\tanh(c+dx)}{6ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(-4),x]
```

```
[Out] x/a^4 - (Sqrt[b]*(35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*ArcTanh[(Sqrt[b]*T
anh[c + d*x])/Sqrt[a + b]])/(16*a^4*(a + b)^(7/2)*d) - (b*Tanh[c + d*x])/(6
*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^3) - (b*(11*a + 6*b)*Tanh[c + d*x]
)/(24*a^2*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2)^2) - (b*(19*a^2 + 22*a*b
+ 8*b^2)*Tanh[c + d*x])/(16*a^3*(a + b)^3*d*(a + b - b*Tanh[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\text{sech}^2(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^4}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^3} - \frac{\text{Subst}\left(\int \frac{-6a-b-5bx^2}{(1-x^2)(a+b-bx^2)^3}\,dx, x, \tanh\right.}{6a(a+b)d}$$

$$= -\frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^3} - \frac{b(11a+6b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b-b\tanh^2(c+\right.}$$

$$= -\frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^3} - \frac{b(11a+6b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b-b\tanh^2(c+\right.}$$

$$= -\frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^3} - \frac{b(11a+6b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b-b\tanh^2(c+\right.}$$

$$= \frac{x}{a^4} - \frac{\sqrt{b}\,(35a^3+70a^2b+56ab^2+16b^3)\tanh^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b}}\right)}{16a^4(a+b)^{7/2}d} - \frac{}{6a(a+b)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.59, size = 1405, normalized size = 6.79

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-4),x]
```

```
[Out] ((35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*(a + 2*b + a*Cosh[2*c + 2*d*x])^4*
Sech[c + d*x]^8*(((I/256)*b*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a
+ b]*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt
[b*Cosh[4*c] - b*Sinh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c
+ d*x])]*Cosh[2*c])/(a^4*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]]) - (
(I/256)*b*ArcTan[Sech[d*x]*(((-1/2*I)*Cosh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4
*c] - b*Sinh[4*c]]) + ((I/2)*Sinh[2*c])/(Sqrt[a + b]*Sqrt[b*Cosh[4*c] - b*S
inh[4*c]]))*(-(a*Sinh[d*x]) - 2*b*Sinh[d*x] + a*Sinh[2*c + d*x])]*Sinh[2*c]
)/(a^4*Sqrt[a + b]*d*Sqrt[b*Cosh[4*c] - b*Sinh[4*c]])))/((a + b)^3*(a + b*S
ech[c + d*x]^2)^4) + ((a + 2*b + a*Cosh[2*c + 2*d*x])*Sech[2*c]*Sech[c + d*
x]^8*(480*a^6*d*x*Cosh[2*c] + 3168*a^5*b*d*x*Cosh[2*c] + 8928*a^4*b^2*d*x*C
osh[2*c] + 14112*a^3*b^3*d*x*Cosh[2*c] + 13248*a^2*b^4*d*x*Cosh[2*c] + 6912
*a*b^5*d*x*Cosh[2*c] + 1536*b^6*d*x*Cosh[2*c] + 360*a^6*d*x*Cosh[2*d*x] + 2
232*a^5*b*d*x*Cosh[2*d*x] + 5688*a^4*b^2*d*x*Cosh[2*d*x] + 7272*a^3*b^3*d*x
```

```
*Cosh[2*d*x] + 4608*a^2*b^4*d*x*Cosh[2*d*x] + 1152*a*b^5*d*x*Cosh[2*d*x] +
360*a^6*d*x*Cosh[4*c + 2*d*x] + 2232*a^5*b*d*x*Cosh[4*c + 2*d*x] + 5688*a^4
*b^2*d*x*Cosh[4*c + 2*d*x] + 7272*a^3*b^3*d*x*Cosh[4*c + 2*d*x] + 4608*a^2*
b^4*d*x*Cosh[4*c + 2*d*x] + 1152*a*b^5*d*x*Cosh[4*c + 2*d*x] + 144*a^6*d*x*
Cosh[2*c + 4*d*x] + 720*a^5*b*d*x*Cosh[2*c + 4*d*x] + 1296*a^4*b^2*d*x*Cosh
[2*c + 4*d*x] + 1008*a^3*b^3*d*x*Cosh[2*c + 4*d*x] + 288*a^2*b^4*d*x*Cosh[2
*c + 4*d*x] + 144*a^6*d*x*Cosh[6*c + 4*d*x] + 720*a^5*b*d*x*Cosh[6*c + 4*d*
x] + 1296*a^4*b^2*d*x*Cosh[6*c + 4*d*x] + 1008*a^3*b^3*d*x*Cosh[6*c + 4*d*x
] + 288*a^2*b^4*d*x*Cosh[6*c + 4*d*x] + 24*a^6*d*x*Cosh[4*c + 6*d*x] + 72*a
^5*b*d*x*Cosh[4*c + 6*d*x] + 72*a^4*b^2*d*x*Cosh[4*c + 6*d*x] + 24*a^3*b^3*
d*x*Cosh[4*c + 6*d*x] + 24*a^6*d*x*Cosh[8*c + 6*d*x] + 72*a^5*b*d*x*Cosh[8*
c + 6*d*x] + 72*a^4*b^2*d*x*Cosh[8*c + 6*d*x] + 24*a^3*b^3*d*x*Cosh[8*c + 6
*d*x] + 870*a^5*b*Sinh[2*c] + 4292*a^4*b^2*Sinh[2*c] + 8792*a^3*b^3*Sinh[2*
c] + 9936*a^2*b^4*Sinh[2*c] + 5824*a*b^5*Sinh[2*c] + 1408*b^6*Sinh[2*c] - 8
70*a^5*b*Sinh[2*d*x] - 3792*a^4*b^2*Sinh[2*d*x] - 6432*a^3*b^3*Sinh[2*d*x]
- 4608*a^2*b^4*Sinh[2*d*x] - 1248*a*b^5*Sinh[2*d*x] + 435*a^5*b*Sinh[4*c +
2*d*x] + 2124*a^4*b^2*Sinh[4*c + 2*d*x] + 3972*a^3*b^3*Sinh[4*c + 2*d*x] +
3072*a^2*b^4*Sinh[4*c + 2*d*x] + 864*a*b^5*Sinh[4*c + 2*d*x] - 435*a^5*b*Si
nh[2*c + 4*d*x] - 1374*a^4*b^2*Sinh[2*c + 4*d*x] - 1248*a^3*b^3*Sinh[2*c +
4*d*x] - 384*a^2*b^4*Sinh[2*c + 4*d*x] + 87*a^5*b*Sinh[6*c + 4*d*x] + 366*a
^4*b^2*Sinh[6*c + 4*d*x] + 408*a^3*b^3*Sinh[6*c + 4*d*x] + 144*a^2*b^4*Sinh
[6*c + 4*d*x] - 87*a^5*b*Sinh[4*c + 6*d*x] - 116*a^4*b^2*Sinh[4*c + 6*d*x]
- 44*a^3*b^3*Sinh[4*c + 6*d*x]))/(3072*a^4*(a + b)^3*d*(a + b*Sech[c + d*x]
^2)^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(191) = 382$.

time = 2.60, size = 545, normalized size = 2.63 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/a^4*((-1/16*a*(29*a^2+26*a*b+8*b^2)/(a+b)*tanh(1/2*d*x+1/2*c)^11-1
/48*(435*a^3+281*a^2*b-66*a*b^2-72*b^3)*a/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*
c)^9-1/8*a*(145*a^4+148*a^3*b+37*a^2*b^2+2*a*b^3+8*b^4)/(a+b)/(a^2+2*a*b+b^
2)*tanh(1/2*d*x+1/2*c)^7-1/8*a*(145*a^4+148*a^3*b+37*a^2*b^2+2*a*b^3+8*b^4)
/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*d*x+1/2*c)^5-1/48*(435*a^3+281*a^2*b-66
*a*b^2-72*b^3)*a/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-1/16*a*(29*a^2+26*a*
b+8*b^2)/(a+b)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^4+b*tanh(1/2*d*x
+1/2*c)^4+2*a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)^2+a+b)^3+1/16*(
35*a^3+70*a^2*b+56*a*b^2+16*b^3)/(a^3+3*a^2*b+3*a*b^2+b^3)*(-1/4/b^(1/2)/(a
+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2+2*tanh(1/2*d*x+1/2*c)*b^(1/2
)+(a+b)^(1/2))+1/4/b^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*d*x+1/2*c)^2
-2*tanh(1/2*d*x+1/2*c)*b^(1/2)+(a+b)^(1/2))))+1/a^4*ln(tanh(1/2*d*x+1/2*c)+
1)-1/a^4*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 718 vs.

$2(200) = 400$.
time = 0.56, size = 718, normalized size = 3.47

$$\frac{\left(35\,a^3b+70\,a^2b^2+56\,ab^3+16\,b^4\right)\log\left(\frac{ae^{(-2\,dx-2\,c)}+a+2\,b-2\,\sqrt{(a+b)b}}{ae^{(-2\,dx-2\,c)}+a+2\,b+2\,\sqrt{(a+b)b}}\right)}{32\,\left(a^7+3\,a^6b+3\,a^5b^2+a^4b^3\right)\sqrt{(a+b)b}\,d}-\frac{87\,a^5b+116\,a^4b^2+44\,a^3b^3+3\,\left(145\,a^5b+458\,a^4b^2+416\,a^3b^3+128\,a^2b^4\right)e^{(-2\,dx-2\,c)}+6\,\left(145\,a^5b+632\,a^4b^2+1072\,a^3b^3+768\,a^2b^4+208\,ab^5\right)e^{(-4\,dx-4\,c)}+2\,\left(435\,a^5b+2146\,a^4b^2+4396\,a^3b^3+4968\,a^2b^4+2912\,ab^5+704\,b^6\right)e^{(-6\,dx-6\,c)}+3\,\left(145\,a^5b+708\,a^4b^2+1324\,a^3b^3+1024\,a^2b^4+288\,ab^5\right)e^{(-8\,dx-8\,c)}+3\,\left(29\,a^5b+122\,a^4b^2+136\,a^3b^3+48\,a^2b^4\right)e^{(-10\,dx-10\,c)}}{24\,\left(a^{10}+3\,a^9b+3\,a^8b^2+a^7b^3+6\,\left(a^{10}+5\,a^9b+9\,a^8b^2+7\,a^7b^3+2\,a^6b^4\right)e^{(-2\,dx-2\,c)}+3\,\left(5\,a^{10}+31\,a^9b+79\,a^8b^2+101\,a^7b^3+64\,a^6b^4+16\,a^5b^5\right)e^{(-4\,dx-4\,c)}+4\,\left(5\,a^{10}+33\,a^9b+93\,a^8b^2+147\,a^7b^3+138\,a^6b^4+72\,a^5b^5+16\,a^4b^6\right)e^{(-6\,dx-6\,c)}+3\,\left(5\,a^{10}+31\,a^9b+79\,a^8b^2+101\,a^7b^3+64\,a^6b^4+16\,a^5b^5\right)e^{(-8\,dx-8\,c)}+6\,\left(a^{10}+5\,a^9b+9\,a^8b^2+7\,a^7b^3+2\,a^6b^4\right)e^{(-10\,dx-10\,c)}+\left(a^{10}+3\,a^9b+3\,a^8b^2+a^7b^3\right)e^{(-12\,dx-12\,c)}\right)d}+\frac{dx+c}{a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/32*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*log((a*e^(-2*d*x - 2*c) +
a + 2*b - 2*sqrt((a + b)*b))/(a*e^(-2*d*x - 2*c) + a + 2*b + 2*sqrt((a + b)
*b)))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*sqrt((a + b)*b)*d) - 1/24*(87*
a^5*b + 116*a^4*b^2 + 44*a^3*b^3 + 3*(145*a^5*b + 458*a^4*b^2 + 416*a^3*b^3
 + 128*a^2*b^4)*e^(-2*d*x - 2*c) + 6*(145*a^5*b + 632*a^4*b^2 + 1072*a^3*b^
3 + 768*a^2*b^4 + 208*a*b^5)*e^(-4*d*x - 4*c) + 2*(435*a^5*b + 2146*a^4*b^2
 + 4396*a^3*b^3 + 4968*a^2*b^4 + 2912*a*b^5 + 704*b^6)*e^(-6*d*x - 6*c) + 3
*(145*a^5*b + 708*a^4*b^2 + 1324*a^3*b^3 + 1024*a^2*b^4 + 288*a*b^5)*e^(-8*
d*x - 8*c) + 3*(29*a^5*b + 122*a^4*b^2 + 136*a^3*b^3 + 48*a^2*b^4)*e^(-10*d
*x - 10*c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3 + 6*(a^10 + 5*a^9*b + 9*
a^8*b^2 + 7*a^7*b^3 + 2*a^6*b^4)*e^(-2*d*x - 2*c) + 3*(5*a^10 + 31*a^9*b +
79*a^8*b^2 + 101*a^7*b^3 + 64*a^6*b^4 + 16*a^5*b^5)*e^(-4*d*x - 4*c) + 4*(5
*a^10 + 33*a^9*b + 93*a^8*b^2 + 147*a^7*b^3 + 138*a^6*b^4 + 72*a^5*b^5 + 16
*a^4*b^6)*e^(-6*d*x - 6*c) + 3*(5*a^10 + 31*a^9*b + 79*a^8*b^2 + 101*a^7*b^
3 + 64*a^6*b^4 + 16*a^5*b^5)*e^(-8*d*x - 8*c) + 6*(a^10 + 5*a^9*b + 9*a^8*b
^2 + 7*a^7*b^3 + 2*a^6*b^4)*e^(-10*d*x - 10*c) + (a^10 + 3*a^9*b + 3*a^8*b^
2 + a^7*b^3)*e^(-12*d*x - 12*c))*d) + (d*x + c)/(a^4*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8503 vs. $2(200) = 400$.
time = 0.58, size = 17283, normalized size = 83.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(96*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cosh(d*x + c)^12 + 1152
*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 +
 96*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*sinh(d*x + c)^12 + 12*(29*a^5
*b + 122*a^4*b^2 + 136*a^3*b^3 + 48*a^2*b^4 + 48*(a^6 + 5*a^5*b + 9*a^4*b^2
 + 7*a^3*b^3 + 2*a^2*b^4)*d*x)*cosh(d*x + c)^10 + 12*(29*a^5*b + 122*a^4*b^
2 + 136*a^3*b^3 + 48*a^2*b^4 + 528*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*
x*cosh(d*x + c)^2 + 48*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*
d*x)*sinh(d*x + c)^10 + 120*(176*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*
cosh(d*x + c)^3 + (29*a^5*b + 122*a^4*b^2 + 136*a^3*b^3 + 48*a^2*b^4 + 48*(
a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*x)*cosh(d*x + c))*sinh
```

(d*x + c)^9 + 12*(145*a^5*b + 708*a^4*b^2 + 1324*a^3*b^3 + 1024*a^2*b^4 + 2
88*a*b^5 + 24*(5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 1
6*a*b^5)*d*x)*cosh(d*x + c)^8 + 12*(3960*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b
^3)*d*x*cosh(d*x + c)^4 + 145*a^5*b + 708*a^4*b^2 + 1324*a^3*b^3 + 1024*a^2
*b^4 + 288*a*b^5 + 24*(5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2
*b^4 + 16*a*b^5)*d*x + 45*(29*a^5*b + 122*a^4*b^2 + 136*a^3*b^3 + 48*a^2*b^
4 + 48*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*x)*cosh(d*x +
c)^2)*sinh(d*x + c)^8 + 96*(792*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*c
osh(d*x + c)^5 + 15*(29*a^5*b + 122*a^4*b^2 + 136*a^3*b^3 + 48*a^2*b^4 + 48
*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*x)*cosh(d*x + c)^3 +
 (145*a^5*b + 708*a^4*b^2 + 1324*a^3*b^3 + 1024*a^2*b^4 + 288*a*b^5 + 24*(5
*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^7 + 8*(435*a^5*b + 2146*a^4*b^2 + 4396*a^3*b^3 +
 4968*a^2*b^4 + 2912*a*b^5 + 704*b^6 + 48*(5*a^6 + 33*a^5*b + 93*a^4*b^2 +
147*a^3*b^3 + 138*a^2*b^4 + 72*a*b^5 + 16*b^6)*d*x)*cosh(d*x + c)^6 + 8*(11
088*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cosh(d*x + c)^6 + 435*a^5*b +
 2146*a^4*b^2 + 4396*a^3*b^3 + 4968*a^2*b^4 + 2912*a*b^5 + 704*b^6 + 315*(2
9*a^5*b + 122*a^4*b^2 + 136*a^3*b^3 + 48*a^2*b^4 + 48*(a^6 + 5*a^5*b + 9*a^
4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*x)*cosh(d*x + c)^4 + 48*(5*a^6 + 33*a^5*b
+ 93*a^4*b^2 + 147*a^3*b^3 + 138*a^2*b^4 + 72*a*b^5 + 16*b^6)*d*x + 42*(145
*a^5*b + 708*a^4*b^2 + 1324*a^3*b^3 + 1024*a^2*b^4 + 288*a*b^5 + 24*(5*a^6
+ 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*d*x)*cosh(d*
x + c)^2)*sinh(d*x + c)^6 + 348*a^5*b + 464*a^4*b^2 + 176*a^3*b^3 + 48*(158
4*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cosh(d*x + c)^7 + 63*(29*a^5*b
+ 122*a^4*b^2 + 136*a^3*b^3 + 48*a^2*b^4 + 48*(a^6 + 5*a^5*b + 9*a^4*b^2 +
7*a^3*b^3 + 2*a^2*b^4)*d*x)*cosh(d*x + c)^5 + 14*(145*a^5*b + 708*a^4*b^2 +
 1324*a^3*b^3 + 1024*a^2*b^4 + 288*a*b^5 + 24*(5*a^6 + 31*a^5*b + 79*a^4*b^
2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*d*x)*cosh(d*x + c)^3 + (435*a^5*b
+ 2146*a^4*b^2 + 4396*a^3*b^3 + 4968*a^2*b^4 + 2912*a*b^5 + 704*b^6 + 48*(5
*a^6 + 33*a^5*b + 93*a^4*b^2 + 147*a^3*b^3 + 138*a^2*b^4 + 72*a*b^5 + 16*b^
6)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 24*(145*a^5*b + 632*a^4*b^2 + 1072
*a^3*b^3 + 768*a^2*b^4 + 208*a*b^5 + 12*(5*a^6 + 31*a^5*b + 79*a^4*b^2 + 10
1*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*d*x)*cosh(d*x + c)^4 + 24*(1980*(a^6 + 3
*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cosh(d*x + c)^8 + 105*(29*a^5*b + 122*a^4
*b^2 + 136*a^3*b^3 + 48*a^2*b^4 + 48*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3
 + 2*a^2*b^4)*d*x)*cosh(d*x + c)^6 + 145*a^5*b + 632*a^4*b^2 + 1072*a^3*b^3
 + 768*a^2*b^4 + 208*a*b^5 + 35*(145*a^5*b + 708*a^4*b^2 + 1324*a^3*b^3 + 1
024*a^2*b^4 + 288*a*b^5 + 24*(5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 +
 64*a^2*b^4 + 16*a*b^5)*d*x)*cosh(d*x + c)^4 + 12*(5*a^6 + 31*a^5*b + 79*a^
4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*d*x + 5*(435*a^5*b + 2146*a^4*
b^2 + 4396*a^3*b^3 + 4968*a^2*b^4 + 2912*a*b^5 + 704*b^6 + 48*(5*a^6 + 33*a
^5*b + 93*a^4*b^2 + 147*a^3*b^3 + 138*a^2*b^4 + 72*a*b^5 + 16*b^6)*d*x)*cos
h(d*x + c)^2)*sinh(d*x + c)^4 + 32*(660*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3)*d*x*cosh(d*x + c)^9 + 45*(29*a^5*b + 122*a^4*b^2 + 136*a^3*b^3 + 48*a^2*
b^4 + 48*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*x)*cosh(d*x

```
+ c)^7 + 21*(145*a^5*b + 708*a^4*b^2 + 1324*a^3*b^3 + 1024*a^2*b^4 + 288*a*
b^5 + 24*(5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b
^5)*d*x)*cosh(d*x + c)^5 + 5*(435*a^5*b + 2146*a^4*b^2 + 4396*a^3*b^3 + 496
8*a^2*b^4 + 2912*a*b^5 + 704*b^6 + 48*(5*a^6 + 33*a^5*b + 93*a^4*b^2 + 147*
a^3*b^3 + 138*a^2*b^4 + 72*a*b^5 + 16*b^6)*d*x)*cosh(d*x + c)^3 + 3*(145*a^
5*b + 632*a^4*b^2 + 1072*a^3*b^3 + 768*a^2*b^4 + 208*a*b^5 + 12*(5*a^6 + 31
*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*d*x)*cosh(d*x +
c))*sinh(d*x + c)^3 + 96*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x + 12*(14
5*a^5*b + 458*a^4*b^2 + 416*a^3*b^3 + 128*a^2*b^4 + 48*(a^6 + 5*a^5*b + 9*a
^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*d*x)*cosh(d*x + c)^2 + 12*(528*(a^6 + 3*a^5
*b + 3*a^4*b^2 + a^3*b^3)*d*x*cosh(d*x + c)^10 + 45*(29*a^5*b + 122*a^4*b^2
 + 136*a^3*b^3 + 48*a^2*b^4 + 48*(a^6 + 5*a^5*b...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2)**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. 2(200) = 400.
time = 0.53, size = 594, normalized size = 2.87

$$-\frac{\frac{3\,(35\,a^3b+70\,a^2b^2+56\,ab^3+16\,b^4)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+a+2\,b}{2\,\sqrt{-ab-b^2}}\right)}{(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\sqrt{-ab-b^2}}-\frac{2\,(87\,a^5be^{(10\,dx+10\,c)}+366\,a^4b^2e^{(10\,dx+10\,c)}+408\,a^3b^3e^{(10\,dx+10\,c)}+144\,a^2b^4e^{(10\,dx+10\,c)}+435\,a^5be^{(8\,dx+8\,c)}+2124\,a^4b^2e^{(8\,dx+8\,c)}+3972\,a^3b^3e^{(8\,dx+8\,c)}+3072\,a^2b^4e^{(8\,dx+8\,c)}+864\,ab^5e^{(8\,dx+8\,c)}+870\,a^5be^{(6\,dx+6\,c)}+4292\,a^4b^2e^{(6\,dx+6\,c)}+8792\,a^3b^3e^{(6\,dx+6\,c)}+9936\,a^2b^4e^{(6\,dx+6\,c)}+5824\,ab^5e^{(6\,dx+6\,c)}+1408\,b^6e^{(6\,dx+6\,c)}+870\,a^5be^{(4\,dx+4\,c)}+3792\,a^4b^2e^{(4\,dx+4\,c)}+6432\,a^3b^3e^{(4\,dx+4\,c)}+4608\,a^2b^4e^{(4\,dx+4\,c)}+1248\,ab^5e^{(4\,dx+4\,c)}+435\,a^5be^{(2\,dx+2\,c)}+1374\,a^4b^2e^{(2\,dx+2\,c)}+1248\,a^3b^3e^{(2\,dx+2\,c)}+384\,a^2b^4e^{(2\,dx+2\,c)}+87\,a^5b+116\,a^4b^2+44\,a^3b^3)}{(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)(ae^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}+4\,be^{(2\,dx+2\,c)}+a)^3}-\frac{48\,(dx+c)}{a^4}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] -1/48*(3*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*arctan(1/2*(a*e^(2*d*x
 + 2*c) + a + 2*b)/sqrt(-a*b - b^2))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)
*sqrt(-a*b - b^2)) - 2*(87*a^5*b*e^(10*d*x + 10*c) + 366*a^4*b^2*e^(10*d*x
+ 10*c) + 408*a^3*b^3*e^(10*d*x + 10*c) + 144*a^2*b^4*e^(10*d*x + 10*c) + 4
35*a^5*b*e^(8*d*x + 8*c) + 2124*a^4*b^2*e^(8*d*x + 8*c) + 3972*a^3*b^3*e^(8
*d*x + 8*c) + 3072*a^2*b^4*e^(8*d*x + 8*c) + 864*a*b^5*e^(8*d*x + 8*c) + 87
0*a^5*b*e^(6*d*x + 6*c) + 4292*a^4*b^2*e^(6*d*x + 6*c) + 8792*a^3*b^3*e^(6*
d*x + 6*c) + 9936*a^2*b^4*e^(6*d*x + 6*c) + 5824*a*b^5*e^(6*d*x + 6*c) + 14
08*b^6*e^(6*d*x + 6*c) + 870*a^5*b*e^(4*d*x + 4*c) + 3792*a^4*b^2*e^(4*d*x
+ 4*c) + 6432*a^3*b^3*e^(4*d*x + 4*c) + 4608*a^2*b^4*e^(4*d*x + 4*c) + 1248
*a*b^5*e^(4*d*x + 4*c) + 435*a^5*b*e^(2*d*x + 2*c) + 1374*a^4*b^2*e^(2*d*x
+ 2*c) + 1248*a^3*b^3*e^(2*d*x + 2*c) + 384*a^2*b^4*e^(2*d*x + 2*c) + 87*a^
5*b + 116*a^4*b^2 + 44*a^3*b^3)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*(a*e
```

```
^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) + 4*b*e^(2*d*x + 2*c) + a)^3) - 48*(d*
x + c)/a^4)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cosh(c+d\,x)^2}\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x)^2)^4,x)
[Out] int(1/(a + b/cosh(c + d*x)^2)^4, x)
```

3.170 $\int \left(1-\operatorname{sech}^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=29

$$\coth(x)\log(\cosh(x))\sqrt{\tanh^2(x)}-\frac{1}{2}\coth(x)\tanh^2(x)^{3/2}$$

```
[Out] coth(x)*ln(cosh(x))*(tanh(x)^2)^(1/2)-1/2*coth(x)*(tanh(x)^2)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {4206, 3739, 3554, 3556}

$$\sqrt{\tanh^2(x)}\coth(x)\log(\cosh(x))-\frac{1}{2}\tanh^2(x)^{3/2}\coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sech[x]^2)^(3/2),x]
```

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2] - (Coth[x]*(Tanh[x]^2)^(3/2))/2
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(1-\operatorname{sech}^2(x)\right)^{3/2}\,dx &= \int \tanh^2(x)^{3/2}\,dx\\
&= \left(\coth(x)\sqrt{\tanh^2(x)}\right)\int \tanh^3(x)\,dx\\
&= -\frac{1}{2}\coth(x)\tanh^2(x)^{3/2}+\left(\coth(x)\sqrt{\tanh^2(x)}\right)\int \tanh(x)\,dx\\
&= \coth(x)\log(\cosh(x))\sqrt{\tanh^2(x)}-\frac{1}{2}\coth(x)\tanh^2(x)^{3/2}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.86

$$\frac{1}{2}(2\coth(x)\log(\cosh(x))+\operatorname{csch}(x)\operatorname{sech}(x))\sqrt{\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sech[x]^2)^(3/2),x]
```

```
[Out] ((2*Coth[x]*Log[Cosh[x]] + Csch[x]*Sech[x])*Sqrt[Tanh[x]^2])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(23) = 46.
time = 1.49, size = 120, normalized size = 4.14

method	result	size
risch	$-\frac{\left(1+{\mathrm e}^{2 x}\right) \sqrt{\frac{\left({\mathrm e}^{2 x}-1\right)^{2}}{\left(1+{\mathrm e}^{2 x}\right)^{2}}}\, x}{{\mathrm e}^{2 x}-1}+\frac{2 \sqrt{\frac{\left({\mathrm e}^{2 x}-1\right)^{2}}{\left(1+{\mathrm e}^{2 x}\right)^{2}}}\, {\mathrm e}^{2 x}}{\left({\mathrm e}^{2 x}-1\right) \left(1+{\mathrm e}^{2 x}\right)}+\frac{\left(1+{\mathrm e}^{2 x}\right) \sqrt{\frac{\left({\mathrm e}^{2 x}-1\right)^{2}}{\left(1+{\mathrm e}^{2 x}\right)^{2}}}\, \ln\left(1+{\mathrm e}^{2 x}\right)}{{\mathrm e}^{2 x}-1}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sech(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(exp(2*x)-1)*(1+exp(2*x))*((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*x+2/(exp
(2*x)-1)/(1+exp(2*x))*((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*exp(2*x)+1/(exp
(2*x)-1)*(1+exp(2*x))*((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*ln(1+exp(2*x))
```

Maxima [A]
time = 0.49, size = 33, normalized size = 1.14

$$-x-\frac{2\,e^{(-2\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}-\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -x - 2*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) - log(e^(-2*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(23) = 46$.
time = 0.38, size = 183, normalized size = 6.31

$$-\frac{x\cosh(x)^4 + 4x\cosh(x)\sinh(x)^3 + x\sinh(x)^4 + 2(x-1)\cosh(x)^2 + 2(3x\cosh(x)^2 + x - 1)\sinh(x)^2 - (\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2(3\cosh(x)^2 + 1)\sinh(x)^2 + 2\cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x))\sinh(x) + 1)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right) + 4(x\cosh(x)^3 + (x-1)\cosh(x))\sinh(x) + x}{\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2(3\cosh(x)^2 + 1)\sinh(x)^2 + 2\cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x))\sinh(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^4 + 4*x*cosh(x)*sinh(x)^3 + x*sinh(x)^4 + 2*(x - 1)*cosh(x)^2 +
 2*(3*x*cosh(x)^2 + x - 1)*sinh(x)^2 - (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + s
inh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cos
h(x))*sinh(x) + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 4*(x*cosh(x)^3 + (x
 - 1)*cosh(x))*sinh(x) + x)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(
x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1 - \operatorname{sech}^2(x)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)**2)**(3/2),x)
```

```
[Out] Integral((1 - sech(x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(23) = 46$.
time = 0.39, size = 72, normalized size = 2.48

$$-x\operatorname{sgn}\left(e^{(4x)} - 1\right) + \log\left(e^{(2x)} + 1\right)\operatorname{sgn}\left(e^{(4x)} - 1\right) - \frac{3e^{(4x)}\operatorname{sgn}\left(e^{(4x)} - 1\right) + 2e^{(2x)}\operatorname{sgn}\left(e^{(4x)} - 1\right) + 3\operatorname{sgn}\left(e^{(4x)} - 1\right)}{2\left(e^{(2x)} + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -x*sgn(e^(4*x) - 1) + log(e^(2*x) + 1)*sgn(e^(4*x) - 1) - 1/2*(3*e^(4*x)*sg
n(e^(4*x) - 1) + 2*e^(2*x)*sgn(e^(4*x) - 1) + 3*sgn(e^(4*x) - 1))/(e^(2*x)
+ 1)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \left(1-\frac{1}{\cosh\left(x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - 1/cosh(x)^2)^(3/2),x)
```

```
[Out] int((1 - 1/cosh(x)^2)^(3/2), x)
```

3.171 $\int \sqrt{1-\mathbf{sech}^2(x)}\, dx$

Optimal. Leaf size=14

$$\coth(x)\log(\cosh(x))\sqrt{\tanh^2(x)}$$

```
[Out] coth(x)*ln(cosh(x))*(tanh(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4206, 3739, 3556}

$$\sqrt{\tanh^2(x)}\coth(x)\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sech[x]^2],x]
```

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \sqrt{1-\operatorname{sech}^2(x)}\,dx = \int \sqrt{\tanh^2(x)}\,dx$$
$$= \left(\coth(x)\sqrt{\tanh^2(x)}\right)\int \tanh(x)\,dx$$
$$= \coth(x)\log(\cosh(x))\sqrt{\tanh^2(x)}$$

<u>**Mathematica**</u> [A]
time = 0.01, size = 14, normalized size = 1.00

$$\coth(x)\log(\cosh(x))\sqrt{\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sech[x]^2],x]
```

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[Tanh[x]^2]
```

<u>**Maple**</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(12) = 24$.
time = 1.50, size = 79, normalized size = 5.64

method	result	size
risch	$-\frac{(1+e^{2x})\sqrt{\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\, x}{e^{2x}-1}+\frac{(1+e^{2x})\sqrt{\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\, \ln(1+e^{2x})}{e^{2x}-1}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(exp(2*x)-1)*(1+exp(2*x))*((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*x+1/(exp
(2*x)-1)*(1+exp(2*x))*((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*ln(1+exp(2*x))
```

<u>**Maxima**</u> [A]
time = 0.48, size = 13, normalized size = 0.93

$$-x-\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -x - log(e^(-2*x) + 1)
```

Fricas [A]
time = 0.36, size = 18, normalized size = 1.29

$$-x+\log\left(\frac{2\cosh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{1-\operatorname{sech}^2\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - sech(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.
time = 0.39, size = 26, normalized size = 1.86

$$-x\operatorname{sgn}\left(e^{(4x)}-1\right)+\log\left(e^{(2x)}+1\right)\operatorname{sgn}\left(e^{(4x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -x*sgn(e^(4*x) - 1) + log(e^(2*x) + 1)*sgn(e^(4*x) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.07

$$\int\sqrt{1-\frac{1}{{\cosh\left(x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - 1/cosh(x)^2)^(1/2),x)
```

```
[Out] int((1 - 1/cosh(x)^2)^(1/2), x)
```

3.172 $\int \frac{1}{\sqrt{1-\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=14

$$\frac{\log(\sinh(x))\tanh(x)}{\sqrt{\tanh^2(x)}}$$

```
[Out] ln(sinh(x))*tanh(x)/(tanh(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4206, 3739, 3556}

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - Sech[x]^2],x]
```

```
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[Tanh[x]^2]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-\operatorname{sech}^2(x)}}\,dx = \int \frac{1}{\sqrt{\tanh^2(x)}}\,dx$$
$$= \frac{\tanh(x)\int \coth(x)\,dx}{\sqrt{\tanh^2(x)}}$$
$$= \frac{\log(\sinh(x))\tanh(x)}{\sqrt{\tanh^2(x)}}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{\log(\sinh(x))\tanh(x)}{\sqrt{\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Sech[x]^2],x]
```

```
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[Tanh[x]^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(12) = 24$.
time = 1.12, size = 79, normalized size = 5.64

method	result	size
risch	$-\frac{(e^{2x}-1)x}{\sqrt{\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\,(1+e^{2x})}+\frac{(e^{2x}-1)\ln(e^{2x}-1)}{\sqrt{\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\,(1+e^{2x})}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/((exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*(exp(2*x)-1)*x+1/((ex
p(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*(exp(2*x)-1)*ln(exp(2*x)-1)
```

Maxima [A]
time = 0.50, size = 22, normalized size = 1.57

$$-x-\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sech(x)^2)^(1/2),x, algorithm="maxima")
[Out] -x - log(e^(-x) + 1) - log(e^(-x) - 1)
```

Fricas [A]
time = 0.41, size = 18, normalized size = 1.29

$$-x+\log\left(\frac{2\,\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sech(x)^2)^(1/2),x, algorithm="fricas")
[Out] -x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{1-\operatorname{sech}^{2}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sech(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(1 - sech(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.
time = 0.39, size = 31, normalized size = 2.21

$$-\frac{x}{\operatorname{sgn}\left(e^{(4\,x)}-1\right)}+\frac{\log\left(\left|e^{(2\,x)}-1\right|\right)}{\operatorname{sgn}\left(e^{(4\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sech(x)^2)^(1/2),x, algorithm="giac")
[Out] -x/sgn(e^(4*x) - 1) + log(abs(e^(2*x) - 1))/sgn(e^(4*x) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.07

$$\int\frac{1}{\sqrt{1-\frac{1}{{\cosh\left(x\right)}^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - 1/cosh(x)^2)^(1/2),x)
[Out] int(1/(1 - 1/cosh(x)^2)^(1/2), x)
```

3.173 $\int \left(-1+\operatorname{sech}^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=34

$$-\coth(x)\log(\cosh(x))\sqrt{-\tanh^2(x)}+\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)}$$

```
[Out] -coth(x)*ln(cosh(x))*(-tanh(x)^2)^(1/2)+1/2*(-tanh(x)^2)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4206, 3739, 3554, 3556}

$$\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)}-\sqrt{-\tanh^2(x)}\coth(x)\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sech[x]^2)^(3/2),x]
```

```
[Out] -(Coth[x]*Log[Cosh[x]]*Sqrt[-Tanh[x]^2]) + (Tanh[x]*Sqrt[-Tanh[x]^2])/2
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \left(-1+\operatorname{sech}^2(x)\right)^{3/2}\,dx &= \int \left(-\tanh^2(x)\right)^{3/2}\,dx\\ &= -\left(\left(\coth(x)\sqrt{-\tanh^2(x)}\right)\int \tanh^3(x)\,dx\right)\\ &= \frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)}-\left(\coth(x)\sqrt{-\tanh^2(x)}\right)\int \tanh(x)\,dx\\ &= -\coth(x)\log(\cosh(x))\sqrt{-\tanh^2(x)}+\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.79

$$-\frac{1}{2}(2\coth(x)\log(\cosh(x))+\operatorname{csch}(x)\operatorname{sech}(x))\sqrt{-\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sech[x]^2)^(3/2),x]
[Out] -1/2*((2*Coth[x]*Log[Cosh[x]] + Csch[x]*Sech[x])*Sqrt[-Tanh[x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(28) = 56.
time = 1.44, size = 123, normalized size = 3.62

method	result	size
risch	$\frac{(1+e^{2x})\sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\,x}{e^{2x}-1}-\frac{2\sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\,e^{2x}}{(e^{2x}-1)(1+e^{2x})}-\frac{(1+e^{2x})\sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\,\ln(1+e^{2x})}{e^{2x}-1}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sech(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/(exp(2*x)-1)*(1+exp(2*x))*(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*x-2/(exp
(2*x)-1)/(1+exp(2*x))*(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*exp(2*x)-1/(ex
p(2*x)-1)*(1+exp(2*x))*(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*ln(1+exp(2*x)
)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 33, normalized size = 0.97

$$i\,x+\frac{2i\,e^{(-2\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}+i\,\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] I*x + 2*I*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + I*log(e^(-2*x) + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\operatorname{sech}^{2}(x) - 1\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)**2)**(3/2),x)
```

```
[Out] Integral((sech(x)**2 - 1)**(3/2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.39, size = 83, normalized size = 2.44

$$-ix\operatorname{sgn}\left(-e^{(4x)}+1\right)+i\log\left(e^{(2x)}+1\right)\operatorname{sgn}\left(-e^{(4x)}+1\right)-\frac{i\left(3e^{(4x)}\operatorname{sgn}\left(-e^{(4x)}+1\right)+2e^{(2x)}\operatorname{sgn}\left(-e^{(4x)}+1\right)+3\operatorname{sgn}\left(-e^{(4x)}+1\right)\right)}{2\left(e^{(2x)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -I*x*sgn(-e^(4*x) + 1) + I*log(e^(2*x) + 1)*sgn(-e^(4*x) + 1) - 1/2*I*(3*e^
(4*x)*sgn(-e^(4*x) + 1) + 2*e^(2*x)*sgn(-e^(4*x) + 1) + 3*sgn(-e^(4*x) + 1)
)/(e^(2*x) + 1)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(\frac{1}{{\mathrm{cosh}\left(x\right)}^2}-1\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(x)^2 - 1)^(3/2),x)
```

```
[Out] int((1/cosh(x)^2 - 1)^(3/2), x)
```

3.174 $\int \sqrt{-1+\text{sech}^2(x)}\,dx$

Optimal. Leaf size=16

$$\coth(x)\log(\cosh(x))\sqrt{-\tanh^2(x)}$$

[Out] coth(x)*ln(cosh(x))*(-tanh(x)^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4206, 3739, 3556}

$$\sqrt{-\tanh^2(x)}\coth(x)\log(\cosh(x))$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + Sech[x]^2],x]

[Out] Coth[x]*Log[Cosh[x]]*Sqrt[-Tanh[x]^2]

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \sqrt{-1+\operatorname{sech}^2(x)}\,dx = \int \sqrt{-\tanh^2(x)}\,dx$$
$$= \left(\coth(x)\sqrt{-\tanh^2(x)}\right)\int \tanh(x)\,dx$$
$$= \coth(x)\log(\cosh(x))\sqrt{-\tanh^2(x)}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\coth(x)\log(\cosh(x))\sqrt{-\tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sech[x]^2],x]
```

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[-Tanh[x]^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(14) = 28$.
time = 1.44, size = 81, normalized size = 5.06

method	result	size
risch	$-\frac{(1+e^{2x})\sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\;x}{e^{2x}-1}+\frac{(1+e^{2x})\sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\;\ln(1+e^{2x})}{e^{2x}-1}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(exp(2*x)-1)*(1+exp(2*x))*(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*x+1/(ex
p(2*x)-1)*(1+exp(2*x))*(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)*ln(1+exp(2*x)
)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 13, normalized size = 0.81

$$-i\,x - i\,\log\left(e^{(-2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*x - I*log(e^(-2*x) + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{sech}^{2}(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)**2)**(1/2),x)
[Out] Integral(sqrt(sech(x)**2 - 1), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.39, size = 31, normalized size = 1.94

$$i\,x\operatorname{sgn}\left(-e^{(4\,x)} + 1\right) - i\,\log\left(e^{(2\,x)} + 1\right)\operatorname{sgn}\left(-e^{(4\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sech(x)^2)^(1/2),x, algorithm="giac")
[Out] I*x*sgn(-e^(4*x) + 1) - I*log(e^(2*x) + 1)*sgn(-e^(4*x) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \sqrt{\frac{1}{\cosh(x)^{2}} - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(x)^2 - 1)^(1/2),x)
[Out] int((1/cosh(x)^2 - 1)^(1/2), x)
```

3.175 $\int \frac{1}{\sqrt{-1+\operatorname{sech}^2(x)}}\,dx$

Optimal. Leaf size=16

$$\frac{\log(\sinh(x))\tanh(x)}{\sqrt{-\tanh^2(x)}}$$

[Out] ln(sinh(x))*tanh(x)/(-tanh(x)^2)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4206, 3739, 3556}

$$\frac{\tanh(x)\log(\sinh(x))}{\sqrt{-\tanh^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-1 + Sech[x]^2],x]

[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[-Tanh[x]^2]

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+\operatorname{sech}^2(x)}}\,dx = \int \frac{1}{\sqrt{-\tanh^2(x)}}\,dx$$
$$= \frac{\tanh(x)\int \coth(x)\,dx}{\sqrt{-\tanh^2(x)}}$$
$$= \frac{\log(\sinh(x))\tanh(x)}{\sqrt{-\tanh^2(x)}}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{\log(\sinh(x))\tanh(x)}{\sqrt{-\tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + Sech[x]^2],x]
```

```
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[-Tanh[x]^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(14) = 28$.
time = 1.35, size = 81, normalized size = 5.06

method	result	size
risch	$-\frac{(e^{2x}-1)x}{\sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\,(1+e^{2x})}+\frac{(e^{2x}-1)\ln(e^{2x}-1)}{\sqrt{-\frac{(e^{2x}-1)^2}{(1+e^{2x})^2}}\,(1+e^{2x})}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-(exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*(exp(2*x)-1)*x+1/(-(
exp(2*x)-1)^2/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*(exp(2*x)-1)*ln(exp(2*x)-1
)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.50, size = 22, normalized size = 1.38

$$i\,x + i\,\log\left(e^{(-x)}+1\right) + i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sech(x)^2)^(1/2),x, algorithm="maxima")
[Out] I*x + I*log(e^(-x) + 1) + I*log(e^(-x) - 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sech(x)^2)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(x) - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sech(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(sech(x)**2 - 1), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.39, size = 37, normalized size = 2.31

$$-\frac{i\,x}{\operatorname{sgn}\left(-e^{(4\,x)}+1\right)}+\frac{i\,\log\left(-i\,e^{(2\,x)}+i\right)}{\operatorname{sgn}\left(-e^{(4\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sech(x)^2)^(1/2),x, algorithm="giac")
[Out] -I*x/sgn(-e^(4*x) + 1) + I*log(-I*e^(2*x) + I)/sgn(-e^(4*x) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{\frac{1}{\cosh(x)^2} - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(x)^2 - 1)^(1/2),x)
[Out] int(1/(1/cosh(x)^2 - 1)^(1/2), x)
```

3.176 $\int \sqrt{a+b\text{sech}^2(x)} \tanh^5(x) \, dx$

Optimal. Leaf size=83

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\text{sech}^2(x)}+\frac{(a+2b)\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2}-\frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b^2}$$

[Out] 1/3*(a+2*b)*(a+b*sech(x)^2)^(3/2)/b^2-1/5*(a+b*sech(x)^2)^(5/2)/b^2+arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-(a+b*sech(x)^2)^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4224, 457, 90, 52, 65, 214}

$$-\frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b^2}+\frac{(a+2b)\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2}-\sqrt{a+b\text{sech}^2(x)}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^5,x]

[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b*Sech[x]^2] + ((a + 2*b)*(a + b*Sech[x]^2)^(3/2))/(3*b^2) - (a + b*Sech[x]^2)^(5/2)/(5*b^2)

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \sqrt{a+b\text{sech}^2(x)}\,\tanh^5(x)\,dx = -\text{Subst}\left(\int \frac{(-1+x^2)^2\sqrt{a+bx^2}}{x}\,dx, x, \text{sech}(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^2\sqrt{a+bx}}{x}\,dx, x, \text{sech}^2(x)\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{(-a-2b)\sqrt{a+bx}}{b} + \frac{\sqrt{a+bx}}{x} + \frac{(a+bx)^{3/2}}{b}\right) dx, x$$

$$= \frac{(a+2b)\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b^2} - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+b}}{x}\right.$$

$$= -\sqrt{a+b\text{sech}^2(x)} + \frac{(a+2b)\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b^2}$$

$$= -\sqrt{a+b\text{sech}^2(x)} + \frac{(a+2b)\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b^2}$$

$$= \sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b\text{sech}^2(x)} + \frac{(a+2b)\left(a+b\text{s\right.}{3b^2}$$

Mathematica [A]
time = 0.49, size = 114, normalized size = 1.37

$$\frac{1}{15}\cosh(x)\sqrt{a+b\text{sech}^2(x)}\left(\frac{15\sqrt{2}\,\sqrt{a}\,\log\left(\sqrt{2}\,\sqrt{a}\,\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)}{\sqrt{a+2b+a\cosh(2x)}} + \left(-15+\frac{2a^2}{b^2}+\frac{10a}{b}\right)\text{sech}(x) + \left(10-\frac{a}{b}\right)\text{sech}^3(x) - 3\text{sech}^5(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^5,x]
```

```
[Out] (Cosh[x]*Sqrt[a + b*Sech[x]^2]*((15*Sqrt[2]*Sqrt[a]*Log[Sqrt[2]*Sqrt[a]*Cos
h[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/Sqrt[a + 2*b + a*Cosh[2*x]] + (-15 + (
2*a^2)/b^2 + (10*a)/b)*Sech[x] + (10 - a/b)*Sech[x]^3 - 3*Sech[x]^5))/15
```

Maple [F]
time = 1.94, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\text{sech}\,(x)^2}\,\left(\tanh^5(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x)
```

```
[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1929 vs. $2(67) = 134$.
time = 0.65, size = 4594, normalized size = 55.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b
^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*
(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*c
osh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2
*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh
(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cos
h(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*
b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^
2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5
+ 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b
^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*
b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2
*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh
(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*
b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (7
0*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^
3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b +
a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3
+ 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x
)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b
^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh
(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b
+ b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b +
 b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*s
inh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(
```

```
x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)
^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*
(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2
 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 1
4*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)
^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3
+ 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*(b^2*cosh
(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9
*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 +
b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sin
h(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2
*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh
(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh
(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*
b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2
+ 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^3 +
 b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 +
a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*co
sh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*sqrt(2)*((2*a^2 + 10*a*b - 15*b^2)*cosh(x)^8 + 8*(2*a^2 + 10*a*b -
15*b^2)*cosh(x)*sinh(x)^7 + (2*a^2 + 10*a*b - 15*b^2)*sinh(x)^8 + 4*(2*a^2
+ 9*a*b - 5*b^2)*cosh(x)^6 + 4*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^2 + 2*a
^2 + 9*a*b - 5*b^2)*sinh(x)^6 + 8*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^3 +
3*(2*a^2 + 9*a*b - 5*b^2)*cosh(x))*sinh(x)^5 + 2*(6*a^2 + 26*a*b - 29*b^2)*
cosh(x)^4 + 2*(35*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^4 + 30*(2*a^2 + 9*a*b -
 5*b^2)*cosh(x)^2 + 6*a^2 + 26*a*b - 29*b^2)*sinh(x)^4 + 8*(7*(2*a^2 + 10*a
*b - 15*b^2)*cosh(x)^5 + 10*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^3 + (6*a^2 + 26
*a*b - 29*b^2)*cosh(x))*sinh(x)^3 + 4*(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^2 + 4
*(7*(2*a^2 + 10*a*b - 15*b^2)*cosh(x)^6 + 15*(2*a^2 + 9*a*b - 5*b^2)*cosh(x
)^4 + 3*(6*a^2 + 26*a*b - 29*b^2)*cosh(x)^2 + 2*a^2 + 9*a*b - 5*b^2)*sinh(x
)^2 + 2*a^2 + 10*a*b - 15*b^2 + 8*((2*a^2 + 10*a*b - 15*b^2)*cosh(x)^7 + 3*
(2*a^2 + 9*a*b - 5*b^2)*cosh(x)^5 + (6*a^2 + 26*a*b - 29*b^2)*cosh(x)^3 + (
2*a^2 + 9*a*b - 5*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^10 + 10
*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x
)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh(x)
)*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 10
*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*c...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\mathrm{sech}^2(x)}\,\tanh^5(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**5,x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x)**5, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^5\,\sqrt{a+\frac{b}{\cosh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^5*(a + b/cosh(x)^2)^(1/2), x)
```

3.177 $\int \sqrt{a+b\,\mathrm{sech}^2(x)}\,\tanh^4(x)\,dx$

Optimal. Leaf size=125

$$-\frac{(a^2+6ab-3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{8b^{3/2}}+\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)+\frac{(a-3b)\,\tan}{}$$

```
[Out] -1/8*(a^2+6*a*b-3*b^2)*arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(3
/2)+arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)+1/8*(a-3*b)*(a
+b-b*tanh(x)^2)^(1/2)*tanh(x)/b-1/4*(a+b-b*tanh(x)^2)^(1/2)*tanh(x)^3
```

Rubi [A]
time = 0.21, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {4226, 2000, 489, 596, 537, 223, 209, 385, 212}

$$-\frac{(a^2+6ab-3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{8b^{3/2}}+\frac{(a-3b)\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{8b}+\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\frac{1}{4}\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^4,x]
```

```
[Out] -1/8*((a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]
^2]])/b^(3/2) + Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]
] + ((a - 3*b)*Tanh[x]*Sqrt[a + b - b*Tanh[x]^2])/(8*b) - (Tanh[x]^3*Sqrt[a
 + b - b*Tanh[x]^2])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
```

```
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \sqrt{a+b\text{sech}^2(x)}\,\tanh^4(x)\,dx = \text{Subst}\left(\int \frac{x^4\sqrt{a+b\left(1-x^2\right)}}{1-x^2}\,dx,x,\tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{x^4\sqrt{a+b-bx^2}}{1-x^2}\,dx,x,\tanh(x)\right)$$
$$= -\frac{1}{4}\tanh^3(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{4}\text{Subst}\left(\int \frac{x^2(3(a+b)+(a-3b)}{(1-x^2)\sqrt{a+b-bx}}\right.$$
$$= \frac{(a-3b)\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b-b\tanh^2(x}$$
$$= \frac{(a-3b)\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b-b\tanh^2(x}$$
$$= \frac{(a-3b)\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b-b\tanh^2(x}$$
$$= -\frac{(a^2+6ab-3b^2)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{8b^{3/2}} + \sqrt{a}\tanh^{-1}\left(\frac{}{\sqrt{a}}\right.$$

Mathematica [A]
time = 0.31, size = 192, normalized size = 1.54

$$-\frac{\cosh(x)\sqrt{a+b\text{sech}^2(x)}\left(\sqrt{2}\left(a^2+6ab-3b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)-8\sqrt{2}\sqrt{a}\,b^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)-(a-5b)\sqrt{b}\sqrt{a+2b+a\cosh(2x)}\,\text{sech}(x)\tanh(x)-2b^{3/2}\sqrt{a+2b+a\cosh(2x)}\,\text{sech}^3(x)\tanh(x)\right)}{8b^{3/2}\sqrt{a+2b+a\cosh(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^4,x]
```

```
[Out] -1/8*(Cosh[x]*Sqrt[a + b*Sech[x]^2]*(Sqrt[2]*(a^2 + 6*a*b - 3*b^2)*ArcTan[(
Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - 8*Sqrt[2]*Sqrt[a]*b
^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - (a
- 5*b)*Sqrt[b]*Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]*Tanh[x] - 2*b^(3/2)*Sqrt
```

[a + 2*b + a*Cosh[2*x]]*Sech[x]^3*Tanh[x]))/(b^(3/2)*Sqrt[a + 2*b + a*Cosh[
2*x]])

Maple [F]
time = 1.66, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\mathrm{sech}\,(x)^2}\ (\tanh^4(x))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x)
```

```
[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1927 vs. 2(103) = 206.
time = 0.68, size = 8852, normalized size = 70.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*c
osh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2
*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x
)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh
(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 +
9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5
 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8
*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*
(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a
*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*
a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sin
h(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*
b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14
```

```
*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*
a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh
(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)
*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*
cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 -
 a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x
))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5
+ (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(
cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sin
h(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^2
 + 6*a*b - 3*b^2)*cosh(x)^8 + 8*(a^2 + 6*a*b - 3*b^2)*cosh(x)*sinh(x)^7 + (
a^2 + 6*a*b - 3*b^2)*sinh(x)^8 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 4*(7*(
a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^6 + 8*(7*(a^2
 + 6*a*b - 3*b^2)*cosh(x)^3 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^5 +
6*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 6*a*b - 3*b^2)*cosh(x)^4 +
 30*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + 3*a^2 + 18*a*b - 9*b^2)*sinh(x)^4 + 8
*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^5 + 10*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 +
3*(a^2 + 6*a*b - 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 6*a*b - 3*b^2)*cosh(x
)^2 + 4*(7*(a^2 + 6*a*b - 3*b^2)*cosh(x)^6 + 15*(a^2 + 6*a*b - 3*b^2)*cosh(
x)^4 + 9*(a^2 + 6*a*b - 3*b^2)*cosh(x)^2 + a^2 + 6*a*b - 3*b^2)*sinh(x)^2 +
 a^2 + 6*a*b - 3*b^2 + 8*((a^2 + 6*a*b - 3*b^2)*cosh(x)^7 + 3*(a^2 + 6*a*b
- 3*b^2)*cosh(x)^5 + 3*(a^2 + 6*a*b - 3*b^2)*cosh(x)^3 + (a^2 + 6*a*b - 3*b
^2)*cosh(x))*sinh(x))*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*
sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x
)^2 + a + 3*b)*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))
*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(
x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) +
 4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)
^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(
x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 +
3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3
+ 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*
cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b
^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x
)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)
*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(2)*((a*b - 5*b^2)*cosh(x)^6 + 6*
(a*b - 5*b^2)*cosh(x)*sinh(x)^5 + (a*b - 5*b^2)*sinh(x)^6 + (a*b + 3*b^2)*c
osh(x)^4 + (15*(a*b - 5*b^2)*cosh(x)^2 + a*b + 3*b^2)*sinh(x)^4 + 4*(5*(a*b
 - 5*b^2)*cosh(x)^3 + (a*b + 3*b^2)*cosh(x))*sinh(x)^3 - (a*b + 3*b^2)*cosh
```

```
(x)^2 + (15*(a*b - 5*b^2)*cosh(x)^4 + 6*(a*b + 3*b^2)*cosh(x)^2 - a*b - 3*b
^2)*sinh(x)^2 - a*b + 5*b^2 + 2*(3*(a*b - 5*b^2)*cosh(x)^5 + 2*(a*b + 3*b^2
)*cosh(x)^3 - (a*b + 3*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)
^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^8
+ 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh
(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(
x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 +
 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*s
inh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}^2(x)} \tanh^4(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**4,x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x)**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^4 \sqrt{a + \frac{b}{\cosh(x)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^4*(a + b/cosh(x)^2)^(1/2), x)
```

3.178 $\int \sqrt{a+b\operatorname{sech}^2(x)} \tanh^3(x)\, dx$

Optimal. Leaf size=59

$$\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\operatorname{sech}^2(x)}+\frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b}$$

```
[Out] 1/3*(a+b*sech(x)^2)^(3/2)/b+arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-
(a+b*sech(x)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4224, 457, 81, 52, 65, 214}

$$\frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b}-\sqrt{a+b\operatorname{sech}^2(x)}+\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^3,x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b*Sech[x]^2] + (a
 + b*Sech[x]^2)^(3/2)/(3*b)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+b\operatorname{sech}^2(x)}\,\tanh^3(x)\,dx &= \operatorname{Subst}\left(\int \frac{(-1+x^2)\sqrt{a+bx^2}}{x}\,dx, x, \operatorname{sech}(x)\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(-1+x)\sqrt{a+bx}}{x}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= \frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= -\sqrt{a+b\operatorname{sech}^2(x)} + \frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b} - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x\right.\\
&= -\sqrt{a+b\operatorname{sech}^2(x)} + \frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b} - \frac{a\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\right.}{b}\\
&= \sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b\operatorname{sech}^2(x)} + \frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/}}{3b}
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 90, normalized size = 1.53

$$\frac{1}{3}\cosh(x)\sqrt{a+b\mathrm{sech}^2(x)}\left(\frac{3\sqrt{2}\sqrt{a}\log\left(\sqrt{2}\sqrt{a}\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)}{\sqrt{a+2b+a\cosh(2x)}}+\left(-3+\frac{a}{b}\right)\mathrm{sech}(x)+\mathrm{sech}^3(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^3,x]
```

```
[Out] (Cosh[x]*Sqrt[a + b*Sech[x]^2]*((3*Sqrt[2]*Sqrt[a]*Log[Sqrt[2]*Sqrt[a]*Cosh
[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/Sqrt[a + 2*b + a*Cosh[2*x]] + (-3 + a/b
)*Sech[x] + Sech[x]^3))/3
```

Maple [F]
time = 1.21, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{sech}\left(x\right)^{2}}\,\left(\tanh^{3}\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x)
```

```
[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 829 vs. 2(47) = 94.
time = 0.46, size = 2394, normalized size = 40.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4
 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x
)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(
```

```
b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a)*log(((a^3 + 2
*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (
a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*co
sh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^
2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3
+ 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a
^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9
*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(
a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a
^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*
a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b
+ b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b +
b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^
2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x
)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 +
2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5
+ (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5
*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3
*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(
x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a)*log(-(a*cosh(x)^4 +
4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)
*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*((a - 3*b)*cosh(x)^4 + 4*(a - 3*b)*
cosh(x)*sinh(x)^3 + (a - 3*b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a - 3
*b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a - 3*b)*cosh(x)^3 + (a - b)*cosh(x)
)*sinh(x) + a - 3*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b
*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(
x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh
(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x)
+ b), -1/6*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh
(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*s
inh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2
+ 6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a)*arctan(
```

```
sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
+ a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*s
inh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*
b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3
+ (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)
^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b
*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*
b*cosh(x)^2 + b)*sinh(x)^2 + 6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*si
nh(x) + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh
(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(
a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*sqrt(2)*((a - 3*b)*cosh(
x)^4 + 4*(a - 3*b)*cosh(x)*sinh(x)^3 + (a - 3*b)*sinh(x)^4 + 2*(a - b)*cosh
(x)^2 + 2*(3*(a - 3*b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a - 3*b)*cosh(x)^
3 + (a - b)*cosh(x))*sinh(x) + a - 3*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a
 + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^6 + 6*b*co
sh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}^2(x)} \tanh^3(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**3,x)
[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tanh(x)^3 \sqrt{a + \frac{b}{\cosh(x)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3*(a + b/cosh(x)^2)^(1/2),x)
[Out] int(tanh(x)^3*(a + b/cosh(x)^2)^(1/2), x)
```

3.179 $\int \sqrt{a+b\,\mathbf{sech}^2(x)}\,\tanh^2(x)\,dx$

Optimal. Leaf size=87

$$-\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{2\sqrt{b}}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)-\frac{1}{2}\tanh(x)\sqrt{a+b-b$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)-1/2*(a-b)*arctan(b
^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(1/2)-1/2*(a+b-b*tanh(x)^2)^(1/2)
*tanh(x)
```

Rubi [A]
time = 0.14, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {4226, 2000, 489, 537, 223, 209, 385, 212}

$$-\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{2\sqrt{b}}-\frac{1}{2}\tanh(x)\sqrt{a-b\tanh^2(x)+b}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x]^2,x]
```

```
[Out] -1/2*((a - b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/Sqrt[b]
+ Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - (Tanh[x]*S
qrt[a + b - b*Tanh[x]^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \sqrt{a+b\text{sech}^2(x)}\,\tanh^2(x)\,dx = \text{Subst}\left(\int \frac{x^2\sqrt{a+b(1-x^2)}}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{x^2\sqrt{a+b-bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)$$
$$= -\frac{1}{2}\tanh(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{a+b+(a-b)x^2}{(1-x^2)\sqrt{a+b-bx^2}}$$
$$= -\frac{1}{2}\tanh(x)\sqrt{a+b-b\tanh^2(x)} + a\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}$$
$$= -\frac{1}{2}\tanh(x)\sqrt{a+b-b\tanh^2(x)} + a\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\text{ta}}{\sqrt{a+b-}}\right.$$
$$= -\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{2\sqrt{b}} + \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,\text{tan}}{\sqrt{a+b-b\,\text{t}}}\right.$$

Mathematica [A]
time = 0.20, size = 150, normalized size = 1.72

$$-\frac{\cosh(x)\sqrt{a+b\text{sech}^2(x)}\left(\sqrt{2}(a-b)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)-2\sqrt{2}\sqrt{a}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)+\sqrt{b}\sqrt{a+2b+a\cosh(2x)}\,\text{sech}(x)\tanh(x)\right)}{2\sqrt{b}\sqrt{a+2b+a\cosh(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x]^2,x]
```

```
[Out] -1/2*(Cosh[x]*Sqrt[a + b*Sech[x]^2]*(Sqrt[2]*(a - b)*ArcTan[(Sqrt[2]*Sqrt[b
]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - 2*Sqrt[2]*Sqrt[a]*Sqrt[b]*ArcTanh
[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] + Sqrt[b]*Sqrt[a +
2*b + a*Cosh[2*x]]*Sech[x]*Tanh[x]))/(Sqrt[b]*Sqrt[a + 2*b + a*Cosh[2*x]])
```

Maple [F]
time = 0.98, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\text{sech}(x)^2}\,\left(\tanh^2(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x)
```

[Out] int((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x, algorithm="maxima")

[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x)^2, x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(69) = 138$.
time = 0.50, size = 4316, normalized size = 49.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x, algorithm="fricas")

```
[Out] [1/4*((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 +
2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*
sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8
- 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^
6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a
^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 -
 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2
- b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(
a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)
^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqr
t(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(
x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cos
h(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh
(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3
 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
+ a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x
)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^
3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x
)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + ((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3
 + (a - b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a - b
)*sinh(x)^2 + 4*((a - b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a - b)*sqrt
(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x
)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 +
 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a
```

```
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)
^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*co
sh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (b*cosh(x)^4 + 4*b*cosh(x
)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2
 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a
*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 +
 a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cos
h(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*(b*cosh(x)^2 + 2*b*cos
h(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh(x)*si
nh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4
*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b), -1/4*(2*((a - b)*cosh(x)^4 + 4*(a
- b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a
- b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a - b)*cosh(x)^3 + (a - b)*cosh(x))
*sinh(x) + a - b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a
*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2
+ 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - (b*cosh(x)^4 + 4*b*co
sh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(
x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a)*log((a*b^2*cosh(x)^
8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6
 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 -
 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 +
 (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2
)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4
*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 +
2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3
 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2
*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 -
 b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*
a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x
)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*s
inh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(
x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) -
(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\mathrm{sech}^2\left(x\right)}\,\tanh^2\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x)**2,x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^2\,\sqrt{a + \frac{b}{\cosh\left(x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^2*(a + b/cosh(x)^2)^(1/2), x)
```

3.180 $\int \sqrt{a+b\text{sech}^2(x)} \tanh(x) \, dx$

Optimal. Leaf size=40

$$\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\text{sech}^2(x)}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-(a+b*sech(x)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4224, 272, 52, 65, 214}

$$\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\text{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2]*Tanh[x],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b*Sech[x]^2]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+b\operatorname{sech}^2(x)}\,\tanh(x)\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x}\,dx, x, \operatorname{sech}(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \operatorname{sech}^2(x)\right)\right)\\
&= -\sqrt{a+b\operatorname{sech}^2(x)} - \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= -\sqrt{a+b\operatorname{sech}^2(x)} - \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{b}\\
&= \sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b\operatorname{sech}^2(x)}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(40) = 80.
time = 0.18, size = 90, normalized size = 2.25

$$
-\frac{\left(a+2b+a\cosh(2x)-\sqrt{2}\,\sqrt{a}\,\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\cosh(x)}{\sqrt{b}}\right)\cosh(x)\sqrt{\frac{a+2b+a\cosh(2x)}{b}}\right)\sqrt{a+b\operatorname{sech}^2(x)}}{a+2b+a\cosh(2x)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2]*Tanh[x],x]
```

```
[Out] -(((a + 2*b + a*Cosh[2*x] - Sqrt[2]*Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[a]*Cosh[x
])/Sqrt[b]]*Cosh[x]*Sqrt[(a + 2*b + a*Cosh[2*x])/b])*Sqrt[a + b*Sech[x]^2])
/(a + 2*b + a*Cosh[2*x]))
```

Maple [A]
time = 0.30, size = 43, normalized size = 1.08

method	result	size
derivativedivides	$-\sqrt{a+b\operatorname{sech}(x)^{2}}+\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\operatorname{sech}(x)^{2}}}{\operatorname{sech}(x)}\right)$	43
default	$-\sqrt{a+b\operatorname{sech}(x)^{2}}+\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\operatorname{sech}(x)^{2}}}{\operatorname{sech}(x)}\right)$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2)*tanh(x),x,method=_RETURNVERBOSE)
```

```
[Out] -(a+b*sech(x)^2)^(1/2)+a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sec
h(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*tanh(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. 2(32) = 64.
time = 0.49, size = 1608, normalized size = 40.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*log(((a^3 + 2
*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (
a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*
```

```
a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*co
sh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^
2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3
+ 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a
^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9
*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(
a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a
^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*
a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b
+ b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b +
b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^
2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x
)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 +
2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5
+ (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*log(-(a*cosh(x)^4 +
 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b
)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(
a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 + 1), -1/2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 + 1)*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh
(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 +
 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*
cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(
2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + (cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*c
osh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 +
 a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 2
*sqrt(2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{sech}^2\left(x\right)}\,\tanh\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2)*tanh(x),x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*tanh(x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2)*tanh(x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B]
time = 1.71, size = 32, normalized size = 0.80

$$\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{{\mathrm{cosh}\left(x\right)}^2}}}{\sqrt{a}}\right)-\sqrt{a+\frac{b}{{\mathrm{cosh}\left(x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] a^(1/2)*atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2)) - (a + b/cosh(x)^2)^(1/2)
```

3.181 $\int \sqrt{a+b\text{sech}^2(x)}\,dx$

Optimal. Leaf size=59

$$\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)+\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)+arctan(b^(1/2)*tan
h(x)/(a+b-b*tanh(x)^2)^(1/2))*b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4213, 399, 223, 209, 385, 212}

$$\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)+\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] + Sqrt[a]*ArcTa
nh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\text{sech}^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{1-x^2}\,dx, x, \tanh(x)\right)\\ &= a\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)+b\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,d\right.\\ &= a\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)+b\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, -\right.\\ &= \sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 134 vs. 2(59) = 118.
time = 0.49, size = 134, normalized size = 2.27

$$\frac{\sqrt{2}\cosh(x)\left(\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\sqrt{a+2b+a\cosh(2x)}+\sqrt{a}\sqrt{a+b}\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)\sqrt{\frac{a+2b+a\cosh(2x)}{a+b}}\right)\sqrt{a+b\text{sech}^2(x)}}{a+2b+a\cosh(2x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[2]*Cosh[x]*(Sqrt[b]*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a
*Cosh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]] + Sqrt[a]*Sqrt[a + b]*ArcSinh[(Sqr
```

```
t[a]*Sinh[x])/Sqrt[a + b]]*Sqrt[(a + 2*b + a*Cosh[2*x])/(a + b)])*Sqrt[a +
b*Sech[x]^2])/(a + 2*b + a*Cosh[2*x])
```

Maple [F]
time = 1.24, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}(x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(1/2),x)
[Out] int((a+b*sech(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(47) = 94$.
time = 0.44, size = 2949, normalized size = 49.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
[Out] [1/4*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(
x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sin
h(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3
+ 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*
b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a
*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3
+ 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*co
sh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2
+ sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*
cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^
2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2
*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh
(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
```

```
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*c
osh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3
+ (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6)) + 1/2*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a
 - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*
(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a +
 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^
4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*s
inh(x) + 1)) + 1/4*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*si
nh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(
2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2)), sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*
sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 +
 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 1/4*sqrt(a)*log((a*b^2
*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*
cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*co
sh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*co
sh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*
cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 +
 (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cos
h(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b
 + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^
6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*co
sh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 -
(a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*
b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*c
osh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b
^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(
x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20
*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(
x)^6)) + 1/4*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^
4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*co
sh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2)), -1/2*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x)
+ b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cos
```

```
h(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sin
h(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 -
 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(
x))) - 1/2*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*
cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh
(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(
a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 1/2*sqrt(-b)*log(-((a - b)
*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a -...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\mathrm{sech}^2\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
itution variable should perhaps be purged.Warning, replacing 0 by ‘ u‘, a s
ubstit
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a+\frac{b}{{\cosh\left(x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int((a + b/cosh(x)^2)^(1/2), x)
```

3.182 $\int \coth(x)\sqrt{a+b\mathbf{sech}^2(x)}\,dx$

Optimal. Leaf size=56

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-arctanh((a+b*sech(x)^2)^(1/2
)/(a+b)^(1/2))*(a+b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4224, 457, 85, 65, 214}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - Sqrt[a + b]*ArcTanh[Sqrt[a
 + b*Sech[x]^2]/Sqrt[a + b]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \coth(x)\sqrt{a+b\operatorname{sech}^2(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(-1+x^2\right)}\,dx, x, \operatorname{sech}(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)x}\,dx, x, \operatorname{sech}^2(x)\right)$$
$$= -\left(\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\right) + \frac{1}{2}(a+b)\text{Subst}\left(\int \frac{}{(-1+}\right.$$
$$= -\frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{b} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,d\right.}{b}$$
$$= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)$$

Mathematica [A]
time = 0.08, size = 111, normalized size = 1.98

$$\frac{\sqrt{2}\cosh(x)\left(-\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{2a+2b}\cosh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)+\sqrt{a}\log\left(\sqrt{2}\sqrt{a}\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)\right)\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+2b+a\cosh(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[2]*Cosh[x]*(-(Sqrt[a + b]*ArcTanh[(Sqrt[2*a + 2*b]*Cosh[x])/Sqrt[a +
2*b + a*Cosh[2*x]]]) + Sqrt[a]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b +
 a*Cosh[2*x]]])*Sqrt[a + b*Sech[x]^2])/Sqrt[a + 2*b + a*Cosh[2*x]]
```

Maple [F]
time = 1.53, size = 0, normalized size = 0.00

$$\int \coth(x)\sqrt{a+b\operatorname{sech}(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)*(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 494 vs. 2(44) = 88.
time = 0.46, size = 3597, normalized size = 64.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(
a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)
*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^
3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 +
 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^
2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x
)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*c
osh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b
 + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*
```

```
a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a
*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b +
 b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*c
osh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(
x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2
 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*
(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*s
qrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b +
 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3
 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)
^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*
sinh(x)^5 + sinh(x)^6)) + 1/2*sqrt(a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a
 + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2
*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cos
h(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sin
h(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(
x)^3 - cosh(x))*sinh(x) + 1)) + 1/4*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)
*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2
+ sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)), sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sin
h(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*
sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 1/4*sqrt(a)
*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)
*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b
^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 +
 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b
 + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*
a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4
 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2
+ b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 +
2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 1
5*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3
+ 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*c
osh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*si
nh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x
)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3
*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(
```

```
a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4
*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b
 + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(
x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
+ 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b
^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*
cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2
 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 +
sinh(x)^6)) + 1/4*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sin
h(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\mathrm{sech}^2(x)}\,\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*coth(x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
itution variable should perhaps be purged.Warning, replacing 0 by ‘ u‘, a s
ubstit
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(x)\,\sqrt{a + \frac{b}{\cosh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)*(a + b/cosh(x)^2)^(1/2), x)
```

3.183 $\int \coth^2(x)\sqrt{a+b\mathbf{sech}^2(x)}\,dx$

Optimal. Leaf size=48

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)-\coth(x)\sqrt{a+b-b\tanh^2(x)}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)-coth(x)*(a+b-b*tan
h(x)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4226, 2000, 486, 12, 385, 212}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - Coth[x]*Sqrt
[a + b - b*Tanh[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
```

```
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)}\,dx = \operatorname{Subst}\left(\int \frac{\sqrt{a+b\left(1-x^2\right)}}{x^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$

$$= \operatorname{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{x^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$

$$= -\coth(x)\sqrt{a+b-b\tanh^2(x)} + \operatorname{Subst}\left(\int \frac{a}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx,$$

$$= -\coth(x)\sqrt{a+b-b\tanh^2(x)} + a\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\,dx$$

$$= -\coth(x)\sqrt{a+b-b\tanh^2(x)} + a\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh}{\sqrt{a+b-b}}\right.$$

$$= \sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) - \coth(x)\sqrt{a+b-b\tanh^2(x)}$$

Mathematica [A]

time = 0.36, size = 75, normalized size = 1.56

$$\left(\frac{\sqrt{2}\,\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{a+b}}\right)\cosh(x)}{\sqrt{a+b}\,\sqrt{\frac{a+2b+a\cosh(2x)}{a+b}}}-\coth(x)\right)\sqrt{a+b\mathrm{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ((Sqrt[2]*Sqrt[a]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]*Cosh[x])/(Sqrt[a +
 b]*Sqrt[(a + 2*b + a*Cosh[2*x])/(a + b)]) - Coth[x])*Sqrt[a + b*Sech[x]^2]
```

Maple [F]
time = 1.55, size = 0, normalized size = 0.00

$$\int \left(\coth^2(x)\right)\sqrt{a+b\,\mathrm{sech}\,(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2*(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(40) = 80.
time = 0.50, size = 1303, normalized size = 27.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*log((a*b^2*co
sh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cos
h(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(
```

```
x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(
x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cos
h(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a
^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x
)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b +
3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 +
 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(
x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^
2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*
sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh
(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)
*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))
*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*co
sh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^
6)) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*log(-(a*cosh(
x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*c
osh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x)
 + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*sqrt((a*cosh
(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), -1/2*((cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*co
sh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a
*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cos
h(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*
cosh(x))*sinh(x))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-
a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*
b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a
+ 2*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 - 1)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}^2(x)}\,\coth^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*coth(x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(x)^2 \sqrt{a+\frac{b}{\cosh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2*(a + b/cosh(x)^2)^(1/2), x)
```

3.184 $\int \coth^3(x)\sqrt{a+b\,\mathrm{sech}^2(x)}\,dx$

Optimal. Leaf size=83

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}}-\frac{1}{2}\coth^2(x)\sqrt{a+b\,\mathrm{sech}^2(x)}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-1/2*(2*a+b)*arctanh((a+b*sec
h(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-1/2*coth(x)^2*(a+b*sech(x)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4224, 457, 101, 162, 65, 214}

$$-\frac{1}{2}\coth^2(x)\sqrt{a+b\,\mathrm{sech}^2(x)}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - ((2*a + b)*ArcTanh[Sqrt[a
+ b*Sech[x]^2]/Sqrt[a + b]])/(2*Sqrt[a + b]) - (Coth[x]^2*Sqrt[a + b*Sech[x
]^2])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \coth^3(x)\sqrt{a+b\text{sech}^2(x)}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\,(-1+x^2)^2}\,dx, x, \text{sech}(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)^2x}\,dx, x, \text{sech}^2(x)\right)\right)$$
$$= -\frac{1}{2}\coth^2(x)\sqrt{a+b\text{sech}^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-a-\frac{bx}{2}}{(-1+x)x\sqrt{a+bx}}\,dx, x, \text{se}$$
$$= -\frac{1}{2}\coth^2(x)\sqrt{a+b\text{sech}^2(x)} - \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)$$
$$= -\frac{1}{2}\coth^2(x)\sqrt{a+b\text{sech}^2(x)} - \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{sech}^2(x)}\right.}{b}$$
$$= \sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}}$$

Mathematica [A]
time = 0.35, size = 156, normalized size = 1.88

$$-\frac{\left(\sqrt{2}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a+b}\,\cosh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\cosh(x)+\sqrt{a+b}\left(\sqrt{a+2b+a\cosh(2x)}\,\coth^2(x)-2\sqrt{2}\,\sqrt{a}\,\cosh(x)\log\left(\sqrt{2}\,\sqrt{a}\,\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)\right)\right)\sqrt{a+b\text{sech}^2(x)}}{2\sqrt{a+b}\,\sqrt{a+2b+a\cosh(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3*Sqrt[a + b*Sech[x]^2],x]
[Out] -1/2*((Sqrt[2]*(2*a + b)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b
 + a*Cosh[2*x]]]*Cosh[x] + Sqrt[a + b]*(Sqrt[a + 2*b + a*Cosh[2*x]]*Coth[x]
^2 - 2*Sqrt[2]*Sqrt[a]*Cosh[x]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b +
 a*Cosh[2*x]]]))*Sqrt[a + b*Sech[x]^2])/(Sqrt[a + b]*Sqrt[a + 2*b + a*Cosh[
2*x]])
```

Maple [F]
time = 1.72, size = 0, normalized size = 0.00

$$\int \left(\coth^3(x)\right)\sqrt{a+b\,\text{sech}(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3*(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3*(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 906 vs. 2(65) = 130.
time = 0.52, size = 5247, normalized size = 63.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
- 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2 + 4*((a +
 b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a)*log(((a^3 + 2*a^2
*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3
+ 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^
6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)
^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x
)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 +
30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*
a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 +
 (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b
)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b
 + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b
^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2
+ 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 +
 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b
+ b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^
2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)
*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 +
a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3
+ (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^
2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6
*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/
```

```
(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*si
nh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((2*
a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*
(2*a + b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((2
*a + b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(a + b)*log((
(2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 +
 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2
- 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b
)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x
)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + ((a + b)*cosh(x
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2
 + 2*(3*(a + b)*cosh(x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a +
b)*cosh(x))*sinh(x) + a + b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x
)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(
2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2)) - 2*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a +
 b)*sinh(x)^2 + a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)
*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)
^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*sinh(x) + a
 + b), 1/4*(2*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a +
 b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a - b)
*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(x) + 2*a + b)
*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1
)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*co
sh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x
)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*
cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + ((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*(a + b)*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 - a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 - (a + b)*cosh(x))*s
inh(x) + a + b)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(
2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 +
 b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2
*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x
)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^
3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b ...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{sech}^2(x)}\,\coth^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3*(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*coth(x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^3\,\sqrt{a+\frac{b}{\cosh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3*(a + b/cosh(x)^2)^(1/2), x)
```

3.185 $\int \coth^4(x)\sqrt{a+b\text{sech}^2(x)}\,dx$

Optimal. Leaf size=84

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)-\frac{(3a+2b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{3(a+b)}-\frac{1}{3}\coth^3(x)\sqrt{a+b-b\tan$$

[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*a^(1/2)-1/3*(3*a+2*b)*coth(x)*(a+b-b*tanh(x)^2)^(1/2)/(a+b)-1/3*coth(x)^3*(a+b-b*tanh(x)^2)^(1/2)

Rubi [A]
time = 0.17, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4226, 2000, 486, 597, 12, 385, 212}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\frac{1}{3}\coth^3(x)\sqrt{a-b\tanh^2(x)+b}-\frac{(3a+2b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{3(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^4*Sqrt[a + b*Sech[x]^2],x]

[Out] Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - ((3*a + 2*b)*Coth[x]*Sqrt[a + b - b*Tanh[x]^2])/(3*(a + b)) - (Coth[x]^3*Sqrt[a + b - b*Tanh[x]^2])/3

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \coth^4(x)\sqrt{a+b\text{sech}^2(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+b\left(1-x^2\right)}}{x^4\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{x^4\left(1-x^2\right)}\,dx, x, \tanh(x)\right)$$
$$= -\frac{1}{3}\coth^3(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{3}\text{Subst}\left(\int \frac{3a+2b-2bx^2}{x^2\left(1-x^2\right)\sqrt{a+b-bx}}\right.$$
$$= -\frac{(3a+2b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{3(a+b)} - \frac{1}{3}\coth^3(x)\sqrt{a+b-b\tanh^2}$$
$$= -\frac{(3a+2b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{3(a+b)} - \frac{1}{3}\coth^3(x)\sqrt{a+b-b\tanh^2}$$
$$= -\frac{(3a+2b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{3(a+b)} - \frac{1}{3}\coth^3(x)\sqrt{a+b-b\tanh^2}$$
$$= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) - \frac{(3a+2b)\coth(x)\sqrt{a+b-b\,\text{ta}}}{3(a+b)}$$

Mathematica [A]
time = 0.40, size = 149, normalized size = 1.77

$$\frac{\sqrt{2}\cosh(x)\sqrt{a+b\text{sech}^2(x)}\sqrt{a+b+a\sinh^2(x)}\left(3\sqrt{a}(a+b)\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)-\sqrt{a+b}\,\text{csch}(x)\left(4a+3b+(a+b)\text{csch}^2(x)\right)\sqrt{\frac{a+b+a\sinh^2(x)}{a+b}}\right)}{3(a+b)^{3/2}\sqrt{a+2b+a\cosh(2x)}\sqrt{\frac{a+b+a\sinh^2(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[2]*Cosh[x]*Sqrt[a + b*Sech[x]^2]*Sqrt[a + b + a*Sinh[x]^2]*(3*Sqrt[a]
*(a + b)*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]] - Sqrt[a + b]*Csch[x]*(4*a
+ 3*b + (a + b)*Csch[x]^2)*Sqrt[(a + b + a*Sinh[x]^2)/(a + b)]))/(3*(a + b)
^(3/2)*Sqrt[a + 2*b + a*Cosh[2*x]]*Sqrt[(a + b + a*Sinh[x]^2)/(a + b)])
```

Maple [F]
time = 1.70, size = 0, normalized size = 0.00

$$\int \left(\coth^4(x)\right)\sqrt{a+b\text{sech}\left(x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4*(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^4*(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(70) = 140.
time = 0.49, size = 2341, normalized size = 27.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)
^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*(5
*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3
*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)
*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) - a - b)*sqrt(a
)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a
*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*
(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b +
 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a
*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)
*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 +
3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a
^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(
b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 +
 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*
sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2
- a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^
2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 -
3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*
a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*si
```

```
nh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(
x)^5 + sinh(x)^6)) + 3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (
a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*si
nh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)
*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^
2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) -
 a - b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2
*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)
^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2)) - 4*sqrt(2)*((4*a + 3*b)*cosh(x)^4 + 4*(4*a + 3*b)*cosh(x)*sinh(x)^3 +
 (4*a + 3*b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(4*a + 3*b)*cosh(x)^2
 - 2*a - b)*sinh(x)^2 + 4*((4*a + 3*b)*cosh(x)^3 - (2*a + b)*cosh(x))*sinh(
x) + 4*a + 3*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x
)^5 + (a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a
- b)*sinh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*
(a + b)*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*s
inh(x)^2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*si
nh(x) - a - b), -1/6*(3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 +
(a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*s
inh(x)^4 + 4*(5*(a + b)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b
)*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)
^2 + 6*((a + b)*cosh(x)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x)
- a - b)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sin
h(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
- 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3
+ a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)
*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) +
3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 - 3*
(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*(5*(a + b
)*cosh(x)^3 - 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a
+ b)*cosh(x)^4 - 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x
)^5 - 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) - a - b)*sqrt(-a)*arct
an(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh
(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*
cosh(x))*sinh(x) + a)) + 2*sqrt(2)*((4*a + 3*b)*cosh(x)^4 + 4*(4*a + 3*b)*c
osh(x)*sinh(x)^3 + (4*a + 3*b)*sinh(x)^4 - 2*(2*a + b)*cosh(x)^2 + 2*(3*(4*
a + 3*b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((4*a + 3*b)*cosh(x)^3 - (2*a +
 b)*cosh(x))*sinh(x) + 4*a + 3*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^6 + 6*(a +
 b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 - 3*(a + b)*cosh(x)^4 + 3*(5*(a +
```

```
b)*cosh(x)^2 - a - b)*sinh(x)^4 + 4*(5*(a + b)...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}^{2}(x)}\coth^{4}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4*(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(x)**2)*coth(x)**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^{4}\sqrt{a + \frac{b}{\cosh(x)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^4*(a + b/cosh(x)^2)^(1/2), x)
```

3.186 $\int \coth^5(x)\sqrt{a+b\mathbf{sech}^2(x)}\,dx$

Optimal. Leaf size=125

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-\frac{(8a^2+12ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}}-\frac{(4a+3b)\coth^2(x)\sqrt{\ldots}}{8(a+b\ldots}$$

[Out] -1/8*(8*a^2+12*a*b+3*b^2)*arctanh((a+b*sech(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)+arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))*a^(1/2)-1/8*(4*a+3*b)*coth(x)^2*(a+b*sech(x)^2)^(1/2)/(a+b)-1/4*coth(x)^4*(a+b*sech(x)^2)^(1/2)

Rubi [A]
time = 0.15, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4224, 457, 101, 156, 162, 65, 214}

$$-\frac{(8a^2+12ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}}-\frac{1}{4}\coth^4(x)\sqrt{a+b\text{sech}^2(x)}-\frac{(4a+3b)\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{8(a+b)}+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^5*Sqrt[a + b*Sech[x]^2],x]

[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - ((8*a^2 + 12*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a + b]])/(8*(a + b)^(3/2)) - ((4*a + 3*b)*Coth[x]^2*Sqrt[a + b*Sech[x]^2])/(8*(a + b)) - (Coth[x]^4*Sqrt[a + b*Sech[x]^2])/4

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\int \coth^5(x)\sqrt{a+b\operatorname{sech}^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(-1+x^2\right)^3}\,dx, x, \operatorname{sech}(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)^3x}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= -\frac{1}{4}\coth^4(x)\sqrt{a+b\operatorname{sech}^2(x)} + \frac{1}{4}\text{Subst}\left(\int \frac{-2a-\frac{3bx}{2}}{(-1+x)^2x\sqrt{a+bx}}\,dx, x, \text{s}\right.\\
&= -\frac{(4a+3b)\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)}}{8(a+b)} - \frac{1}{4}\coth^4(x)\sqrt{a+b\operatorname{sech}^2(x)} - \frac{\text{S}}{}\\
&= -\frac{(4a+3b)\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)}}{8(a+b)} - \frac{1}{4}\coth^4(x)\sqrt{a+b\operatorname{sech}^2(x)} - \frac{1}{2}\\
&= -\frac{(4a+3b)\coth^2(x)\sqrt{a+b\operatorname{sech}^2(x)}}{8(a+b)} - \frac{1}{4}\coth^4(x)\sqrt{a+b\operatorname{sech}^2(x)} - \frac{a}{}\\
&= \sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right) - \frac{(8a^2+12ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a}}\right.}{8(a+b)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.61, size = 191, normalized size = 1.53

$$
-\frac{\cosh(x)\left(\sqrt{2}\,(8a^2+12ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)+\sqrt{a+b}\left(\frac{1}{2}\sqrt{a+2b+a\cosh(2x)}\,(-2a-b+(6a+5b)\cosh(2x))\coth(x)\operatorname{csch}^3(x)-8\sqrt{2}\sqrt{a}\,(a+b)\log\left(\sqrt{2}\sqrt{a}\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)\right)\right)\sqrt{a+b\operatorname{sech}^2(x)}}{8(a+b)^{3/2}\sqrt{a+2b+a\cosh(2x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5*Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] -1/8*(Cosh[x]*(Sqrt[2]*(8*a^2 + 12*a*b + 3*b^2)*ArcTanh[(Sqrt[2]*Sqrt[a + b
]*Cosh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] + Sqrt[a + b]*((Sqrt[a + 2*b + a*Co
sh[2*x]]*(-2*a - b + (6*a + 5*b)*Cosh[2*x])*Coth[x]*Csch[x]^3)/2 - 8*Sqrt[2
]*Sqrt[a]*(a + b)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]
]))*Sqrt[a + b*Sech[x]^2])/((a + b)^(3/2)*Sqrt[a + 2*b + a*Cosh[2*x]])
```

Maple [F]
time = 1.69, size = 0, normalized size = 0.00

$$
\int \left(\coth^5(x)\right)\sqrt{a+b\operatorname{sech}(x)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5*(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^5*(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5*(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(x)^2 + a)*coth(x)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2732 vs. 2(103) = 206.
time = 0.69, size = 12548, normalized size = 100.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5*(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sin
h(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 - 4*(a^2 + 2*a*b + b^2)*cosh(x)^6 +
4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^6 + 8*(7*(a
^2 + 2*a*b + b^2)*cosh(x)^3 - 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^5 + 6*
(a^2 + 2*a*b + b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 - 30*(a
^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2
+ 2*a*b + b^2)*cosh(x)^5 - 10*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*
b + b^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 4*(7*(a^2 +
 2*a*b + b^2)*cosh(x)^6 - 15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 9*(a^2 + 2*a*b
 + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 8*((
a^2 + 2*a*b + b^2)*cosh(x)^7 - 3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 3*(a^2 + 2
*a*b + b^2)*cosh(x)^3 - (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x))*sqrt(a)*log((
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(
x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b
^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*
a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a
*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b
+ 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(
14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)
*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a
```

```
^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a
^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a
^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)
^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^
6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 +
a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2
+ 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 +
2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*
sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2
)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 +
 a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2
*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*co
sh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x
))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*
cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x
)^6)) + ((8*a^2 + 12*a*b + 3*b^2)*cosh(x)^8 + 8*(8*a^2 + 12*a*b + 3*b^2)*co
sh(x)*sinh(x)^7 + (8*a^2 + 12*a*b + 3*b^2)*sinh(x)^8 - 4*(8*a^2 + 12*a*b +
3*b^2)*cosh(x)^6 + 4*(7*(8*a^2 + 12*a*b + 3*b^2)*cosh(x)^2 - 8*a^2 - 12*a*b
 - 3*b^2)*sinh(x)^6 + 8*(7*(8*a^2 + 12*a*b + 3*b^2)*cosh(x)^3 - 3*(8*a^2 +
12*a*b + 3*b^2)*cosh(x))*sinh(x)^5 + 6*(8*a^2 + 12*a*b + 3*b^2)*cosh(x)^4 +
 2*(35*(8*a^2 + 12*a*b + 3*b^2)*cosh(x)^4 - 30*(8*a^2 + 12*a*b + 3*b^2)*cos
h(x)^2 + 24*a^2 + 36*a*b + 9*b^2)*sinh(x)^4 + 8*(7*(8*a^2 + 12*a*b + 3*b^2)
*cosh(x)^5 - 10*(8*a^2 + 12*a*b + 3*b^2)*cosh(x)^3 + 3*(8*a^2 + 12*a*b + 3*
b^2)*cosh(x))*sinh(x)^3 - 4*(8*a^2 + 12*a*b + 3*b^2)*cosh(x)^2 + 4*(7*(8*a^
2 + 12*a*b + 3*b^2)*cosh(x)^6 - 15*(8*a^2 + 12*a*b + 3*b^2)*cosh(x)^4 + 9*(
8*a^2 + 12*a*b + 3*b^2)*cosh(x)^2 - 8*a^2 - 12*a*b - 3*b^2)*sinh(x)^2 + 8*a
^2 + 12*a*b + 3*b^2 + 8*((8*a^2 + 12*a*b + 3*b^2)*cosh(x)^7 - 3*(8*a^2 + 12
*a*b + 3*b^2)*cosh(x)^5 + 3*(8*a^2 + 12*a*b + 3*b^2)*cosh(x)^3 - (8*a^2 + 1
2*a*b + 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(((2*a + b)*cosh(x)^4 + 4*(
2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2
+ 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x
)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*
cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*
sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(co
sh(x)^3 - cosh(x))*sinh(x) + 1)) + 4*((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^
2 + 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 - 4*(a^2
 + 2*a*b + b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*
b - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 - 3*(a^2 + 2*a*b +
b^2)*cosh(x))*sinh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*
a*b + b^2)*cosh(x)^4 - 30*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 6*a*b + 3
*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 - 10*(a^2 + 2*a*b + b^
2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + 2*a*b +
b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 - 15*(a^2 + 2*a*b + b^2
)*cosh(x)^4 + 9*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^
```

```
2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5*(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5*(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^5 \sqrt{a + \frac{b}{\cosh(x)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5*(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^5*(a + b/cosh(x)^2)^(1/2), x)
```

3.187 $\int \left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}\tanh^3(x)\,dx$

Optimal. Leaf size=76

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)-a\sqrt{a+b\,\mathrm{sech}^2(x)}-\frac{1}{3}\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}+\frac{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}{5b}$$

```
[Out] a^(3/2)*arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))-1/3*(a+b*sech(x)^2)^(3/2)+1/
5*(a+b*sech(x)^2)^(5/2)/b-a*(a+b*sech(x)^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4224, 457, 81, 52, 65, 214}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)+\frac{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}{5b}-\frac{1}{3}\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}-a\sqrt{a+b\,\mathrm{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^3,x]
```

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - a*Sqrt[a + b*Sech[x]^2] -
(a + b*Sech[x]^2)^(3/2)/3 + (a + b*Sech[x]^2)^(5/2)/(5*b)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^3(x)\,dx = \text{Subst}\left(\int \frac{(-1+x^2)\left(a+bx^2\right)^{3/2}}{x}\,dx, x, \text{sech}(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)(a+bx)^{3/2}}{x}\,dx, x, \text{sech}^2(x)\right)$$
$$= \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b} - \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \text{sech}^2(x)\right)$$
$$= -\frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2} + \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b} - \frac{1}{2}a\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\right.$$
$$= -a\sqrt{a+b\text{sech}^2(x)} - \frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2} + \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b} - \frac{1}{2}a$$
$$= -a\sqrt{a+b\text{sech}^2(x)} - \frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2} + \frac{\left(a+b\text{sech}^2(x)\right)^{5/2}}{5b} - \frac{a^2}{}$$
$$= a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right) - a\sqrt{a+b\text{sech}^2(x)} - \frac{1}{3}\left(a+b\text{sech}^2\right.$$

Mathematica [A]
time = 0.59, size = 129, normalized size = 1.70

$$\cosh^3(x)\left(a+b\text{sech}^2(x)\right)^{3/2}\left(\frac{2\sqrt{2}\,a^{3/2}\log\left(\sqrt{2}\sqrt{a}\,\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)}{(a+2b+a\cosh(2x))^{3/2}} + \frac{2(a(3a-20b)\text{sech}(x)+(6a-5b)b\text{sech}^3(x)+3b^2\text{sech}^5(x))}{15b(a+2b+a\cosh(2x))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^3,x]
```

```
[Out] Cosh[x]^3*(a + b*Sech[x]^2)^(3/2)*((2*Sqrt[2]*a^(3/2)*Log[Sqrt[2]*Sqrt[a]*C
osh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/(a + 2*b + a*Cosh[2*x])^(3/2) + (2*(
a*(3*a - 20*b)*Sech[x] + (6*a - 5*b)*b*Sech[x]^3 + 3*b^2*Sech[x]^5))/(15*b*
(a + 2*b + a*Cosh[2*x])))
```

Maple [F]
time = 1.11, size = 0, normalized size = 0.00

$$\int \left(a+b\,\text{sech}(x)^2\right)^{\frac{3}{2}}\left(\tanh^3(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x)
```

```
[Out] int((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x, algorithm="maxima")
[Out] integrate((b*sech(x)^2 + a)^(3/2)*tanh(x)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1745 vs. $2(60) = 120$.
time = 0.63, size = 4226, normalized size = 55.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x, algorithm="fricas")
[Out] [1/60*(15*(a*b*cosh(x)^10 + 10*a*b*cosh(x)*sinh(x)^9 + a*b*sinh(x)^10 + 5*a
*b*cosh(x)^8 + 5*(9*a*b*cosh(x)^2 + a*b)*sinh(x)^8 + 10*a*b*cosh(x)^6 + 40*
(3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^7 + 10*(21*a*b*cosh(x)^4 + 14*a*b*c
osh(x)^2 + a*b)*sinh(x)^6 + 10*a*b*cosh(x)^4 + 4*(63*a*b*cosh(x)^5 + 70*a*b
*cosh(x)^3 + 15*a*b*cosh(x))*sinh(x)^5 + 10*(21*a*b*cosh(x)^6 + 35*a*b*cosh
(x)^4 + 15*a*b*cosh(x)^2 + a*b)*sinh(x)^4 + 5*a*b*cosh(x)^2 + 40*(3*a*b*cos
h(x)^7 + 7*a*b*cosh(x)^5 + 5*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^3 + 5*(9*
a*b*cosh(x)^8 + 28*a*b*cosh(x)^6 + 30*a*b*cosh(x)^4 + 12*a*b*cosh(x)^2 + a*
b)*sinh(x)^2 + a*b + 10*(a*b*cosh(x)^9 + 4*a*b*cosh(x)^7 + 6*a*b*cosh(x)^5
+ 4*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b
^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*
b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2
*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh
(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*
b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (7
0*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^
3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b +
a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3
+ 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x
)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b
^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh
(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b
+ b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b +
 b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*s
inh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(
```

```
x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)
^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*
(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2
 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 1
4*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)
^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3
+ 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*(a*b*cosh
(x)^10 + 10*a*b*cosh(x)*sinh(x)^9 + a*b*sinh(x)^10 + 5*a*b*cosh(x)^8 + 5*(9
*a*b*cosh(x)^2 + a*b)*sinh(x)^8 + 10*a*b*cosh(x)^6 + 40*(3*a*b*cosh(x)^3 +
a*b*cosh(x))*sinh(x)^7 + 10*(21*a*b*cosh(x)^4 + 14*a*b*cosh(x)^2 + a*b)*sin
h(x)^6 + 10*a*b*cosh(x)^4 + 4*(63*a*b*cosh(x)^5 + 70*a*b*cosh(x)^3 + 15*a*b
*cosh(x))*sinh(x)^5 + 10*(21*a*b*cosh(x)^6 + 35*a*b*cosh(x)^4 + 15*a*b*cosh
(x)^2 + a*b)*sinh(x)^4 + 5*a*b*cosh(x)^2 + 40*(3*a*b*cosh(x)^7 + 7*a*b*cosh
(x)^5 + 5*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x)^3 + 5*(9*a*b*cosh(x)^8 + 28*
a*b*cosh(x)^6 + 30*a*b*cosh(x)^4 + 12*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b
+ 10*(a*b*cosh(x)^9 + 4*a*b*cosh(x)^7 + 6*a*b*cosh(x)^5 + 4*a*b*cosh(x)^3 +
 a*b*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 +
a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*co
sh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*sqrt(2)*((3*a^2 - 20*a*b)*cosh(x)^8 + 8*(3*a^2 - 20*a*b)*cosh(x)*si
nh(x)^7 + (3*a^2 - 20*a*b)*sinh(x)^8 + 4*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^6
 + 4*(7*(3*a^2 - 20*a*b)*cosh(x)^2 + 3*a^2 - 14*a*b - 5*b^2)*sinh(x)^6 + 8*
(7*(3*a^2 - 20*a*b)*cosh(x)^3 + 3*(3*a^2 - 14*a*b - 5*b^2)*cosh(x))*sinh(x)
^5 + 2*(9*a^2 - 36*a*b + 4*b^2)*cosh(x)^4 + 2*(35*(3*a^2 - 20*a*b)*cosh(x)^
4 + 30*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^2 + 9*a^2 - 36*a*b + 4*b^2)*sinh(x)
^4 + 8*(7*(3*a^2 - 20*a*b)*cosh(x)^5 + 10*(3*a^2 - 14*a*b - 5*b^2)*cosh(x)^
3 + (9*a^2 - 36*a*b + 4*b^2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 - 14*a*b - 5*b^2
)*cosh(x)^2 + 4*(7*(3*a^2 - 20*a*b)*cosh(x)^6 + 15*(3*a^2 - 14*a*b - 5*b^2)
*cosh(x)^4 + 3*(9*a^2 - 36*a*b + 4*b^2)*cosh(x)^2 + 3*a^2 - 14*a*b - 5*b^2)
*sinh(x)^2 + 3*a^2 - 20*a*b + 8*((3*a^2 - 20*a*b)*cosh(x)^7 + 3*(3*a^2 - 14
*a*b - 5*b^2)*cosh(x)^5 + (9*a^2 - 36*a*b + 4*b^2)*cosh(x)^3 + (3*a^2 - 14*
a*b - 5*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^10 + 10*b*cosh(x)*s
inh(x)^9 + b*sinh(x)^10 + 5*b*cosh(x)^8 + 5*(9*b*cosh(x)^2 + b)*sinh(x)^8 +
 40*(3*b*cosh(x)^3 + b*cosh(x))*sinh(x)^7 + 10*b*cosh(x)^6 + 10*(21*b*cosh(
x)^4 + 14*b*cosh(x)^2 + b)*sinh(x)^6 + 4*(63*b*cosh(x)^5 + 70*b*cosh(x)^3 +
 15*b*cosh(x))*sinh(x)^5 + 10*b*cosh(x)^4 + 10*(21*b*cosh(x)^6 + 35*b*cosh(
x)^4 + 15*b*cosh(x)^2 + b)*sinh(x)^4 + 40*(3*b*...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^2(x)\right)^{\frac{3}{2}} \tanh^3(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(3/2)*tanh(x)**3,x)
```

```
[Out] Integral((a + b*sech(x)**2)**(3/2)*tanh(x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^3 \left(a + \frac{b}{\cosh(x)^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3*(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^3*(a + b/cosh(x)^2)^(3/2), x)
```

3.188 $\int \left(a+b\,\mathbf{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx$

Optimal. Leaf size=125

$$-\frac{(3a^2-6ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{8\sqrt{b}}+a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)-\frac{1}{8}(5a+b)\tan$$

```
[Out] a^(3/2)*arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))-1/8*(3*a^2-6*a*b-b
^2)*arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(1/2)-1/8*(5*a+b)*(a+
b-b*tanh(x)^2)^(1/2)*tanh(x)+1/4*b*(a+b-b*tanh(x)^2)^(1/2)*tanh(x)^3
```

Rubi [A]

time = 0.24, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {4226, 2000, 488, 596, 537, 223, 209, 385, 212}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-\frac{(3a^2-6ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{8\sqrt{b}}-\frac{1}{8}(5a+b)\tanh(x)\sqrt{a-b\tanh^2(x)+b}+\frac{1}{4}b\tanh^3(x)\sqrt{a-b\tanh^2(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^2,x]
```

```
[Out] -1/8*((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]
^2]])/Sqrt[b] + a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]
] - ((5*a + b)*Tanh[x]*Sqrt[a + b - b*Tanh[x]^2])/8 + (b*Tanh[x]^3*Sqrt[a +
 b - b*Tanh[x]^2])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh^2(x)\,dx = \text{Subst}\left(\int \frac{x^2\left(a+b\left(1-x^2\right)\right)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^2\left(a+b-bx^2\right)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{4}b\tanh^3(x)\sqrt{a+b-b\tanh^2(x)} - \frac{1}{4}\text{Subst}\left(\int \frac{x^2(-(a+b)(4a+b) +}{\left(1-x^2\right)\sqrt{a+b}}\right.$$

$$= -\frac{1}{8}(5a+b)\tanh(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{4}b\tanh^3(x)\sqrt{a+b-b\,\text{ta}}$$

$$= -\frac{1}{8}(5a+b)\tanh(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{4}b\tanh^3(x)\sqrt{a+b-b\,\text{ta}}$$

$$= -\frac{1}{8}(5a+b)\tanh(x)\sqrt{a+b-b\tanh^2(x)} + \frac{1}{4}b\tanh^3(x)\sqrt{a+b-b\,\text{ta}}$$

$$= -\frac{\left(3a^2-6ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{8\sqrt{b}} + a^{3/2}\tanh^{-1}\left(\frac{}{\sqrt{}}\right.$$

Mathematica [A]
time = 0.56, size = 197, normalized size = 1.58

$$-\frac{\cosh^3(x)\left(a+b\text{sech}^2(x)\right)^{3/2}\left(\sqrt{2}\left(3a^2-6ab-b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)-8\sqrt{2}a^{3/2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)+(5a-b)\sqrt{b}\sqrt{a+2b+a\cosh(2x)}\,\text{sech}(x)\tanh(x)+2b^{3/2}\sqrt{a+2b+a\cosh(2x)}\,\text{sech}^3(x)\tanh(x)\right)}{4\sqrt{b}\left(a+2b+a\cosh(2x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(3/2)*Tanh[x]^2,x]
```

```
[Out] -1/4*(Cosh[x]^3*(a + b*Sech[x]^2)^(3/2)*(Sqrt[2]*(3*a^2 - 6*a*b - b^2)*ArcT
an[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] - 8*Sqrt[2]*a^(3/
2)*Sqrt[b]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]] +
 (5*a - b)*Sqrt[b]*Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]*Tanh[x] + 2*b^(3/2)*
Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]^3*Tanh[x]))/(Sqrt[b]*(a + 2*b + a*Cosh[
2*x])^(3/2))
```

Maple [F]
time = 0.92, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{sech}(x)^2\right)^{\frac{3}{2}} \left(\tanh^2(x)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x)
[Out] int((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x, algorithm="maxima")
[Out] integrate((b*sech(x)^2 + a)^(3/2)*tanh(x)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1826 vs. 2(103) = 206.
time = 0.69, size = 8582, normalized size = 68.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x, algorithm="fricas")
[Out] [1/16*(4*(a*b*cosh(x)^8 + 8*a*b*cosh(x)*sinh(x)^7 + a*b*sinh(x)^8 + 4*a*b*c
osh(x)^6 + 4*(7*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7*a*b
*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^5 + 2*(35*a*b*cosh(x)^4 + 30*a*b*cosh(x
)^2 + 3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*cosh
(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 4*(7*a*b*cosh(x)^6 + 15*a*b*cosh(x)^4 +
9*a*b*cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 8*(a*b*cosh(x)^7 + 3*a*b*cosh(x)^5
 + 3*a*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8
*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*
(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a
*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*
a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sin
h(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*
b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14
*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*
a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh
(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)
```

```
*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*
cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 -
 a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x
))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5
+ (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(
cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sin
h(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((3*a
^2 - 6*a*b - b^2)*cosh(x)^8 + 8*(3*a^2 - 6*a*b - b^2)*cosh(x)*sinh(x)^7 + (
3*a^2 - 6*a*b - b^2)*sinh(x)^8 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 4*(7*(
3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^6 + 8*(7*(3*a
^2 - 6*a*b - b^2)*cosh(x)^3 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^5 +
6*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 + 2*(35*(3*a^2 - 6*a*b - b^2)*cosh(x)^4 +
 30*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 9*a^2 - 18*a*b - 3*b^2)*sinh(x)^4 + 8
*(7*(3*a^2 - 6*a*b - b^2)*cosh(x)^5 + 10*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 +
3*(3*a^2 - 6*a*b - b^2)*cosh(x))*sinh(x)^3 + 4*(3*a^2 - 6*a*b - b^2)*cosh(x
)^2 + 4*(7*(3*a^2 - 6*a*b - b^2)*cosh(x)^6 + 15*(3*a^2 - 6*a*b - b^2)*cosh(
x)^4 + 9*(3*a^2 - 6*a*b - b^2)*cosh(x)^2 + 3*a^2 - 6*a*b - b^2)*sinh(x)^2 +
 3*a^2 - 6*a*b - b^2 + 8*((3*a^2 - 6*a*b - b^2)*cosh(x)^7 + 3*(3*a^2 - 6*a*
b - b^2)*cosh(x)^5 + 3*(3*a^2 - 6*a*b - b^2)*cosh(x)^3 + (3*a^2 - 6*a*b - b
^2)*cosh(x))*sinh(x))*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*
sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x
)^2 + a + 3*b)*sinh(x)^2 + 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))
*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(
x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) +
 4*(a*b*cosh(x)^8 + 8*a*b*cosh(x)*sinh(x)^7 + a*b*sinh(x)^8 + 4*a*b*cosh(x)
^6 + 4*(7*a*b*cosh(x)^2 + a*b)*sinh(x)^6 + 6*a*b*cosh(x)^4 + 8*(7*a*b*cosh(
x)^3 + 3*a*b*cosh(x))*sinh(x)^5 + 2*(35*a*b*cosh(x)^4 + 30*a*b*cosh(x)^2 +
3*a*b)*sinh(x)^4 + 4*a*b*cosh(x)^2 + 8*(7*a*b*cosh(x)^5 + 10*a*b*cosh(x)^3
+ 3*a*b*cosh(x))*sinh(x)^3 + 4*(7*a*b*cosh(x)^6 + 15*a*b*cosh(x)^4 + 9*a*b*
cosh(x)^2 + a*b)*sinh(x)^2 + a*b + 8*(a*b*cosh(x)^7 + 3*a*b*cosh(x)^5 + 3*a
*b*cosh(x)^3 + a*b*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x
)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)
*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*((5*a*b - b^2)*cosh(x)^6 + 6*
(5*a*b - b^2)*cosh(x)*sinh(x)^5 + (5*a*b - b^2)*sinh(x)^6 + (5*a*b + 7*b^2)
*cosh(x)^4 + (15*(5*a*b - b^2)*cosh(x)^2 + 5*a*b + 7*b^2)*sinh(x)^4 + 4*(5*
(5*a*b - b^2)*cosh(x)^3 + (5*a*b + 7*b^2)*cosh(x))*sinh(x)^3 - (5*a*b + 7*b
^2)*cosh(x)^2 + (15*(5*a*b - b^2)*cosh(x)^4 + 6*(5*a*b + 7*b^2)*cosh(x)^2 -
 5*a*b - 7*b^2)*sinh(x)^2 - 5*a*b + b^2 + 2*(3*(5*a*b - b^2)*cosh(x)^5 + 2*
(5*a*b + 7*b^2)*cosh(x)^3 - (5*a*b + 7*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(
```

```
x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))
/(b*cosh(x)^8 + 8*b*cosh(x)*sinh(x)^7 + b*sinh(x)^8 + 4*b*cosh(x)^6 + 4*(7*
b*cosh(x)^2 + b)*sinh(x)^6 + 8*(7*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^5 + 6*
b*cosh(x)^4 + 2*(35*b*cosh(x)^4 + 30*b*cosh(x)^2 + 3*b)*sinh(x)^4 + 8*(7*b*
cosh(x)^5 + 10*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 4*b*cosh(x)^2 + 4*(7*
b*cosh(x)^6 + 15*b*cosh(x)^4 + 9*b*cosh(x)^2 + ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^{2}\left(x\right)\right)^{\frac{3}{2}}\tanh^{2}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(3/2)*tanh(x)**2,x)
```

```
[Out] Integral((a + b*sech(x)**2)**(3/2)*tanh(x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^2\left(a+\frac{b}{\cosh\left(x\right)^2}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2*(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^2*(a + b/cosh(x)^2)^(3/2), x)
```

3.189 $\int \left(a+b\text{sech}^2(x)\right)^{3/2} \tanh(x)\,dx$

Optimal. Leaf size=57

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-a\sqrt{a+b\text{sech}^2(x)}-\frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}$$

```
[Out] a^(3/2)*arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))-1/3*(a+b*sech(x)^2)^(3/2)-a*
(a+b*sech(x)^2)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4224, 272, 52, 65, 214}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-a\sqrt{a+b\text{sech}^2(x)}-\frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(3/2)*Tanh[x],x]
```

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - a*Sqrt[a + b*Sech[x]^2] -
(a + b*Sech[x]^2)^(3/2)/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\text{sech}^2(x)\right)^{3/2}\tanh(x)\,dx &= -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x}\,dx,x,\text{sech}(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,\text{sech}^2(x)\right)\right)\\
&= -\frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}-\frac{1}{2}a\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,\text{sech}^2(x)\right)\\
&= -a\sqrt{a+b\text{sech}^2(x)}-\frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}-\frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,d\right.\\
&= -a\sqrt{a+b\text{sech}^2(x)}-\frac{1}{3}\left(a+b\text{sech}^2(x)\right)^{3/2}-\frac{a^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{\right.}}{b}\\
&= a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)-a\sqrt{a+b\text{sech}^2(x)}-\frac{1}{3}\left(a+b\text{sech}^2(x\right.
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.08, size = 65, normalized size = 1.14

$$-\frac{2b\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{a\cosh^2(x)}{b}\right)\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3\sqrt{1+\frac{a\cosh^2(x)}{b}}\,(a+2b+a\cosh(2x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(3/2)*Tanh[x],x]
```

```
[Out] (-2*b*Hypergeometric2F1[-3/2, -3/2, -1/2, -((a*Cosh[x]^2)/b)]*(a + b*Sech[x
]^2)^(3/2))/(3*Sqrt[1 + (a*Cosh[x]^2)/b]*(a + 2*b + a*Cosh[2*x]))
```

Maple [A]
time = 0.22, size = 58, normalized size = 1.02

method	result	siz
derivativedivides	$-\frac{\left(a+b\operatorname{sech}(x)^2\right)^{\frac{3}{2}}}{3}-a\left(\sqrt{a+b\operatorname{sech}(x)^2}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\operatorname{sech}(x)^2}}{\operatorname{sech}(x)}\right)\right)$	58
default	$-\frac{\left(a+b\operatorname{sech}(x)^2\right)^{\frac{3}{2}}}{3}-a\left(\sqrt{a+b\operatorname{sech}(x)^2}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\operatorname{sech}(x)^2}}{\operatorname{sech}(x)}\right)\right)$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(3/2)*tanh(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a+b*sech(x)^2)^(3/2)-a*((a+b*sech(x)^2)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/
2)*(a+b*sech(x)^2)^(1/2))/sech(x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*tanh(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 788 vs. 2(45) = 90.
time = 0.58, size = 2312, normalized size = 40.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4
 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x
)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(
a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*log(((a^3 + 2
*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (
a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*co
sh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^
2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3
+ 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a
^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9
*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(
a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a
^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*
a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b
+ b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b +
b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^
2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x
)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 +
2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5
+ (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(
x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^
3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) +
3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5
*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3
*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(
x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*log(-(a*cosh(x)^4 +
4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)
*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2)) - 16*sqrt(2)*(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)
^3 + a*sinh(x)^4 + (2*a + b)*cosh(x)^2 + (6*a*cosh(x)^2 + 2*a + b)*sinh(x)^
2 + 2*(2*a*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + a)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(
x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*
```

```
cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh
(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*
sinh(x) + 1), -1/6*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 +
3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*co
sh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*si
nh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(-a)
*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*si
nh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*c
osh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(
a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*co
sh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + 3*(a*cosh(x)^6 + 6*a*cosh(x)
*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4
+ 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x
)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cos
h(x))*sinh(x) + a)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x
)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
+ a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)
^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 8*sqrt(2)*(a*cosh(
x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + (2*a + b)*cosh(x)^2 + (6*a*cos
h(x)^2 + 2*a + b)*sinh(x)^2 + 2*(2*a*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x)
 + a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sin
h(x) + sinh(x)^2)))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cos
h(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3
 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 ...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(x\right)\right)^{\frac{3}{2}} \tanh\left(x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(3/2)*tanh(x),x)
```

```
[Out] Integral((a + b*sech(x)**2)**(3/2)*tanh(x), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2)*tanh(x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B]
time = 3.36, size = 45, normalized size = 0.79

$$a^{3/2}\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right)-\frac{\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}}{3}-a\sqrt{a+\frac{b}{\cosh(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] a^(3/2)*atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2)) - (a + b/cosh(x)^2)^(3/2)/3
- a*(a + b/cosh(x)^2)^(1/2)
```

3.190 $\int \left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=88

$$\frac{1}{2}\sqrt{b}\,(3a+b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)+a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)+\frac{1}{2}b\tanh(x)\sqrt{a+b$$

```
[Out] a^(3/2)*arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))+1/2*(3*a+b)*arctan
(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))*b^(1/2)+1/2*b*(a+b-b*tanh(x)^2)^(
1/2)*tanh(x)
```

Rubi [A]

time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {4213, 427, 537, 223, 209, 385, 212}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)+\frac{1}{2}\sqrt{b}\,(3a+b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)+\frac{1}{2}b\tanh(x)\sqrt{a-b\tanh^2(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/2 +
 a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] + (b*Tanh[x]*
Sqrt[a + b - b*Tanh[x]^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(x)\right)^{3/2}\,dx = \text{Subst}\left(\int \frac{(a+b-bx^2)^{3/2}}{1-x^2}\,dx, x, \tanh(x)\right)$$

$$= \frac{1}{2}b\tanh(x)\sqrt{a+b-b\tanh^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{-(a+b)(2a+b)+b(3a+b)x^2}{(1-x^2)\sqrt{a+b-bx^2}}\right.$$

$$= \frac{1}{2}b\tanh(x)\sqrt{a+b-b\tanh^2(x)} + a^2\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \text{ta}\right.$$

$$= \frac{1}{2}b\tanh(x)\sqrt{a+b-b\tanh^2(x)} + a^2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh}}\right.$$

$$= \frac{1}{2}\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x}{\sqrt{a+b-b\tanh}}\right.$$

Mathematica [**A**]
time = 0.18, size = 152, normalized size = 1.73

$$\frac{(b+a\cosh^2(x))\operatorname{sech}(x)\sqrt{a+b\operatorname{sech}^2(x)}\left(\sqrt{2}\sqrt{b}\,(3a+b)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\cosh^2(x)+2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\cosh^2(x)+b\sqrt{a+2b+a\cosh(2x)}\sinh(x)\right)}{(a+2b+a\cosh(2x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ((b + a*Cosh[x]^2)*Sech[x]*Sqrt[a + b*Sech[x]^2]*(Sqrt[2]*Sqrt[b]*(3*a + b)
*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Cosh[x]^2 +
2*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2
*x]]]*Cosh[x]^2 + b*Sqrt[a + 2*b + a*Cosh[2*x]]*Sinh[x]))/(a + 2*b + a*Cosh
[2*x])^(3/2)
```

Maple [**F**]
time = 1.22, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{sech}(x)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int((a+b*sech(x)^2)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(70) = 140$.
time = 0.52, size = 4140, normalized size = 47.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 +
2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*
sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8
- 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^
6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a
^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 -
 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2
- b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(
a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)
^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqr
t(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(
x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cos
h(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh
(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3
 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
+ a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x
)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^
3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x
)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + ((3*a + b)*cosh(x)^4 + 4*(3*a + b)*cosh(x)*sinh(
x)^3 + (3*a + b)*sinh(x)^4 + 2*(3*a + b)*cosh(x)^2 + 2*(3*(3*a + b)*cosh(x)
^2 + 3*a + b)*sinh(x)^2 + 4*((3*a + b)*cosh(x)^3 + (3*a + b)*cosh(x))*sinh(
x) + 3*a + b)*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^
3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a
+ 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1
)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x)
 + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1
)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (a*cosh
(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x
)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*log(-
```

```
(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 +
2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x
) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))
*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(2)*(b*c
osh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt((a*cosh(x)^2 + a*sin
h(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(
x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1), 1/4*(2*((3*a + b)*cosh(x)^4 +
4*(3*a + b)*cosh(x)*sinh(x)^3 + (3*a + b)*sinh(x)^4 + 2*(3*a + b)*cosh(x)^2
 + 2*(3*(3*a + b)*cosh(x)^2 + 3*a + b)*sinh(x)^2 + 4*((3*a + b)*cosh(x)^3 +
 (3*a + b)*cosh(x))*sinh(x) + 3*a + b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
+ a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*
cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2
+ a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) +
(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*a
*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a
)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a
*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*
(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b +
 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a
*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)
*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 +
3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a
^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(
b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 +
 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*
sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2
- a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^
2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 -
3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*
a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*si
nh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(
x)^5 + sinh(x)^6)) + (a*cosh(x)^4 + 4*a*cosh(x)...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{sech}^2\left(x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sech(x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{\cosh\left(x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] int((a + b/cosh(x)^2)^(3/2), x)
```

3.191 $\int \coth(x)\left(a+b\operatorname{sech}^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=70

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)-(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)+b\sqrt{a+b\operatorname{sech}^2(x)}$$

```
[Out] a^(3/2)*arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))-(a+b)^(3/2)*arctanh((a+b*sec
h(x)^2)^(1/2)/(a+b)^(1/2))+b*(a+b*sech(x)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.400$, Rules used = {4224, 457, 86, 162, 65, 214}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)+b\sqrt{a+b\operatorname{sech}^2(x)}-(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] a^(3/2)*ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]] - (a + b)^(3/2)*ArcTanh[Sqrt
[a + b*Sech[x]^2]/Sqrt[a + b]] + b*Sqrt[a + b*Sech[x]^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \coth(x)\left(a+b\operatorname{sech}^2(x)\right)^{3/2}\,dx &= \operatorname{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x(-1+x^2)}\,dx,x,\operatorname{sech}(x)\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{(-1+x)x}\,dx,x,\operatorname{sech}^2(x)\right)\\
&= b\sqrt{a+b\operatorname{sech}^2(x)}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{a^2+b(2a+b)x}{(-1+x)x\sqrt{a+bx}}\,dx,x,\operatorname{sech}^2(x)\right)\\
&= b\sqrt{a+b\operatorname{sech}^2(x)}-\frac{1}{2}a^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\operatorname{sech}^2(x)\right)+\frac{1}{2}(a+\\
&= b\sqrt{a+b\operatorname{sech}^2(x)}-\frac{a^2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\operatorname{sech}^2(x)}\right)}{b}+\frac{(a+}{}\\
&= a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)-(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right.
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 159 vs. $2(70) = 140$.
time = 0.33, size = 159, normalized size = 2.27

$$-\frac{2(b+a\cosh^2(x))\left(\sqrt{2}(a+b)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\cosh(x)-\sqrt{a+b}\left(b\sqrt{a+2b+a\cosh(2x)}+\sqrt{2}a^{3/2}\cosh(x)\log\left(\sqrt{2}\sqrt{a}\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)\right)\right)\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}(a+2b+a\cosh(2x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (-2*(b + a*Cosh[x]^2)*(Sqrt[2]*(a + b)^2*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[
x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Cosh[x] - Sqrt[a + b]*(b*Sqrt[a + 2*b + a*
Cosh[2*x]] + Sqrt[2]*a^(3/2)*Cosh[x]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a +
 2*b + a*Cosh[2*x]]]))*Sqrt[a + b*Sech[x]^2])/(Sqrt[a + b]*(a + 2*b + a*Cos
h[2*x])^(3/2))
```

Maple [F]
time = 1.56, size = 0, normalized size = 0.00

$$\int \coth(x)\left(a+b\,\text{sech}(x)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)*(a+b*sech(x)^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*coth(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(56) = 112$.
time = 0.60, size = 4123, normalized size = 58.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(a)*log(((a
^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)
^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3
)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^
2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^
3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b
^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b +
9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14
*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*c
osh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3
 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2
*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6
 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6
+ 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^
2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 +
2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*
a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*si
nh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a
*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(
a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))
*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*co
sh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^
6)) + 2*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
+ a + b)*sqrt(a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)
^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)
^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*
cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(
x) + 1)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(a)*lo
g(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(
3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)
/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*b*sqrt((a*cosh(x)
^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1), 1/4*(4*((a + b)*cosh(x)^2 +
 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(-a - b)*arctan
(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(
(a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh
(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*co
```

```
sh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b
)*cosh(x))*sinh(x) + a)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2
 + a)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a
*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2
)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a
^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4
+ 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2
*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(
x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*
b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2
*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*
a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b
+ b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh
(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^
2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6
*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cos...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(56) = 112.
time = 0.60, size = 134, normalized size = 1.91

$$-\frac{4\left(\left(\sqrt{a}\,e^{(2\,x)}-\sqrt{ae^{(4\,x)}+2\,ae^{(2\,x)}+4\,be^{(2\,x)}+a}\right)b^2-\sqrt{a}\,b^2\right)}{\left(\sqrt{a}\,e^{(2\,x)}-\sqrt{ae^{(4\,x)}+2\,ae^{(2\,x)}+4\,be^{(2\,x)}+a}\right)^2+2\left(\sqrt{a}\,e^{(2\,x)}-\sqrt{ae^{(4\,x)}+2\,ae^{(2\,x)}+4\,be^{(2\,x)}+a}\right)\sqrt{a}+a+4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -4*((sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a))*b^2
 - sqrt(a)*b^2)/((sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2
```

```
*x) + a))^2 + 2*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*
x) + a))*sqrt(a) + a + 4*b)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a + b/cosh(x)^2)^(3/2),x)
[Out] int(coth(x)*(a + b/cosh(x)^2)^(3/2), x)
```

3.192 $\int \coth^2(x) \left(a + b\mathbf{sech}^2(x)\right)^{3/2} dx$

Optimal. Leaf size=81

$$-b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)+a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)-(a+b)\coth(x)\sqrt{a+b-b}$$

```
[Out] -b^(3/2)*arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))+a^(3/2)*arctanh(a^
(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))-(a+b)*coth(x)*(a+b-b*tanh(x)^2)^(1/2
)
```

Rubi [A]
time = 0.15, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {4226, 2000, 485, 537, 223, 209, 385, 212}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)-(a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] -(b^(3/2)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]) + a^(3/2)*Ar
cTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]] - (a + b)*Coth[x]*Sqrt[a
 + b - b*Tanh[x]^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$
\begin{aligned}
\int \coth^2(x)\left(a+b\text{sech}^2(x)\right)^{3/2}\,dx &= \text{Subst}\left(\int \frac{\left(a+b(1-x^2)\right)^{3/2}}{x^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right) \\
&= \text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{3/2}}{x^2\left(1-x^2\right)}\,dx, x, \tanh(x)\right) \\
&= -(a+b)\coth(x)\sqrt{a+b-b\tanh^2(x)} + \text{Subst}\left(\int \frac{a^2-b^2+b^2x^2}{\left(1-x^2\right)\sqrt{a+b-}} \right. \\
&= -(a+b)\coth(x)\sqrt{a+b-b\tanh^2(x)} + a^2\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b}} \right. \\
&= -(a+b)\coth(x)\sqrt{a+b-b\tanh^2(x)} + a^2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, - \right. \\
&= -b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) + a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(}{\sqrt{a+b-b\tan}} \right.
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 144, normalized size = 1.78

$$
-\frac{2\left(b+a\cosh^2(x)\right)\left(\sqrt{2}\,b^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\cosh(x)-\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\cosh(x)+(a+b)\sqrt{a+2b+a\cosh(2x)}\coth(x)\right)\sqrt{a+b\text{sech}^2(x)}}{\left(a+2b+a\cosh(2x)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (-2*(b + a*Cosh[x]^2)*(Sqrt[2]*b^(3/2)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqr
t[a + 2*b + a*Cosh[2*x]]]*Cosh[x] - Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a
]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Cosh[x] + (a + b)*Sqrt[a + 2*b + a*
Cosh[2*x]]*Coth[x])*Sqrt[a + b*Sech[x]^2])/(a + 2*b + a*Cosh[2*x])^(3/2)
```

Maple [F]
time = 1.50, size = 0, normalized size = 0.00

$$
\int \left(\coth^2(x)\right)\left(a+b\text{sech}(x)^2\right)^{\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)^2*(a+b*sech(x)^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(3/2)*coth(x)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(67) = 134$.
time = 0.49, size = 3349, normalized size = 41.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a)*log((a*
b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^
3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2
*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)
*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^
3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^
3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*
cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^
2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(
x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2
*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3
 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4
*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b
)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cos
h(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2
- b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*co
sh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 +
 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + si
nh(x)^6)) + 2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(-b
)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4
 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 + 2*
sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*co
sh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(
x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (a*cosh(x)^2 + 2*a*cosh(x)*s
```

```
inh(x) + a*sinh(x)^2 - a)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2
+ sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
+ a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1), -1/4*(4*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) +
 b*sinh(x)^2 - b)*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a
*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2
+ 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - (a*cosh(x)^2 + 2*a*co
sh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cos
h(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*
cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3
)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh
(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4
*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^
2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cos
h(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*
a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x
)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4
 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2
+ (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(
3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)
*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4
*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6
+ 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 1
5*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - (a*cosh(x)^2 +
2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh
(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a +
b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt
(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^
2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*(a + b)*sqrt((a*cosh(x)^2 +
 a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh
(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), -1/2*((a*cosh(x)^2 + 2*a*cosh(x
)*sinh(x) + a*sinh(x)^2 - a)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cos
h(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(67) = 134.
time = 0.56, size = 222, normalized size = 2.74

$$\frac{4\left(\left(\sqrt{a}\,e^{(2x)}-\sqrt{ae^{(4x)}+2\,ae^{(2x)}+4\,be^{(2x)}+a}\right)a^2+2\left(\sqrt{a}\,e^{(2x)}-\sqrt{ae^{(4x)}+2\,ae^{(2x)}+4\,be^{(2x)}+a}\right)ab+\left(\sqrt{a}\,e^{(2x)}-\sqrt{ae^{(4x)}+2\,ae^{(2x)}+4\,be^{(2x)}+a}\right)b^2+a^{\frac{5}{2}}+2\,a^{\frac{3}{2}}b+\sqrt{a}\,b^2\right)}{\left(\sqrt{a}\,e^{(2x)}-\sqrt{ae^{(4x)}+2\,ae^{(2x)}+4\,be^{(2x)}+a}\right)^2-2\left(\sqrt{a}\,e^{(2x)}-\sqrt{ae^{(4x)}+2\,ae^{(2x)}+4\,be^{(2x)}+a}\right)\sqrt{a}-3\,a-4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 4*((sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a))*a^2
+ 2*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a))*a*b
 + (sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) + 2*a*e^(2*x) + 4*b*e^(2*x) + a))*b^2
+ a^(5/2) + 2*a^(3/2)*b + sqrt(a)*b^2)/((sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) +
 2*a*e^(2*x) + 4*b*e^(2*x) + a))^2 - 2*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) +
2*a*e^(2*x) + 4*b*e^(2*x) + a))*sqrt(a) - 3*a - 4*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^2\left(a+\frac{b}{\cosh\left(x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)^2*(a + b/cosh(x)^2)^(3/2), x)
```

3.193 $\int \left(a+b\operatorname{sech}^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=170

$$\frac{\sqrt{b}\,(15a^2+10ab+3b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a+b-b\tanh^2(c+dx)}}\right)}{8d}+\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a+b-b\tanh^2(c+dx)}}\right)}{d}$$

```
[Out] a^(5/2)*arctanh(a^(1/2)*tanh(d*x+c)/(a+b-b*tanh(d*x+c)^2)^(1/2))/d+1/8*(15*
a^2+10*a*b+3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/(a+b-b*tanh(d*x+c)^2)^(1/2))*b
^(1/2)/d+1/8*b*(7*a+3*b)*(a+b-b*tanh(d*x+c)^2)^(1/2)*tanh(d*x+c)/d+1/4*b*ta
nh(d*x+c)*(a+b-b*tanh(d*x+c)^2)^(3/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4213, 427, 542, 537, 223, 209, 385, 212}

$$\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{d}+\frac{\sqrt{b}\,(15a^2+10ab+3b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{8d}+\frac{b\tanh(c+dx)\,(a-b\tanh^2(c+dx)+b)^{3/2}}{4d}+\frac{b(7a+3b)\tanh(c+dx)\sqrt{a-b\tanh^2(c+dx)+b}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a +
b - b*Tanh[c + d*x]^2]])/(8*d) + (a^(5/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/S
qrt[a + b - b*Tanh[c + d*x]^2]])/d + (b*(7*a + 3*b)*Tanh[c + d*x]*Sqrt[a +
b - b*Tanh[c + d*x]^2])/(8*d) + (b*Tanh[c + d*x]*(a + b - b*Tanh[c + d*x]^2
)^(3/2))/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\operatorname{sech}^2(c+dx)\right)^{5/2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{5/2}}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)\left(a+b-b\tanh^2(c+dx)\right)^{3/2}}{4d} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+b-bx^2}\,((a+}{1}\right.}{}$$

$$= \frac{b(7a+3b)\tanh(c+dx)\sqrt{a+b-b\tanh^2(c+dx)}}{8d} + \frac{b\tanh(c+dx)\left(a+\right.}{}$$

$$= \frac{b(7a+3b)\tanh(c+dx)\sqrt{a+b-b\tanh^2(c+dx)}}{8d} + \frac{b\tanh(c+dx)\left(a+\right.}{}$$

$$= \frac{b(7a+3b)\tanh(c+dx)\sqrt{a+b-b\tanh^2(c+dx)}}{8d} + \frac{b\tanh(c+dx)\left(a+\right.}{}$$

$$= \frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a+b-b\tanh^2(c+dx)}}\right)}{8d} + \frac{a^{5/2}\tanh}{}$$

Mathematica [A]
time = 8.22, size = 280, normalized size = 1.65

$$\frac{\left(\sqrt{b}\left(15a^2+10ab+3b^2\right)\operatorname{ArcTan}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a+b+a\sinh^2(c+dx)}}\right)+8a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b+a\sinh^2(c+dx)}}\right)\right)\cosh^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^{5/2}}{\sqrt{2}\,d(a+2b+a\cosh(2c+2dx))^{5/2}} + \frac{\cosh^5(c+dx)\left(a+b\operatorname{sech}^2(c+dx)\right)^{5/2}\left(\frac{b^2\operatorname{sech}(c)\operatorname{sech}^4(c+dx)\sinh(dx)}{d}+\frac{3\operatorname{sech}(c)\operatorname{sech}^2(c+dx)\left(3ab\sinh(dx)+b^2\sinh(dx)\right)}{2d}+\frac{3b(3a+b)\operatorname{sech}(c+dx)\tanh(c)}{2d}+\frac{b^2\operatorname{sech}^3(c+dx)\tanh(c)}{d}\right)}{(a+2b+a\cosh(2c+2dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(5/2),x]
[Out] ((Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a +
 b + a*Sinh[c + d*x]^2]] + 8*a^(5/2)*ArcTanh[(Sqrt[a]*Sinh[c + d*x])/Sqrt[a
 + b + a*Sinh[c + d*x]^2]])*Cosh[c + d*x]^5*(a + b*Sech[c + d*x]^2)^(5/2))/
(Sqrt[2]*d*(a + 2*b + a*Cosh[2*c + 2*d*x])^(5/2)) + (Cosh[c + d*x]^5*(a + b
*Sech[c + d*x]^2)^(5/2)*((b^2*Sech[c]*Sech[c + d*x]^4*Sinh[d*x])/d + (3*Sec
h[c]*Sech[c + d*x]^2*(3*a*b*Sinh[d*x] + b^2*Sinh[d*x]))/(2*d) + (3*b*(3*a +
 b)*Sech[c + d*x]*Tanh[c])/(2*d) + (b^2*Sech[c + d*x]^3*Tanh[c])/d))/(a + 2
*b + a*Cosh[2*c + 2*d*x])^2
```

Maple [F]
time = 2.83, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{sech}(dx+c)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c)^2)^(5/2),x)
```

```
[Out] int((a+b*sech(d*x+c)^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2712 vs. $2(148) = 296$.
time = 0.90, size = 12452, normalized size = 73.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*s
inh(d*x + c)^8 + 4*a^2*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x + c)^2 + a^2)*si
nh(d*x + c)^6 + 6*a^2*cosh(d*x + c)^4 + 8*(7*a^2*cosh(d*x + c)^3 + 3*a^2*co
sh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cosh(d*x + c)^4 + 30*a^2*cosh(d*x
+ c)^2 + 3*a^2)*sinh(d*x + c)^4 + 4*a^2*cosh(d*x + c)^2 + 8*(7*a^2*cosh(d*x
 + c)^5 + 10*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4
*(7*a^2*cosh(d*x + c)^6 + 15*a^2*cosh(d*x + c)^4 + 9*a^2*cosh(d*x + c)^2 +
a^2)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 + 3*a^2*cosh(d*x + c)^5
 + 3*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(a)*log((a
*b^2*cosh(d*x + c)^8 + 8*a*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + a*b^2*sinh(d
*x + c)^8 - 2*(a*b^2 - b^3)*cosh(d*x + c)^6 + 2*(14*a*b^2*cosh(d*x + c)^2 -
 a*b^2 + b^3)*sinh(d*x + c)^6 + 4*(14*a*b^2*cosh(d*x + c)^3 - 3*(a*b^2 - b^
3)*cosh(d*x + c))*sinh(d*x + c)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(d*x + c)
^4 + (70*a*b^2*cosh(d*x + c)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(14*a*b^2*cosh(d*x + c)^5 - 10*(a*b^2
 - b^3)*cosh(d*x + c)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(d*x + c)^2 + 2*(14*a*b^2*cosh(d*x +
c)^6 - 15*(a*b^2 - b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b
+ 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + sqrt(2)*(b^2*cosh(d*x + c)^6
+ 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*
x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(
```

```
d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 - (a^2 + 4*a*b)*cosh(d*x
+ c)^2 + (15*b^2*cosh(d*x + c)^4 - 18*b^2*cosh(d*x + c)^2 - a^2 - 4*a*b)*si
nh(d*x + c)^2 - a^2 + 2*(3*b^2*cosh(d*x + c)^5 - 6*b^2*cosh(d*x + c)^3 - (a
^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a)*sqrt((a*cosh(d*x + c)^2 +
 a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x +
 c) + sinh(d*x + c)^2)) + 4*(2*a*b^2*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*cosh
(d*x + c)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(d*x + c)^3 + (a^3 + 3*a^2*b)*c
osh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^6 + 6*cosh(d*x + c)^5*sinh(d*x
+ c) + 15*cosh(d*x + c)^4*sinh(d*x + c)^2 + 20*cosh(d*x + c)^3*sinh(d*x + c
)^3 + 15*cosh(d*x + c)^2*sinh(d*x + c)^4 + 6*cosh(d*x + c)*sinh(d*x + c)^5
+ sinh(d*x + c)^6)) + ((15*a^2 + 10*a*b + 3*b^2)*cosh(d*x + c)^8 + 8*(15*a^
2 + 10*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^2 + 10*a*b + 3*b^
2)*sinh(d*x + c)^8 + 4*(15*a^2 + 10*a*b + 3*b^2)*cosh(d*x + c)^6 + 4*(7*(15
*a^2 + 10*a*b + 3*b^2)*cosh(d*x + c)^2 + 15*a^2 + 10*a*b + 3*b^2)*sinh(d*x
+ c)^6 + 8*(7*(15*a^2 + 10*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(15*a^2 + 10*a*
b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(15*a^2 + 10*a*b + 3*b^2)*cos
h(d*x + c)^4 + 2*(35*(15*a^2 + 10*a*b + 3*b^2)*cosh(d*x + c)^4 + 30*(15*a^2
 + 10*a*b + 3*b^2)*cosh(d*x + c)^2 + 45*a^2 + 30*a*b + 9*b^2)*sinh(d*x + c)
^4 + 8*(7*(15*a^2 + 10*a*b + 3*b^2)*cosh(d*x + c)^5 + 10*(15*a^2 + 10*a*b +
 3*b^2)*cosh(d*x + c)^3 + 3*(15*a^2 + 10*a*b + 3*b^2)*cosh(d*x + c))*sinh(d
*x + c)^3 + 4*(15*a^2 + 10*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*(7*(15*a^2 + 10
*a*b + 3*b^2)*cosh(d*x + c)^6 + 15*(15*a^2 + 10*a*b + 3*b^2)*cosh(d*x + c)^
4 + 9*(15*a^2 + 10*a*b + 3*b^2)*cosh(d*x + c)^2 + 15*a^2 + 10*a*b + 3*b^2)*
sinh(d*x + c)^2 + 15*a^2 + 10*a*b + 3*b^2 + 8*((15*a^2 + 10*a*b + 3*b^2)*co
sh(d*x + c)^7 + 3*(15*a^2 + 10*a*b + 3*b^2)*cosh(d*x + c)^5 + 3*(15*a^2 + 1
0*a*b + 3*b^2)*cosh(d*x + c)^3 + (15*a^2 + 10*a*b + 3*b^2)*cosh(d*x + c))*s
inh(d*x + c))*sqrt(-b)*log(-((a - b)*cosh(d*x + c)^4 + 4*(a - b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a - b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2
 + 2*(3*(a - b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 - 2*sqrt(2)*(cos
h(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b
)*sqrt((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + a + 2*b)/(cosh(d*x + c)^2 -
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 4*((a - b)*cosh(d*x +
c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + a - b)/(cosh(d*x + c)^4 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1
)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))
*sinh(d*x + c) + 1)) + 4*(a^2*cosh(d*x + c)^8 + 8*a^2*cosh(d*x + c)*sinh(d*
x + c)^7 + a^2*sinh(d*x + c)^8 + 4*a^2*cosh(d*x + c)^6 + 4*(7*a^2*cosh(d*x
+ c)^2 + a^2)*sinh(d*x + c)^6 + 6*a^2*cosh(d*x + c)^4 + 8*(7*a^2*cosh(d*x +
 c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*cosh(d*x + c)^4 +
30*a^2*cosh(d*x + c)^2 + 3*a^2)*sinh(d*x + c)^4 + 4*a^2*cosh(d*x + c)^2 + 8
*(7*a^2*cosh(d*x + c)^5 + 10*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(7*a^2*cosh(d*x + c)^6 + 15*a^2*cosh(d*x + c)^4 + 9*a^2*co
sh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 8*(a^2*cosh(d*x + c)^7 + 3*a^2
*cosh(d*x + c)^5 + 3*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c)
)*sqrt(a)*log(-(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*s
```

```
inh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{sech}^{2}\left(c + dx\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.53Error: Bad Argum
ent Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{b}{{\mathrm{cosh}\left(c+d\,x\right)}^2}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x)^2)^(5/2),x)
```

```
[Out] int((a + b/cosh(c + d*x)^2)^(5/2), x)
```

3.194 $\int \frac{\tanh^5(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

Optimal. Leaf size=66

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{(a+2b)\sqrt{a+b\text{sech}^2(x)}}{b^2}-\frac{\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2}$$

```
[Out] -1/3*(a+b*sech(x)^2)^(3/2)/b^2+arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1/
2)+(a+2*b)*(a+b*sech(x)^2)^(1/2)/b^2
```

Rubi [A]
time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4224, 457, 90, 65, 214}

$$-\frac{\left(a+b\text{sech}^2(x)\right)^{3/2}}{3b^2}+\frac{(a+2b)\sqrt{a+b\text{sech}^2(x)}}{b^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] + ((a + 2*b)*Sqrt[a + b*Sech
[x]^2])/b^2 - (a + b*Sech[x]^2)^(3/2)/(3*b^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{(-1+x^2)^2}{x\sqrt{a+bx^2}}\,dx, x, \operatorname{sech}(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^2}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{-a-2b}{b\sqrt{a+bx}}+\frac{1}{x\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)\,dx, x, \operatorname{sech}^2(x)\right)\right)\\
&= \frac{(a+2b)\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}-\frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b^2}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sec}
\end{aligned}$$

$$= \frac{(a+2b)\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}-\frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b^2}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{se}}\right.}{b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{(a+2b)\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}-\frac{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}{3b^2}$$

Mathematica [A]

time = 0.33, size = 109, normalized size = 1.65

$$\frac{\operatorname{sech}(x)\left(\frac{\sqrt{2}\sqrt{a+2b+a\cosh(2x)}\log\left(\sqrt{2}\sqrt{a}\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)}{\sqrt{a}}+\frac{(a+2b+a\cosh(2x))\operatorname{sech}(x)\left(2a+6b-b\operatorname{sech}^2(x)\right)}{3b^2}\right)}{2\sqrt{a+b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^5/Sqrt[a + b*Sech[x]^2],x]

[Out] (Sech[x]*((Sqrt[2]*Sqrt[a + 2*b + a*Cosh[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/Sqrt[a] + ((a + 2*b + a*Cosh[2*x])*Sech[x]*(2*a + 6*b - b*Sech[x]^2))/(3*b^2)))/(2*Sqrt[a + b*Sech[x]^2])

Maple [**F**]
time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\operatorname{sech}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x)

[Out] int(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^5/sqrt(b*sech(x)^2 + a), x)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 971 vs. 2(54) = 108.
time = 0.49, size = 2678, normalized size = 40.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")

[Out] [1/12*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 + 4*(5*b^2

```
*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2*cosh(x)^
2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^2*cosh(x)
)*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3
+ 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b +
a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 +
 (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(
x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*co
sh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 +
5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*
sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3
*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^
2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2
 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2
*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b +
b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)
*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cos
h(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^
5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqr
t(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh
(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 +
(2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6)) + 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5
+ b^2*sinh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3
*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*c
osh(x)^4 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^
2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)
*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2
+ sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)) + 8*sqrt(2)*((a^2 + 3*a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)*cos
h(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(x)^4 + 2*(a^2 + 2*a*b)*cosh(x)^2 + 2*(3
*(a^2 + 3*a*b)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 3*a*b + 4*((a^2 +
 3*a*b)*cosh(x)^3 + (a^2 + 2*a*b)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*s
inh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b^2*co
sh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 + 3*a*b^2*cosh(x)^4 +
 3*a*b^2*cosh(x)^2 + 3*(5*a*b^2*cosh(x)^2 + a*b^2)*sinh(x)^4 + 4*(5*a*b^2*c
osh(x)^3 + 3*a*b^2*cosh(x))*sinh(x)^3 + a*b^2 + 3*(5*a*b^2*cosh(x)^4 + 6*a*
b^2*cosh(x)^2 + a*b^2)*sinh(x)^2 + 6*(a*b^2*cosh(x)^5 + 2*a*b^2*cosh(x)^3 +
 a*b^2*cosh(x))*sinh(x)), -1/6*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5
```

```
+ b^2*sinh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3
*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*c
osh(x)^4 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^
2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x
)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*
sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 +
3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(
x))*sinh(x))) + 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6
+ 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 +
 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2
*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^
2*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x
)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 4*sqrt(2)*((a^2 +
 3*a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(
x)^4 + 2*(a^2 + 2*a*b)*cosh(x)^2 + 2*(3*(a^2 + ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\sqrt{a + b\,\mathrm{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**5/sqrt(a + b*sech(x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^5}{\sqrt{a+\frac{b}{\cosh(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + b/cosh(x)^2)^(1/2),x)
[Out] int(tanh(x)^5/(a + b/cosh(x)^2)^(1/2), x)
```

3.195 $\int \frac{\tanh^4(x)}{\sqrt{a+b\,\mathrm{sech}^2(x)}}\,dx$

Optimal. Leaf size=90

$$-\frac{(a+3b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{2b^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}}+\frac{\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{2b}$$

```
[Out] -1/2*(a+3*b)*arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(3/2)+arctan
h(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(1/2)+1/2*(a+b-b*tanh(x)^2)^(1
/2)*tanh(x)/b
```

Rubi [A]
time = 0.16, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {4226, 2000, 490, 537, 223, 209, 385, 212}

$$-\frac{(a+3b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{2b^{3/2}}+\frac{\tanh(x)\sqrt{a-b\tanh^2(x)+b}}{2b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] -1/2*((a + 3*b)*ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]])/b^(3/2
) + ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a] + (Tanh[x]
*Sqrt[a + b - b*Tanh[x]^2])/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \text{Subst}\left(\int \frac{x^4}{(1-x^2)\sqrt{a+b(1-x^2)}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^4}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)$$

$$= \frac{\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{2b} - \frac{\text{Subst}\left(\int \frac{a+b+(-a-3b)x^2}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{2b}$$

$$= \frac{\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{2b} - \frac{(a+3b)\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{2b}$$

$$= \frac{\tanh(x)\sqrt{a+b-b\tanh^2(x)}}{2b} - \frac{(a+3b)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2}}\right)}{2b}$$

$$= -\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{2b^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.36, size = 169, normalized size = 1.88

$$\frac{\text{sech}(x)\left(2\sqrt{2}\,b^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\sqrt{a+2b+a\cosh(2x)}+\sqrt{a}\left(-\sqrt{2}(a+3b)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\sqrt{a+2b+a\cosh(2x)}+\sqrt{b}(a+2b+a\cosh(2x))\text{sech}(x)\tanh(x)\right)\right)}{4\sqrt{a}\,b^{3/2}\sqrt{a+b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sech[x]*(2*Sqrt[2]*b^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b
+ a*Cosh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]] + Sqrt[a]*(-(Sqrt[2]*(a + 3*b)*
ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Sqrt[a + 2*b
+ a*Cosh[2*x]]) + Sqrt[b]*(a + 2*b + a*Cosh[2*x])*Sech[x]*Tanh[x])))/(4*Sqr
t[a]*b^(3/2)*Sqrt[a + b*Sech[x]^2])
```

Maple [F]
time = 1.71, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\,\text{sech}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/sqrt(b*sech(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 822 vs. 2(72) = 144.
time = 0.58, size = 4569, normalized size = 50.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh
(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*
cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7
+ a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b
^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh
(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4
*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh
(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sin
h(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*
b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^
2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(
x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cos
h(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(
x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5
 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^
2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*s
inh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sin
h(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - ((a^2 + 3*a*b)*cosh(x)^4 + 4*(
a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(x)^4 + 2*(a^2 + 3*a*b)*
```

```
cosh(x)^2 + 2*(3*(a^2 + 3*a*b)*cosh(x)^2 + a^2 + 3*a*b)*sinh(x)^2 + a^2 + 3
*a*b + 4*((a^2 + 3*a*b)*cosh(x)^3 + (a^2 + 3*a*b)*cosh(x))*sinh(x))*sqrt(-b
)*log(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4
 + 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*
sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*co
sh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(
x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (b^2*cosh(x)^4 + 4*b^2*cosh(
x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*
sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log(-(a*
cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(
3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*si
nh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 2*sqrt(2)*(a*b*co
sh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - a*b)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b^2
*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + 2*a*b^2*cosh(x)^
2 + a*b^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 +
a*b^2*cosh(x))*sinh(x)), -1/4*(2*((a^2 + 3*a*b)*cosh(x)^4 + 4*(a^2 + 3*a*b)
*cosh(x)*sinh(x)^3 + (a^2 + 3*a*b)*sinh(x)^4 + 2*(a^2 + 3*a*b)*cosh(x)^2 +
2*(3*(a^2 + 3*a*b)*cosh(x)^2 + a^2 + 3*a*b)*sinh(x)^2 + a^2 + 3*a*b + 4*((a
^2 + 3*a*b)*cosh(x)^3 + (a^2 + 3*a*b)*cosh(x))*sinh(x))*sqrt(b)*arctan(sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)
^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a
*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 +
2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))
*sinh(x) + a)) - (b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 +
 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(
x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)
*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh
(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*co
sh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^
4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14
*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*c
osh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)
^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^
2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5
+ b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4
*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (1
5*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^
2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2
```

```
*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\sqrt{a + b\,\text{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**4/sqrt(a + b*sech(x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^4}{\sqrt{a + \frac{b}{\cosh(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^4/(a + b/cosh(x)^2)^(1/2), x)
```

3.196 $\int \frac{\tanh^3(x)}{\sqrt{a+b\operatorname{sech}^2(x)}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1/2)+(a+b*sech(x)^2)^(1/2)/b
```

Rubi [A]

time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4224, 457, 81, 65, 214}

$$\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2/Sqrt[a]]/Sqrt[a] + Sqrt[a + b*Sech[x]^2]/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx &= \text{Subst}\left(\int \frac{-1+x^2}{x\sqrt{a+bx^2}}\,dx, x, \text{sech}(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)\\
&= \frac{\sqrt{a+b\text{sech}^2(x)}}{b} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)\\
&= \frac{\sqrt{a+b\text{sech}^2(x)}}{b} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{sech}^2(x)}\right)}{b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\sqrt{a+b\text{sech}^2(x)}}{b}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 105 vs. 2(42) = 84.
time = 0.14, size = 105, normalized size = 2.50

$$
\frac{\sqrt{a+2b+a\cosh(2x)}\,\log\left(\sqrt{2}\sqrt{a}\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)\text{sech}(x)}{\sqrt{2}\sqrt{a}\sqrt{a+b\text{sech}^2(x)}} + \frac{(a+2b+a\cosh(2x))\text{sech}^2(x)}{2b\sqrt{a+b\text{sech}^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a
*Cosh[2*x]]]*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b*Sech[x]^2]) + ((a + 2*b +
 a*Cosh[2*x])*Sech[x]^2)/(2*b*Sqrt[a + b*Sech[x]^2])
```

Maple [**F**]
time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\sqrt{a + b\operatorname{sech}(x)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^3/sqrt(b*sech(x)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(34) = 68$.
time = 0.47, size = 1650, normalized size = 39.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a)*log(((a
^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)
^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3
)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^
2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^
3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b
^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b +
9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14
```

```
*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*c
osh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3
 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2
*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6
 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6
+ 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^
2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 +
2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*
a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*si
nh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a
*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(
a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh
(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))
*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*co
sh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^
6)) + (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a)*log(-(a
*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*c
osh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*co
sh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cos
h(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*a*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b*c
osh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 + a*b), -1/2*((b*cosh(x)^2
 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(-a)*arctan(sqrt(2)*((a + b)*
cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 +
 a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*
a^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)
*cosh(x))*sinh(x))) + (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)
*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sq
rt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*
(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^
3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - 2*sqrt(2)*a*sqrt((a*cosh(x)^2 + a*si
nh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*b*cosh(
x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 + a*b)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\sqrt{a + b\,\mathrm{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**3/sqrt(a + b*sech(x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^3}{\sqrt{a+\frac{b}{\cosh\left(x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^3/(a + b/cosh(x)^2)^(1/2), x)
```

3.197 $\int \frac{\tanh^2(x)}{\sqrt{a+b\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=60

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(1/2)-arctan(b^(1/2)*tan
h(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4226, 2000, 494, 223, 209, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[b]) + ArcTanh[(S
qrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \text{Subst}\left(\int \frac{x^2}{(1-x^2)\sqrt{a+b(1-x^2)}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^2}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)$$

$$= -\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right) + \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx\right.$$

$$= \text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right) - \text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{}{\sqrt{a}}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.12, size = 107, normalized size = 1.78

$$\frac{\left(-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)}{\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)}{\sqrt{a}}\right)\sqrt{a+2b+a\cosh(2x)}\,\text{sech}(x)}{\sqrt{2}\sqrt{a+b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ((-(ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]/Sqrt[b])
+ ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]/Sqrt[a])*S
qrt[a + 2*b + a*Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[a + b*Sech[x]^2])
```

Maple [F]
time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\,\text{sech}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x)
```

[Out] int(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^2/sqrt(b*sech(x)^2 + a), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(48) = 96$.
time = 0.49, size = 2856, normalized size = 47.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")

```
[Out] [1/4*(sqrt(a)*b*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*si
nh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*
sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a
^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9
*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10
*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a
^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)
*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)
^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b
^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3
*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*
b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*c
osh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*si
nh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^
2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)
^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) - 2*a*sqrt(-b)*log(-((a - b)*cosh(x)^4 + 4
*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(a + 3*b)*cosh(x)^2 + 2*
(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2
*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a - b)*cosh(x)^3 + (
a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(
x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x)
)*sinh(x) + 1)) + sqrt(a)*b*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*s
```

```
inh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)
^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)))/(a*b), -1/4*(4*a*sqrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a
+ 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh
(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a
+ 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) - sqrt
(a)*b*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 -
2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6
+ 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2
*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 3
0*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 -
b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^
3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4
 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(
2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)
^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(
x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x
)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 -
 (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^
7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3
+ 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^
4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*s
inh(x)^5 + sinh(x)^6)) - sqrt(a)*b*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2
 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*
cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2)))/(a*b), -1/2*(sqrt(-a)*b*arctan(sqrt(2)*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*s
inh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(
x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 +
(6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2
 + 3*a*b)*cosh(x))*sinh(x))) + sqrt(-a)*b*arctan(sqrt(2)*sqrt(-a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)) + a*sqrt(-b)*log
(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*
(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(
2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 -...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\sqrt{a + b\,\mathrm{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)**2/sqrt(a + b*sech(x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^2}{\sqrt{a + \frac{b}{\cosh(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(tanh(x)^2/(a + b/cosh(x)^2)^(1/2), x)
```

3.198 $\int \frac{\tanh(x)}{\sqrt{a+b\mathbf{sech}^2(x)}} \, dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4224, 272, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \text{sech}(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}^2(x)\right)\right)\\ &= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{sech}^2(x)}\right)}{b}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 70 vs. 2(25) = 50.
time = 0.05, size = 70, normalized size = 2.80

$$\frac{\sqrt{a+2b+a\cosh(2x)}\,\log\left(\sqrt{2}\sqrt{a}\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)\text{sech}(x)}{\sqrt{2}\sqrt{a}\sqrt{a+b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Sech[x]^2],x]
[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a
*Cosh[2*x]]]*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b*Sech[x]^2])
```

Maple [A]
time = 0.68, size = 30, normalized size = 1.20

method	result	size

derivativedivides	$\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\,\mathrm{sech}\,(x)^2}}{\mathrm{sech}(x)}\right)}{\sqrt{a}}$	30
default	$\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\,\mathrm{sech}\,(x)^2}}{\mathrm{sech}(x)}\right)}{\sqrt{a}}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sech(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(b*sech(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(19) = 38$.
time = 0.43, size = 1430, normalized size = 57.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a
*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2
)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a
^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4
+ 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2
*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(
x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*
```

```
cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*
b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2
*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*
a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b
+ b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh
(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^
2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6
*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b
+ 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^
3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x
)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)
*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2
+ a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(
a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh
(x)^2)))/a, -1/2*(sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*co
sh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a*b)*c
osh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2
 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x)^2 +
 a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x))) + sq
rt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(
-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a
+ 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 +
 (a + 2*b)*cosh(x))*sinh(x) + a)))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{a + b\,\mathrm{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(tanh(x)/sqrt(a + b*sech(x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B]
time = 1.67, size = 19, normalized size = 0.76

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh\left(x\right)^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

3.199 $\int \frac{1}{\sqrt{a+b\,\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4213, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.
time = 0.03, size = 62, normalized size = 2.14

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b+a\sinh^2(x)}}\right)\sqrt{a+2b+a\cosh(2x)}\,\text{sech}(x)}{\sqrt{2}\sqrt{a}\sqrt{a+b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sech[x]^2],x]
[Out] (ArcTanh[(Sqrt[a]*Sinh[x])/Sqrt[a + b + a*Sinh[x]^2]]*Sqrt[a + 2*b + a*Cosh
[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b*Sech[x]^2])
```

Maple [F]
time = 1.69, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\text{sech}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(x)^2)^(1/2),x)
[Out] int(1/(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

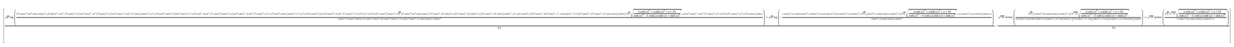
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sech(x)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(23) = 46$.
time = 0.40, size = 1059, normalized size = 36.52



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh
(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*si
nh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3
 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a
*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(
a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3
 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*c
osh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2
 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2
*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b
^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^
2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cos
h(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh
(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*
cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3
 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*
cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*c
osh(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*si
nh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sin
h(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2)))/a, -1/2*(sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*s
inh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(
x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 +
(6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2
 + 3*a*b)*cosh(x))*sinh(x))) + sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
)/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{sech}^2\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sech(x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{a+\frac{b}{{\cosh\left(x\right)}^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(1/(a + b/cosh(x)^2)^(1/2), x)
```

3.200 $\int \frac{\coth(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

Optimal. Leaf size=56

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1/2)-arctanh((a+b*sech(x)^2)^(1/2
)/(a+b)^(1/2))/(a+b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4224, 457, 88, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] - ArcTanh[Sqrt[a + b*Sech[x]
^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] && !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)\sqrt{a+bx^2}}\,dx,x,\text{sech}(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)x\sqrt{a+bx}}\,dx,x,\text{sech}^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx,x,\text{sech}^2(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\text{se}\right.\\
&= \frac{\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{sech}^2(x)}\right)}{b}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{sech}^2}\right.}{b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 124 vs. 2(56) = 112.

time = 0.16, size = 124, normalized size = 2.21

$$
\frac{\sqrt{a+2b+a\cosh(2x)}\left(-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a+b}\,\cosh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)+\sqrt{a+b}\,\log\left(\sqrt{2}\,\sqrt{a}\,\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)\right)\text{sech}(x)}{\sqrt{2}\,\sqrt{a}\,\sqrt{a+b}\,\sqrt{a+b\text{sech}^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*(-(Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x
])/Sqrt[a + 2*b + a*Cosh[2*x]]]) + Sqrt[a + b]*Log[Sqrt[2]*Sqrt[a]*Cosh[x]
+ Sqrt[a + 2*b + a*Cosh[2*x]]])*Sech[x])/(Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[
a + b*Sech[x]^2])
```

Maple [**F**]
time = 1.68, size = 0, normalized size = 0.00

$$\int \frac{\coth (x)}{\sqrt{a+b\mathrm{sech}(x)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)/(a+b*sech(x)^2)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(b*sech(x)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(44) = 88.
time = 0.48, size = 3663, normalized size = 65.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*
a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b
^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b
 + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^
5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*co
sh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)
```

```
*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x
))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b +
 a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3
+ 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(
(a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (
a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2
+ 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b
 + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a
*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*
cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(
x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*
sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*s
inh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 +
5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3
 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 1
5*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6
*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 2*sqrt(a + b)*a*log(((2*a + b)*cosh(x)^4
 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh
(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a
 + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 +
 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + (a + b)*sqrt(a)*log(-(a*cosh(x)^4
+ 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 +
b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt
(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2)))/(a^2 + a*b), 1/4*(4*a*sqrt(-a - b)*arctan(sq
rt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*
cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(
x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*c
osh(x))*sinh(x) + a)) + (a + b)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x
)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)
*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*
(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)
*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cos
h(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b
 + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(1
4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*
```

```
cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sin
h(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cos
h(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh
(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 +
 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x
)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a
^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 +
2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*
cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(
x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b +
9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cos
h(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(
x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a + b)*sqrt(a)*log(-(
a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{a + b\operatorname{sech}^2(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)**2)**(1/2),x)
[Out] Integral(coth(x)/sqrt(a + b*sech(x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth(x)}{\sqrt{a + \frac{b}{\cosh(x)^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b/cosh(x)^2)^(1/2),x)
[Out] int(coth(x)/(a + b/cosh(x)^2)^(1/2), x)
```

3.201 $\int \frac{\coth^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

Optimal. Leaf size=53

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}}-\frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(1/2)-coth(x)*(a+b-b*tan
h(x)^2)^(1/2)/(a+b)
```

Rubi [A]

time = 0.13, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4226, 2000, 491, 12, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{\sqrt{a}}-\frac{\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/Sqrt[a] - (Coth[x]*Sqr
t[a + b - b*Tanh[x]^2])/(a + b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\coth^2(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x^2(1-x^2)\sqrt{a+b(1-x^2)}}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{x^2(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)$$
$$= -\frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b} + \frac{\text{Subst}\left(\int \frac{a+b}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{a+b}$$
$$= -\frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b} + \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)$$
$$= -\frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b} + \text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{\sqrt{a}} - \frac{\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a+b}$$

Mathematica [A]
time = 0.08, size = 94, normalized size = 1.77

$$\frac{\sqrt{a+2b+a\cosh(2x)}\,\text{sech}(x)\left((a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b+a\sinh^2(x)}}\right) - \sqrt{a}\,\text{csch}(x)\sqrt{a+b+a\sinh^2(x)}\right)}{\sqrt{2}\sqrt{a}(a+b)\sqrt{a+b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cosh[2*x]]*Sech[x]*((a + b)*ArcTanh[(Sqrt[a]*Sinh[x])/Sqr
t[a + b + a*Sinh[x]^2]] - Sqrt[a]*Csch[x]*Sqrt[a + b + a*Sinh[x]^2]))/(Sqrt
[2]*Sqrt[a]*(a + b)*Sqrt[a + b*Sech[x]^2])
```

Maple [F]
time = 1.80, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\sqrt{a+b\text{sech}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^2/(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/sqrt(b*sech(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(45) = 90$.
time = 0.45, size = 1365, normalized size = 25.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
a - b)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sin
h(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*s
inh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^
3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*
a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*
(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^
3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*
cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^
2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^
2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*
b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b
^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*co
sh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sin
h(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2
*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^
3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15
*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*
cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*si
nh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*
sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*s
inh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
```

```
 sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*a*sqrt((a*cosh(x)^2 + a*sinh(x)
^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*b)*co
sh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 - a^2 - a*b
), -1/2*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
 - a - b)*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*si
nh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2
 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3
 + a*b*sinh(x)^4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b
)*sinh(x)^2 - a^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) +
 ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b
)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*cosh(x)*
sinh(x) + a*sinh(x)^2 + a)) + 2*sqrt(2)*a*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*b)*cosh(x
)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 - a^2 - a*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\sqrt{a + b\,\mathrm{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(coth(x)**2/sqrt(a + b*sech(x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth(x)^2}{\sqrt{a + \frac{b}{\cosh(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b/cosh(x)^2)^(1/2),x)
[Out] int(coth(x)^2/(a + b/cosh(x)^2)^(1/2), x)
```

3.202 $\int \frac{\coth^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx$

Optimal. Leaf size=90

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}}-\frac{\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{2(a+b)}$$

```
[Out] -1/2*(2*a+3*b)*arctanh((a+b*sech(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)+arcta
nh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(1/2)-1/2*coth(x)^2*(a+b*sech(x)^2)^(1/
2)/(a+b)
```

Rubi [A]
time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4224, 457, 105, 162, 65, 214}

$$-\frac{\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{2(a+b)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/Sqrt[a] - ((2*a + 3*b)*ArcTanh[Sqrt[
a + b*Sech[x]^2]/Sqrt[a + b]])/(2*(a + b)^(3/2)) - (Coth[x]^2*Sqrt[a + b*Se
ch[x]^2])/(2*(a + b))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(x)}{\sqrt{a+b\text{sech}^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)^2\sqrt{a+bx^2}}\,dx,x,\text{sech}(x)\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)^2x\sqrt{a+bx}}\,dx,x,\text{sech}^2(x)\right)\right) \\
&= -\frac{\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{2(a+b)} + \frac{\text{Subst}\left(\int \frac{a+b+\frac{bx}{2}}{(-1+x)x\sqrt{a+bx}}\,dx,x,\text{sech}^2(x)\right)}{2(a+b)} \\
&= -\frac{\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{2(a+b)} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\text{sech}^2(x)\right) + \frac{(2a+3b}{} \\
&= -\frac{\coth^2(x)\sqrt{a+b\text{sech}^2(x)}}{2(a+b)} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{sech}^2(x)}\right)}{b} + \frac{(2a+}{} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}} - \frac{\coth^2(x}{}
\end{aligned}$$

Mathematica [A]
time = 0.51, size = 159, normalized size = 1.77

$$\frac{-\left((a+2b+a\cosh(2x))\text{csch}^2(x)\right)+\frac{\sqrt{2}\sqrt{a+2b+a\cosh(2x)}\left(-\sqrt{a}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)+2(a+b)^{3/2}\log\left(\sqrt{2}\sqrt{a}\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)\right)\text{sech}(x)}{\sqrt{a}\sqrt{a+b}}}{4(a+b)\sqrt{a+b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/Sqrt[a + b*Sech[x]^2],x]
```

```
[Out] (-((a + 2*b + a*Cosh[2*x])*Csch[x]^2) + (Sqrt[2]*Sqrt[a + 2*b + a*Cosh[2*x]
]*(-(Sqrt[a]*(2*a + 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b
 + a*Cosh[2*x]]]) + 2*(a + b)^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a +
2*b + a*Cosh[2*x]]])*Sech[x])/(Sqrt[a]*Sqrt[a + b]))/(4*(a + b)*Sqrt[a + b*
Sech[x]^2])
```

Maple [F]
time = 1.83, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{\sqrt{a+b\,\text{sech}(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*sech(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3/(a+b*sech(x)^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^3/sqrt(b*sech(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1213 vs. $2(72) = 144$.
time = 0.62, size = 6475, normalized size = 71.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x
)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(
3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*
b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*a*b + b^2)*cosh(x))*s
inh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b +
 a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 +
5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 1
4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b
^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6
*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^
4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(
x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a
^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sin
h(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2
)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^
2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 +
 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 +
2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*
b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2
)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*co
sh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x
```

```
)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 +
 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a
)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
 + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*
a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh
(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(
x)*sinh(x)^5 + sinh(x)^6)) + ((2*a^2 + 3*a*b)*cosh(x)^4 + 4*(2*a^2 + 3*a*b)
*cosh(x)*sinh(x)^3 + (2*a^2 + 3*a*b)*sinh(x)^4 - 2*(2*a^2 + 3*a*b)*cosh(x)^
2 + 2*(3*(2*a^2 + 3*a*b)*cosh(x)^2 - 2*a^2 - 3*a*b)*sinh(x)^2 + 2*a^2 + 3*a
*b + 4*((2*a^2 + 3*a*b)*cosh(x)^3 - (2*a^2 + 3*a*b)*cosh(x))*sinh(x))*sqrt(
a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)
*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b
)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqr
t(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(
x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + ((a
^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^3 + (a^
2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(3*(a^2 +
2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x))*s
qrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x
)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sin
h(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 2*sqrt(2)*((a^2 +
a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 + a^
2 + a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)))/((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*
a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2 - 2*(a^3 + 2*a^2*b + a*
b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b +
a*b^2)*cosh(x)^3 - (a^3 + 2*a^2*b + a*b^2)*cosh(x))*sinh(x)), 1/4*(2*((2*a^
2 + 3*a*b)*cosh(x)^4 + 4*(2*a^2 + 3*a*b)*cosh(x)*sinh(x)^3 + (2*a^2 + 3*a*b
)*sinh(x)^4 - 2*(2*a^2 + 3*a*b)*cosh(x)^2 + 2*(3*(2*a^2 + 3*a*b)*cosh(x)^2
- 2*a^2 - 3*a*b)*sinh(x)^2 + 2*a^2 + 3*a*b + 4*((2*a^2 + 3*a*b)*cosh(x)^3 -
 (2*a^2 + 3*a*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh
(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4
+ 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh
(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) +
a)) + ((a^2 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x
)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4 - 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(
3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a^2 - 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*
```

```
b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(x)^3 - (a...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{\sqrt{a + b\,\mathrm{sech}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(coth(x)**3/sqrt(a + b*sech(x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^3}{\sqrt{a + \frac{b}{\cosh(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b/cosh(x)^2)^(1/2),x)
```

```
[Out] int(coth(x)^3/(a + b/cosh(x)^2)^(1/2), x)
```

3.203 $\int \frac{\tanh^5(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(a+b)^2}{ab^2\sqrt{a+b\text{sech}^2(x)}}-\frac{\sqrt{a+b\text{sech}^2(x)}}{b^2}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(3/2)-(a+b)^2/a/b^2/(a+b*sech(x)^2
)^(1/2)-(a+b*sech(x)^2)^(1/2)/b^2
```

Rubi [A]

time = 0.10, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4224, 457, 89, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(a+b)^2}{ab^2\sqrt{a+b\text{sech}^2(x)}}-\frac{\sqrt{a+b\text{sech}^2(x)}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - (a + b)^2/(a*b^2*Sqrt[a +
b*Sech[x]^2]) - Sqrt[a + b*Sech[x]^2]/b^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^5(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x\left(a+bx^2\right)^{3/2}}\,dx, x, \operatorname{sech}(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^2}{x(a+bx)^{3/2}}\,dx, x, \operatorname{sech}^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{(a+b)^2}{ab(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}+\frac{1}{ax\sqrt{a+bx}}\right)\,dx, x, \operatorname{sech}^2(x)\right)\right)\\
&= -\frac{(a+b)^2}{ab^2\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)}{2a}\\
&= -\frac{(a+b)^2}{ab^2\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{ab}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(a+b)^2}{ab^2\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.32, size = 129, normalized size = 1.90

$$\frac{\operatorname{sech}^3(x)\left(\frac{\sqrt{2}(a+2b+a\cosh(2x))^{3/2}\log\left(\sqrt{2}\sqrt{a}\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)}{a^{3/2}}-\frac{(a+2b+a\cosh(2x))(2a^2+4ab+b^2+(2a^2+2ab+b^2)\cosh(2x))\operatorname{sech}(x)}{ab^2}\right)}{4\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (Sech[x]^3*((Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh
[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/a^(3/2) - ((a + 2*b + a*Cosh[2*x])*(2*a
^2 + 4*a*b + b^2 + (2*a^2 + 2*a*b + b^2)*Cosh[2*x])*Sech[x])/(a*b^2)))/(4*(
a + b*Sech[x]^2)^(3/2))
```

Maple [F]
time = 1.68, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\left(a+b\operatorname{sech}(x)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^5/(b*sech(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1312 vs. 2(58) = 116.
time = 0.54, size = 3360, normalized size = 49.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 + (3*a
*b^2 + 4*b^3)*cosh(x)^4 + (15*a*b^2*cosh(x)^2 + 3*a*b^2 + 4*b^3)*sinh(x)^4
+ 4*(5*a*b^2*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 + a*b^2 + (3*
a*b^2 + 4*b^3)*cosh(x)^2 + (15*a*b^2*cosh(x)^4 + 3*a*b^2 + 4*b^3 + 6*(3*a*b
^2 + 4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a*b^2*cosh(x)^5 + 2*(3*a*b^2 + 4*b^
3)*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^
2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3
 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2
*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(
x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 +
 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3
+ (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*
b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*
b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*
b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2
 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2
+ 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b
 + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b
^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2
)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 +
 a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3
 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (
6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))
/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*s
inh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*
b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 + (3*a*b^2 + 4*
b^3)*cosh(x)^4 + (15*a*b^2*cosh(x)^2 + 3*a*b^2 + 4*b^3)*sinh(x)^4 + 4*(5*a*
b^2*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 + a*b^2 + (3*a*b^2 + 4
*b^3)*cosh(x)^2 + (15*a*b^2*cosh(x)^4 + 3*a*b^2 + 4*b^3 + 6*(3*a*b^2 + 4*b^
3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a*b^2*cosh(x)^5 + 2*(3*a*b^2 + 4*b^3)*cosh(x
)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*c
osh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh
(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*((2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4
*(2*a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 2*a^2*b + a*b^2)*si
nh(x)^4 + 2*a^3 + 2*a^2*b + a*b^2 + 2*(2*a^3 + 4*a^2*b + a*b^2)*cosh(x)^2 +
 2*(2*a^3 + 4*a^2*b + a*b^2 + 3*(2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x
)^2 + 4*((2*a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (2*a^3 + 4*a^2*b + a*b^2)*co
```

```
sh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2*cosh(x)^6 + 6*a^3*b^2*cosh(x)*sinh(x
)^5 + a^3*b^2*sinh(x)^6 + a^3*b^2 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x)^4 + (15
*a^3*b^2*cosh(x)^2 + 3*a^3*b^2 + 4*a^2*b^3)*sinh(x)^4 + 4*(5*a^3*b^2*cosh(x
)^3 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x)^3 + (3*a^3*b^2 + 4*a^2*b^3)*
cosh(x)^2 + (15*a^3*b^2*cosh(x)^4 + 3*a^3*b^2 + 4*a^2*b^3 + 6*(3*a^3*b^2 +
4*a^2*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*a^3*b^2*cosh(x)^5 + 2*(3*a^3*b^2 + 4
*a^2*b^3)*cosh(x)^3 + (3*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x)), -1/2*((a*b
^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 + (3*a*b^2 + 4*b
^3)*cosh(x)^4 + (15*a*b^2*cosh(x)^2 + 3*a*b^2 + 4*b^3)*sinh(x)^4 + 4*(5*a*b
^2*cosh(x)^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 + a*b^2 + (3*a*b^2 + 4*
b^3)*cosh(x)^2 + (15*a*b^2*cosh(x)^4 + 3*a*b^2 + 4*b^3 + 6*(3*a*b^2 + 4*b^3
)*cosh(x)^2)*sinh(x)^2 + 2*(3*a*b^2*cosh(x)^5 + 2*(3*a*b^2 + 4*b^3)*cosh(x)
^3 + (3*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt
((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 +
a*b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a
^2 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*
cosh(x))*sinh(x))) + (a*b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*s
inh(x)^6 + (3*a*b^2 + 4*b^3)*cosh(x)^4 + (15*a*b^2*cosh(x)^2 + 3*a*b^2 + 4*
b^3)*sinh(x)^4 + 4*(5*a*b^2*cosh(x)^3 + (3*a*b^...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**5/(a + b*sech(x)**2)**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^5}{\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + b/cosh(x)^2)^(3/2),x)
[Out] int(tanh(x)^5/(a + b/cosh(x)^2)^(3/2), x)
```

3.204 $\int \frac{\tanh^4(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{b^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}}-\frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}}$$

```
[Out] arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(3/2)+arctanh(a^(1/2)*tan
h(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(3/2)-(a+b)*tanh(x)/a/b/(a+b-b*tanh(x)^2)^(
1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$,
Rules used = {4226, 2000, 481, 537, 223, 209, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}+\frac{\mathrm{ArcTan}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{b^{3/2}}-\frac{(a+b)\tanh(x)}{ab\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/b^(3/2) + ArcTanh[(Sqrt
[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - ((a + b)*Tanh[x])/(a*b*Sq
rt[a + b - b*Tanh[x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^4(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a+b\left(1-x^2\right)\right)^{3/2}}\,dx, x, \tanh(x)\right)\\ &= \text{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)\\ &= -\frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{a+b-ax^2}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{ab}\\ &= -\frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{a} + \frac{\text{Su}}{}\\ &= -\frac{(a+b)\tanh(x)}{ab\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a} + \frac{\text{Su}}{}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{b^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}} - \frac{(a\ }{ab\sqrt{a+}}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 169, normalized size = 1.97

$$-\frac{(a+2b+a\cosh(2x))\text{sech}^3(x)\left(-\sqrt{2}\,b^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\sqrt{a+2b+a\cosh(2x)}+\sqrt{a}\left(-\sqrt{2}\,a\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\sqrt{a+2b+a\cosh(2x)}+2\sqrt{b}\,(a+b)\sinh(x)\right)\right)}{4a^{3/2}b^{3/2}\left(a+b\text{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cosh[2*x])*Sech[x]^3*(-(Sqrt[2]*b^(3/2)*ArcTanh[(Sqrt[2]
*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cosh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]])
 + Sqrt[a]*(-(Sqrt[2]*a*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2*b + a*C
osh[2*x]]]*Sqrt[a + 2*b + a*Cosh[2*x]]) + 2*Sqrt[b]*(a + b)*Sinh[x])))/(a^(
3/2)*b^(3/2)*(a + b*Sech[x]^2)^(3/2))
```

Maple [F]
time = 1.56, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\left(a+b\,\text{sech}(x)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/(b*sech(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1007 vs. $2(72) = 144$.
time = 0.60, size = 5170, normalized size = 60.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(x)^3 + a*b^2*sinh(x)^4 + a*b^
2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)^2 + a*b^2 + 2*b^3)*sin
h(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(a)*log
((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2
- b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a
*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*
b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2
- b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh
(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2
*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 +
3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*c
osh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5
*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(
x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2
 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4
*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*
b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b
)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)
^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5
+ sinh(x)^6)) - 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4
```

```
+ a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*s
inh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(-b)*lo
g(-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2
*(a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 + 2*sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
 4*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*
cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2
 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + (a*b^2*cosh(x)^4 + 4*a*b^2*cosh(
x)*sinh(x)^3 + a*b^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3
*a*b^2*cosh(x)^2 + a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 +
 2*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2
+ sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*c
osh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*(a^2*b + a*b^2 - (a^2*b + a*b^2)*cosh(x)
^2 - 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) - (a^2*b + a*b^2)*sinh(x)^2)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)))/(a^3*b^2*cosh(x)^4 + 4*a^3*b^2*cosh(x)*sinh(x)^3 + a^3*b^2*sinh(x)^4
 + a^3*b^2 + 2*(a^3*b^2 + 2*a^2*b^3)*cosh(x)^2 + 2*(3*a^3*b^2*cosh(x)^2 + a
^3*b^2 + 2*a^2*b^3)*sinh(x)^2 + 4*(a^3*b^2*cosh(x)^3 + (a^3*b^2 + 2*a^2*b^3
)*cosh(x))*sinh(x)), 1/4*(4*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*
sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 +
2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*s
qrt(b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(
b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a +
 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 +
(a + 2*b)*cosh(x))*sinh(x) + a)) + (a*b^2*cosh(x)^4 + 4*a*b^2*cosh(x)*sinh(
x)^3 + a*b^2*sinh(x)^4 + a*b^2 + 2*(a*b^2 + 2*b^3)*cosh(x)^2 + 2*(3*a*b^2*c
osh(x)^2 + a*b^2 + 2*b^3)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + (a*b^2 + 2*b^3)*
cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7
+ a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b
^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh
(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4
*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh
(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sin
h(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*
b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^
2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(
x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cos
h(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(
x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5
 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x
```

```
)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{(a + b\,\mathrm{sech}^2(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*sech(x)**2)**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^4}{\left(a + \frac{b}{\cosh(x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^4/(a + b/cosh(x)^2)^(3/2), x)
```

3.205 $\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}} dx$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{a+b}{ab\sqrt{a+b\operatorname{sech}^2(x)}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(3/2)+(-a-b)/a/b/(a+b*sech(x)^2)^(
1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4224, 457, 79, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{a+b}{ab\sqrt{a+b\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - (a + b)/(a*b*Sqrt[a + b*Se
ch[x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
```

```
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{-1+x^2}{x\left(a+bx^2\right)^{3/2}}\,dx, x, \operatorname{sech}(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{x(a+bx)^{3/2}}\,dx, x, \operatorname{sech}^2(x)\right)\\
&= -\frac{a+b}{ab\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)}{2a}\\
&= -\frac{a+b}{ab\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{ab}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{a+b}{ab\sqrt{a+b\operatorname{sech}^2(x)}}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 103 vs. $2(49) = 98$.
time = 0.15, size = 103, normalized size = 2.10

$$\frac{\left(-\frac{2\sqrt{a}(a+b)\cosh(x)(a+2b+a\cosh(2x))}{b}+\sqrt{2}\,(a+2b+a\cosh(2x))^{3/2}\log\left(\sqrt{2}\,\sqrt{a}\,\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)\right)\operatorname{sech}^3(x)}{4a^{3/2}\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (((-2*Sqrt[a]*(a + b)*Cosh[x]*(a + 2*b + a*Cosh[2*x]))/b + Sqrt[2]*(a + 2*b
 + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2
*x]]])*Sech[x]^3)/(4*a^(3/2)*(a + b*Sech[x]^2)^(3/2))
```

Maple [F]
time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\left(a+b\operatorname{sech}(x)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^3/(b*sech(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(41) = 82$.
time = 0.44, size = 2194, normalized size = 44.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 + 2*(a*b + 2
*b^2)*cosh(x)^2 + 2*(3*a*b*cosh(x)^2 + a*b + 2*b^2)*sinh(x)^2 + a*b + 4*(a*
```

```
b*cosh(x)^3 + (a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b +
 a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*
a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 +
2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*
sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b +
4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4
+ (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(
2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*
a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*co
sh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4
*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*
cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*
a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a
*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^
2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*c
osh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cos
h(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2
+ 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3
*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a
 + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b
+ a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3
 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cos
h(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x
)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*b*cos
h(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 + 2*(a*b + 2*b^2)*cosh(x)^
2 + 2*(3*a*b*cosh(x)^2 + a*b + 2*b^2)*sinh(x)^2 + a*b + 4*(a*b*cosh(x)^3 +
(a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sin
h(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sq
rt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(
x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))
+ 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2)) - 4*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sin
h(x) + (a^2 + a*b)*sinh(x)^2 + a^2 + a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b*cosh(x)^4 +
4*a^3*b*cosh(x)*sinh(x)^3 + a^3*b*sinh(x)^4 + a^3*b + 2*(a^3*b + 2*a^2*b^2)
*cosh(x)^2 + 2*(3*a^3*b*cosh(x)^2 + a^3*b + 2*a^2*b^2)*sinh(x)^2 + 4*(a^3*b
*cosh(x)^3 + (a^3*b + 2*a^2*b^2)*cosh(x))*sinh(x)), -1/2*((a*b*cosh(x)^4 +
4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 + 2*(a*b + 2*b^2)*cosh(x)^2 + 2*(3*
a*b*cosh(x)^2 + a*b + 2*b^2)*sinh(x)^2 + a*b + 4*(a*b*cosh(x)^3 + (a*b + 2*
b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*((a + b)*cosh(x)^2 + 2*(a +
b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a^2 + a
*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (
2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + 3*a*b)*sinh(x
```

```
)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*cosh(x))*sinh(x)))
 + (a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 + 2*(a*b + 2*b^
2)*cosh(x)^2 + 2*(3*a*b*cosh(x)^2 + a*b + 2*b^2)*sinh(x)^2 + a*b + 4*(a*b*c
osh(x)^3 + (a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh
(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4
+ 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh
(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) +
a)) + 2*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a
^2 + a*b)*sinh(x)^2 + a^2 + a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b*cosh(x)^4 + 4*a^3*b*c
osh(x)*sinh(x)^3 + a^3*b*sinh(x)^4 + a^3*b + 2*(a^3*b + 2*a^2*b^2)*cosh(x)^
2 + 2*(3*a^3*b*cosh(x)^2 + a^3*b + 2*a^2*b^2)*sinh(x)^2 + 4*(a^3*b*cosh(x)^
3 + (a^3*b + 2*a^2*b^2)*cosh(x))*sinh(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{(a + b\,\mathrm{sech}^2(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*sech(x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
itution variable should perhaps be purged.Warning, replacing 0 by ‘ u‘, a s
ubstit
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^3}{\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^3/(a + b/cosh(x)^2)^(3/2), x)
```

3.206 $\int \frac{\tanh^2(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}}-\frac{\tanh(x)}{a\sqrt{a+b-b\tanh^2(x)}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(3/2)-tanh(x)/a/(a+b-b*t
anh(x)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4226, 2000, 482, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{\tanh(x)}{a\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - Tanh[x]/(a*S
qrt[a + b - b*Tanh[x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)\\
&= -\frac{\tanh(x)}{a\sqrt{a+b-b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right)}{a}\\
&= -\frac{\tanh(x)}{a\sqrt{a+b-b\tanh^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}}-\frac{\tanh(x)}{a\sqrt{a+b-b\tanh^2(x)}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 128 vs. 2(51) = 102.
time = 0.50, size = 128, normalized size = 2.51

$$\frac{(a+2b+a\cosh(2x))\mathrm{sech}^2(x)\left(\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)(a+2b+a\cosh(2x))\mathrm{sech}(x)-2\sqrt{a}\sqrt{a+b}\sqrt{\frac{a+b+a\sinh^2(x)}{a+b}}\tanh(x)\right)}{4a^{3/2}\sqrt{a+b}\left(a+b\mathrm{sech}^2(x)\right)^{3/2}\sqrt{\frac{a+b+a\sinh^2(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*x])*Sech[x]^2*(ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]*
(a + 2*b + a*Cosh[2*x])*Sech[x] - 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + b + a*Sin
h[x]^2)/(a + b)]*Tanh[x]))/(4*a^(3/2)*Sqrt[a + b]*(a + b*Sech[x]^2)^(3/2)*S
qrt[(a + b + a*Sinh[x]^2)/(a + b)])
```

Maple [F]
time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\left(a+b\,\mathrm{sech}(x)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^2/(b*sech(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(43) = 86.
time = 0.43, size = 1733, normalized size = 33.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")

[Out] [1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh
(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*
cosh(x))*sinh(x) + a)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x
)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 -
 a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*
sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3
 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*
cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))
*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15
*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh
(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*s
inh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2
*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*c
osh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(
x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*co
sh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2
)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*
a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)
^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2
*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*cosh(x)^4 + 4*a*cosh(x)
*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2
*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(a)*lo
g(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2
 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(
x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(
a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt((a*cosh(x)^2 + a*
sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*cos
h(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*
cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^
3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x)), -1/2*((a*cosh(x)^4 + 4*a*cosh(x)*sin
h(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*
sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(-a)*arcta
n(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^4 - (a^2
+ 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a^2 + 2*(2
*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + (a*cosh(x)^4 + 4*a*cosh
(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a
+ 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(-a
)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*s
qrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
```

```
sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b
)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a +
 2*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x)
 + a*sinh(x)^2 - a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 -
 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3
+ a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 +
a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh
(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{(a + b\operatorname{sech}^2(x))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*sech(x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
itution variable should perhaps be purged.Warning, replacing 0 by ‘ u‘, a s
ubstit
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^2}{\left(a + \frac{b}{\cosh(x)^2}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] int(tanh(x)^2/(a + b/cosh(x)^2)^(3/2), x)
```

3.207 $\int \frac{\tanh(x)}{\left(a+b\mathbf{sech}^2(x)\right)^{3/2}} \, dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\sqrt{a+b\text{sech}^2(x)}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(3/2)-1/a/(a+b*sech(x)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4224, 272, 53, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\sqrt{a+b\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - 1/(a*Sqrt[a + b*Sech[x]^2]
)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx, x, \operatorname{sech}(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \operatorname{sech}^2(x)\right)\right)\\
&= -\frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2(x)\right)}{2a}\\
&= -\frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{sech}^2(x)}\right)}{ab}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\sqrt{a+b\operatorname{sech}^2(x)}}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(43) = 86.

time = 0.23, size = 98, normalized size = 2.28

$$-\frac{(a+2b+a\cosh(2x))\left(2\sqrt{a}\,\cosh(x)-\sqrt{2}\,\sqrt{a+2b+a\cosh(2x)}\,\log\left(\sqrt{2}\,\sqrt{a}\,\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)\right)\operatorname{sech}^3(x)}{4a^{3/2}\left(a+b\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] -1/4*((a + 2*b + a*Cosh[2*x])*(2*Sqrt[a]*Cosh[x] - Sqrt[2]*Sqrt[a + 2*b + a
*Cosh[2*x]]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])*Sec
h[x]^3)/(a^(3/2)*(a + b*Sech[x]^2)^(3/2))
```

Maple [A]
time = 0.51, size = 46, normalized size = 1.07

method	result	size
derivativedivides	$-\frac{1}{a\sqrt{a+b\operatorname{sech}(x)^2}}+\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\operatorname{sech}(x)^2}}{\operatorname{sech}(x)}\right)}{a^{\frac{3}{2}}}$	46
default	$-\frac{1}{a\sqrt{a+b\operatorname{sech}(x)^2}}+\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\operatorname{sech}(x)^2}}{\operatorname{sech}(x)}\right)}{a^{\frac{3}{2}}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sech(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/(a+b*sech(x)^2)^(1/2)+1/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2
))/sech(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/(b*sech(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(35) = 70.
time = 0.46, size = 2034, normalized size = 47.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh
(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*
cosh(x))*sinh(x) + a)*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a
^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8
 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a
*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 +
 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*
sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a
*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^
2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 1
0*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2
)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2
*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4
+ 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + s
qrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(
x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*
(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2
 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a
^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b
 + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^
2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*s
inh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(
2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*c
osh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sin
h(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(
x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (a*cosh(x)^4 + 4*a*cosh(x)*sinh(
x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*si
nh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(a)*log(-(a*
cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*co
sh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh
(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a*cosh(x)^2 + 2*a*cosh
(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)
*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*c
osh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*co
sh(x))*sinh(x)), -1/2*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 +
 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(
x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(-a)*arctan(sqrt(2)*((a + b)*cos
```

```
h(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a)*sqrt(-a)*sqrt((
a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*
b)*sinh(x)^4 + (2*a^2 + 3*a*b)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2
 + 3*a*b)*sinh(x)^2 + a^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + 3*a*b)*co
sh(x))*sinh(x))) + (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(
a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3
 + (a + 2*b)*cosh(x))*sinh(x) + a)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*co
sh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 +
a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 2*
sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt((a*cosh(
x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))
/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 +
2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3
*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))]
```

Sympy [**A**]
time = 3.37, size = 44, normalized size = 1.02

$$-\frac{1}{a\sqrt{a+b\,\mathrm{sech}^2\left(x\right)}}-\frac{\mathrm{atan}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2\left(x\right)}}{\sqrt{-a}}\right)}{a\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] -1/(a*sqrt(a + b*sech(x)**2)) - atan(sqrt(a + b*sech(x)**2)/sqrt(-a))/(a*sq
rt(-a))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ` u`, a subst
itution variable should perhaps be purged.Warning, replacing 0 by ` u`, a s
ubstit
```

Mupad [B]
time = 1.74, size = 35, normalized size = 0.81

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(x)^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\sqrt{a+\frac{b}{\cosh(x)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b/cosh(x)^2)^(3/2),x)
[Out] atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2))/a^(3/2) - 1/(a*(a + b/cosh(x)^2)^(1/
2))
```

3.208 $\int \frac{1}{\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}}-\frac{b\tanh(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}}$$

[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(3/2)-b*tanh(x)/a/(a+b)/(a+b-b*tanh(x)^2)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4213, 390, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{b\tanh(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sech[x]^2)^(-3/2),x]

[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - (b*Tanh[x])/(a*(a + b)*Sqrt[a + b - b*Tanh[x]^2])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{b\tanh(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x)\right.}{a}$$

$$= -\frac{b\tanh(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right.}{a}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}} - \frac{b\tanh(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}}$$

Mathematica [A]
time = 0.56, size = 107, normalized size = 1.88

$$\frac{(a+2b+a\cosh(2x))\text{sech}^3(x)\left((a+b)^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)\sqrt{\frac{a+2b+a\cosh(2x)}{a+b}}-\sqrt{2}\sqrt{a}\,b\sinh(x)\right)}{2\sqrt{2}\,a^{3/2}(a+b)\left(a+b\text{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(-3/2),x]
```

```
[Out] ((a + 2*b + a*Cosh[2*x])*Sech[x]^3*((a + b)^(3/2)*ArcSinh[(Sqrt[a]*Sinh[x])
/Sqrt[a + b]]*Sqrt[(a + 2*b + a*Cosh[2*x])/(a + b)] - Sqrt[2]*Sqrt[a]*b*Sin
h[x]))/(2*Sqrt[2]*a^(3/2)*(a + b)*(a + b*Sech[x]^2)^(3/2))
```

Maple [**F**]
time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{sech}\left(x\right)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(1/(a+b*sech(x)^2)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(-3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 801 vs. 2(49) = 98.
time = 0.51, size = 2095, normalized size = 36.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b
)*sinh(x)^4 + 2*(a^2 + 3*a*b + 2*b^2)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^
2 + a^2 + 3*a*b + 2*b^2)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 +
 (a^2 + 3*a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a
*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(1
4*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b
^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*
b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(
x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b
+ 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a
*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^
2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x
```

```
)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*s
inh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*co
sh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a
^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))
*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 +
(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(co
sh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(
x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^2 +
 a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 +
 2*(a^2 + 3*a*b + 2*b^2)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 + 3*a
*b + 2*b^2)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 + 3*a*b
 + 2*b^2)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)
^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^
2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x)
 + a*b*sinh(x)^2 - a*b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^4 + a^3*b)*cosh(x)^4 + 4*(a^4 + a
^3*b)*cosh(x)*sinh(x)^3 + (a^4 + a^3*b)*sinh(x)^4 + a^4 + a^3*b + 2*(a^4 +
3*a^3*b + 2*a^2*b^2)*cosh(x)^2 + 2*(a^4 + 3*a^3*b + 2*a^2*b^2 + 3*(a^4 + a^
3*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 + a^3*b)*cosh(x)^3 + (a^4 + 3*a^3*b + 2
*a^2*b^2)*cosh(x))*sinh(x)), -1/2*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*c
osh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2 + 3*a*b + 2*b^2)*cosh(x)^
2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 + 3*a*b + 2*b^2)*sinh(x)^2 + a^2 + a*b
 + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 + 3*a*b + 2*b^2)*cosh(x))*sinh(x))*sqrt(
-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sq
rt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2))/(a*b*cosh(x)^4 + 4*a*b*cosh(x)*sinh(x)^3 + a*b*sinh(x)^
4 - (a^2 + 3*a*b)*cosh(x)^2 + (6*a*b*cosh(x)^2 - a^2 - 3*a*b)*sinh(x)^2 - a
^2 + 2*(2*a*b*cosh(x)^3 - (a^2 + 3*a*b)*cosh(x))*sinh(x))) + ((a^2 + a*b)*c
osh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2
 + 3*a*b + 2*b^2)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 + 3*a*b + 2*
b^2)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 + 3*a*b + 2*b^
2)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2 + 1)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^
3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + 2*b)*sinh(
x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) + 2*sqrt(2)*(a*b*c
osh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - a*b)*sqrt((a*cosh(x)^2 +
 a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^4
 + a^3*b)*cosh(x)^4 + 4*(a^4 + a^3*b)*cosh(x)*sinh(x)^3 + (a^4 + a^3*b)*sin
h(x)^4 + a^4 + a^3*b + 2*(a^4 + 3*a^3*b + 2*a^2*b^2)*cosh(x)^2 + 2*(a^4 + 3
*a^3*b + 2*a^2*b^2 + 3*(a^4 + a^3*b)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 + a^3*b
```

```
)*cosh(x)^3 + (a^4 + 3*a^3*b + 2*a^2*b^2)*cosh(x))*sinh(x))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\operatorname{sech}^{2}\left(x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sech(x)**2)**(-3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
itution variable should perhaps be purged.Warning, replacing 0 by ‘ u‘, a s
ubstit
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a+\frac{b}{{\mathrm{cosh}\left(x\right)}^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b/cosh(x)^2)^(3/2), x)
```

3.209 $\int \frac{\coth(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{b}{a(a+b)\sqrt{a+b\operatorname{sech}^2(x)}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(3/2)-arctanh((a+b*sech(x)^2)^(1/2
)/(a+b)^(1/2))/(a+b)^(3/2)-b/a/(a+b)/(a+b*sech(x)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4224, 457, 87, 162, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{b}{a(a+b)\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(3/2) - ArcTanh[Sqrt[a + b*Sech[x]
^2]/Sqrt[a + b]]/(a + b)^(3/2) - b/(a*(a + b)*Sqrt[a + b*Sech[x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth(x)}{\left(a+b\mathrm{sech}^2(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \mathrm{sech}(x)\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \mathrm{sech}^2(x)\right) \\
&= -\frac{b}{a(a+b)\sqrt{a+b\mathrm{sech}^2(x)}} + \frac{\text{Subst}\left(\int \frac{a+b-bx}{(-1+x)x\sqrt{a+bx}}\,dx, x, \mathrm{sech}^2(x)\right)}{2a(a+b)} \\
&= -\frac{b}{a(a+b)\sqrt{a+b\mathrm{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \mathrm{sech}^2(x)\right)}{2a} + \frac{\text{Subst}\left(\int\right.}{} \\
&= -\frac{b}{a(a+b)\sqrt{a+b\mathrm{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\mathrm{sech}^2(x)}\right)}{ab} + \frac{\text{Subs}}{} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{b}{a(a+b)\sqrt{a+}}
\end{aligned}
$$

Mathematica [A]
time = 0.42, size = 155, normalized size = 1.96

$$
\frac{\mathrm{sech}^2(x)\left(-2b(a+2b+a\cosh(2x)) + \frac{\sqrt{2}\,(a+2b+a\cosh(2x))^{3/2}\left(-a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)+(a+b)^{3/2}\log\left(\sqrt{2}\sqrt{a}\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)\right)\mathrm{sech}(x)}{\sqrt{a}\sqrt{a+b}}\right)}{4a(a+b)\left(a+b\mathrm{sech}^2(x)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (Sech[x]^2*(-2*b*(a + 2*b + a*Cosh[2*x]) + (Sqrt[2]*(a + 2*b + a*Cosh[2*x])
^(3/2)*(-(a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b + a*Co
sh[2*x]]]) + (a + b)^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*C
osh[2*x]]])*Sech[x])/(Sqrt[a]*Sqrt[a + b])))/(4*a*(a + b)*(a + b*Sech[x]^2)
^(3/2))
```

Maple [F]
time = 1.54, size = 0, normalized size = 0.00

$$
\int \frac{\coth(x)}{\left(a+b\,\mathrm{sech}(x)^2\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sech(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)/(a+b*sech(x)^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/(b*sech(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1330 vs. 2(65) = 130.
time = 0.57, size = 6939, normalized size = 87.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2
*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 +
 2*b^3 + 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b
 + a*b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*s
qrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*c
osh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b +
 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 +
2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(
x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14
*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3
 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sin
h(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*
a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 +
a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)
^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(
6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b +
b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b
^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*
cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)
^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 +
```

```
(15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a
^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 +
 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a
*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x
)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b
^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a
^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sin
h(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x
)^5 + sinh(x)^6)) + 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x
)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*
b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(a
+ b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*s
inh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a + 3*b)*
sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(
a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*sinh(x)
 + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 -
 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + ((a^3
 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^3
 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2*(a^3 + 4*a
^2*b + 5*a*b^2 + 2*b^3)*cosh(x)^2 + 2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3 + 3*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b + a*b^2)*c
osh(x)^3 + (a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(
-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*
a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2
*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*(a^2*b + a*b^2 + (a
^2*b + a*b^2)*cosh(x)^2 + 2*(a^2*b + a*b^2)*cosh(x)*sinh(x) + (a^2*b + a*b^
2)*sinh(x)^2)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2)))/(a^5 + 2*a^4*b + a^3*b^2 + (a^5 + 2*a^4*b + a^3
*b^2)*cosh(x)^4 + 4*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 + 2*
a^4*b + a^3*b^2)*sinh(x)^4 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*cosh
(x)^2 + 2*(a^5 + 4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3 + 3*(a^5 + 2*a^4*b + a^3*b
^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^3 + (a^5 +
4*a^4*b + 5*a^3*b^2 + 2*a^2*b^3)*cosh(x))*sinh(x)), 1/4*(4*(a^3*cosh(x)^4 +
 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + a^3 + 2*(a^3 + 2*a^2*b)*cosh(x)^
2 + 2*(3*a^3*cosh(x)^2 + a^3 + 2*a^2*b)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + (a^3
 + 2*a^2*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^4 + 4*a
*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*a*cosh(x)^2
 + a + 2*b)*sinh(x)^2 + 4*(a*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a)) +
 ((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sin
h(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 + 2*a^2*b + a*b^2 + 2*(a^3
```

```
 + 4*a^2*b + 5*a*b^2 + 2*b^3)*cosh(x)^2 + 2*(a^...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{(a+b\operatorname{sech}^2(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral(coth(x)/(a + b*sech(x)**2)**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)}{\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)/(a + b/cosh(x)^2)^(3/2), x)
```

3.210 $\int \frac{\coth^2(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}}-\frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}}-\frac{(a-b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a(a+b)^2}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(3/2)-b*coth(x)/a/(a+b)/
(a+b-b*tanh(x)^2)^(1/2)-(a-b)*coth(x)*(a+b-b*tanh(x)^2)^(1/2)/a/(a+b)^2
```

Rubi [A]
time = 0.18, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4226, 2000, 483, 597, 12, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{3/2}}-\frac{(a-b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{a(a+b)^2}-\frac{b\coth(x)}{a(a+b)\sqrt{a-b\tanh^2(x)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(3/2) - (b*Coth[x])/
(a*(a + b)*Sqrt[a + b - b*Tanh[x]^2]) - ((a - b)*Coth[x]*Sqrt[a + b - b*Tan
h[x]^2])/(a*(a + b)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\coth^2(x)}{\left(a+b\text{sech}^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{x^2\,(1-x^2)\,(a+b\,(1-x^2))^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{1}{x^2\,(1-x^2)\,(a+b-bx^2)^{3/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{-a+b-2bx^2}{x^2(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \tanh(x\right.}{a(a+b)}$$

$$= -\frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a(a+b)^2} + \frac{\text{Sub}}{}$$

$$= -\frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a(a+b)^2} + \frac{\text{Sub}}{}$$

$$= -\frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-b)\coth(x)\sqrt{a+b-b\tanh^2(x)}}{a(a+b)^2} + \frac{\text{Sub}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{3/2}} - \frac{b\coth(x)}{a(a+b)\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-b)\,\text{c}}{}$$

Mathematica [A]
time = 0.30, size = 120, normalized size = 1.36

$$\frac{\text{sech}^3(x)\left(\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)(a+2b+a\cosh(2x))^{3/2}}{a^{3/2}} - \frac{(a+2b+a\cosh(2x))\left(a(a+2b+a\cosh(2x))\text{csch}(x)+2b^2\sinh(x)\right)}{a(a+b)^2}\right)}{4\left(a+b\text{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Sech[x]^2)^(3/2),x]
```

```
[Out] (Sech[x]^3*((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cos
h[2*x]]]*(a + 2*b + a*Cosh[2*x])^(3/2))/a^(3/2) - ((a + 2*b + a*Cosh[2*x])*
```

```
(a*(a + 2*b + a*Cosh[2*x])*Csch[x] + 2*b^2*Sinh[x]))/(a*(a + b)^2)))/(4*(a
+ b*Sech[x]^2)^(3/2))
```

Maple [**F**]
time = 1.62, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{(a+b\operatorname{sech}(x)^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sech(x)^2)^(3/2),x)
[Out] int(coth(x)^2/(a+b*sech(x)^2)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(coth(x)^2/(b*sech(x)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1724 vs. 2(78) = 156.
time = 0.52, size = 3941, normalized size = 44.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="fricas")
[Out] [1/4*(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)*sinh(x)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^6 + (a^3 + 6*a^2*b + 9*a*b^2
+ 4*b^3)*cosh(x)^4 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 + 15*(a^3 + 2*a^2*b +
 a*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^
3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 - a^3 - 2*a^2*b - a*b^2 -
 (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2 + (15*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^4 - a^3 - 6*a^2*b - 9*a*b^2 - 4*b^3 + 6*(a^3 + 6*a^2*b + 9*a*b^2 +
4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 2*(a
^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^
3)*cosh(x))*sinh(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)
^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 -
a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*s
inh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3
```

```
+ 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*c
osh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*
sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*
(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(
x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*si
nh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*
cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*co
sh(x)^4 - 18*b^2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x
)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cos
h(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)
) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a
*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^
5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*
sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^3 + 2*a^2*b + a*b^2)*co
sh(x)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^5 + (a^3 + 2*a^2*b + a*
b^2)*sinh(x)^6 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^4 + (a^3 + 6*a^2
*b + 9*a*b^2 + 4*b^3 + 15*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^4 + 4*
(5*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*co
sh(x))*sinh(x)^3 - a^3 - 2*a^2*b - a*b^2 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3
)*cosh(x)^2 + (15*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 - a^3 - 6*a^2*b - 9*a*b
^2 - 4*b^3 + 6*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(
3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 2*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*c
osh(x)^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(
-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 +
 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cos
h(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x)
)*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*((a
^3 + a*b^2)*cosh(x)^4 + 4*(a^3 + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + a*b^2)*s
inh(x)^4 + a^3 + a*b^2 + 2*(a^3 + 2*a^2*b - a*b^2)*cosh(x)^2 + 2*(a^3 + 2*a
^2*b - a*b^2 + 3*(a^3 + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a*b^2)*cosh
(x)^3 + (a^3 + 2*a^2*b - a*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sin
h(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^5 + 2*a
^4*b + a^3*b^2)*cosh(x)^6 + 6*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)*sinh(x)^5 +
 (a^5 + 2*a^4*b + a^3*b^2)*sinh(x)^6 - a^5 - 2*a^4*b - a^3*b^2 + (a^5 + 6*a
^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^4 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^
2*b^3 + 15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 + 2*a
^4*b + a^3*b^2)*cosh(x)^3 + (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)
)*sinh(x)^3 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^2 - (a^5 + 6*
a^4*b + 9*a^3*b^2 + 4*a^2*b^3 - 15*(a^5 + 2*a^4*b + a^3*b^2)*cosh(x)^4 - 6*
(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^5 +
2*a^4*b + a^3*b^2)*cosh(x)^5 + 2*(a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*co
sh(x)^3 - (a^5 + 6*a^4*b + 9*a^3*b^2 + 4*a^2*b^3)*cosh(x))*sinh(x)), -1/2*(
((a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh
(x)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^6 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^
```

```
3)*cosh(x)^4 + (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 + 15*(a^3 + 2*a^2*b + a*b^2
)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + (a^3 + 6*
a^2*b + 9*a*b^2 + 4*b^3)*cosh(x))*sinh(x)^3 - a^3 - 2*a^2*b - a*b^2 - (a^3
+ 6*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^2 + (15*(a^3 + 2*a^2*b + a*b^2)*cosh(x
)^4 - a^3 - 6*a^2*b - 9*a*b^2 - 4*b^3 + 6*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)
*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 2*(a^3 + 6
*a^2*b + 9*a*b^2 + 4*b^3)*cosh(x)^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos
h(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) +
 b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\left(a + b\operatorname{sech}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sech(x)**2)**(3/2),x)
```

```
[Out] Integral(coth(x)**2/(a + b*sech(x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^2}{\left(a + \frac{b}{\cosh(x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b/cosh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)^2/(a + b/cosh(x)^2)^(3/2), x)
```

3.211 $\int \frac{\tanh^6(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{b^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}}-\frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}}-\frac{(}{\sqrt{a}}$$

[Out] -arctan(b^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/b^(5/2)+arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(5/2)-(1/a^2-1/b^2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2)-1/3*(a+b)*tanh(x)^3/a/b/(a+b-b*tanh(x)^2)^(3/2)

Rubi [A]
time = 0.23, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {4226, 2000, 481, 592, 537, 223, 209, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{b^{5/2}}-\frac{(a+b)\tanh^3(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^6/(a + b*Sech[x]^2)^(5/2),x]

[Out] -(ArcTan[(Sqrt[b]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/b^(5/2)) + ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - ((a + b)*Tanh[x]^3)/(3*a*b*(a + b - b*Tanh[x]^2)^(3/2)) - ((a^(-2) - b^(-2))*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^6(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^6}{(1-x^2)\left(a+b\left(1-x^2\right)\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^6}{(1-x^2)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)-3ax^2\right)}{(1-x^2)(a+b-bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3ab}$$

$$= -\frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} - \frac{\text{Subst}\left(\int \frac{3(a^2-b^2)-\cdots}{(1-x^2)\sqrt{a+\cdots}}\right.}{3\cdots}$$

$$= -\frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+\cdots}}\right.}{\cdots}$$

$$= -\frac{(a+b)\tanh^3(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \cdots\right.}{\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{b^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{\cdots}{3ab\,(\cdots}$$

Mathematica [A]

time = 0.49, size = 178, normalized size = 1.51

$$\frac{\text{sech}^5(x)\left(\frac{\sqrt{2}\left(-a^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)+b^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)\right)(a+2b+a\cosh(2x))^{5/2}}{a^{5/2}b^{5/2}}+\frac{2(a+b)(a+2b+a\cosh(2x))(3a^2+4ab-6b^2+a(3a-4b)\cosh(2x))\sinh(x)}{3a^2b^2}\right)}{8\left(a+b\text{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^6/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (Sech[x]^5*((Sqrt[2]*(-(a^(5/2)*ArcTan[(Sqrt[2]*Sqrt[b]*Sinh[x])/Sqrt[a + 2
*b + a*Cosh[2*x]]]) + b^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*
b + a*Cosh[2*x]]])*(a + 2*b + a*Cosh[2*x])^(5/2))/(a^(5/2)*b^(5/2)) + (2*(a
 + b)*(a + 2*b + a*Cosh[2*x])*(3*a^2 + 4*a*b - 6*b^2 + a*(3*a - 4*b)*Cosh[2
*x])*Sinh[x])/(3*a^2*b^2)))/(8*(a + b*Sech[x]^2)^(5/2))
```

Maple [F]
time = 1.57, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{6}(x)}{\left(a+b\,\mathrm{sech}\,(x)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^6/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2666 vs. 2(100) = 200.
time = 0.87, size = 11939, normalized size = 101.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^3*cosh(x)^8 + 8*a^2*b^3*cosh(x)*sinh(x)^7 + a^2*b^3*sinh(x)
^8 + 4*(a^2*b^3 + 2*a*b^4)*cosh(x)^6 + 4*(7*a^2*b^3*cosh(x)^2 + a^2*b^3 + 2
*a*b^4)*sinh(x)^6 + 8*(7*a^2*b^3*cosh(x)^3 + 3*(a^2*b^3 + 2*a*b^4)*cosh(x))
*sinh(x)^5 + a^2*b^3 + 2*(3*a^2*b^3 + 8*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(35*a^
2*b^3*cosh(x)^4 + 3*a^2*b^3 + 8*a*b^4 + 8*b^5 + 30*(a^2*b^3 + 2*a*b^4)*cosh
(x)^2)*sinh(x)^4 + 8*(7*a^2*b^3*cosh(x)^5 + 10*(a^2*b^3 + 2*a*b^4)*cosh(x)^
```

```
3 + (3*a^2*b^3 + 8*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^3 + 4*(a^2*b^3 + 2*a*b^4
)*cosh(x)^2 + 4*(7*a^2*b^3*cosh(x)^6 + a^2*b^3 + 2*a*b^4 + 15*(a^2*b^3 + 2*
a*b^4)*cosh(x)^4 + 3*(3*a^2*b^3 + 8*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^2 + 8
*(a^2*b^3*cosh(x)^7 + 3*(a^2*b^3 + 2*a*b^4)*cosh(x)^5 + (3*a^2*b^3 + 8*a*b^
4 + 8*b^5)*cosh(x)^3 + (a^2*b^3 + 2*a*b^4)*cosh(x))*sinh(x))*sqrt(a)*log((a
*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a*b^2 - b
^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*(14*a*b^
2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2
)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a*b^2 - b
^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)*cosh(x)
^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 + 3*a^2*b)
*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a^3 + 3*a
^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh
(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^
2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^
3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 -
4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 + 4*a*
b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(co
sh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 - 3*(a*b^2
 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*a^2*b)*c
osh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2
+ 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + s
inh(x)^6)) - 6*(a^5*cosh(x)^8 + 8*a^5*cosh(x)*sinh(x)^7 + a^5*sinh(x)^8 + 4
*(a^5 + 2*a^4*b)*cosh(x)^6 + 4*(7*a^5*cosh(x)^2 + a^5 + 2*a^4*b)*sinh(x)^6
+ 8*(7*a^5*cosh(x)^3 + 3*(a^5 + 2*a^4*b)*cosh(x))*sinh(x)^5 + a^5 + 2*(3*a^
5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^4 + 2*(35*a^5*cosh(x)^4 + 3*a^5 + 8*a^4*b
+ 8*a^3*b^2 + 30*(a^5 + 2*a^4*b)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^5*cosh(x)^5
+ 10*(a^5 + 2*a^4*b)*cosh(x)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x))*sin
h(x)^3 + 4*(a^5 + 2*a^4*b)*cosh(x)^2 + 4*(7*a^5*cosh(x)^6 + a^5 + 2*a^4*b +
 15*(a^5 + 2*a^4*b)*cosh(x)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^2)*
sinh(x)^2 + 8*(a^5*cosh(x)^7 + 3*(a^5 + 2*a^4*b)*cosh(x)^5 + (3*a^5 + 8*a^4
*b + 8*a^3*b^2)*cosh(x)^3 + (a^5 + 2*a^4*b)*cosh(x))*sinh(x))*sqrt(-b)*log(
-((a - b)*cosh(x)^4 + 4*(a - b)*cosh(x)*sinh(x)^3 + (a - b)*sinh(x)^4 + 2*(
a + 3*b)*cosh(x)^2 + 2*(3*(a - b)*cosh(x)^2 + a + 3*b)*sinh(x)^2 - 2*sqrt(2
)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt((a*cosh(x)^
2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*((a - b)*cosh(x)^3 + (a + 3*b)*cosh(x))*sinh(x) + a - b)/(cosh(x)^4 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 +
 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 3*(a^2*b^3*cosh(x)^8 + 8*a^2*b^3*c
osh(x)*sinh(x)^7 + a^2*b^3*sinh(x)^8 + 4*(a^2*b^3 + 2*a*b^4)*cosh(x)^6 + 4*
(7*a^2*b^3*cosh(x)^2 + a^2*b^3 + 2*a*b^4)*sinh(x)^6 + 8*(7*a^2*b^3*cosh(x)^
3 + 3*(a^2*b^3 + 2*a*b^4)*cosh(x))*sinh(x)^5 + a^2*b^3 + 2*(3*a^2*b^3 + 8*a
*b^4 + 8*b^5)*cosh(x)^4 + 2*(35*a^2*b^3*cosh(x)^4 + 3*a^2*b^3 + 8*a*b^4 + 8
*b^5 + 30*(a^2*b^3 + 2*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^2*b^3*cosh(x)^5
 + 10*(a^2*b^3 + 2*a*b^4)*cosh(x)^3 + (3*a^2*b^3 + 8*a*b^4 + 8*b^5)*cosh(x)
```

```
)*sinh(x)^3 + 4*(a^2*b^3 + 2*a*b^4)*cosh(x)^2 + 4*(7*a^2*b^3*cosh(x)^6 + a^
2*b^3 + 2*a*b^4 + 15*(a^2*b^3 + 2*a*b^4)*cosh(x)^4 + 3*(3*a^2*b^3 + 8*a*b^4
 + 8*b^5)*cosh(x)^2)*sinh(x)^2 + 8*(a^2*b^3*cosh(x)^7 + 3*(a^2*b^3 + 2*a*b^
4)*cosh(x)^5 + (3*a^2*b^3 + 8*a*b^4 + 8*b^5)*cosh(x)^3 + (a^2*b^3 + 2*a*b^4
)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*s
inh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt
(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)
^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2)) + 4*sqrt(2)*((3*a^4*b - a^3*b^2 - 4*a^2*b^3)*cosh(x)^6 + 6
*(3*a^4*b - a^3*b^2 - 4*a^2*b^3)*cosh(x)*sinh(x)^5 + (3*a^4*b - a^3*b^2 - 4
*a^2*b^3)*sinh(x)^6 - 3*a^4*b + a^3*b^2 + 4*a^2*b^3 + 3*(a^4*b + 5*a^3*b^2
- 4*a*b^4)*cosh(x)^4 + 3*(a^4*b + 5*a^3*b^2 - 4*a*b^4 + 5*(3*a^4*b - a^3*b^
2 - 4*a^2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(3*a^4*b - a^3*b^2 - 4*a^2*b^3)*
cosh(x)^3 + 3*(a^4*b + 5*a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x)^3 - 3*(a^4*b +
 5*a^3*b^2 - 4*a*b^4)*cosh(x)^2 - 3*(a^4*b + 5*...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^6(x)}{(a + b\operatorname{sech}^2(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**6/(a+b*sech(x)**2)**(5/2),x)
[Out] Integral(tanh(x)**6/(a + b*sech(x)**2)**(5/2), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^6}{\left(a+\frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a + b/cosh(x)^2)^(5/2),x)
[Out] int(tanh(x)^6/(a + b/cosh(x)^2)^(5/2), x)
```

3.212 $\int \frac{\tanh^5(x)}{\left(a+b\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{(a+b)^2}{3ab^2\left(a+b\text{sech}^2(x)\right)^{3/2}}-\frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\text{sech}^2(x)}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(5/2)-1/3*(a+b)^2/a/b^2/(a+b*sech(
x)^2)^(3/2)+(-1/a^2+1/b^2)/(a+b*sech(x)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4224, 457, 89, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\text{sech}^2(x)}}-\frac{(a+b)^2}{3ab^2\left(a+b\text{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - (a + b)^2/(3*a*b^2*(a + b*
Sech[x]^2)^(3/2)) - (a^(-2) - b^(-2))/Sqrt[a + b*Sech[x]^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tanh^5(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x\left(a+bx^2\right)^{5/2}}\,dx, x, \operatorname{sech}(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^2}{x(a+bx)^{5/2}}\,dx, x, \operatorname{sech}^2(x)\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{(a+b)^2}{ab(a+bx)^{5/2}} + \frac{a^2-b^2}{a^2b(a+bx)^{3/2}} + \frac{1}{a^2x\sqrt{a+bx}}\right) dx, x, \operatorname{sech}^2\right.\right.$$

$$= -\frac{(a+b)^2}{3ab^2\left(a+b\operatorname{sech}^2(x)\right)^{3/2}} - \frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sech}^2\right.}{2a^2}$$

$$= -\frac{(a+b)^2}{3ab^2\left(a+b\operatorname{sech}^2(x)\right)^{3/2}} - \frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{se}}\right.}{a^2b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{(a+b)^2}{3ab^2\left(a+b\operatorname{sech}^2(x)\right)^{3/2}} - \frac{\frac{1}{a^2}-\frac{1}{b^2}}{\sqrt{a+b\operatorname{sech}^2(x)}}$$

Mathematica [A]
time = 0.37, size = 126, normalized size = 1.66

$$\frac{\left(\frac{4(a+b)\cosh(x)(a+2b+a\cosh(2x))(a^2+ab-3b^2+a(a-2b)\cosh(2x))}{3a^2b^2}+\frac{\sqrt{2}\,(a+2b+a\cosh(2x))^{5/2}\log\left(\sqrt{2}\,\sqrt{a}\,\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)}{a^{5/2}}\right)\mathrm{sech}^5(x)}{8\left(a+b\mathrm{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (((4*(a + b)*Cosh[x]*(a + 2*b + a*Cosh[2*x])*(a^2 + a*b - 3*b^2 + a*(a - 2*
b)*Cosh[2*x]))/(3*a^2*b^2) + (Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(5/2)*Log[Sqr
t[2]*Sqrt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])/a^(5/2))*Sech[x]^5)/(8
*(a + b*Sech[x]^2)^(5/2))
```

Maple [F]
time = 1.53, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\left(a+b\,\mathrm{sech}(x)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^5/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2224 vs. 2(64) = 128.
time = 0.63, size = 5184, normalized size = 68.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^2*cosh(x)^8 + 8*a^2*b^2*cosh(x)*sinh(x)^7 + a^2*b^2*sinh(x)
^8 + 4*(a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(7*a^2*b^2*cosh(x)^2 + a^2*b^2 + 2
*a*b^3)*sinh(x)^6 + 8*(7*a^2*b^2*cosh(x)^3 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x))
*sinh(x)^5 + 2*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(35*a^2*b^2*cosh
(x)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 30*(a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sin
h(x)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(x)^5 + 10*(a^2*b^2 + 2*a*b^3)*cosh(x)^
3 + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^2*b^2 + 2*a*b^3
)*cosh(x)^2 + 4*(7*a^2*b^2*cosh(x)^6 + 15*(a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a
^2*b^2 + 2*a*b^3 + 3*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8
*(a^2*b^2*cosh(x)^7 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^2*b^2 + 8*a*b^
3 + 8*b^4)*cosh(x)^3 + (a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(((
a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x
)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^
3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a^3 + 2*a^2*b + a*b
^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(2*a
^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*
b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 6*a^3 + 14*a^2*b +
 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(1
4*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*
cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(2*a^
3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(2*a^
3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b + 3*(6*a^3 + 14*a^
2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a*b + b^2)*cosh(x)^
6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*b + b^2)*sinh(x)^6
 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a
^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 +
 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2
*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*s
inh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)
*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 +
a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cos
h(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x)
)*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*c
osh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)
^6)) + 3*(a^2*b^2*cosh(x)^8 + 8*a^2*b^2*cosh(x)*sinh(x)^7 + a^2*b^2*sinh(x)
^8 + 4*(a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(7*a^2*b^2*cosh(x)^2 + a^2*b^2 + 2
*a*b^3)*sinh(x)^6 + 8*(7*a^2*b^2*cosh(x)^3 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x))
*sinh(x)^5 + 2*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(35*a^2*b^2*cosh
(x)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 30*(a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sin
h(x)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(x)^5 + 10*(a^2*b^2 + 2*a*b^3)*cosh(x)^
3 + (3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^2*b^2 + 2*a*b^3
)*cosh(x)^2 + 4*(7*a^2*b^2*cosh(x)^6 + 15*(a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a
^2*b^2 + 2*a*b^3 + 3*(3*a^2*b^2 + 8*a*b^3 + 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8
*(a^2*b^2*cosh(x)^7 + 3*(a^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^2*b^2 + 8*a*b^
```

```
3 + 8*b^4)*cosh(x)^3 + (a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(a)*log(-(
a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*
cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 8*sqrt(2)*((a^4 - a^3*b - 2*a^2
*b^2)*cosh(x)^6 + 6*(a^4 - a^3*b - 2*a^2*b^2)*cosh(x)*sinh(x)^5 + (a^4 - a^
3*b - 2*a^2*b^2)*sinh(x)^6 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^
4 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + 5*(a^4 - a^3*b - 2*a^2*b^2)*cosh
(x)^2)*sinh(x)^4 + a^4 - a^3*b - 2*a^2*b^2 + 4*(5*(a^4 - a^3*b - 2*a^2*b^2)
*cosh(x)^3 + 3*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x))*sinh(x)^3 + 3*(
a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^2 + 3*(5*(a^4 - a^3*b - 2*a^2*b^
2)*cosh(x)^4 + a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3 + 6*(a^4 + a^3*b - 2*a^2*b
^2 - 2*a*b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^4 - a^3*b - 2*a^2*b^2)*cosh(x)^5
 + 2*(a^4 + a^3*b - 2*a^2*b^2 - 2*a*b^3)*cosh(x)^3 + (a^4 + a^3*b - 2*a^2*b
^2 - 2*a*b^3)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5*b^2*cosh(x)^8 + 8*a^5*b^
2*cosh(x)*sinh(x)^7 + a^5*b^2*sinh(x)^8 + a^5*b^2 + 4*(a^5*b^2 + 2*a^4*b^3)
*cosh(x)^6 + 4*(7*a^5*b^2*cosh(x)^2 + a^5*b^2 + 2*a^4*b^3)*sinh(x)^6 + 8*(7
*a^5*b^2*cosh(x)^3 + 3*(a^5*b^2 + 2*a^4*b^3)*cosh(x))*sinh(x)^5 + 2*(3*a^5*
b^2 + 8*a^4*b^3 + 8*a^3*b^4)*cosh(x)^4 + 2*(35*a^5*b^2*cosh(x)^4 + 3*a^5*b^
2 + 8*a^4*b^3 + 8*a^3*b^4 + 30*(a^5*b^2 + 2*a^4...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{(a + b\,\mathrm{sech}^2(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**5/(a + b*sech(x)**2)**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
itution variable should perhaps be purged.Warning, replacing 0 by ‘ u‘, a s
ubstit
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^5}{\left(a+\frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + b/cosh(x)^2)^(5/2),x)
[Out] int(tanh(x)^5/(a + b/cosh(x)^2)^(5/2), x)
```

3.213 $\int \frac{\tanh^4(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=90

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}}-\frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}}+\frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(5/2)+1/3*(a-3*b)*tanh(x
)/a^2/b/(a+b-b*tanh(x)^2)^(1/2)-1/3*(a+b)*tanh(x)/a/b/(a+b-b*tanh(x)^2)^(3/
2)
```

Rubi [A]
time = 0.17, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4226, 2000, 481, 541, 12, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}+\frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a-b\tanh^2(x)+b}}-\frac{(a+b)\tanh(x)}{3ab\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - ((a + b)*Tan
h[x])/(3*a*b*(a + b - b*Tanh[x]^2)^(3/2)) + ((a - 3*b)*Tanh[x])/(3*a^2*b*Sq
rt[a + b - b*Tanh[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^4(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a+b\left(1-x^2\right)\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b+(-a+2b)x^2}{(1-x^2)(a+b-bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3ab}$$

$$= -\frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}} - \frac{\text{Subst}\left(\int -\frac{}{(1-x^2)}\right.}{}$$

$$= -\frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{}{(1-x^2)\sqrt{}}\right.}{}$$

$$= -\frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{(a-3b)\tanh(x)}{3a^2b\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,d\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{(a+b)\tanh(x)}{3ab\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{(a-3}{3a^2b\sqrt{a+}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 290 vs. $2(90) = 180$.

time = 1.38, size = 290, normalized size = 3.22

$$\frac{\text{sech}^4(x)\left(\frac{\sqrt{2}\,(a+2b+a\cosh(2x))^{5/2}\text{csch}(x)\text{sech}(x)\left(\frac{\sinh^2(x)}{a+b}+\frac{12\sinh^4(x)}{a+b}+\frac{2\sinh^2(x)\left(a+b+a\sinh^2(x)\right)}{(a+b)^2}-\frac{16\left(a+b+a\sinh^2(x)\right)\left(1+\frac{a\sinh^2(x)}{a+b}\right)\left(\frac{a^2(a+b)\sinh^4(x)}{\left(a+b+a\sinh^2(x)\right)^2}+\frac{3a(a+b)\sinh^2(x)}{a+b+a\sinh^2(x)}-\frac{3\sqrt{a}\sqrt{a+b}\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)\sinh(x)}{\sqrt{\frac{a+b+a\sinh^2(x)}{a+b}}}\right)}{a^3}\right)}{\left(a+b+a\sinh^2(x)\right)^{3/2}}+\frac{8(a+2b+a\cosh(2x))(2a+3b+a\cosh(2x))\tanh(x)}{(a+b)^2}-\frac{12(a+2b+a\cosh(2x))(b+(3a+2b)\cosh(2x))\tanh(x)}{(a+b)^2}\right)}{384\left(a+b\text{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (Sech[x]^4*((Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(5/2)*Csch[x]*Sech[x]*(Sinh[x]
^2/(a + b) + (12*Sinh[x]^4)/(a + b) + (2*Sinh[x]^2*(a + b + a*Sinh[x]^2))/(
a + b)^2 - (16*(a + b + a*Sinh[x]^2)*(1 + (a*Sinh[x]^2)/(a + b))*((a^2*(a +
 b)*Sinh[x]^4)/(a + b + a*Sinh[x]^2)^2 + (3*a*(a + b)*Sinh[x]^2)/(a + b + a
*Sinh[x]^2) - (3*Sqrt[a]*Sqrt[a + b]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]
*Sinh[x])/Sqrt[(a + b + a*Sinh[x]^2)/(a + b)]))/a^3))/(a + b + a*Sinh[x]^2)
^(3/2) + (8*(a + 2*b + a*Cosh[2*x])*(2*a + 3*b + a*Cosh[2*x])*Tanh[x])/(a +
 b)^2 - (12*(a + 2*b + a*Cosh[2*x])*(b + (3*a + 2*b)*Cosh[2*x])*Tanh[x])/(a
 + b)^2))/(384*(a + b*Sech[x]^2)^(5/2))
```

Maple [F]
time = 1.55, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\left(a+b\,\mathrm{sech}(x)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^4/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1500 vs. $2(76) = 152$.
time = 0.63, size = 3559, normalized size = 39.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*(a^2
+ 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^6 + 8*(7*a^2
*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)
*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cosh(x)^2 + 3*a^2 + 8*a
```

*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 + 2*a*b)*cosh(x)^3 + (
3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*a*b)*cosh(x)^2 + 4*(
7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*co
sh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 + 3*(a^2 + 2*a*b)
*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 + 2*a*b)*cosh(x))*sin
h(x))*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh
(x)^8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*si
nh(x)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3
 + 4*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a
b^2 - 30(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(
a*b^2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3
 + 2*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*c
osh(x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2
 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2
cosh(x)^4 + 3(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b
^2*cosh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^
2*cosh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cos
h(x)^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh
(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*
cosh(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3
 + (a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*
cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*c
osh(x)*sinh(x)^5 + sinh(x)^6)) + 3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7
 + a^2*sinh(x)^8 + 4*(a^2 + 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2
*a*b)*sinh(x)^6 + 8*(7*a^2*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 +
 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*
b)*cosh(x)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(
a^2 + 2*a*b)*cosh(x)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^
2 + 2*a*b)*cosh(x)^2 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*
(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*c
osh(x)^7 + 3*(a^2 + 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 +
(a^2 + 2*a*b)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sin
h(x)^3 + a*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh
(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqr
t((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) + 4*(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2)) - 16*sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*si
nh(x)^5 + a^2*sinh(x)^6 + 3*a*b*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a*b)*sinh(
x)^4 - 3*a*b*cosh(x)^2 + 4*(5*a^2*cosh(x)^3 + 3*a*b*cosh(x))*sinh(x)^3 + 3*
(5*a^2*cosh(x)^4 + 6*a*b*cosh(x)^2 - a*b)*sinh(x)^2 - a^2 + 6*(a^2*cosh(x)^
5 + 2*a*b*cosh(x)^3 - a*b*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2
 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5*cosh(x)^8 +
8*a^5*cosh(x)*sinh(x)^7 + a^5*sinh(x)^8 + 4*(a^5 + 2*a^4*b)*cosh(x)^6 + 4*(
7*a^5*cosh(x)^2 + a^5 + 2*a^4*b)*sinh(x)^6 + 8*(7*a^5*cosh(x)^3 + 3*(a^5 +
2*a^4*b)*cosh(x))*sinh(x)^5 + a^5 + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)

```
^4 + 2*(35*a^5*cosh(x)^4 + 3*a^5 + 8*a^4*b + 8*a^3*b^2 + 30*(a^5 + 2*a^4*b)
*cosh(x)^2)*sinh(x)^4 + 8*(7*a^5*cosh(x)^5 + 10*(a^5 + 2*a^4*b)*cosh(x)^3 +
 (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^5 + 2*a^4*b)*cosh(
x)^2 + 4*(7*a^5*cosh(x)^6 + a^5 + 2*a^4*b + 15*(a^5 + 2*a^4*b)*cosh(x)^4 +
3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 8*(a^5*cosh(x)^7 + 3
*(a^5 + 2*a^4*b)*cosh(x)^5 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^3 + (a^5
 + 2*a^4*b)*cosh(x))*sinh(x)), -1/6*(3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(
x)^7 + a^2*sinh(x)^8 + 4*(a^2 + 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2
 + 2*a*b)*sinh(x)^6 + 8*(7*a^2*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)
^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 +
2*a*b)*cosh(x)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 +
10*(a^2 + 2*a*b)*cosh(x)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4
*(a^2 + 2*a*b)*cosh(x)^2 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4
+ 3*(3*a^2 + 8*a*b + 8*b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a
^2*cosh(x)^7 + 3*(a^2 + 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^
3 + (a^2 + 2*a*b)*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(b*cosh(x)^2 +
2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + a)*sqrt(-a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\left(a + b\operatorname{sech}^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*sech(x)**2)**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
itution variable should perhaps be purged.Warning, replacing 0 by ‘ u‘, a s
ubstit
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\tanh\left(x\right)}^4}{{\left(a+\frac{b}{{\cosh\left(x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b/cosh(x)^2)^(5/2),x)
[Out] int(tanh(x)^4/(a + b/cosh(x)^2)^(5/2), x)
```

3.214 $\int \frac{\tanh^3(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=68

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{a+b}{3ab\left(a+b\text{sech}^2(x)\right)^{3/2}}-\frac{1}{a^2\sqrt{a+b\text{sech}^2(x)}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(5/2)+1/3*(-a-b)/a/b/(a+b*sech(x)^
2)^(3/2)-1/a^2/(a+b*sech(x)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4224, 457, 79, 53, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{1}{a^2\sqrt{a+b\text{sech}^2(x)}}-\frac{a+b}{3ab\left(a+b\text{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - (a + b)/(3*a*b*(a + b*Sech
[x]^2)^(3/2)) - 1/(a^2*Sqrt[a + b*Sech[x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tanh^3(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{-1+x^2}{x\left(a+bx^2\right)^{5/2}}\,dx, x, \text{sech}(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{x(a+bx)^{5/2}}\,dx, x, \text{sech}^2(x)\right)$$
$$= -\frac{a+b}{3ab\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \text{sech}^2(x)\right)}{2a}$$
$$= -\frac{a+b}{3ab\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{1}{a^2\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{sech}\right.}{2a^2}$$
$$= -\frac{a+b}{3ab\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{1}{a^2\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bs}\right.}{a^2b}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{a+b}{3ab\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{1}{a^2\sqrt{a+b\text{sech}^2(x)}}$$

Mathematica [A]
time = 0.62, size = 124, normalized size = 1.82

$$-\frac{(a+2b+a\cosh(2x))\left(3\sqrt{a}\,(a+2b)^2\cosh(x)+a^{3/2}(a+4b)\cosh(3x)-3\sqrt{2}\,b(a+2b+a\cosh(2x))^{3/2}\log\left(\sqrt{2}\,\sqrt{a}\,\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)\right)\text{sech}^5(x)}{24a^{5/2}b\left(a+b\text{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] -1/24*((a + 2*b + a*Cosh[2*x])*(3*Sqrt[a]*(a + 2*b)^2*Cosh[x] + a^(3/2)*(a
 + 4*b)*Cosh[3*x] - 3*Sqrt[2]*b*(a + 2*b + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sq
rt[a]*Cosh[x] + Sqrt[a + 2*b + a*Cosh[2*x]]])*Sech[x]^5)/(a^(5/2)*b*(a + b*
Sech[x]^2)^(5/2))
```

Maple [F]
time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\left(a+b\,\text{sech}(x)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^3/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1954 vs. $2(56) = 112$.
time = 0.60, size = 4644, normalized size = 68.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b*cosh(x)^8 + 8*a^2*b*cosh(x)*sinh(x)^7 + a^2*b*sinh(x)^8 + 4
*(a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(7*a^2*b*cosh(x)^2 + a^2*b + 2*a*b^2)*sinh
(x)^6 + 8*(7*a^2*b*cosh(x)^3 + 3*(a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(
3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*a^2*b*cosh(x)^4 + 3*a^2*b + 8*
a*b^2 + 8*b^3 + 30*(a^2*b + 2*a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^2*b*cosh
(x)^5 + 10*(a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x
))*sinh(x)^3 + a^2*b + 4*(a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*a^2*b*cosh(x)^6
 + 15*(a^2*b + 2*a*b^2)*cosh(x)^4 + a^2*b + 2*a*b^2 + 3*(3*a^2*b + 8*a*b^2
+ 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*(a^2*b*cosh(x)^7 + 3*(a^2*b + 2*a*b^2)*co
sh(x)^5 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^3 + (a^2*b + 2*a*b^2)*cosh(x)
)*sinh(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*
b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3
 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3
+ 14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b +
a*b^2)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 +
 (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(
x)^4 + 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*co
sh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 +
5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*
sinh(x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*
b^2)*cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3
*a^2*b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^
2 + 2*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2
```

+ 2*a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 3*(a^2*b*cosh(x)^8 + 8*a^2*b*cosh(x)*sinh(x)^7 + a^2*b*sinh(x)^8 + 4*(a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(7*a^2*b*cosh(x)^2 + a^2*b + 2*a*b^2)*sinh(x)^6 + 8*(7*a^2*b*cosh(x)^3 + 3*(a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*a^2*b*cosh(x)^4 + 3*a^2*b + 8*a*b^2 + 8*b^3 + 30*(a^2*b + 2*a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^2*b*cosh(x)^5 + 10*(a^2*b + 2*a*b^2)*cosh(x)^3 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 4*(a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*a^2*b*cosh(x)^6 + 15*(a^2*b + 2*a*b^2)*cosh(x)^4 + a^2*b + 2*a*b^2 + 3*(3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*(a^2*b*cosh(x)^7 + 3*(a^2*b + 2*a*b^2)*cosh(x)^5 + (3*a^2*b + 8*a*b^2 + 8*b^3)*cosh(x)^3 + (a^2*b + 2*a*b^2)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*((a^3 + 4*a^2*b)*cosh(x)^6 + 6*(a^3 + 4*a^2*b)*cosh(x)*sinh(x)^5 + (a^3 + 4*a^2*b)*sinh(x)^6 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^4 + 3*(a^3 + 4*a^2*b + 4*a*b^2 + 5*(a^3 + 4*a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + 4*a^2*b)*cosh(x)^3 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 4*a^2*b + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^2 + 3*(5*(a^3 + 4*a^2*b)*cosh(x)^4 + a^3 + 4*a^2*b + 4*a*b^2 + 6*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^3 + 4*a^2*b)*cosh(x)^5 + 2*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^3 + (a^3 + 4*a^2*b + 4*a*b^2)*cosh(x))*sinh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5*b*cosh(x)^8 + 8*a^5*b*cosh(x)*sinh(x)^7 + a^5*b*sinh(x)^8 + 4*(a^5*b + 2*a^4*b^2)*cosh(x)^6 + 4*(7*a^5*b*cosh(x)^2 + a^5*b + 2*a^4*b^2)*sinh(x)^6 + a^5*b + 8*(7*a^5*b*cosh(x)^3 + 3*(a^5*b + 2*a^4*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*cosh(x)^4 + 2*(35*a^5*b*cosh(x)^4 + 3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3 + 30*(a^5*b + 2*a^4*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^5*b*cosh(x)^5 + 10*(a^5*b + 2*a^4*b^2)*cosh(x)^3 + (3*a^5*b + 8*a^4*b^2 + 8*a^3*b^3)*cosh(x))*sinh(x)^3 + 4*(a^5*b + 2*a^4*b^2)*cosh(x)^2 + 4*(7*a^5*b*cosh(x)^6 + a^5*b + 2*a^4*b^2 + 15*(a^5*b + 2*a^4*b^2)*cosh(x)^4 + 3*(3*a^5*b + 8*a^4*b^2 + 8*a^3*b...

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{(a + b\,\mathrm{sech}^2(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*sech(x)**2)**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
itution variable should perhaps be purged.Warning, replacing 0 by ‘ u‘, a s
ubstit
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^3}{\left(a + \frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b/cosh(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^3/(a + b/cosh(x)^2)^(5/2), x)
```

3.215 $\int \frac{\tanh^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}} \, dx$

Optimal. Leaf size=88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}}-\frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}}-\frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a+b-b\tanh^2(x)}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(5/2)-1/3*(2*a+3*b)*tanh
(x)/a^2/(a+b)/(a+b-b*tanh(x)^2)^(1/2)-1/3*tanh(x)/a/(a+b-b*tanh(x)^2)^(3/2)
```

Rubi [A]

time = 0.17, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4226, 2000, 482, 541, 12, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a-b\tanh^2(x)+b}}-\frac{\tanh(x)}{3a\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - Tanh[x]/(3*a
*(a + b - b*Tanh[x]^2)^(3/2)) - ((2*a + 3*b)*Tanh[x])/(3*a^2*(a + b)*Sqrt[a
 + b - b*Tanh[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\tanh^2(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^2}{(1-x^2)\left(a+b\left(1-x^2\right)\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{x^2}{(1-x^2)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1+2x^2}{(1-x^2)(a+b-bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3a}$$

$$= -\frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a+b-b\tanh^2(x)}} - \frac{\text{Subst}\left(\int -\frac{}{(1-}\right.}{}$$

$$= -\frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{}{(1-x^2}\right.}{}$$

$$= -\frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{(2a+3b)\tanh(x)}{3a^2(a+b)\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{1-ax}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{\tanh(x)}{3a\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{(2a+3}{3a^2(a+b)\sqrt{a}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 290 vs. 2(88) = 176.

time = 0.77, size = 290, normalized size = 3.30

$$\frac{\text{sech}^4(x)\left(\frac{\sqrt{2}\,(a+2b+a\cosh(2x))^{5/2}\text{csch}(x)\text{sech}(x)\left(\frac{\sinh^2(x)}{a+b}+\frac{12\sinh^4(x)}{a+b}+\frac{2\sinh^2(x)\left(a+b+a\sinh^2(x)\right)}{(a+b)^2}-\frac{16\left(a+b+a\sinh^2(x)\right)\left(1+\frac{a\sinh^2(x)}{a+b}\right)\left(\frac{a^2(a+b)\sinh^4(x)}{\left(a+b+a\sinh^2(x)\right)^2}+\frac{3a(a+b)\sinh^2(x)}{a+b+a\sinh^2(x)}-\frac{3\sqrt{a}\sqrt{a+b}\sinh^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{a+b}}\right)\sinh(x)}{\sqrt{\frac{a+b+a\sinh^2(x)}{a+b}}}\right)}{a^3}\right)}{\left(a+b+a\sinh^2(x)\right)^{3/2}}-\frac{8(a+2b+a\cosh(2x))(2a+3b+a\cosh(2x))\tanh(x)}{(a+b)^2}+\frac{4(a+2b+a\cosh(2x))(b+(3a+2b)\cosh(2x))\tanh(x)}{(a+b)^2}\right)}{384\left(a+b\text{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (Sech[x]^4*((Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(5/2)*Csch[x]*Sech[x]*(Sinh[x]
^2/(a + b) + (12*Sinh[x]^4)/(a + b) + (2*Sinh[x]^2*(a + b + a*Sinh[x]^2))/(
a + b)^2 - (16*(a + b + a*Sinh[x]^2)*(1 + (a*Sinh[x]^2)/(a + b))*((a^2*(a +
 b)*Sinh[x]^4)/(a + b + a*Sinh[x]^2)^2 + (3*a*(a + b)*Sinh[x]^2)/(a + b + a
*Sinh[x]^2) - (3*Sqrt[a]*Sqrt[a + b]*ArcSinh[(Sqrt[a]*Sinh[x])/Sqrt[a + b]]
*Sinh[x])/Sqrt[(a + b + a*Sinh[x]^2)/(a + b)]))/a^3))/(a + b + a*Sinh[x]^2)
^(3/2) - (8*(a + 2*b + a*Cosh[2*x])*(2*a + 3*b + a*Cosh[2*x])*Tanh[x])/(a +
 b)^2 + (4*(a + 2*b + a*Cosh[2*x])*(b + (3*a + 2*b)*Cosh[2*x])*Tanh[x])/(a
+ b)^2))/(384*(a + b*Sech[x]^2)^(5/2))
```

Maple [F]
time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\left(a + b\,\mathrm{sech}(x)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)^2/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2215 vs. 2(74) = 148.
time = 0.62, size = 4989, normalized size = 56.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^
3 + a^2*b)*sinh(x)^8 + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^6 + 4*(a^3 + 3*a
^2*b + 2*a*b^2 + 7*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^3 + a^2*b)*
cosh(x)^3 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^3 + 11*
```

```
a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*(a^3 + a^2*b)*cosh(x)^4 + 3*a^3
 + 11*a^2*b + 16*a*b^2 + 8*b^3 + 30*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2)*si
nh(x)^4 + 8*(7*(a^3 + a^2*b)*cosh(x)^5 + 10*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(
x)^3 + (3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x))*sinh(x)^3 + a^3 + a^2
*b + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*(a^3 + a^2*b)*cosh(x)^6 +
 15*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^4 + a^3 + 3*a^2*b + 2*a*b^2 + 3*(3*a^
3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^3 + a^2*b)*co
sh(x)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^5 + (3*a^3 + 11*a^2*b + 16*a*
b^2 + 8*b^3)*cosh(x)^3 + (a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x))*sqrt(a
)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 - 2*(a
*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6 + 4*
(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^2*b +
 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*(a
*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 - b^3)
*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a^3 +
3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^4 + a
^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(
b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 +
 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*
sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2
- a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^
2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a +
2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)^7 -
3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3 + 3*
a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*si
nh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(
x)^5 + sinh(x)^6)) + 3*((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*s
inh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^6
+ 4*(a^3 + 3*a^2*b + 2*a*b^2 + 7*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 8*(7*
(a^3 + a^2*b)*cosh(x)^3 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x)^5 +
2*(3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^4 + 2*(35*(a^3 + a^2*b)*cos
h(x)^4 + 3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3 + 30*(a^3 + 3*a^2*b + 2*a*b^2)
*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^3 + a^2*b)*cosh(x)^5 + 10*(a^3 + 3*a^2*b +
2*a*b^2)*cosh(x)^3 + (3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x))*sinh(x)
^3 + a^3 + a^2*b + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^2 + 4*(7*(a^3 + a^2*
b)*cosh(x)^6 + 15*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^4 + a^3 + 3*a^2*b + 2*a
*b^2 + 3*(3*a^3 + 11*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a
^3 + a^2*b)*cosh(x)^7 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(x)^5 + (3*a^3 + 11
*a^2*b + 16*a*b^2 + 8*b^3)*cosh(x)^3 + (a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*s
inh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2
*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(
x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a*cosh(x)
^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2)) - 4*sqrt(2)*((3*a^3 + 4*a^2*b)*cosh(x)^6 + 6*(3*a^3 + 4*a^2*b)*cosh(x)
```

```
*sinh(x)^5 + (3*a^3 + 4*a^2*b)*sinh(x)^6 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh
(x)^4 + 3*(a^3 + 4*a^2*b + 4*a*b^2 + 5*(3*a^3 + 4*a^2*b)*cosh(x)^2)*sinh(x)
^4 + 4*(5*(3*a^3 + 4*a^2*b)*cosh(x)^3 + 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)
)*sinh(x)^3 - 3*a^3 - 4*a^2*b - 3*(a^3 + 4*a^2*b + 4*a*b^2)*cosh(x)^2 + 3*(
5*(3*a^3 + 4*a^2*b)*cosh(x)^4 - a^3 - 4*a^2*b - 4*a*b^2 + 6*(a^3 + 4*a^2*b
+ 4*a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^3 + 4*a^2*b)*cosh(x)^5 + 2*(a^3 +
 4*a^2*b + 4*a*b^2)*cosh(x)^3 - (a^3 + 4*a^2*b + 4*a*b^2)*cosh(x))*sinh(x))
*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)))/((a^6 + a^5*b)*cosh(x)^8 + 8*(a^6 + a^5*b)*cosh(x)*sinh(x)^7
 + (a^6 + a^5*b)*sinh(x)^8 + 4*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^6 + 4*(a
^6 + 3*a^5*b + 2*a^4*b^2 + 7*(a^6 + a^5*b)*cosh(x)^2)*sinh(x)^6 + a^6 + a^5
*b + 8*(7*(a^6 + a^5*b)*cosh(x)^3 + 3*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x))*
sinh(x)^5 + 2*(3*a^6 + 11*a^5*b + 16*a^4*b^2 + 8*a^3*b^3)*cosh(x)^4 + 2*(3*
a^6 + 11*a^5*b + 16*a^4*b^2 + 8*a^3*b^3 + 35*(a^6 + a^5*b)*cosh(x)^4 + 30*(
a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^6 + a^5*b)*cosh(x
)^5 + 10*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x)^3 + (3*a^6 + 11*a^5*b + 16*a^4
*b^2 + 8*a^3*b^3)*cosh(x))*sinh(x)^3 + 4*(a^6 + 3*a^5*b + 2*a^4*b^2)*cosh(x
)^2 + 4*(7*(a^6 + a^5*b)*cosh(x)^6 + a^6 + 3*a^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\left(a + b\operatorname{sech}^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sech(x)**2)**(5/2),x)
[Out] Integral(tanh(x)**2/(a + b*sech(x)**2)**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
itution variable should perhaps be purged.Warning, replacing 0 by ‘ u‘, a s
ubstit
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^2}{\left(a + \frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b/cosh(x)^2)^(5/2),x)
[Out] int(tanh(x)^2/(a + b/cosh(x)^2)^(5/2), x)
```

3.216 $\int \frac{\tanh(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=62

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{1}{3a\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}-\frac{1}{a^2\sqrt{a+b\,\mathrm{sech}^2(x)}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(5/2)-1/3/a/(a+b*sech(x)^2)^(3/2)-
1/a^2/(a+b*sech(x)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4224, 272, 53, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{1}{a^2\sqrt{a+b\,\mathrm{sech}^2(x)}}-\frac{1}{3a\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - 1/(3*a*(a + b*Sech[x]^2)^(
3/2)) - 1/(a^2*Sqrt[a + b*Sech[x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\tanh(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx, x, \operatorname{sech}(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \operatorname{sech}^2(x)\right)\right)$$
$$= -\frac{1}{3a\left(a+b\operatorname{sech}^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \operatorname{sech}^2(x)\right)}{2a}$$
$$= -\frac{1}{3a\left(a+b\operatorname{sech}^2(x)\right)^{3/2}} - \frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \operatorname{sec}\right.}{2a^2}$$
$$= -\frac{1}{3a\left(a+b\operatorname{sech}^2(x)\right)^{3/2}} - \frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b}\right.}{a^2b}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{1}{3a\left(a+b\operatorname{sech}^2(x)\right)^{3/2}} - \frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}}$$

Mathematica [A]
time = 0.47, size = 112, normalized size = 1.81

$$-\frac{(a+2b+a\cosh(2x))\left(12\sqrt{a}\,(a+b)\cosh(x)+4a^{3/2}\cosh(3x)-3\sqrt{2}\,(a+2b+a\cosh(2x))^{3/2}\log\left(\sqrt{2}\,\sqrt{a}\,\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)\right)\operatorname{sech}^5(x)}{24a^{5/2}\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] -1/24*((a + 2*b + a*Cosh[2*x])*(12*Sqrt[a]*(a + b)*Cosh[x] + 4*a^(3/2)*Cosh
[3*x] - 3*Sqrt[2]*(a + 2*b + a*Cosh[2*x])^(3/2)*Log[Sqrt[2]*Sqrt[a]*Cosh[x]
 + Sqrt[a + 2*b + a*Cosh[2*x]]])*Sech[x]^5)/(a^(5/2)*(a + b*Sech[x]^2)^(5/2
))
```

Maple [A]
time = 0.52, size = 67, normalized size = 1.08

method	result	size

derivativedivides	$-\frac{1}{3a\left(a+b\,\text{sech}(x)^2\right)^{\frac{3}{2}}}-\frac{\frac{1}{a\sqrt{a+b\,\text{sech}(x)^2}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\,\text{sech}(x)^2}}{\text{sech}(x)}\right)}{a^{\frac{3}{2}}}}{a}$	67
default	$-\frac{1}{3a\left(a+b\,\text{sech}(x)^2\right)^{\frac{3}{2}}}-\frac{\frac{1}{a\sqrt{a+b\,\text{sech}(x)^2}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\,\text{sech}(x)^2}}{\text{sech}(x)}\right)}{a^{\frac{3}{2}}}}{a}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sech(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/(a+b*sech(x)^2)^(3/2)-1/a*(1/a/(a+b*sech(x)^2)^(1/2)-1/a^(3/2)*ln((2
*a+2*a^(1/2)*(a+b*sech(x)^2)^(1/2))/sech(x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1629 vs. $2(50) = 100$.
time = 0.61, size = 3994, normalized size = 64.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*(a^2
+ 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^6 + 8*(7*a^2
*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)
*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cosh(x)^2 + 3*a^2 + 8*a
*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 + 2*a*b)*cosh(x)^3 + (
```

```
3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*a*b)*cosh(x)^2 + 4*(
7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2 + 8*a*b + 8*b^2)*co
sh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 + 3*(a^2 + 2*a*b)
*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 + 2*a*b)*cosh(x))*sin
h(x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a
*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*
a^2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*
(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2
)*cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a
^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4
+ 6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)
^2)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2
*b + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(
x)^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*
b + 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2
*a*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*
a*b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b
+ b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*
cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh
(x)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^
2 + 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6
*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b
+ 4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^
3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x
)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)
*sinh(x)^5 + sinh(x)^6)) + 3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2
*sinh(x)^8 + 4*(a^2 + 2*a*b)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*
sinh(x)^6 + 8*(7*a^2*cosh(x)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*
a^2 + 8*a*b + 8*b^2)*cosh(x)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*cos
h(x)^2 + 3*a^2 + 8*a*b + 8*b^2)*sinh(x)^4 + 8*(7*a^2*cosh(x)^5 + 10*(a^2 +
2*a*b)*cosh(x)^3 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 + 2*
a*b)*cosh(x)^2 + 4*(7*a^2*cosh(x)^6 + 15*(a^2 + 2*a*b)*cosh(x)^4 + 3*(3*a^2
 + 8*a*b + 8*b^2)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)
^7 + 3*(a^2 + 2*a*b)*cosh(x)^5 + (3*a^2 + 8*a*b + 8*b^2)*cosh(x)^3 + (a^2 +
 2*a*b)*cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3
 + a*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*a*cosh(x)^2 + b)*sinh(x)^2 + sqrt(2)*
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*sqrt((a*cosh(x)^2 +
 a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(a
*cosh(x)^3 + b*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(
x)^2)) - 16*sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^
6 + 3*(a^2 + a*b)*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2 + a*b)*sinh(x)^4 + 4
*(5*a^2*cosh(x)^3 + 3*(a^2 + a*b)*cosh(x))*sinh(x)^3 + 3*(a^2 + a*b)*cosh(x
```

```
)^2 + 3*(5*a^2*cosh(x)^4 + 6*(a^2 + a*b)*cosh(x)^2 + a^2 + a*b)*sinh(x)^2 +
 a^2 + 6*(a^2*cosh(x)^5 + 2*(a^2 + a*b)*cosh(x)^3 + (a^2 + a*b)*cosh(x))*si
nh(x))*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)))/(a^5*cosh(x)^8 + 8*a^5*cosh(x)*sinh(x)^7 + a^5*sinh(x)
^8 + 4*(a^5 + 2*a^4*b)*cosh(x)^6 + 4*(7*a^5*cosh(x)^2 + a^5 + 2*a^4*b)*sinh
(x)^6 + 8*(7*a^5*cosh(x)^3 + 3*(a^5 + 2*a^4*b)*cosh(x))*sinh(x)^5 + a^5 + 2
*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^4 + 2*(35*a^5*cosh(x)^4 + 3*a^5 + 8*
a^4*b + 8*a^3*b^2 + 30*(a^5 + 2*a^4*b)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^5*cosh
(x)^5 + 10*(a^5 + 2*a^4*b)*cosh(x)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x
))*sinh(x)^3 + 4*(a^5 + 2*a^4*b)*cosh(x)^2 + 4*(7*a^5*cosh(x)^6 + a^5 + 2*a
^4*b + 15*(a^5 + 2*a^4*b)*cosh(x)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(
x)^2)*sinh(x)^2 + 8*(a^5*cosh(x)^7 + 3*(a^5 + 2*a^4*b)*cosh(x)^5 + (3*a^5 +
 8*a^4*b + 8*a^3*b^2)*cosh(x)^3 + (a^5 + 2*a^4*b)*cosh(x))*sinh(x)), -1/6*(
3*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*(a^2 + 2*a*b
)*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2 + 2*a*b)*sinh(x)^6 + 8*(7*a^2*cosh(x
)^3 + 3*(a^2 + 2*a*b)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 8*a*b + 8*b^2)*cosh(x
)^4 + 2*(35*a^2*cosh(x)^4 + 30*(a^2 + 2*a*b)*co...
```

Sympy [**A**]
time = 7.70, size = 65, normalized size = 1.05

$$-\frac{1}{3a\left(a+b\operatorname{sech}^2(x)\right)^{\frac{3}{2}}}-\frac{1}{a^2\sqrt{a+b\operatorname{sech}^2(x)}}-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\operatorname{sech}^2(x)}}{\sqrt{-a}}\right)}{a^2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)**2)**(5/2),x)
[Out] -1/(3*a*(a + b*sech(x)**2)**(3/2)) - 1/(a**2*sqrt(a + b*sech(x)**2)) - atan
(sqrt(a + b*sech(x)**2)/sqrt(-a))/(a**2*sqrt(-a))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
itution variable should perhaps be purged.Warning, replacing 0 by ‘ u‘, a s
ubstit
```

Mupad [B]
time = 3.07, size = 50, normalized size = 0.81

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh\left(x\right)^2}}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{1}{3\,a}+\frac{a+\frac{b}{\cosh(x)^2}}{a^2}}{\left(a+\frac{b}{\cosh(x)^2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b/cosh(x)^2)^(5/2),x)
```

```
[Out] atanh((a + b/cosh(x)^2)^(1/2)/a^(1/2))/a^(5/2) - (1/(3*a) + (a + b/cosh(x)^
2)/a^2)/(a + b/cosh(x)^2)^(3/2)
```

3.217 $\int \frac{1}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}}-\frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}}-\frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}}$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(5/2)-1/3*b*(5*a+3*b)*ta
nh(x)/a^2/(a+b)^2/(a+b-b*tanh(x)^2)^(1/2)-1/3*b*tanh(x)/a/(a+b)/(a+b-b*tanh
(x)^2)^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4213, 425, 541, 12, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a-b\tanh^2(x)+b}}-\frac{b\tanh(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[x]^2)^(-5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - (b*Tanh[x])/
(3*a*(a + b)*(a + b - b*Tanh[x]^2)^(3/2)) - (b*(5*a + 3*b)*Tanh[x])/(3*a^2*
(a + b)^2*Sqrt[a + b - b*Tanh[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a-b-2bx^2}{(1-x^2)(a+b-bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3a(a+b)}$$

$$= -\frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}}{}$$

$$= -\frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}}{}$$

$$= -\frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(5a+3b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{b\tanh(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{}{3a^2(a+}$$

Mathematica [A]
time = 0.30, size = 130, normalized size = 1.37

$$\frac{\operatorname{sech}^5(x)\left(\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{a}\,\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)(a+2b+a\cosh(2x))^{5/2}}{a^{5/2}} - \frac{4b(a+2b+a\cosh(2x))\left(3a^2+7ab+3b^2+a(3a+2b)\cosh(2x)\right)\sinh(x)}{3a^2(a+b)^2}\right)}{8\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[x]^2)^(-5/2),x]
```

```
[Out] (Sech[x]^5*((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cos
h[2*x]]]*(a + 2*b + a*Cosh[2*x])^(5/2))/a^(5/2) - (4*b*(a + 2*b + a*Cosh[2*
x])*(3*a^2 + 7*a*b + 3*b^2 + a*(3*a + 2*b)*Cosh[2*x])*Sinh[x])/(3*a^2*(a +
b)^2)))/(8*(a + b*Sech[x]^2)^(5/2))
```

Maple [F]
time = 1.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{sech}\left(x\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(1/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(x)^2 + a)^(-5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2870 vs. $2(81) = 162$.
time = 0.58, size = 6299, normalized size = 66.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)
*cosh(x)*sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(x)^8 + 4*(a^4 + 4*a^3*b
 + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3
+ 7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 2*a^3*b +
a^2*b^2)*cosh(x)^3 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(
x)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^4 + 2*(
35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^4 + 3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24
*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^2)*sinh(x
)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^5 +
10*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^3 + (3*a^4 + 14*a^3*b + 27
*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 + 4*a^3*b + 5*a^2*
b^2 + 2*a*b^3)*cosh(x)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^6 + 15*(a
^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a^4 + 4*a^3*b + 5*a^2*b^2 +
 2*a*b^3 + 3*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^2)*
sinh(x)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^7 + 3*(a^4 + 4*a^3*b + 5*a
```

```
^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8
*b^4)*cosh(x)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*s
qrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^8 -
 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x)^6
 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4*a^
2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2 -
30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^2 -
 b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*(a
^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(x)^
4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt
(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x
)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh
(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(
x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3
- (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 +
 a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh(x)
^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (a^3
 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)
^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*
sinh(x)^5 + sinh(x)^6)) + 3*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^8 + 8*(a^4 +
 2*a^3*b + a^2*b^2)*cosh(x)*sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(x)^8
 + 4*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^6 + 4*(a^4 + 4*a^3*b + 5
*a^2*b^2 + 2*a*b^3 + 7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^2)*sinh(x)^6 + 8*(
7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*
b^3)*cosh(x))*sinh(x)^5 + 2*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b
^4)*cosh(x)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^4 + 3*a^4 + 14*a^3*
b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4 + 30*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3
)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a^
2*b^2)*cosh(x)^5 + 10*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^3 + (3*
a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 + 8*b^4)*cosh(x))*sinh(x)^3 + 4*(a^4
 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^2 + 4*(7*(a^4 + 2*a^3*b + a^2*b^2
)*cosh(x)^6 + 15*(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^4 + a^4 + 4*
a^3*b + 5*a^2*b^2 + 2*a*b^3 + 3*(3*a^4 + 14*a^3*b + 27*a^2*b^2 + 24*a*b^3 +
 8*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^7 + 3*(
a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cosh(x)^5 + (3*a^4 + 14*a^3*b + 27*a^2
*b^2 + 24*a*b^3 + 8*b^4)*cosh(x)^3 + (a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3)*
cosh(x))*sinh(x))*sqrt(a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sin
h(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*a*cosh(x)^2 + a + b)*sinh(x)^2 + sqrt(2
)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt((a*cosh(x)^2
 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*
(a*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a)/(cosh(x)^2 + 2*cosh(x)*sinh(x)
 + sinh(x)^2)) - 8*sqrt(2)*((3*a^3*b + 2*a^2*b^2)*cosh(x)^6 + 6*(3*a^3*b +
2*a^2*b^2)*cosh(x)*sinh(x)^5 + (3*a^3*b + 2*a^2*b^2)*sinh(x)^6 + 3*(a^3*b +
 4*a^2*b^2 + 2*a*b^3)*cosh(x)^4 + 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3 + 5*(3*a^3
*b + 2*a^2*b^2)*cosh(x)^2)*sinh(x)^4 - 3*a^3*b - 2*a^2*b^2 + 4*(5*(3*a^3*b
```

```
+ 2*a^2*b^2)*cosh(x)^3 + 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x)^3
 - 3*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x)^2 + 3*(5*(3*a^3*b + 2*a^2*b^2)*c
osh(x)^4 - a^3*b - 4*a^2*b^2 - 2*a*b^3 + 6*(a^3*b + 4*a^2*b^2 + 2*a*b^3)*co
sh(x)^2)*sinh(x)^2 + 6*((3*a^3*b + 2*a^2*b^2)*cosh(x)^5 + 2*(a^3*b + 4*a^2*
b^2 + 2*a*b^3)*cosh(x)^3 - (a^3*b + 4*a^2*b^2 + 2*a*b^3)*cosh(x))*sinh(x))*
sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2)))/((a^7 + 2*a^6*b + a^5*b^2)*cosh(x...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\operatorname{sech}^2(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sech(x)**2)**(-5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
itution variable should perhaps be purged.Warning, replacing 0 by ‘ u‘, a s
ubstit
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{b}{{\mathrm{cosh}\left(x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b/cosh(x)^2)^(5/2), x)
```

3.218 $\int \frac{\coth(x)}{\left(a+b\,\mathrm{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{b}{3a(a+b)\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}-\frac{b(2a+}{a^2(a+b)^2\sqrt{a+}}$$

```
[Out] arctanh((a+b*sech(x)^2)^(1/2)/a^(1/2))/a^(5/2)-arctanh((a+b*sech(x)^2)^(1/2
)/(a+b)^(1/2))/(a+b)^(5/2)-1/3*b/a/(a+b)/(a+b*sech(x)^2)^(3/2)-b*(2*a+b)/a^
2/(a+b)^2/(a+b*sech(x)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {4224, 457, 87, 157, 162, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\,\mathrm{sech}^2(x)}}-\frac{b}{3a(a+b)\left(a+b\,\mathrm{sech}^2(x)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sech[x]^2]/Sqrt[a]]/a^(5/2) - ArcTanh[Sqrt[a + b*Sech[x]
^2]/Sqrt[a + b]]/(a + b)^(5/2) - b/(3*a*(a + b)*(a + b*Sech[x]^2)^(3/2)) -
(b*(2*a + b))/(a^2*(a + b)^2*Sqrt[a + b*Sech[x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] && LtQ[p, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\int \frac{\coth(x)}{\left(a+b\text{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \text{sech}(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)x(a+bx)^{5/2}}\,dx, x, \text{sech}^2(x)\right)$$

$$= -\frac{b}{3a(a+b)\left(a+b\text{sech}^2(x)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b-bx}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \text{sech}^2(x)\right)}{2a(a+b)}$$

$$= -\frac{b}{3a(a+b)\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(a+b)}{(-1+x)x} \right.}{}$$

$$= -\frac{b}{3a(a+b)\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+}} \right.}{}$$

$$= -\frac{b}{3a(a+b)\left(a+b\text{sech}^2(x)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\text{sech}^2(x)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} \right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{b}{3a(a+b)\left(a+b\right.}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 242 vs. 2(109) = 218.

time = 0.73, size = 242, normalized size = 2.22

$$\frac{\left(-\frac{2b\cosh(x)(a+2b+a\cosh(2x))\left(7a^2+16ab+6b^2+a(7a+4b)\cosh(2x)\right)}{3a^2(a+b)^2} - \frac{(a+2b+a\cosh(2x))^{5/2}\left(\sqrt{a}\left(a^2-2ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\cosh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)+(a+b)^2\left(\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2a+2b}\cosh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)-2\sqrt{a+b}\log\left(\sqrt{2}\sqrt{a}\cosh(x)+\sqrt{a+2b+a\cosh(2x)}\right)\right)\right)}{\sqrt{2}a^{5/2}(a+b)^{5/2}}\right)\text{sech}^5(x)}{8\left(a+b\text{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (((-2*b*Cosh[x]*(a + 2*b + a*Cosh[2*x])*(7*a^2 + 16*a*b + 6*b^2 + a*(7*a +
4*b)*Cosh[2*x]))/(3*a^2*(a + b)^2) - ((a + 2*b + a*Cosh[2*x])^(5/2)*(Sqrt[a
]*(a^2 - 2*a*b - b^2)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Cosh[x])/Sqrt[a + 2*b +
a*Cosh[2*x]]] + (a + b)^2*(Sqrt[a]*ArcTanh[(Sqrt[2*a + 2*b]*Cosh[x])/Sqrt[a
 + 2*b + a*Cosh[2*x]]] - 2*Sqrt[a + b]*Log[Sqrt[2]*Sqrt[a]*Cosh[x] + Sqrt[a
```

```
+ 2*b + a*Cosh[2*x]]])))/(Sqrt[2]*a^(5/2)*(a + b)^(5/2)))*Sech[x]^5)/(8*(a
+ b*Sech[x]^2)^(5/2))
```

Maple [F]
time = 1.64, size = 0, normalized size = 0.00

$$\int \frac{\coth (x)}{\left(a+b\operatorname{sech}(x)^{2}\right)^{\frac{5}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sech(x)^2)^(5/2),x)
[Out] int(coth(x)/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(coth(x)/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4236 vs. 2(91) = 182.
time = 0.95, size = 18563, normalized size = 170.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
[Out] [1/12*(3*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^8 + 8*(a^5 + 3*a^4*
b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^7 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a
^2*b^3)*sinh(x)^8 + 4*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cos
h(x)^6 + 4*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 7*(a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^5 + 3*a^4*b + 3*a
^3*b^2 + a^2*b^3)*cosh(x)^3 + 3*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*
a*b^4)*cosh(x))*sinh(x)^5 + a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + 2*(3*a^5
+ 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(3*a
^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5 + 35*(a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^4 + 30*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^
2*b^3 + 2*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^5 + 3*a^4*b + 3*a^3*b^2 + a
^2*b^3)*cosh(x)^5 + 10*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*co
sh(x)^3 + (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*c
```

```
osh(x))*sinh(x)^3 + 4*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cos
h(x)^2 + 4*(7*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^6 + a^5 + 5*a^4
*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 15*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^
2*b^3 + 2*a*b^4)*cosh(x)^4 + 3*(3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3
+ 32*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((a^5 + 3*a^4*b + 3*a^3*b^2 +
a^2*b^3)*cosh(x)^7 + 3*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*co
sh(x)^5 + (3*a^5 + 17*a^4*b + 41*a^3*b^2 + 51*a^2*b^3 + 32*a*b^4 + 8*b^5)*c
osh(x)^3 + (a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh(x))*sinh(
x))*sqrt(a)*log(((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b
^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 2*(2*a^3 + 5*a^
2*b + 4*a*b^2 + b^3)*cosh(x)^6 + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3 + 14*(a
^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*
cosh(x)^3 + 3*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x))*sinh(x)^5 + (6*a^3
 + 14*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 +
6*a^3 + 14*a^2*b + 9*a*b^2 + 30*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^2
)*sinh(x)^4 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(2*a^3 + 5*a^2*b
 + 4*a*b^2 + b^3)*cosh(x)^3 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)
^3 + a^3 + 2*(2*a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*co
sh(x)^6 + 15*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)*cosh(x)^4 + 2*a^3 + 3*a^2*b
+ 3*(6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*((a^2 + 2*a
*b + b^2)*cosh(x)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^5 + (a^2 + 2*a*
b + b^2)*sinh(x)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 3*(5*(a^2 + 2*a*b +
b^2)*cosh(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*co
sh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + (3*a^2 + 4*a*b)*cosh(x
)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 18*(a^2 + 2*a*b + b^2)*cosh(x)^2
+ 3*a^2 + 4*a*b)*sinh(x)^2 + a^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 6*(
a^2 + 2*a*b + b^2)*cosh(x)^3 + (3*a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sq
rt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2)) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(2*a^3 + 5*a^2*b +
4*a*b^2 + b^3)*cosh(x)^5 + (6*a^3 + 14*a^2*b + 9*a*b^2)*cosh(x)^3 + (2*a^3
+ 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^
4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*s
inh(x)^5 + sinh(x)^6)) + 6*(a^5*cosh(x)^8 + 8*a^5*cosh(x)*sinh(x)^7 + a^5*s
inh(x)^8 + 4*(a^5 + 2*a^4*b)*cosh(x)^6 + 4*(7*a^5*cosh(x)^2 + a^5 + 2*a^4*b
)*sinh(x)^6 + 8*(7*a^5*cosh(x)^3 + 3*(a^5 + 2*a^4*b)*cosh(x))*sinh(x)^5 + a
^5 + 2*(3*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^4 + 2*(35*a^5*cosh(x)^4 + 3*a^
5 + 8*a^4*b + 8*a^3*b^2 + 30*(a^5 + 2*a^4*b)*cosh(x)^2)*sinh(x)^4 + 8*(7*a^
5*cosh(x)^5 + 10*(a^5 + 2*a^4*b)*cosh(x)^3 + (3*a^5 + 8*a^4*b + 8*a^3*b^2)*
cosh(x))*sinh(x)^3 + 4*(a^5 + 2*a^4*b)*cosh(x)^2 + 4*(7*a^5*cosh(x)^6 + a^5
 + 2*a^4*b + 15*(a^5 + 2*a^4*b)*cosh(x)^4 + 3*(3*a^5 + 8*a^4*b + 8*a^3*b^2)
*cosh(x)^2)*sinh(x)^2 + 8*(a^5*cosh(x)^7 + 3*(a^5 + 2*a^4*b)*cosh(x)^5 + (3
*a^5 + 8*a^4*b + 8*a^3*b^2)*cosh(x)^3 + (a^5 + 2*a^4*b)*cosh(x))*sinh(x))*s
qrt(a + b)*log(((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a
+ b)*sinh(x)^4 + 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a +
 3*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

```
*sqrt(a + b)*sqrt((a*cosh(x)^2 + a*sinh(x)^2 + a + 2*b)/(cosh(x)^2 - 2*cosh
(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a + 3*b)*cosh(x))*s
inh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(
x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) +
 3*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^8 + 8*(a^5 + 3*a^4*b + 3*
a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^7 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3
)*sinh(x)^8 + 4*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*cosh(x)^6
 + 4*(a^5 + 5*a^4*b + 9*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 7*(a^5 + 3*a^4*b +
3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)^6 + 8*(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)}{\left(a+\frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b/cosh(x)^2)^(5/2),x)
```

```
[Out] int(coth(x)/(a + b/cosh(x)^2)^(5/2), x)
```

3.219 $\int \frac{\coth^2(x)}{\left(a+b\,\mathbf{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}}-\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}}-\frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}}-\frac{(a$$

```
[Out] arctanh(a^(1/2)*tanh(x)/(a+b-b*tanh(x)^2)^(1/2))/a^(5/2)-1/3*b*(7*a+3*b)*co
th(x)/a^2/(a+b)^2/(a+b-b*tanh(x)^2)^(1/2)-1/3*(a-3*b)*(3*a+b)*coth(x)*(a+b-
b*tanh(x)^2)^(1/2)/a^2/(a+b)^3-1/3*b*coth(x)/a/(a+b)/(a+b-b*tanh(x)^2)^(3/2
)
```

Rubi [A]
time = 0.25, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {4226, 2000, 483, 593, 597, 12, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a-b\tanh^2(x)+b}}\right)}{a^{5/2}}-\frac{(a-3b)(3a+b)\coth(x)\sqrt{a-b\tanh^2(x)+b}}{3a^2(a+b)^3}-\frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a-b\tanh^2(x)+b}}-\frac{b\coth(x)}{3a(a+b)\left(a-b\tanh^2(x)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b - b*Tanh[x]^2]]/a^(5/2) - (b*Coth[x])/
(3*a*(a + b)*(a + b - b*Tanh[x]^2)^(3/2)) - (b*(7*a + 3*b)*Coth[x])/(3*a^2*
(a + b)^2*Sqrt[a + b - b*Tanh[x]^2]) - ((a - 3*b)*(3*a + b)*Coth[x]*Sqrt[a
+ b - b*Tanh[x]^2])/(3*a^2*(a + b)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 2000

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 4226

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2)
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rubi steps

$$\int \frac{\coth^2(x)}{\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+b\left(1-x^2\right)\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= \text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$

$$= -\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a+b-4bx^2}{x^2(1-x^2)(a+b-bx^2)^{3/2}}\,dx, x, \tanh(x)\right)}{3a(a+b)}$$

$$= -\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} + \frac{\text{Subst}}{}$$

$$= -\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-3}{}$$

$$= -\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-3}{}$$

$$= -\frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{b(7a+3b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b-b\tanh^2(x)}} - \frac{(a-3}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b-b\tanh^2(x)}}\right)}{a^{5/2}} - \frac{b\coth(x)}{3a(a+b)\left(a+b-b\tanh^2(x)\right)^{3/2}} - \frac{}{3a^2(a+}$$

Mathematica [A]
time = 0.54, size = 155, normalized size = 1.17

$$\frac{\operatorname{sech}^5(x)\left(\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sinh(x)}{\sqrt{a+2b+a\cosh(2x)}}\right)(a+2b+a\cosh(2x))^{5/2}}{a^{5/2}} - \frac{(a+2b+a\cosh(2x))\left(3a^2(a+2b+a\cosh(2x))^2\operatorname{csch}(x)-4b^3(a+b)\sinh(x)+2b^2(9a+4b)(a+2b+a\cosh(2x))\sinh(x)\right)}{3a^2(a+b)^3}\right)}{8\left(a+b\operatorname{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Sech[x]^2)^(5/2),x]
```

```
[Out] (Sech[x]^5*((Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[a]*Sinh[x])/Sqrt[a + 2*b + a*Cos
h[2*x]]]*(a + 2*b + a*Cosh[2*x])^(5/2))/a^(5/2) - ((a + 2*b + a*Cosh[2*x])*
(3*a^2*(a + 2*b + a*Cosh[2*x])^2*Csch[x] - 4*b^3*(a + b)*Sinh[x] + 2*b^2*(9
*a + 4*b)*(a + 2*b + a*Cosh[2*x])*Sinh[x]))/(3*a^2*(a + b)^3)))/(8*(a + b*S
ech[x]^2)^(5/2))
```

Maple [F]
time = 1.68, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\left(a+b\,\mathrm{sech}(x)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sech(x)^2)^(5/2),x)
```

```
[Out] int(coth(x)^2/(a+b*sech(x)^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/(b*sech(x)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5323 vs. 2(115) = 230.
time = 0.99, size = 11205, normalized size = 84.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^10 + 10*(a^5 + 3*a^
4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^9 + (a^5 + 3*a^4*b + 3*a^3*b^2 +
 a^2*b^3)*sinh(x)^10 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^
4)*cosh(x)^8 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4 + 45*(
a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)^8 + 8*(15*(a^5 + 3*
a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 2
```

```
7*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x)^7 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 3
7*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^6 + 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 3
7*a^2*b^3 + 28*a*b^4 + 8*b^5 + 105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*co
sh(x)^4 + 14*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)
^2)*sinh(x)^6 + 4*(63*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^5 + 14*
(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^3 + 3*(a^5 +
 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^5 -
 a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3 - 2*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a
^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(105*(a^5 + 3*a^4*b + 3*a^3*b^2 +
a^2*b^3)*cosh(x)^6 - a^5 - 7*a^4*b - 23*a^3*b^2 - 37*a^2*b^3 - 28*a*b^4 - 8
*b^5 + 35*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^4
+ 15*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^2
)*sinh(x)^4 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^7 + 7*(3*
a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^5 + 5*(a^5 + 7*
a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^3 - (a^5 + 7*a^
4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^3 - (3*a
^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^2 + (45*(a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^8 + 28*(3*a^5 + 17*a^4*b + 33*a^3*b^2
 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^6 - 3*a^5 - 17*a^4*b - 33*a^3*b^2 - 27*a^2
*b^3 - 8*a*b^4 + 30*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8
*b^5)*cosh(x)^4 - 12*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 +
8*b^5)*cosh(x)^2)*sinh(x)^2 + 2*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*co
sh(x)^9 + 4*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^
7 + 6*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^
5 - 4*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^
3 - (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x)
)*sqrt(a)*log((a*b^2*cosh(x)^8 + 8*a*b^2*cosh(x)*sinh(x)^7 + a*b^2*sinh(x)^
8 - 2*(a*b^2 - b^3)*cosh(x)^6 + 2*(14*a*b^2*cosh(x)^2 - a*b^2 + b^3)*sinh(x
)^6 + 4*(14*a*b^2*cosh(x)^3 - 3*(a*b^2 - b^3)*cosh(x))*sinh(x)^5 + (a^3 + 4
*a^2*b + 9*a*b^2)*cosh(x)^4 + (70*a*b^2*cosh(x)^4 + a^3 + 4*a^2*b + 9*a*b^2
 - 30*(a*b^2 - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*a*b^2*cosh(x)^5 - 10*(a*b^
2 - b^3)*cosh(x)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2
*(a^3 + 3*a^2*b)*cosh(x)^2 + 2*(14*a*b^2*cosh(x)^6 - 15*(a*b^2 - b^3)*cosh(
x)^4 + a^3 + 3*a^2*b + 3*(a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^2)*sinh(x)^2 + s
qrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cos
h(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*c
osh(x))*sinh(x)^3 - (a^2 + 4*a*b)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*co
sh(x)^2 - a^2 - 4*a*b)*sinh(x)^2 - a^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)
^3 - (a^2 + 4*a*b)*cosh(x))*sinh(x))*sqrt(a)*sqrt((a*cosh(x)^2 + a*sinh(x)^
2 + a + 2*b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*a*b^2*cosh
(x)^7 - 3*(a*b^2 - b^3)*cosh(x)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(x)^3 + (
a^3 + 3*a^2*b)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh
(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(
x)*sinh(x)^5 + sinh(x)^6)) + 3*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(
x)^10 + 10*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^9 + (a^5 +
```

```
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sinh(x)^10 + (3*a^5 + 17*a^4*b + 33*a^3*b^2
 + 27*a^2*b^3 + 8*a*b^4)*cosh(x)^8 + (3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^
2*b^3 + 8*a*b^4 + 45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(
x)^8 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (3*a^5 + 17*
a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4)*cosh(x))*sinh(x)^7 + 2*(a^5 + 7*
a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^6 + 2*(a^5 + 7*
a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5 + 105*(a^5 + 3*a^4*b + 3
*a^3*b^2 + a^2*b^3)*cosh(x)^4 + 14*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*
b^3 + 8*a*b^4)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^
2*b^3)*cosh(x)^5 + 14*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3 + 8*a*b^4
)*cosh(x)^3 + 3*(a^5 + 7*a^4*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5
)*cosh(x))*sinh(x)^5 - a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3 - 2*(a^5 + 7*a^4
*b + 23*a^3*b^2 + 37*a^2*b^3 + 28*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(105*(a^5 +
3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^6 - a^5 - 7*a^4*b - 23*a^3*b^2 - 37*
a^2*b^3 - 28*a*b^4 - 8*b^5 + 35*(3*a^5 + 17*a^4*b + 33*a^3*b^2 + 27*a^2*b^3
 + 8*a*b^4)*cosh(x)^4 + 15*(a^5 + 7*a^4*b + 23*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sech(x)**2)**(5/2),x)
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)^2)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^2}{\left(a+\frac{b}{\cosh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b/cosh(x)^2)^(5/2),x)
[Out] int(coth(x)^2/(a + b/cosh(x)^2)^(5/2), x)
```

3.220 $\int \frac{1}{\left(a+b\,\mathrm{sech}^2(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=183

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a+b-b\tanh^2(c+dx)}}\right)}{a^{7/2}d}-\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}}-\frac{b(9a+5b)\tanh(}{15a^2(a+b)^2d\left(a+b-b\,t\right.}$$

```
[Out] arctanh(a^(1/2)*tanh(d*x+c)/(a+b-b*tanh(d*x+c)^2)^(1/2))/a^(7/2)/d-1/15*b*(
33*a^2+40*a*b+15*b^2)*tanh(d*x+c)/a^3/(a+b)^3/d/(a+b-b*tanh(d*x+c)^2)^(1/2)
-1/5*b*tanh(d*x+c)/a/(a+b)/d/(a+b-b*tanh(d*x+c)^2)^(5/2)-1/15*b*(9*a+5*b)*t
anh(d*x+c)/a^2/(a+b)^2/d/(a+b-b*tanh(d*x+c)^2)^(3/2)
```

Rubi [A]

time = 0.14, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4213, 425, 541, 12, 385, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a-b\tanh^2(c+dx)+b}}\right)}{a^{7/2}d}-\frac{b(9a+5b)\tanh(c+dx)}{15a^2d(a+b)^2\left(a-b\tanh^2(c+dx)+b\right)^{3/2}}-\frac{b(33a^2+40ab+15b^2)\tanh(c+dx)}{15a^3d(a+b)^3\sqrt{a-b\tanh^2(c+dx)+b}}-\frac{b\tanh(c+dx)}{5ad(a+b)\left(a-b\tanh^2(c+dx)+b\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x]^2)^(-7/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + b - b*Tanh[c + d*x]^2]]/(a^(7/2)*d
) - (b*Tanh[c + d*x])/(5*a*(a + b)*d*(a + b - b*Tanh[c + d*x]^2)^(5/2)) - (
b*(9*a + 5*b)*Tanh[c + d*x])/(15*a^2*(a + b)^2*d*(a + b - b*Tanh[c + d*x]^2
)^(3/2)) - (b*(33*a^2 + 40*a*b + 15*b^2)*Tanh[c + d*x])/(15*a^3*(a + b)^3*d
*Sqrt[a + b - b*Tanh[c + d*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 4213

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\operatorname{sech}^2(c+dx)\right)^{7/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^{7/2}}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{\operatorname{Subst}\left(\int \frac{-5a-b-4bx^2}{(1-x^2)(a+b-bx^2)^{5/2}}\,dx, x\right.}{5a(a+b)d}$$

$$= -\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tanh(c+d}{15a^2(a+b)^2d\left(a+b-b\tanh^2(\right.}$$

$$= -\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tanh(c+d}{15a^2(a+b)^2d\left(a+b-b\tanh^2(\right.}$$

$$= -\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tanh(c+d}{15a^2(a+b)^2d\left(a+b-b\tanh^2(\right.}$$

$$= -\frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tanh(c+d}{15a^2(a+b)^2d\left(a+b-b\tanh^2(\right.}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+b-b\tanh^2(c+dx)}}\right)}{a^{7/2}d} - \frac{b\tanh(c+dx)}{5a(a+b)d\left(a+b-b\tanh^2(c+d\right.}$$

<u>Mathematica</u> [A]
time = 4.95, size = 330, normalized size = 1.80

$$\frac{\operatorname{sech}^7(c+dx)\left(\frac{15}{4}e^{-7(c+dx)}\left(4be^{2(c+dx)}+a\left(1+e^{2(c+dx)}\right)^2\right)^{7/2}\left(\tanh^{-1}\left(\frac{a+2b+ae^{2(c+dx)}}{\sqrt{a}\sqrt{4be^{2(c+dx)}+a\left(1+e^{2(c+dx)}\right)^2}}\right)-\tanh^{-1}\left(\frac{a+ae^{2(c+dx)}+2be^{2(c+dx)}}{\sqrt{a}\sqrt{4be^{2(c+dx)}+a\left(1+e^{2(c+dx)}\right)^2}}\right)\right)-\frac{4\sqrt{a}\,b(a+2b+a\cosh(2(c+dx)))(135a^4+480a^3b+709a^2b^2+460ab^3+120b^4+4a(45a^3+135a^2b+117ab^2+35b^3)\cosh(2(c+dx))+a^2(45a^2+60ab+23b^2)\cosh(4(c+dx)))\sinh(c+dx)}{(a+b)^3}\right)}{960a^{7/2}d\left(a+b\operatorname{sech}^2(c+dx)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x]^2)^(-7/2),x]
```

```
[Out] (Sech[c + d*x]^7*((15*(4*b*E^(2*(c + d*x)) + a*(1 + E^(2*(c + d*x)))^2)^(7/
2)*(ArcTanh[(a + 2*b + a*E^(2*(c + d*x)))/(Sqrt[a]*Sqrt[4*b*E^(2*(c + d*x))
 + a*(1 + E^(2*(c + d*x)))^2])] - ArcTanh[(a + a*E^(2*(c + d*x)) + 2*b*E^(2
*(c + d*x)))/(Sqrt[a]*Sqrt[4*b*E^(2*(c + d*x)) + a*(1 + E^(2*(c + d*x)))^2]
)]))/(4*E^(7*(c + d*x))) - (4*Sqrt[a]*b*(a + 2*b + a*Cosh[2*(c + d*x)])*(13
```

```
5*a^4 + 480*a^3*b + 709*a^2*b^2 + 460*a*b^3 + 120*b^4 + 4*a*(45*a^3 + 135*a
^2*b + 117*a*b^2 + 35*b^3)*Cosh[2*(c + d*x)] + a^2*(45*a^2 + 60*a*b + 23*b^
2)*Cosh[4*(c + d*x)])*Sinh[c + d*x])/(a + b)^3))/(960*a^(7/2)*d*(a + b*Sech
[c + d*x]^2)^(7/2))
```

Maple [**F**]
time = 2.99, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{sech}\left(dx+c\right)^{2}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)^2)^(7/2),x)
```

```
[Out] int(1/(a+b*sech(d*x+c)^2)^(7/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c)^2 + a)^(-7/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8940 vs. $2(165) = 330$.
time = 1.66, size = 18565, normalized size = 101.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^12 + 12*(a^6
 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (a^6 + 3
*a^5*b + 3*a^4*b^2 + a^3*b^3)*sinh(d*x + c)^12 + 6*(a^6 + 5*a^5*b + 9*a^4*b
^2 + 7*a^3*b^3 + 2*a^2*b^4)*cosh(d*x + c)^10 + 6*(a^6 + 5*a^5*b + 9*a^4*b^2
 + 7*a^3*b^3 + 2*a^2*b^4 + 11*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^10 + 20*(11*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*c
osh(d*x + c)^3 + 3*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*cosh
(d*x + c))*sinh(d*x + c)^9 + 3*(5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3
 + 64*a^2*b^4 + 16*a*b^5)*cosh(d*x + c)^8 + 3*(5*a^6 + 31*a^5*b + 79*a^4*b^
2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5 + 165*(a^6 + 3*a^5*b + 3*a^4*b^2 +
a^3*b^3)*cosh(d*x + c)^4 + 90*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^
```

2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 24*(33*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^5 + 30*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*cosh(d*x + c)^3 + (5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(5*a^6 + 33*a^5*b + 93*a^4*b^2 + 147*a^3*b^3 + 138*a^2*b^4 + 72*a*b^5 + 16*b^6)*cosh(d*x + c)^6 + 4*(231*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^6 + 5*a^6 + 33*a^5*b + 93*a^4*b^2 + 147*a^3*b^3 + 138*a^2*b^4 + 72*a*b^5 + 16*b^6 + 315*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*cosh(d*x + c)^4 + 21*(5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 + 24*(33*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^7 + 63*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*cosh(d*x + c)^5 + 7*(5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*cosh(d*x + c)^3 + (5*a^6 + 33*a^5*b + 93*a^4*b^2 + 147*a^3*b^3 + 138*a^2*b^4 + 72*a*b^5 + 16*b^6)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*cosh(d*x + c)^4 + 3*(165*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^8 + 420*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*cosh(d*x + c)^6 + 5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5 + 70*(5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*cosh(d*x + c)^4 + 20*(5*a^6 + 33*a^5*b + 93*a^4*b^2 + 147*a^3*b^3 + 138*a^2*b^4 + 72*a*b^5 + 16*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(55*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^9 + 180*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*cosh(d*x + c)^7 + 42*(5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*cosh(d*x + c)^5 + 20*(5*a^6 + 33*a^5*b + 93*a^4*b^2 + 147*a^3*b^3 + 138*a^2*b^4 + 72*a*b^5 + 16*b^6)*cosh(d*x + c)^3 + 3*(5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*cosh(d*x + c)^2 + 6*(11*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^10 + 45*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*cosh(d*x + c)^8 + 14*(5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*cosh(d*x + c)^6 + a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4 + 10*(5*a^6 + 33*a^5*b + 93*a^4*b^2 + 147*a^3*b^3 + 138*a^2*b^4 + 72*a*b^5 + 16*b^6)*cosh(d*x + c)^4 + 3*(5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cosh(d*x + c)^11 + 5*(a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*cosh(d*x + c)^9 + 2*(5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*cosh(d*x + c)^7 + 2*(5*a^6 + 33*a^5*b + 93*a^4*b^2 + 147*a^3*b^3 + 138*a^2*b^4 + 72*a*b^5 + 16*b^6)*cosh(d*x + c)^5 + (5*a^6 + 31*a^5*b + 79*a^4*b^2 + 101*a^3*b^3 + 64*a^2*b^4 + 16*a*b^5)*cosh(d*x + c)^3 + (a^6 + 5*a^5*b + 9*a^4*b^2 + 7*a^3*b^3 + 2*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a)*log((a*b^2*cosh(d*x + c)^8 + 8*a*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + a*b^2*sinh(d*x + c)^8 - 2*(a*b^2 - b^3)*cosh(d*x + c)^6 + 2*(14*a*b^2*cosh(d*x + c)^2 - a*b^2 + b^3)*sinh(d*x + c)^6 + 4*(14*a*b^2*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (a^3 + 4*a^2*b + 9*a*b^2)*

```
cosh(d*x + c)^4 + (70*a*b^2*cosh(d*x + c)^4 + a^3 + 4*a^2*b + 9*a*b^2 - 30*
(a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(14*a*b^2*cosh(d*x + c)^
5 - 10*(a*b^2 - b^3)*cosh(d*x + c)^3 + (a^3 + 4*a^2*b + 9*a*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^3 + a^3 + 2*(a^3 + 3*a^2*b)*cosh(d*x + c)^2 + 2*(14*a*b^
2*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*(a
^3 + 4*a^2*b + 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + sqrt(2)*(b^2*cos
h(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 -
3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4 + 4
*(5*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 - (a^2 + 4*a
*b)*cosh(d*x + c)^2 + (15*b^2*cosh(d*x + c)^4 - 18*b^2*cosh(d*x + c)^2 - a^
2 - 4*a*b)*sinh(d*x + c)^2 - a^2 + 2*(3*b^2*cos...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\,\mathrm{sech}^2\left(c + dx\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)**2)**(7/2),x)
```

```
[Out] Integral((a + b*sech(c + d*x)**2)**(-7/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{b}{{\mathrm{cosh}\left(c+d\,x\right)}^2}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x)^2)^(7/2),x)
```

```
[Out] int(1/(a + b/cosh(c + d*x)^2)^(7/2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
      ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstar
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
      return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```